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Environmental Fiscal Policy in an Endogenous Growth Model with Human Capital.

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Abstract

This paper analyzes environmental …scal policy within a two-sector endogenous growth model with elastic labor supply. Pollution is modelled as a side product of production. The framework allows us to analyze the consequences of an environmental tax on the economic dynamics. Both transitional dynamics and balanced growth path are computed and the response to an environmental tax change is explored. Short-run and long-run welfare costs are computed too. We show that an environmental tax change induces a sharp contrast between short-run and long-run e¤ects. The magnitude of this contrast depends on the agents' aptitude to substitute studying time for leisure.

Keywords: Endogenous Growth, Human Capital, Environmental Externality, Environmental Tax, Transitional Dynamics, Welfare Cost.

JEL classi…cation: E62, I21, H22, Q28, O41, D62.

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1 Introduction

Recent growth theory has made progress in analyzing the dynamic e¤ects of taxes. Until recently, economic models that could o¤er insight into this question were lacking. The bulk of the growth literature focused on steady states with constant per capita output, whilst those that did consider sustained growth focused on exogenous trends. By de…nition, such taxation cannot impact on this long-run exogenous growth path. It is only since the development of endogenous growth theory that a tool has existed for investigating how taxation a¤ects growth. These new models explicitly model the processes through which growth is generated and, by doing so, can trace out the e¤ects of taxations upon the underlying individual decisions. Thus, taxation incidences on growth can be rigorously understood and predicted. This is also true for e¢cient instruments like a Pigouvian tax that internalize environmental externalities.

How environmental tax a¤ects economic growth is an ambiguous issue. In the simplest endogenous growth model, the AK model, the growth e¤ect of environmental policy is negative. This is shown by both Gradus and Smulders (1993) for a centrally planned economy with varying pollution's weight in the utility function and Ligthart and van der Ploeg (1994) for a decentralized economy. In the literature on endogenous growth with human capital, it is shown that a tighter environmental policy might have a stimulating growth e¤ect. In a Uzawa-Lucas setup augmented with an explicit treatment of the environment, Gradus and Smulders (1993) …nd that the optimal growth rate is independent from environmental care. Only by assuming that pollution also negatively a¤ects the e¢ciency in the human-capital sector, did they detect positive growth e¤ects.

Bovenberg and Smulders (1995) consider a two-sector model consisting of a consumption/capital good and an R&D sector generating knowledge about pollution-augmenting techniques. Since better environmental quality improves factor productivity in the consumption-good sector, positive growth e¤ects of a tighter environmental policy are possible. In a pure human capital variant of the two-sector Lucas model, van Ewijk and van Wijnbergen (1995) …nd positive growth e¤ects of a tighter environmental policy also by assuming that pollution negatively a¤ects the production process.

Hence, the existing literature can only explain positive growth e¤ects of tighter environmental policy by assuming direct positive productivity e¤ects - positive environmental externalities in production - either in the education or in the consumption-good sector. In contrast with this conclusion, we show in this paper that in a two-sector endogenous growth model with leisure, a higher environmental tax might a¤ect the long-run growth rate. The reason for this is as follows. Due to an increased environmental tax, …rms increase their abatement activities, which reduces …nal output net of abatement at the expense of households' consumption. Households substitute education time for leisure time so as to counteract reduced consumption, and this …nally boosts growth up[1].

Whereas most endogenous growth models dealing with environmental concerns restrict the analysis to the steady state, little has been said so far on the short-run e¤ects of taxation. There are a few exceptions in the literature. Van der Ploeg and Ligthart (1994) derive the transitional dynamics of linear growth model augmented with a renewable environmental resource. By increasing the disutility parameter of pollution of the representative agent, they …nd that the fall in the short-run growth rate in the centrally planned economy is bigger than the long-run growth rate. Note that in this linear framework, the evolution of the environmental stock is responsible for and solely determines the transitional dynamics of the economy. Bovenberg and Smulders (1996) analytically compute the transitional dynamics of a two sector model consisting of a consumption/capital goods sector and a research and development sector that generates knowledge

[1] In a similar model, Hettich (1998) shows analytically that changes in leisure may create a link between growth and environment. But he does not study the short run e¤ects in this structure.

about pollution-augmenting techniques. The renewable environmental resource acts both as public consumption and as a public input into production, where the latter is identical to a productive environmental spillover. They find that if the environment acts mainly as a consumption good, then a tighter environmental policy reduces growth in both the long-run and the short-run. But if the environment acts mainly as a public investment good, then long-run growth rises, while short-run growth declines[2].

In this paper, we compute the entire dynamic adjustment path towards a balanced growth path. Our analysis of the transition reveals the short-run impact of environmental tax. Furthermore, this analysis of the dynamic adjustment path enables us to perform welfare calculations. In particular, we make explicit the trade-off between the short-run and the long-run costs of environmental policy.

