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Inflation and Welfare in an OLG Economy with a Privately Provided Public Good*

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Abstract

In this paper we study the welfare effects of monetary policy in a simple overlapping generation economy in which agents voluntarily contribute to a public good. Inflation has two effects at equilibrium: it increases voluntary contributions and it misallocates private consumption across time. We show that the aggregate effect is welfare-improving for "not too large" inflation rates. Moreover, there exists an optimal inflation rate.

Keywords: *Optimal Inflation, Public Goods, Voluntary Contributions.*

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1 Introduction

In this paper, we analyze optimal monetary policy in an overlapping generations economy with no capital, in which a public good is *privately and voluntarily* provided by the economic agents. Private provision refers to situations in which, either by the intrinsic nature of the public good or because of the specific institutional arrangements, provisions rely entirely or in part on individual decentralized actions. This case has been extensively studied as the "contribution game" (see, for instance, Bergstrom, Blume and Varian (1986)); examples range from donations to charities (Andreoni (1988)), to the funding of political parties, to the willingness to pay taxes (Cowell and Gordon (1988)) or to preserve the environmental quality, to the private investment in education. We show that in these situations positive inflation rates can mitigate the inefficiency associated with the equilibrium under-accumulation of the public good in the contribution game, and that this generates a welfare improvement for small inflation rates.

The economy we consider is a variation of the standard pure-exchange overlapping generations model (Samuelson (1958)). There is a unique, perishable commodity and a linear technology to produce a (possibly durable) public good out of this commodity. Agents have time-separable inter-temporal preferences defined over the private and the public goods and an endowment of private commodity only when young. Money allows individuals to save. The public good is produced through voluntary contributions from the individuals in both periods of life. Thus, agents interact (as price-takers) *via* the private good market and (as quantity-takers) *via* their public good contributions. This second interaction occurs both within periods (at each period, a young and an old agent contribute simultaneously) and across periods (when the public good is durable, young generations inherit the public good provided by the old). A central authority controls the rate of monetary expansion (or contraction) through *lump-sum* transfers.

We consider *steady state equilibria* in which public good stationary contributions are mutually consistent and private good markets clear. We then study the welfare properties of alternative monetary policies by performing comparative statics around the constant money supply steady state. Since at the constant money supply equilibrium marginal utilities of private consumption are equalized across periods, the inter-temporal allocation of the private good maximizes steady state welfare (a sort of *golden rule* is attained), *given* the level of public good. On the other hand, this equilibrium is affected by the typical under-provision of the public good associated with the voluntary contribution game: social returns to public good provision exceed private returns. Since the private good is efficiently allocated, any infinitesimal resources reallocation involves a steady state welfare improvement *if and only if* it increases public good production. In other terms, the first order gain due to increased contributions always out-weights the second order loss due to any misallocation of private consumption. As a consequence, any policy change attains a welfare improvement if and only if it induces agents to contribute more at equilibrium.

Inflation induces a substitution of second period with first period private good consumption. According to the above argument, it is welfare improving if and only if this substitution is associated with an increase in the public good contribution. When the public good decays instantaneously, this is directly implied by normality, discounting and time-separability of preferences: in equilibrium, individuals contribute only in their first period of life and, as long as the first period private good consumption increases, the marginal valuation of first period provision increases, and with it the private benefit from public good provision. This may not be true when the public good decays gradually in time. In this case, the overall marginal valuation of the public good provision is affected by both the increase in first period private consumption and by the decrease in second period private consumption, so that the total effect is ambiguous. For this case, we need to invoke net-substitutability of first period public good contribution and second period private good consumption as an additional assumption.

This paper is organized as follows. In the next section we discuss our result in relation to the existing literature on optimal monetary policy. In section

3, the model is presented and the individual maximization problem is solved. In section 4, we prove existence of a steady state equilibrium and present our results on the welfare implications of an inflationary policy. Section 5 concludes the paper.

2 Related Literature

The issue of optimal monetary policy has been extensively debated in the literature. Much of the effort has been devoted to the study of economies in which money provides liquidity services in addition to being a store of value (money-in-the-utility-function models such as Sidrausky (1967), Brock (1974), Turnowsky and Brock (1980), and Summers (1981)) or in which some transaction or liquidity constraint is present (cash-in-advance models such as Clower (1967) and Lucas-Stockey(1983)). In these models, when there are no other inefficiencies, the well known Friedman's recommendation of a zero nominal interest rate has been validated: a positive nominal interest rate is the only distortion in the equilibrium allocation of resources and, when the monetary policy can control inflation through non-distortionary transfers policy and individual can be satiated in real money balances, it is in general optimal to withdraw money for circulation at the same rate of individuals' time preference, so to male zero the nominal interest rate at equilibrium.

In monetary overlapping generations economies in which money has no transaction role (the case we take up in this paper), a treatment of the optimal monetary policy in terms of nominal interest rate and satiation of individuals in real monetary balances is meaningless. Money is demanded in equilibrium solely because it is a store of value allowing for trade between generations and, more importantly, no arbitrage always implies a zero nominal interest rate, so that money is not dominated by any other asset. In this setting, any consideration on the optimal inflation rate should not be interpreted as a validation or a rejection of Friedman's rule. The focus must instead be shifted on the inter-temporal effect of inflation on (long-run) prices, more in the spirit of Balasko and Shell (1980a, 1980b). When there are no other sources of inefficiency coming from non-convexities or transaction costs and the monetary authority can operate lump-sum transfers, an optimal monetary policy in general requires price stability.