The remainder of the paper is organized as follows. In section 2 the general model is laid out and both market and central-planner solution are derived. The optimal environmental tax rate in a first-best setting is analyzed. Section 3 proposes a numerical exercise: we calibrate the model at the steady state, compute the transitional dynamics and comment the short-run dynamics. Section 4 computes short-run and long-run welfare costs of taxation. Section 5, summarizes the main findings.

2 The model

We consider an economy populated with an infinitely-lived representative household. The household owns the stock of physical capital in the economy, K_t, and is endowed with a (normalized) unit time. The time endowment can be allocated between work (remunerated at the current competitive wage rate), leisure and schooling. The capital stock used in production causes a negative environmental externality as a side product. Pollution is assumed to affect individuals' utility. We introduce government in minimal fashion, its task consists solely to correct the market failure caused by the environmental externality.

2.1 Preferences, technology and pollution

The behavior of the rational household is guided by the maximization of the discounted lifetime utility

$$W_0 = \sum_{t=0}^{\infty} \beta^t u(C_t; l_t; P_t) \tag{1}$$

where

$$u(C_t; l_t; P_t) = \log C_t + \phi_l \log l_t - \phi_P \log P_t \tag{2}$$

C_t is consumption, l_t represents hours spent away from leisure, $0 < \beta < 1$ is the discount factor and P_t is the net pollution flow. The parameters ϕ_l and ϕ_P represent the weights of leisure and pollution in utility. The consumer budget constraint can be written as follows

$$K_t = (1 + r_t - \delta_K) K_{t-1} + w_t u_t H_{t-1} - C_t + T_t \tag{3}$$

where r_t is the return to physical capital and w_t is the gross wage rate per effective unit of human capital $u_t H_{t-1}$. T_t represents transfers from the public sector, and u_t is the supply of working time. δ_K denotes the rate of depreciation for physical capital.

[2] Without taking the environment into account, Caballé and Santos (1993), Mulligan and Sala-i-Martin (1993), Devereux and Love (1994), Faig (1995) and Lardon-de-Guevara and al.(1997) investigate the transitional dynamics within similar models.

The representative agent can increase his human capital stock H_t, by devoting time to schooling. We assume that this activity takes place outside the market, and new human capital can only be obtained by spending time. Thus, the law of motion for human capital is given by the constraint

$$H_t - H_{t-1} = (Bv_t - \delta_H)H_{t-1} \tag{4}$$

where B is the marginal productivity of schooling time v_t and δ_H denotes the rate of human capital depreciation.

The household is endowed with a (normalized) time unit which can be allocated either to work, leisure or schooling

$$1 = u_t + v_t + l_t \tag{5}$$

The physical capital used in production is the source of the pollution flow P. This flow can be reduced by means of private abatement activities D which in turn consume a part of output, in line with the flow resource constraint. The net pollution function has the form[3]:

$$P_t = \left(\frac{K_{t-1}}{D_t}\right)^{\chi} \tag{6}$$

where $\chi > 0$ is the exogenous elasticity of P with respect to K/D.

2.2 Firms

The economy consists of a large number of identical and competitive firms. They rent capital and hire effective labor from the households at the interest rate r and the wage rate w respectively. They use the following constant-returns Cobb-Douglas technology

$$Y_t = AK_{t-1}^{\alpha}(u_tH_{t-1})^{1-\alpha} \tag{7}$$

where $A > 0$ and $0 < \alpha < 1$.

Firms must pay a pollution tax τ^P according to their net pollution P. Abatement is assumed to be a privative good which enables firms to increase output without causing more pollution. Firms are assumed to maximize their market value, which is equal to the appropriately discounted sum of profits flows, the later is given by

$$\pi_t = Y_t - rK_{t-1} - w_tu_tH_{t-1} - D_t - \tau_t^P P_t$$

Profits maximization implies that in equilibrium, firms pay each production factor at its marginal productivity.

$$r_t = \alpha\frac{Y_t}{K_{t-1}} - \tau_t^P\chi\frac{P_t}{K_{t-1}} \tag{8}$$

$$w_t = (1-\alpha)\frac{Y_t}{u_tH_{t-1}} \tag{9}$$

$$\tau^P\chi P_t = D_t \tag{10}$$

[3] The same specification is used by Gradus and Smulders (1993).

Without a pollution tax, firms would neglect the negative side product of physical capital in the production process and abatement activities would be zero. The market clearing condition for the goods market is

$$Y_t = C_t + D_t + K_t - (1 - \delta_K) K_{t-1} \tag{11}$$

The government budget constraint implies that all revenue is lump-sum transferred back to households in every period $\left(\tau_t^P P_t = T_t\right)$.

2.3 The market solution

Definition 1 A competitive equilibrium for this economy is a set of allocations $\{C_t, D_t, u_t, l_t, K_t, H_t, P_t, T_t\}$, a price system $\{r_t, w_t\}$ and an environmental tax τ^P, such that, taking the price system and fiscal policy as given, $\{C_t, D_t, u_t, l_t\}$ maximizes (1), subject to (3), (4) and (5), and the path $\{u_t, K_t, H_t, r_t, w_t\}$ satisfies equations (8), (9), (10) and (11).

So as to characterize the competitive equilibrium, let us focus on the different trade-offs faced by the household. After eliminating the shadow prices for physical and human capital, the first order conditions for the household problem write

$$l_t = \frac{\phi_l}{w_t} \frac{C_t}{H_{t-1}} \tag{12}$$

$$\frac{C_{t+1}}{C_t} = \beta \left[1 + r_{t+1} - \delta_K\right] \tag{13}$$

$$\frac{C_{t+1}}{C_t} = \beta \frac{w_{t+1}}{w_t} \left[1 + B \left(1 - l_{t+1}\right) - \delta_H\right] \tag{14}$$

Equation (12) equates the marginal rate of substitution between consumption and leisure to the real wage. Equation (13) and (14) are the Euler conditions determining the optimal accumulation of physical and human capital. It is obvious that environmental tax affects only the intertemporal incentive to invest in physical capital, as described by equation (13).

These conditions, along with equations (3), (4), (5), (8), (9), (10) and (11) constitute a dynamical system in C, D, u, v, l, K and H which, together with the transversality conditions[4] and initial $K(0)$ and $H(0)$, fully describe the dynamic behavior of the economy along an interior equilibrium.

2.4 The central-planner solution

In contrast to a market solution, the central planner maximizes the utility of the representative economic agent and takes into account pollution. The central planner maximizes life-time utility by choosing time paths for C, D, K, H, u and l, subject to the flow-resource constraint (11) and human capital accumulation constraint (4). After eliminating the shadow prices, the first-order conditions of the central-planner solution are given by:

$$D_t = \phi_P \psi C_t \tag{15}$$

[4] These conditions are standard and impose that the present discounted value of both capital stocks tends to zero at the infinity.

$$\frac{C_t}{Y_t} = \frac{(1-\alpha)}{\phi_l}\frac{l_t}{u_t} \tag{16}$$

$$K_t = Y_t + (1-\delta_K)K_{t-1} - D_t - C_t \tag{17}$$

$$H_t = [1 + B(1 - l_t - u_t) - \delta_H] H_{t-1} \tag{18}$$

$$\frac{C_{t+1}}{C_t} = \beta\left[1 + \alpha Y_{t+1} - \frac{D_{t+1}}{K_t} - \delta_K\right] \tag{19}$$

$$\frac{C_{t+1}}{C_t} = \beta\frac{Y_{t+1}/(u_{t+1}H_t)}{Y_t/(u_t H_{t-1})}[1 + B(1 - l_{t+1}) - \delta_H] \tag{20}$$

The central-planner solution only differs from the market solution through equation (15). It shows that for a social optimum, the marginal utility of consumption and abatement must be equalized. Equation (19) is also known as the Keynes-Ramsey rule describing the optimal consumption path over time. The right-hand side consists of the private marginal product of physical, corrected by the term D/K, the depreciation rate of the physical capital stock δ_K, and the rate of time preference β. There is a wedge between private and social return to physical capital. The term D/K can be seen as the marginal damage of physical capital. Consumption grows, remains constant, or declines if the social return to physical capital is larger than, equal to, or smaller than the sum of the rate of depreciation and the rate of time preference.

2.5 The balanced growth path

In this section we will focus on the dynamic properties of the balanced growth path.

Definition 2 A balanced growth path (or steady state) is an allocation $\{C_t, D_t, u_t, v_t, l_t, K_t, H_t, P_t, T_t\}$, a price system $\{r_t, w_t\}$ and an environmental tax τ^P satisfying Definition 1, and such that for some initial conditions $K(0) = K_0$ and $H(0) = H_0$, the paths $\{C_t, D_t, K_t, H_t, T_t\}$ grow at the constant rate g, and u_t, v_t, l_t and P_t remain constant.

Following this definition, we have that in a balanced growth path[5]

$$\frac{\dot C_t}{C_t} = \frac{\dot D_t}{D_t} = \frac{\dot K_t}{K_t} = \frac{\dot H_t}{H_t} = g$$
$$\frac{\dot u_t}{u_t} = \frac{\dot v_t}{v_t} = \frac{\dot l_t}{l_t} = \frac{\dot P_t}{P_t} = 0$$

For analytical convenience we use the following transformed variables: $h_t = H_t/K_t$, $\tilde\tau_t^P = \tau_t^P/K_{t-1}$, $c_t = C_t/K_{t-1}$, $y_t = Y_t/K_{t-1}$, $d_t = D_t/K_{t-1}$ and $g_t = K_t/K_{t-1}$.