The two issues mentioned above overlap in Weiss (1980), who studies a standard overlapping generations model with production introducing real balances in individuals' utilities. Weiss' result (obtained under rather strong hypothesis, as pointed out by Gahavary (1988)) might seem to contradict Friedman's recommendation: inflation improves steady state welfare even though it augments the distortion related to an increase in the opportunity cost of holding money. This conclusion is crucially driven by the overlapping generations structure: even though the (present value) inter-temporal distribution of transfers to an individual does not effect his budget set, it does affect savings (in particular, the amount of savings invested in real assets). Following Gahavary's analysis, we interpret Weiss's proposition as a qualification of Friedman's recommendation: provided that preferences can be satiated in real monetary balances and the monetary authority can carry out an appropriate lump-sum transfers policy (basically, a stationary transfer from old to young individuals), then the steady state welfare is maximal when the nominal interest rate is set to zero. Putting aside Weiss's model from the perspective of Friedman's rule analysis, we shall below examine the relation between the welfare improving mechanism in Weiss's paper with the one we obtain in our work.

This having been said, we will relate our work to two lines of research proposing arguments for an optimal positive inflation rate: (a) economies in which some distortionary taxation is used to finance a public budget and (b) economies in which some productive asset is under-provided at the zero inflation equilibrium.

The first theoretical argument has developed as a by-product of the theory of optimal indirect taxation. In line with the observation that inflation imposes a tax on money holdings, the problem of optimal monetary policy has been addressed as the problem of designing the optimal tax mix in economies where only distortionary tax instruments are available to finance an *exogenous* public budget (see, for instance, Phelps, 1973, Siegel, 1978, Helpman and Sadka, 1979, Drazen, 1980, Chamley (1985)). According to a "Ramsey's rule" argument, the structure of demand for money may well be such that the optimal tax mix includes a positive inflation tax. We here point out that our result can hardly be related to this first approach. Indeed, as will become apparent after the formal presentation of our economy, the second best taxation problem does not arise in the present paper: if the government could itself provide the public good and finance it *via* commodity taxation, the first best allocation could be achieved by the appropriate uniform taxation of both periods' consumption. Indeed, this would be equivalent to levying a lump sum tax and the optimal inflation rate for this economy would still be zero. More importantly, in our model, the direct effect of the use of the inflation tax on the economy (namely, the effect of the public good on the utility of economic agents) determines the welfare result: it is through the induced increase in the public good contributions that every agent ends up being better off under inflationary policy. This effect of public expenditure on agents' utilities is usually not taken into account in the optimal taxation literature: inflation is welfare improving there only because it reduces the distortionary burden of the other tax instruments.[1]

The contribution enhancement mechanism observed in the present paper is more akin to the second line of research, stressing the relation between money creation and capital accumulation (be it private or public). This literature is mainly devoted to showing that inflation may have positive welfare effects by increasing the equilibrium investment in some under-provided productive assets. It has mainly developed within the framework of overlapping generations models with production where money has a transactional role (*e.g.*, Weiss (1980) and Gahavari (1988)). In a money-in-utility-function setting, Weiss (1980) reproduces a sort of Tobin's effect in a life cycle model with separable preferences. He first shows that, as long as agents are non-satiated in the consumption of money and marginal returns to capital are affected by capital accumulation, the

[1] Iohri (1996, Chapter 7) studies a monetary overlapping generations model with distortionary taxation and capital accumulation, and his results contain elements of both literatures under discussion here.

steady state equilibrium associated with a zero inflation rate (assuming no population growth) exhibits under-accumulation of physical capital. In this setting, the substitution of capital for money induced by money creation increases the steady state consumption of each agent (and therefore welfare) around the zero inflation equilibrium. This substitution effect, responsible for the welfare result, is shown by Gahavari (1988) to crucially depend on the separability assumption on preferences. More precisely, he shows that this effect always holds only when money and second period consumption are *net substitutes*, a condition which is trivially satisfied given the separability assumption in Weiss (1980). [2]

Comparing this result with our model's, we note that, in our economy, the demand for money exclusively derives from the need to finance second period's consumption. We can therefore say that money and second period consumption are *perfect complements*. Indeed, the substitution occurring in our model is of a completely different nature from the one responsible for Weiss and Gahavari's results. Agents substitute second period private consumption with first period private consumption and public good contribution. As a result of this difference, we obtain our welfare result under a perfect complementarity assumption, the opposite case to Weiss and Gahavari. In our case, it is crucial that the introduction of some price distortion allows for a reduction of the static inefficiency resulting in the under-provision of the public good.

The idea that static inefficiencies can be corrected *via* monetary policy is already present in Ferreira (1999), where money financed public expenditure

[2] The mechanism responsible for the result can be roughly explained as follows. Inflation increases the cost of holding money, inducing a reallocation of consumption from money to other commodities. If second period consumption is a net substitute for money balances, then utility maximization requires a higher investment in the productive asset in order to raise consumption in the second period without paying the inflation tax. This portfolio adjustment (much in the spirit of the so-called Tobin's effect) allows for an increase of the steady state stock of capital. Since capital is under-provided with respect to the golden rule at the zero inflation equilibrium, a higher stock is beneficial. When money and second period consumption are net complements, the latter would decline with the former if inflation increases. If the induced decrease in savings is greater than the reduction in money demand, then capital would itself decrease.

has a direct effect on labor productivity, and in Azariadis and Reichlin (1994), where capital accumulation has an external effect in production.[3] Both these papers consider situations in which social returns to capital (private or public) exceed private returns, making an expansion of investments socially desirable (at least in the long run). These papers differ from ours in two main respects.

In Ferreira (1999), a positive inflation is beneficial only because it is the only source of funding for the public budget. Inter-temporal price distortion is not responsible for the result and is a kind of dead weight. Any other non-distortionary tax would be strictly preferred by the planner. The same would of course be true in our model, if the government were be able to provide the public good, but (we remark once again) the sort of public good we consider can only be provided by the private sector.

In Azariadis and Reichlin (1994), instead, the welfare improvement is a genuine general equilibrium effect passing through the alteration of relative prices across periods. At the heart of their conclusion stands a static inefficiency: at the constant prices equilibrium social returns to capital exceed private returns. If inflation enhances capital accumulation at the steady state, welfare increases. Here, the steady state reaction of capital to inflation depends only on technology. By no-arbitrage, a positive inflation requires private returns to capital to decrease and can be consistent with an increase in steady state capital stock only if private returns to capital decrease with social capital. Savings reactions to the real interest rate play no role and, indeed, the conclusion would be consistent also with savings (locally) decreasing in the real interest rate.

Differently from these works, our paper examines the equilibrium change in the provision of the public good in response to money creation; the choice of monetary policy is here more similar to the design of a mechanism: by changing

[3]It should be mentioned that Azariadis and Reichlin (1994) do not address the issue of monetary policy. In their paper, they study the welfare properties of different levels of national debt in the standard overlapping generations economy with positive externality in production. Their proposition 7 is related to our paper in that it shows the social desirability of substitution from national debt to physical capital. We reinterpret this result in terms of monetary policy for the case of bounded growth.

the payoff structure of the contribution game, higher inflation rates lead to higher contribution levels. Our clear-cut result for the case of a perishable public good is related to the structure of equilibrium contributions (only young agents provide the public good). Without this specific structure of equilibria, the distortion prices in favor of the first period would not directly imply an increase of the level of produced public good.

3 The Economy

3.1 Fundamentals

We consider an overlapping generations model with infinite time horizon (*e.g.*, Samuelson 1958). Time, indexed by t, goes from negative to positive infinity. In each period, there are two commodities: a private good x and a public good Q. In each period, a single agent is born with one unit of private good as endowment. The public good is produced through a linear technology which uses as input the private good. Agents live for two periods.

Individuals' preferences are represented by the time-separable utility function

$$U(x_0, x_1, Q_0, Q_1) = u(x_0, Q_0) + \beta u(x_1, Q_1),$$

where x_0 and x_1 denote the private good consumed by the agent when young and old, respectively, Q_0 and Q_1 denote the consumption of public good when young and old, respectively, and β, $0 < \beta < 1$, is the discount factor.

We make the following assumptions on preferences.[4]

Assumption 1 (Convexity) *The utility function $u : \mathbb{R}^2_{++} \rightarrow \mathbb{R}$ is smooth, smoothly strictly increasing and smoothly strictly concave (*i.e.*, the quadratic form associated with its Hessian is negative definite) and it satisfies the strong Inada's conditions:*

$$\lim_{x \to 0} u(x, Q) = -\infty, \textit{ for all positive } Q;$$

[4] We use u_x, u_Q and u_{xx}, u_{xQ}, u_{QQ} to denote the first and second derivatives of u, respectively.

$$\lim_{Q \to 0} u(x, Q) = -\infty, \text{ for all positive } x.$$

Assumption 2 (Normality) *The utility function* $u : \mathbb{R}^2_{++} \to \mathbb{R}$ *is such that the cross-derivative* $u_{xQ}(x, Q)$ *is non-negative for all positive* (x, Q).

Assumption 1, in addition to convexity and continuity of preferences, basically requires that both goods, private and public, are demanded when prices are strictly positive, and that the marginal rate of substitution changes smoothly along indifference curves. Assumption 2 implies that the two goods are normal and is quite standard in the literature. We remark that the strong version of the Inada's conditions, jointly with concavity, implies that marginal utilities tend to positive infinity on the boundary. Note as well that, although U is only defined on positive vectors, preferences are continuous on the space of all non-negative vectors: in practice, the boundary of the inter-temporal consumption space coincides with the lowest indifference curve. Unboundedness of the utility function is only needed for the proof of our proposition 3, whereas unboundedness of marginal utilities turns out to be useful for characterization.

We assume that agents are allowed to trade in a purely monetary asset having no intrinsic value. The non-negative aggregate quantity of this asset at time t is denoted by M_t. The nominal money supply M_t grows at a steady pre-announced rate σ, with $1+\sigma > 0$; the additional money $H_t = \sigma M_{t-1}$ is injected into the economy through a monetary transfer to the young agent at time t.[5] We denote real balances and transfer in period t by m_t and h_t, respectively.