Using this change of variables, we obtain the following dynamical system

$$\frac{l_t}{u_t} = \frac{\phi_l}{(1-\alpha)}\frac{c_t}{y_t} \tag{21}$$

[5] A constant level of pollution is in accord with ecological sustainability of growth, see Smulders (2000).

$$d_t = \left(\theta \tau_t^P\right)^{1/1+\theta} \quad (22)$$

$$g_t = 1 + y_t - d_t - c_t - \delta_K \quad (23)$$

$$g_t \frac{h_t}{h_{t-1}} = 1 + B\,(1 - u_t - l_t) - \delta_H \quad (24)$$

$$g_t \frac{c_{t+1}}{c_t} = \beta\,[1 + r_{t+1} - \delta_K] \quad (25)$$

$$g_t \frac{c_{t+1}}{c_t} = \beta \frac{w_{t+1}}{w_t}\,[1 + B\,(1 - l_{t+1}) - \delta_H] \quad (26)$$

Steady-state values c, d, l, u, P and g are obtained by eliminating the index t. From the linearization of the above system one can show that, independently of the size of tax rate, the model displays a saddle path dynamic structure[6]. Thus, unlike other models presented in the literature [Benhabib and Perli (1994), Bond and al. (1996), Xie (1994)] our model is unable to generate the indeterminacy phenomenon typical of distorted economies[7].

2.6 Optimal Environmental tax rate

To determine the first-best environmental tax, we compare the first-order conditions of the market solution and their central-planner counterparts. Particularly, by comparing (10) and (15) we can compute the optimal pollution-tax rule:

$$\left(\tau_t^P\right)^{op} = K_{t-1}\frac{1}{\theta}\left(\frac{D_t}{K_{t-1}}\right)^{1+\theta} = K_{t-1}\frac{(\psi_P\theta)^{1+\theta}}{\theta}\left(\frac{C_t}{K_{t-1}}\right)^{1+\theta} \quad (27)$$

The optimal environmental tax $\left(\tau^P\right)^{op}$ must be equal to the product of the current physical capital stock with the optimal consumption-capital ratio (C/K) of the central planner solution. The ratio C/K is constant along a balanced growth path. But K increases over time.

For that reason the pigouvian tax rate must increase over time with the growth rate of the economy. This result becomes intuitive by remembering that P must be constant along a balanced growth path. To keep the level of pollution constant, $\left(\tau^P\right)^{op}$ must rise over time, because the physical capital stock, which is responsible for the pollution, accumulates over time. Firms only increase abatement activities over time if they have an incentive to do so via an increasing pollution tax. Therefore, we can separate trend and level of the pollution tax rate. To do so, we normalize it by the physical capital stock and define $\tilde{\tau}^P = \tau_t^P/K_{t-1}$, which is constant along a balanced growth path.

[6] See Hettich (1998) for a similar model with tax on consumption, labor income and capital income.

[7] In Bond and al. (1996) indeterminacy emerges from the presence of taxes in a model with physical capital as an input in the educational sector. As we assume that physical capital is only productive in the output sector, the condition for general instability or indeterminacy is never satisfied. In Benhabib and Perli (1994) and Xie (1994) indeterminacy arises from knowledge spill-overs.

3 Numerical results

3.1 Calibration

In this section we derive a full numerical solution for the model. For this calibration exercise we cannot really hope to be as precise as those who employ the same model without environmental externality, since we lack strong empirical evidence concerning the nature of the environmental preferences and pollution function. Nevertheless, to the greatest possible extent, we follow the recent literature.

The parameter values needed are (i) preferences parameters, ¯, $Á_l$ and $Á_P$, (ii) technology parameters ®, A, B, $\pm_K$ $\pm_H$ and Â; (iii) environmental tax $¿^P$. We proceed by choosing parameters according to the arguments below to pin down a benchmark economy.

We calibrate the model in two di¤erent steps. First, we consider that the economy is initially on the equilibrium growth path where polluted emissions are taxed at a lower rate[8]. To compute the steady state variables values, we resort to common parameters values already used in two-sector endogenous growth models. Additionally, the calibration is made so as to capture a pollution abatement as a percentage of GDP of 1:8%, which correspond to the average of environmental protection expenses in OCED countries. Second, we compute the …nal steady state values corresponding to the optimal rate of environmental tax. We suppose that this optimal rate is twice as high as that initially used. Then, this optimal rate is used to determine the environmental preferences' weight [see Eq. (27)][9].

3.1.1 Parameters choice

The calibration is made in order to capture a quarterly equilibrium growth rate of 0:3% which correspond to 1:2 % per annum. This rate is plausible for most developed countries. Our choice of parameters closely follows the literature on simulated two sector endogenous growth models that are similar to ours. According to Barro and Sala-i-Martin (1995, p. 37), the measured depreciation rate for the overall stock of structure and equipments is around 5 % per year which correspond to 1:25 % per quarter. Taking this as a proxy for the industrialized economies, the rate of depreciation of the physical and human capital stock are assumed to be $\pm_K = \pm_H$ = 0:0125. The share of physical capital in …nal good production ® = 0:25 is taken from Lucas (1988). In addition, we set the discount factor to 0:99 so that the quarterly equilibrium interest rate is equal to 1 %. Following Devereux and Love (1994), we set the weight of leisure in utility $Á_l$ to 1:24. Finally, we consider a share of pollution abatement in production of 1:8 % and, in an ad hoc manner, we set the elasticity of pollution with respect to the ratio (K=D), Â to 0:1.