The public good originates through the voluntary contributions of agents, who can devote part of their endowments (when young) or of their savings (when old) to its production. The contribution of young and old agents are denoted by z_0 and z_1, respectively. The public good, once produced, depreciates at the constant rate $(1 - \delta)$ *per* period, with $0 \leq \delta < 1$; the private good is perishable.

Throughout our analysis, we will denote by (e, σ) an economy where preferences, technology, endowments and the market structure are according to the

[5] If the proceeds from money creation were distributed to the old agent, our qualitative results would not be altered.

previous description and the constant nominal money growth rate is σ. We allow such a rate to be negative and, whenever it is strictly positive, the economy is referred to as inflationary.

3.2 Optimal Plans

Agents choose optimal consumption and contribution plans taking as given prices and other agents' contributions. As we are concerned with steady state analysis, we assume a constant growth rate σ of nominal prices and a constant real monetary transfer h; furthermore, we suppose that all other agents have a constant plan of contributions $(\bar{z}_0, \bar{z}_1)$ to the public good. Given all parameters $(\bar{z}_0, \bar{z}_1, h, \sigma)$, each agent's optimal plan (x_0, x_1, z_0, z_1) is the solution to the program

$$\sup u(x_0, Q_0) + \beta u(x_1, Q_1)$$

subject to

$$\begin{aligned} \left(\frac{1}{1-\delta}\right)(\bar{z}_1 + \delta\bar{z}_0) + z_0 &= Q_0, &(1) \\ \left(\frac{\delta}{1-\delta}\right)[\bar{z}_1 + \delta\bar{z}_1 + (1-\delta) z_0] + \bar{z}_0 + z_1 &= Q_1, &(2) \\ x_0 + z_0 + (1+\sigma)(x_1 + z_1) - 1 - h &\leq 0, &(3) \\ -(x_0, x_1, z_0, z_1) &\leq 0. &(4) \end{aligned}$$

Constraints (1)-(2) are merely definitory. The inter-temporal budget constraint is given by inequality (3). Notice that, even though money does not appear explicitly for notational convenience, the reader should always interpret the quantity $(1 - x_0 - z_0)$ as the excess demand of real money balances by the young agent in his first period of life.

The strict concavity of utility and a simple application of the Maximum Theorem guarantee that the optimal plan is a continuous, single-valued map of parameters $(\bar{z}_0, \bar{z}_1, h, \sigma)$. We denote the optimal plan mapping by $(x_0, x_1, z_0, z_1) = \mathbf{f}(\bar{z}_0, \bar{z}_1, h, \sigma)$, where $\mathbf{f} = (\mathbf{x}_0, \mathbf{x}_1, \mathbf{z}_0, \mathbf{z}_1)$. To clarify, the terms $\mathbf{z}_0(\bar{z}_0, \bar{z}_1, h, \sigma)$ and $\mathbf{z}_1(\bar{z}_0, \bar{z}_1, h, \sigma)$ stand for the optimal first and second period contributions to the public good, while the terms $\mathbf{x}_0(\bar{z}_0, \bar{z}_1, h, \sigma)$ and $\mathbf{x}_1(\bar{z}_0, \bar{z}_1, h, \sigma)$ are the demand for private consumption in the first and second period, given that all other agents contribute according to the plan $(\bar{z}_0, \bar{z}_1)$, the real monetary transfer is h and the nominal prices grow at the constant rate σ.

By our assumptions on preferences, an optimal plan is characterized by the following conditions:

$$
\begin{aligned}
(1+\sigma)\, u_x(x_0, Q_0) - \beta u_x(x_1, Q_1) &= 0, && (5)\\
u_Q(x_0, Q_0) + (\beta\delta)\, u_Q(x_1, Q_1) - u_x(x_0, Q_0) + \lambda_0 &= 0, && (6)\\
\beta u_Q(x_1, Q_1) - (1+\sigma)\, u_x(x_0, Q_0) + \lambda_1 &= 0, && (7)\\
x_0 + z_0 + (1+\sigma)(x_1 + z_1) - 1 - h &= 0, && (8)\\
\lambda_0 z_0 + \lambda_1 z_1 &= 0, && (9)
\end{aligned}
$$

plus non-negative constraints on (x_0, x_1, z_0, z_1) and the Lagrange multipliers (λ_0, λ_1) and definitions (1)-(2). Condition (5) requires that the marginal rate of substitution between private consumption in the first and second periods equate the price ratio. Conditions (6)-(7) state that in each period the marginal utilities of the private good and the public good contribution are equal when the latter is positive.

3.3 Monetary Equilibrium

We consider an equilibrium concept combining Walrasian and strategic elements. A competitive equilibrium of the economy (e, σ) is a sequence of prices, money demands, private good consumptions and public good contributions such that each agent maximizes his utility (given prices *and* the other agents' contributions) and all markets clear. To limit our notation, we propose only a steady state definition.