Table 1: Baseline Parameter values

¯	= 0:99	discount factor
g	= 1:003	quarterly growth rate
$\pm_K = \pm_H$	= 0:0125	depreciation rate
®	= 0:25	physical capital share in production
D=Y	= 0:018	share of pollution abatement in production
$Á_l$	= 1:24	weights of leisure in utility
Â	= 0:1	elasticity of pollution with respect to the ratio K=D

[8] The environmental tax rate is exogenously …xed by the government. Both abatement expenses and pollution are induced by this tax rate.

[9] Obviously, this is an arbitrary choice, which means that current abatement e¤orts do not correspond to the environmental agents' preferences.

We suppose that in period t = 0, the government decides to double the environmental tax rate in order to reach an optimal level. Thus, the share of pollution abatement expenses (D=Y = 1; 8 %) does not match the agent's preferences. We suppose that doubling $¿^P$ allows to reach an optimal share D=Y.

3.1.2 Results:

We have chosen the following variables and parameters values ¯, g, ¾, $±_K$, $±_H$, ®, $Á_l$, d=y. Values of the remaining parameters and variables are solution to the system (21)-(26). Initial steady state (SS1) values are summarized in the following table

Table 2.a: Calibration Results at SS1

y	= 0:1105	…nal output per unit of physical capital stock
$¿^P$	= 0:0107	environmental tax rate per unit of physical capital stock
c	= 0:0930	consumption per unit of physical capital stock
B	= 0:0397	human capital productivity
l	= 0:3549	leisure
u	= 0:2550	working hours
h	= 15:3346	H=K ratio

We suppose that in period t = 0 the government doubles the environmental tax rate. This environmental policy change a¤ects the steady state (SS1) and initiates transitional dynamics to a new steady state. During the transitional dynamics, variables grow di¤erently, which re‡ects the responses of agents to the environmental policy shock. The new steady state (SS2) is directly deduced from the equilibrium system (21)-(26).

Table 2.b: Steady state change

	g	y	c=y	d=y	u	l
SS1	1:0030	0:1105	0:8417	0:0180	0:2550	0:3549
SS2	1:0031	0:1178	0:8360	0:0317	0:2550	0:3525
variation (%)	0:0111	0:7430	¡ 0:5720	1:3690	0:0024	¡ 0:2379

We get a slight growth rate increase and both consumption and leisure decrease. Both the share of pollution abatement in GDP and the ratio y = Y=K increase. Thus, environmental tax increase stimulates long-run growth rate since labor supply is endogenous. Therefore, an abatement increase a¤ects negatively both consumption and investment. In order to compensate for the decrease in the share of consumption in GDP, agents reduce their leisure and devote more time for schooling, which …nally improves human capital accumulation and boosts the long-run growth rate up.

Additionally, the environmental tax change induces a factorial substitution process whereby production becomes more human capital intensive (clean factor). Notice that the working time u is almost not sensitive to an environmental tax variation.

3.2 Transitional dynamics

To compute the transitional dynamics we log-linearize the dynamic system (21)-(26) to make the equations approximately linear in the log-deviations from the steady state (see appendix). After doing this, we solve for the recursive equilibrium law of motion via the methods of undetermined coefficients.

The recursive equilibrium law of motion can be defined by two relations:

- On the one hand, a relation between state variables in t and their values in $t-1$ (states dynamics).
- On the other hand, a relation between jump variables in t and state variables in $t-1$ (jump-state dynamics).

Let us collect the state variables in the vector X_t, i.e. $X_t = (h_t; g_t)$, and the jump variables in the vector Y_t; i.e. $Y_t = (y_t; c_t; d_t; u_t; l_t; r_t; w_t; P_t)$. Then the recursive equilibrium law of motion write

$$\begin{aligned} X_t &= PX_{t-1} \\ Y_t &= QX_{t-1} \end{aligned} \tag{28}$$

where P and Q are matrixes of partial elasticities.

The linear dynamic system can be represented in the following matrix formula

$$\begin{aligned} 0 &= AX_t + BX_{t-1} + CY_t \\ 0 &= FX_{t+1} + GX_t + HX_{t-1} + JY_{t+1} + KY_t \end{aligned} \tag{29}$$

Using the method of undetermined coefficients we can compute the P and Q matrixes[10].

The simulation of the transitional dynamics starts in period 0, where the government suddenly doubles the environmental tax rate. This environmental policy shock induces an instantaneous reaction of all economic variables. We then observe different impacts on the variables, which leave their initial level at (SS1) and reach at different rates their new level at (SS2). Table 2.b. summarizes the changes values.

In fact, the environmental tax change induces three effects:

- A crowding out effect caused by the increase in abatement expenses, which negatively affects consumption and investment.
- A factorial reallocation effect, which reduces the intensity of physical capital in production.
- A reallocation of available time, whereby schooling time increases.

The pace at which the economy reaches the new steady state (SS2) is the result of the interaction between these three effects. In the short-run, the stock of physical capital decreases, but inherits an increased trend after a while, and finally its growth rate reaches a new level on SS2, which is slightly higher than its initial SS1 level.

The short-run behavior of the economy is described by the growth rate transitional dynamics.