Definition 1 (Equilibrium) *A* monetary steady-state equilibrium *(or, simply, an* equilibrium*) of the economy (e, σ) is a plan $(x_0^*, x_1^*, z_0^*, z_1^*)$ such that*

$$(x_0^*, x_1^*, z_0^*, z_1^*) = \mathbf{f}(z_0^*, z_1^*, h^*, \sigma), \tag{10}$$

$$h^* = \sigma(x_1^* + z_1^*). \tag{11}$$

Condition (10) states that the plan is optimal for an individual; in addition, it prescribes that, given a contribution plan by all other individuals, each individual finds the same plan optimal. Condition (**??**), in turn, guarantees that all markets clear: in fact, using the individual budget constraint (and strict monotonicity of preferences),

$$x_0^* + x_1^* + z_0^* + z_1^* = 1 + h^* - \sigma\left(x_1^* + z_1^*\right) = 1.$$

The real value of money at equilibrium is $m^* = x_1^* + z_1^*$ and naturally equals the amount of resources commanded by an old individual.

Loosely speaking, the above definition of equilibrium simultaneously imposes a Nash requirement on public good contributions and a competitive equilibrium requirement on private good consumptions. It is worth pointing out that our analysis is not truly dynamical, in that we define agents' strategies as pairs of contribution levels rather than functions mapping from histories to actions. By doing this, we do not allow agents to optimally revise their plans at every point in time: the sequence of equilibrium contributions is indeed such that each agent chooses once and for all his lifetime contributions so to maximizes his utility, given the contributions sequences of the other agents. In other terms, we only consider stationary open-loop strategies; a more interesting and complex approach would be to allow for closed loop strategies, yielding a leader-follower game structure in which each player takes as given the action of preceding players and the *reaction functions* of following players. Perfect stationary (Markovian) equilibria of this new game would be fixed points in the space of such reaction functions.

4 Welfare and Inflation

In this section, we address the issue of optimal monetary policy for our economy. We first establish existence of equilibria and characterize equilibrium plans. We then study the welfare effect of inflationary policy by means of a comparative statics exercise around the no inflation equilibrium. Although small inflation rates are welfare improving, large inflation rates are Pareto inferior.

We start by proving existence of equilibrium. The proof of the proposition follows Debreu (1952), but it also takes into account the strategic interaction among agents.

Proposition 1 (Existence) *Every economy* (e, σ) *admits an equilibrium.*

Proof. Consider the analytical construction described below for a given σ. There are two agents A and B: A can be interpreted as a representative agent of our economy, B plays the role of the traditional auctioneer. Let s be an element of $S = S_{\text{A}} \times S_{\text{B}}$, where

$$S_{\text{A}} = \{(x_0, x_1, z_0, z_1) \geq 0 : x_0 + z_0 + (1+\sigma)(x_1 + z_1) \leq 1 + \sigma\}$$

and $S_{\text{B}} = [0,1]$. Each agent is given an $\bar{s}$ in S and chooses an element of his strategy set to maximize his preferences.

Let the correspondence $B_{\text{A}} : S \mapsto S_{\text{A}}$ be defined by

$$B_{\text{A}}(\bar{s}) = \{s_{\text{A}} \in S_{\text{A}} : x_0 + z_0 + (1+\sigma)(x_1 + z_1) \leq 1 + \sigma \bar{s}_{\text{B}}\}.$$

It can be easily checked that B_{A} is continuous, convex and compact valued. Agent A's preferences $\succeq_{\text{A}}$ on S are induced by

$$u\left(x_0, z_0 + \bar{Q}_0\right) + \beta u\left(x_1, z_1 + \delta z_0 + \bar{Q}_1\right),$$

where $\bar{Q}_0 = (1-\delta)^{-1}(\bar{z}_1 + \delta \bar{z}_0)$ and $\bar{Q}_1 = \delta \bar{Q}_0 + \bar{z}_0$. Agent A chooses an action so to maximize his preferences on $B_{\text{A}}(\bar{s})$. By the maximum theorem, the solution of the above problem yields a non-empty, upper-hemicontinuous and convex valued correspondence $f_{\text{A}} : S \mapsto S_i$.

Agent B chooses an element s_{B} in S_{B} and his preferences $\succeq_{\text{A}}$ on S are induced by

$$-\left|s_{\text{B}} - \bar{x}' - \bar{z}'\right|.$$

The optimal choice correspondence $f_{\text{B}} : S \mapsto S_{\text{B}}$ is non-empty, upper-hemicontinuous and convex valued.

We apply Kakutani Fixed Point Theorem to the correspondence $f = f_{\text{A}} \times f_{\text{B}} : S \mapsto S$ and we denote by $s^* \in f(s^*)$ a fixed point. We observe now that s^*

must be such that

$$s_{\text{B}}^* = x_1^* + z_1^*. \tag{12}$$

If not, since s_{B}^* is such that B maximizes his utility, we would obtain

$$s_{\text{B}}^* = 1 < x_1^* + z_1^*;$$

and so, by the budget constraint above,

$$0 \leq x_0^* + z_0^* \leq (1+\sigma)(1 - x_1^* - z_1^*) < 0,$$

a contradiction.

Any fixed point of f is clearly a steady state equilibrium of our economy. In fact, set $h^* = \sigma s_{\text{B}}^*$: when an agent in our economy is given a real money transfer h^*, σ as inflation rate and (z_0^*, z_1^*) as contributions to the public good coming from all other agents, his optimal plan is exactly $(x_0^*, x_1^*, z_0^*, z_1^*)$. Condition (12) guarantees that all markets clear. ■

We now present a lemma that characterizes equilibrium contributions. When the return on real money balances exceeds the rate of time preference, it turns out that there are no second period contributions. This fact facilitates the analysis.