A higher environmental tax reduces the physical capital-human capital ratio ($k=h$) because the clean input factor H is substituted for the dirty input factor K. The productivity of physical

[10] We use the Matlab routines to compute P and Q. Starting from this solution one can easily reconstitute the times series from the SS1 variable values.

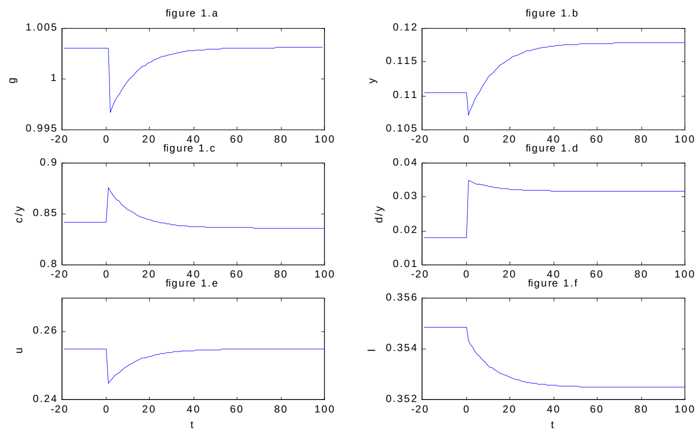

Figure 1: Transitional dynamics 1

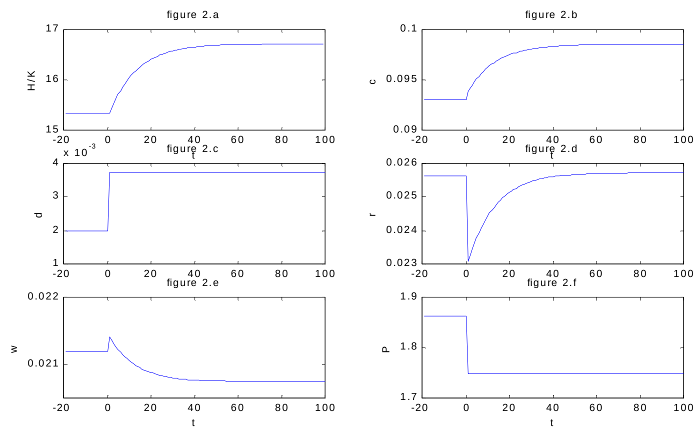

Figure 2: Transitional dynamics 2

capital increase and boosts growth up [see Eqs. (8) and (25)]. But there is also an e¤ect working in the opposite direction: a higher environmental tax reduces the productivity of physical capital and lowers growth. Due to the increased environmental tax, …rms increase their abatement activities, which reduces …nal output net of abatement at the expense of households' consumption. Households increase their marginal utility of leisure by substituting schooling time for leisure to counteract reduced consumption. From equation (24) it can be seen that a higher studying time enhances growth. A higher environmental tax increases the ratio $d=y$, hence lowers pollution [see …gures (1.d) and (2.f)].

In the beginning of the transitional dynamics, the crowding out e¤ect of abatement reduces both the growth rate (see …gure 1.a) and the ratio of physical capital to production (see …gure 1.b). As mentioned before, increased environmental tax leads to a more human capital intensive …nal output and to a higher studying time. The immediate response to the environmental tax increase is a sectorial reallocation of resources, which reduces the physical capital-human capital ratio. Households reduce their leisure time (see …gure 1.f) and increase their studying time. The time spent at work decreases in the beginning of the transitional dynamics but after some periods increases to reach a slightly higher level than before. As soon as the sectorial reallocation becomes important, the crowding out e¤ect is reduced. As a …nal result the higher environmental boosts long-run growth.

The main conclusion to be drawn is that an unanticipated increase in the environmental tax leads to a slight increase in the long-run growth rate, but also leads to a negative physical capital growth rate in the short-run. The aptitude of agents to substitute their schooling time for leisure determines the amplitude of the short-run and long-run e¤ects.

4 Welfare costs

The results presented in the previous section evidence the role played by an environmental tax change in the dynamic behavior of the economy. In this section we compute the welfare cost associated to this environmental tax change. We suppose that at $t = 0$, the economy transits from an initial optimal situation (with $¿^P = ¿^{op}$) to an non-optimal situation[11]. Evidently, this environmental policy shock changes the steady state, initiates transitional dynamics and generates a welfare cost. Two types of welfare variations can be distinguished: a …rst variation is associated with the short-run dynamics and a second variation is related to the change of steady state.

4.1 Welfare decomposition

We decompose welfare into transitional welfare (also referred to as the short-run welfare) $W_{1 ! 2}$ corresponding to the economy's transition from SS1 to SS2, and welfare related to the new steady state W_2. So as to get a numerical result, we suppose that the transition from a steady state to another is achieved in a …nite amount of periods, and we simply denote T the date at which we consider that the economy has numerically reached its new rest point. The total welfare associated to the environmental policy change W^{Tot} is equal to the sum of utility ‡ows, from $t = 0$ to 1, which can be written as the sum of $W_{1 ! 2}$ and $\bar{\ }^{T+1}W_2$:

$$W^{Tot} = W_{1 ! 2} + \bar{\ }^{T+1}W_2 \qquad (30)$$

[11] We supposed earlier that by doubling the environmental tax, an optimal ratio $(D=Y)$ can be reached. This assumption allows us to calculate $Á_P$.