Lemma 1 (No second-period contribution) *In any equilibrium, private-good consumption and the aggregate amount of public good are strictly positive. Furthermore, if*

$$1 + \sigma > \beta, \tag{13}$$

there is no second-period contribution to the public good.

Proof. The fact that x_0^*, x_1^* and

$$Q^* = \left(\frac{z_0^* + z_1^*}{1 - \delta}\right)$$

are strictly positive is implied by Inada's conditions on preferences stated in assumption 1. We then consider the contributions pattern. In order for contributions by old agents to occur, the Lagrange multiplier λ_1^* in conditions (5)-(9)

must be zero; hence,

$$\beta u_Q(x_1^*, Q^*) - (1+\sigma) u_x(x_0^*, Q^*) = 0.$$

Using conditions (6) and (13) this yields

$$u_Q(x_0^*, Q^*) \leq \beta \left(\frac{1}{1+\sigma} - \delta \right) u_Q(x_1^*, Q^*) < u_Q(x_1^*, Q^*).$$

Note now that, by conditions (5) and (13), $x_1^* < x_0^*$, which in turns implies, by the normality assumption (assumption 2), that $u_Q(x_0^*, Q^*) \geq u_Q(x_1^*, Q^*)$, a contradiction. ■

A positive inflation rate induces an inter-temporal substitution in private good consumption: since holding real balances is relatively costly, agents anticipate private good consumption. By our assumption 2 on preferences, the public good is more valuable in the first period than in the second; as a consequence, agents find it optimal not to contribute when old (*i.e.*, the non-negativity constraint is binding).

We discuss now the welfare properties of alternative monetary policies. The monetary authority can alter the real return on money (or, equivalently, the inflation rate) by varying the nominal size of monetary transfer to young individuals (or, equivalently, the nominal rate of money growth). Our aim is to assess the effect of small inflation rates by performing a comparative statics exercise around the constant-price equilibrium. The criterion of optimality is the welfare of an individual in the steady state.

Proposition 2 (Welfare improving inflation) *Welfare is smoothly increasing in inflation at the constant price equilibrium* if and only if *first-period public good contribution and second-period private good consumption are (strictly)* net-substitutes.[6] *In particular, welfare is smoothly increasing in inflation at the constant price equilibrium in the instant decay case.*

Proof. We evaluate the marginal welfare effect of a change in inflation at equilibrium. For small inflation (or deflation) rates, equilibria are the zeros of

[6] Here we intend that the Hicksian (or compensated) demand for z_0 smoothly increases with σ.

the map below (notice that, by lemma **??**, there is no second-period contribution at the zero-inflation equilibrium; by continuity of the optimal plan, second-period contribution is still zero for small changes in all parameters of $\mathbf{f}$):

$$\mathbf{G}(x_0, x_1, z_0; \sigma) = \begin{pmatrix} 1 - x_0 - x_1 - z_0 \\ x_1 - \mathbf{x}_1(z_0, 0, \sigma x_1, \sigma) \\ z_0 - \mathbf{z}_0(z_0, 0, \sigma x_1, \sigma) \end{pmatrix}.$$

In a small open neighborhood of $(x_0^*, x_1^*, z_0^*; 0)$, the constant price equilibrium, the two mappings $\mathbf{z}_0$ and $\mathbf{x}_1$ are differentiable, and so is $\mathbf{G}$, by the implicit function theorem applied to the first-order conditions (this is a well-established result in consumer demand theory; indeed, our assumption 1 guarantees that the objective function of each individual is smoothly strictly concave). We denote by $(\mathbf{x}_0^*, \mathbf{x}_1^*, \mathbf{z}_0^*)$ the map which gives steady state equilibria as a smooth function of σ around zero. We then proceed with simple computations and obtain the linear system of equations

$$\frac{\partial \mathbf{x}_0^*}{\partial \sigma} + \frac{\partial \mathbf{x}_1^*}{\partial \sigma} + \frac{\partial \mathbf{z}_0^*}{\partial \sigma} = 0, \tag{14}$$

$$\frac{\partial \mathbf{x}_1^*}{\partial \sigma} - \frac{\partial \mathbf{x}_1}{\partial z_0}\frac{\partial \mathbf{z}_0^*}{\partial \sigma} = x_1^* \frac{\partial \mathbf{x}_1}{\partial h} + \frac{\partial \mathbf{x}_1}{\partial \sigma}, \tag{15}$$

$$\left(1 - \frac{\partial \mathbf{z}_0}{\partial z_0}\right)\frac{\partial \mathbf{z}_0^*}{\partial \sigma} = x_1^* \frac{\partial \mathbf{z}_0}{\partial h} + \frac{\partial \mathbf{z}_0}{\partial \sigma}, \tag{16}$$

where all derivatives are evaluated at the constant price equilibrium.

The above equations are now used to estimate the welfare effect of marginal changes in inflation at a constant price equilibrium. Consider the quantity

$$u_x^0 \frac{\partial \mathbf{x}_0^*}{\partial \sigma} + \beta u_x^1 \frac{\partial \mathbf{x}_1^*}{\partial \sigma} + \left[\left(\frac{1}{1-\delta}\right)\left(u_Q^0 + \beta u_Q^1\right)\right]\frac{\partial \mathbf{z}_0^*}{\partial \sigma}$$

or, equivalently—see (5) and (14),

$$\left[\left(\frac{1}{1-\delta}\right)\left(u_Q^0 + \beta u_Q^1\right) - u_x^0\right]\frac{\partial \mathbf{z}_0^*}{\partial \sigma}. \tag{17}$$

Expression (17) gives the change in utility along the equilibrium locus due to a marginal change in inflation. It is easy to check that, by condition (6), the first term is positive. The sign of expression (17) is then given by the sign of its second term. It can also be easily shown that, as agents' contributions are perfect substitutes, the inequality

$$1-\frac{\partial \mathbf{z}_0}{\partial z_0}>0$$

is always satisfied. Therefore, by condition (16),

$$\operatorname{sign}\frac{\partial \mathbf{z}_*}{\partial \sigma}=\operatorname{sign}\left[x_1^*\frac{\partial \mathbf{z}_0}{\partial h}+\frac{\partial \mathbf{z}_0}{\partial \sigma}\right].$$

The first term is the net substitution effect, or the derivative of the Hicksian demand for z_0 with respect to the price of x_1, *i.e.,* with respect to σ. Thus, the result follows by our assumption on net-substitution effects in the statement of the proposition.

To see that the above result holds in the instant decay rate, it is sufficient to formulate the net substitutability condition in terms of excess demand properties. Using techniques from standard demand theory, net substitution obtains if and only if

$$u_{xx}^0-u_{xQ}^0+\beta\delta u_{xQ}^1<0, \tag{18}$$

where all derivatives are evaluated at the zero inflation equilibrium. From inspection of (18), we see that inflation is welfare improving in the full depreciation case, or for a high enough depreciation rate. The same effect emerges when individuals are patient enough, since in this case the private consumption in the two periods will be close at equilibrium. ■

The mechanics underlying the welfare result of Proposition 2 can be summarized as follows. A positive inflation rate induces a shift of real resources from the second to the first period. First, private consumption when young always increases (see condition (5)), with the traditionally associated welfare loss due to the violation of the golden rule; this is a second order loss, since at the zero inflation equilibrium private consumption is allocated efficiently. Second, if some of the new resources shifted to the first period are devoted to the provision of the public good, then the inefficiency due to the under-provision of the public good is mitigated and a (first order) welfare gain results. This happens *if and only if* second period private consumption and first period provision are net substitutes in the individual maximization problem. Note that this is an equilibrium phenomenon: although individual reaction functions are negatively sloped in the contribution game, *individual contributions at equilibrium are positively correlated as σ changes.* It is interesting to stress the nature of the welfare effects at work in Proposition 2: by raising the inflation rate, we introduce a source of *dynamic* inefficiency in the economy in order to correct the *static* inefficiency characterizing the constant price equilibrium. For small enough inflation rates, the first order welfare gain of increased contributions outweights the second order welfare loss due to the violation of the golden rule.

Although the net substitutability condition may seem a strong one, once normality, discounting and time-separability of preferences are assumed, it does not impose any additional requirement in the case of a perishable public good. In fact, proposition 2 states that the welfare effect always holds in this case. This can be better explained in terms of marginal valuations. By time-separability, the choice of providing the public good when young has no effect on the utility of consumption when old; therefore, the marginal valuation of providing the public good is affected only by the private consumption when young, which, as we saw, always increases. By normality, this effect is positive. On the contrary, when the public good is durable, the decrease of private consumption when old lowers the return to the consumption of public good when old and, therefore, the return to provisions when young, according to the decay rate. By normality, the total marginal valuation of contributing when young is therefore affected positively by the increase of private consumption when young and negatively by the decrease of private consumption when old. The total effect is ambiguous, and finally rests on condition (18).

Our welfare comparison is based on steady state utilities and neglects the consequences of a policy change only on the first old generation, when the public good is perishable, and on some transitional generations, when the public good is durable. If non-stationary equilibria (locally) converge to the steady state, then at most a finite number of generations can be made worse off by a slightly inflationary monetary policy. We expect an adverse welfare effect on initial generations. Consider, for instance, the welfare change of the first old generation after a change in money growth in the case of complete depreciation of the public good. This can be evaluated, at the constant prices equilibrium, using the formula

$$u_x^1 \frac{\partial \mathbf{x}_1^*}{\partial \sigma} + u_Q^1 \frac{\partial \mathbf{z}_0^*}{\partial \sigma}.$$

The first terms captures the loss due to real balances depreciation, whereas the second term measures the benefit coming from an increase in the public good provision. From first-order conditions for an optimal plan, we know that $u_x^1 - u_Q^1 > 0$ and, from condition (14) and the fact that first period consumption is increasing in inflation, we have $\partial \mathbf{x}_1^* / \partial \sigma + \partial \mathbf{z}_0^* / \partial \sigma < 0$. Therefore, all benefits from a higher inflation are enjoyed when young and the first old individual faces a loss in welfare.

A final question we wish to address is whether there exists a critical level of inflation above which further inflation would decrease welfare. The existence of such a critical level, which we prove in the next proposition, endows our result with a rather reasonable property of the equilibrium effects of inflationary policy.