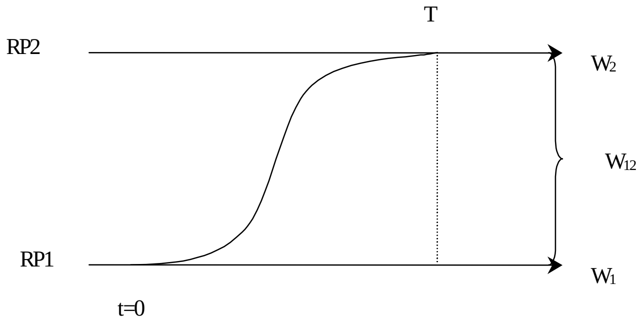

Figure 3: Welfare decomposition

Note that the economy converges only asymptotically to the steady state, and we therefore truncate the transitional dynamics in the effective computation at the horizon T. This horizon is chosen so that for all $t > T$, the difference between the value of physical capital stock at T (k_T) and its value at SS2 (k_2) is numerically very small[12].

Formally, the transitional welfare can be written:

$$W_{1\to 2} = \sum_{t=0}^{\infty} \beta^t \left[\log c_t + \sum_{i=1}^{t-1} \log g_i + \psi_l \log l_t - \psi_P \log P_t \right] \qquad (31)$$

the welfare related to the new steady state (SS2) is given by:

$$W_2 = \frac{\log c_2 + \psi_l \log l_2 - \psi_1 \log P_2}{1-\beta} + \frac{\beta \log g_2}{(1-\beta)^2} \qquad (32)$$

and the welfare related to the initial steady state (SS1) is given by

$$W_1 = \frac{\log c_1 + \psi_l \log l_1 - \psi_1 \log P_1}{1-\beta} + \frac{\beta \log g_1}{(1-\beta)^2}$$

Fixed at its optimal level, the environmental tax has a zero welfare cost. Conversely, the transition from a non optimal tax rate (τ^P) to an optimal tax rate generates a welfare benefit.

To obtain a meaningful evaluation of the welfare cost associated to our policy change, we express all welfare measures as percentage point of the permanent consumption that generates an equivalent welfare in the benchmark case. Thus, our welfare cost measures the compensation in consumption terms that leaves the consumer indifferent between the non optimal taxed stationary consumption path and the consumption path corresponding to the optimal tax rate[13]. We propose a dissociation of welfare cost related to transitional dynamics from the total welfare cost.

Total welfare cost:

Let us define $\mathbf{e}(\tau^P)$ as the constant flow of consumption that gives a welfare $W^{Tot}\left(\tau^P\right)$ when agents work as in the benchmark steady state, pollution disutility and growth rate are constant.

[12] We tolerate a difference between k_T and k_2 smaller than 10^{-10}.

[13] See Hairault and al. (1998).

$$e\left(\tau^p\right) = \exp\left[(1-\beta)\, W^{Tot}\left(\tau^p\right) - \frac{\beta}{1-\beta}\log g_1 - \chi_l \log l_1 + \chi_P \log P_1\right] \tag{33}$$

The total welfare cost is given by

$$\lambda = \frac{e(\tau^{op})}{e(\tau^p)} - 1 \tag{34}$$

where

$$e(\tau^{op}) = \exp\left[(1-\beta)\, W^{Tot}(\tau^{op}) - \frac{\beta}{1-\beta}\log g_1 - \chi_l \log l_1 + \chi_P \log P_1\right] \tag{35}$$

Transitional welfare cost

We suppose that the economy can instantaneously jump on the new steady state (without transition). Let us define $e^{rp}(\tau^p)$ as the constant flow of consumption that gives the same welfare in this hypothetical scenario. We obtain

$$e^{rp}(\tau^p) = \exp\left\{(1-\beta)\, W_2 - \frac{\beta}{1-\beta}\log g_1 - \chi_l \log l_1 + \chi_1 \log P_1\right\}$$

The welfare cost of transition expressed in consumption terms is then

$$\lambda^{dyn} = \frac{e^{rp}(\tau^p) - e(\tau^p)}{e(\tau^{op})} \tag{36}$$

4.2 Welfare costs simulation

We propose to compute these two measures of the welfare cost for many environmental tax ratio belonging to the interval $[0.5 \times \tau^{op}; 1.5 \times \tau^{op}]$. When environmental tax is equal to its optimal level, the total welfare cost is equal to zero. Therefore, with environmental tax rate lower (higher) than the optimal rate, welfare cost decreases (increases). One can obtain a "U shaped" curve which relates the total welfare cost and different tax rate levels near the optimal rate (see figure 4.a). In the same manner, we compute the transitional welfare cost for different environmental tax rates near the optimal one. We obtain a decreasing relation. The higher the environmental tax, the higher the transitional welfare cost.