Proposition 3 (Optimal inflation rate) *There is a positive rate of inflation which dominates (in terms of welfare) all non-negative rates of inflation.*

Proof. We again remark that the second-period contribution to the public good is always zero when the economy is inflationary and so an equilibrium allocation is simply characterized by a vector $a = (x_0, x_1, z_0)$. To simplify notation, we write

$$w(a) = u\left(x_0, \frac{z_0}{1-\delta}\right) + \beta u\left(x_1, \frac{z_0}{1-\delta}\right).$$

Fix any equilibrium a^0 for the zero inflation rate. If there is no optimal inflation rate, then we can find an increasing sequence of inflation rates $\{\sigma^n\}$ and an associated sequence of equilibria $\{a^n\}$ involving an increasing welfare. By construction, $w(a^n) \geq w\left(a^0\right)$ for all n. Since inter-temporal utility is unbounded from below, the sequence $\{a^n\}$ is bounded away from zero and, as it lies in a compact set, we can assume that it converges to a limit $a^\infty = (x_0^\infty, x_1^\infty, z_0^\infty)$.

By means of the first-order conditions for an optimal plan, it can be simply verified that $x_1^{\infty} = 0$, a contradiction. ■

Our result in proposition 3 relies on a rather strong assumption on preferences: inter-temporal indifference curves do not intersect the boundary of the inter-temporal consumption space. Such a requirement is far from being minimal. However, we stress here that weak Inada's conditions alone do not seem sufficient to exclude the case of welfare increasing monotonically with inflation.

5 Conclusion

This paper studies the welfare properties of inflation in an economy affected by underprovision of a public good at the constant price equilibrium. We show that, set aside all effects of inflation other than the distortion of inter-temporal allocation of private consumption, expansionary policies can have positive welfare effect by stimulating the voluntary contributions to the public good. Although we have not stressed any particular interpretation for the public good, the mechanism of voluntary contributions is consistent we various real life examples. Since inflation is typically a policy tool available to national authorities, such examples cannot include the largely debated problem of international public goods. It includes, however, relevant national public goods problems such as, for instance, compliance to environmental standards, tax evasion, investment in human capital, provision of local public goods. In the presence of such externalities, the problem of optimal monetary policy, much as any problem of optimal tax reform, should take into account the equilibrium reaction of involved agents to alternative inflation rates. As shown in the paper, these reactions may well outweight the welfare loss associated with inflation.

References

[1] Andreoni, J., (1988) "Privately provided public goods in a large economy: the limits of altruism", *Journal of Public Economics,* **35**, *57-73.*

[2] Balasko and Shell (1980a), "The overlapping generations model I: the case of pure exchange without money" *Journal of Economic Theory* **23(3)**, 281-306.

[3] Balasko and Shell (1980b), "Existence of competitive equilibrium in a general overlapping generations model", *Journal of Economic Theory* **23(3)**, 307-322.

[4] Bergstrom, T.C., L. Blume and H.L. Varian (1986) "On the private provision of public goods" *Journal of Public Economics,* **29**, 25-49.

[5] Brock, W.A. (1974) "Money and growth:the case of long run perfect foresight, *International Economic Review* **15**, 750-777.

[6] Chamley, C. (1985) "On a simple rule for the optimal inflation rate in second best taxation" *Journal of Public Economics* **26**, 35-50.

[7] Clower, R. W. (1967) "A reconsideration of the microfoundations of monetary theory, *Western Economic Journal* **6**, 1-9.

[8] Cowell, F.A. and J. Gordon (1988), "Unwillingness to pay: tax evasion and public good provision", *Journal of Public Economy* **36**, 305-321.

[9] Debreu, G. (1952) "A social equilibrium existence theorem" *Proceedings of the National Academy Of Science of the USA*, **38**, 886-893.

[10] Friedman, M. (1969) *The Optimum Quantity of Money and other Essays*, Aldine, Chicago, 1969.

[11] Helpman, E., and E. Sadka (1979) "Optimal financing of the government's budget: taxes, bonds or money?" *American Economic Review* **68**, 152-160.

[12] Ihori, T., (1996), *Public Finance in an Overlapping Generations Economy,* MacMillian, London and St. Martin, New York.

[13] Lucas, R.J. and N.L. Stockey (1983) "Money and interest in a cash in advance economy" Working Paper 1618, NBER, Chicago.

[14] Phelps, E.S. (1973) "Inflation in the theory of public finance" *Swedish Journal of Economics,* **75***,* 67-82*.*

[15] Samuelson, P. A. (1958) "An exact consumption loan model of interest with or without the social contrivance of money" *Journal of Political Economy*, **66**, 1002-1011.

[16] Sidrausky M. (1967) "Rational choice and patterns of growth in a monetary economy, *American Economic Review* **57(2)**, 434-544.

[17] Siegel, J.J. (1978) "Notes on optimal taxation and the optimal rate of inflation" *Journal of Monetary Economics*, **4**, 297-305.

[18] Summers, L.H. (1981) "Optimal inflation policy" *Journal of Monetary Economics* **7**, 175-194.

[19] Tobin, J. (1965) "Money and economic growth" *Econometrica*, **33**, 671-684.

[20] Turnowsky, S.J. and W.A. Brock (1980) "Time consistency and optimal government policies in perfect foresight equilibrium" *Journal of Public Economics* **13**, 183-212.

[21] Weiss, L. (1980) "The effects of money supply on economics welfare in the steady state" *Econometrica*, **48**, 565-576.