Now we propose to compute total and transitional welfare costs for different values of the weight of leisure in households' preferences (χ_l). Thus, for a share of abatement expenses in production of 1.8 %, we obtain the welfare costs values summarized in the following table.

Table n°3 : Welfare costs sensitive to leisure preferences weight

	$\chi_l = 0$	$\chi_l = 1.24$	$\chi_l = 2.48$
λ %	0.611	0.666	0.672
λ^{dyn} %	-6.913	-7.262	-7.641

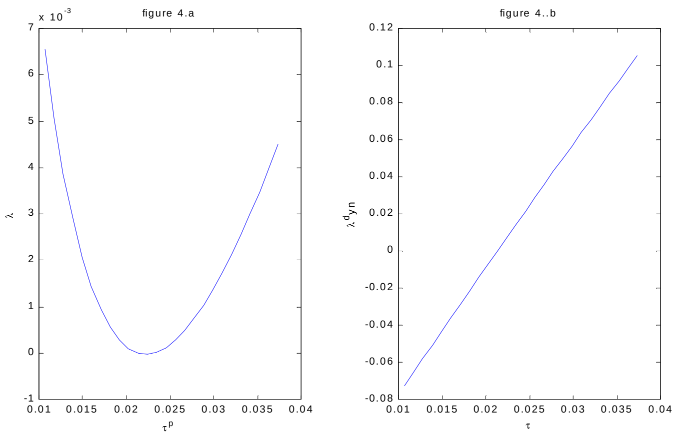

Figure 4: Environmental tax welfare costs

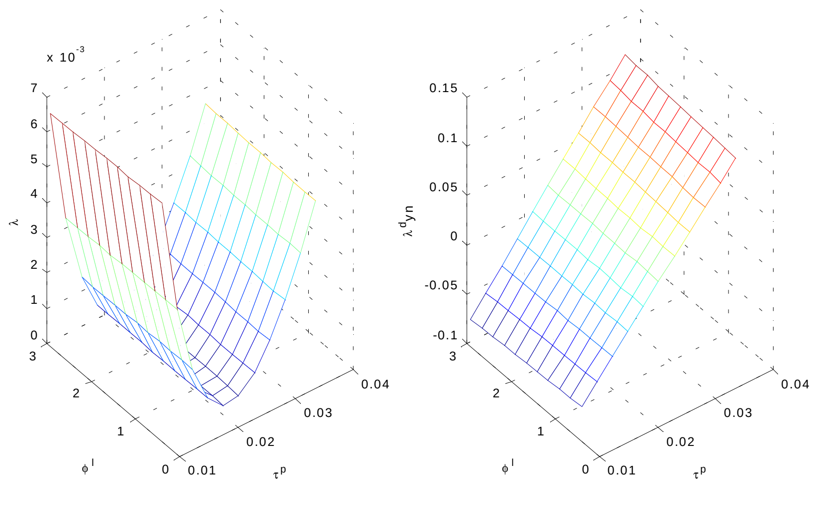

Figure 5: Welfare costs sensitivities to ¿^{p} et Á$_{l}$

From this sensitivity analysis we note that total welfare cost (λ) is positive and very low. When the government applies an optimal environmental policy, the economy can realize a slight welfare gain[14]. In the short-run, the transitional welfare cost is negative and relatively important. Thus, reaching the optimal environmental policy induces an important welfare cost in the short term[15].

We note too that λ has a lower sensitivity with respect to leisure preference weight (φ_l) and that λ^{dyn} has a higher sensitivity to φ_l. When agents' preferences for leisure are important, an increased environmental tax will induce sizable short-run movements, which induce an important transitional welfare cost. In illustrative terms, one can compute total and transitional welfare costs as functions of environmental tax changes near the optimal rate and of the weight of leisure in preference. The three-dimensional figure reported below confirms the above results (see figure 5).

When government increases the environmental tax rate to reach an optimal one, the economy experiences a slight welfare gain in the long-run and a relatively important welfare cost in the short-run. The magnitude of the welfare cost is related to the marginal utility of leisure. In an economy where agents give a higher weight to leisure, the short-run welfare cost will be very important.

5 Conclusion

We have studied in this paper the short-run and long-run behavior of an economy responding to the environmental fiscal policy effects. The model we used is a version of a two sector endogenous growth model. This model allows us to consider the environmental tax effect on the growth rate according to the preferences' weight for leisure.

Our ambition was to bring further development to the literature's results concerning the transitional effects of an environmental tax. This literature emphasized the effect of an environmental tax on the equilibrium growth rate by showing that pollution impacts on factors productivity or on human capital accumulation. By introducing leisure into the utility function, we have established a link between environmental tax and long-run growth rate without making the above mentioned assumptions. Furthermore we have contributed to the short-run dynamic study of such a tax reform.

The existing calibration exercises concerning two-sector endogenous growth models inspired us to develop our numerical study. Two steps were necessary to calibrate our model: first, we calculated the environmental tax rate in order to be sure that the pollution abatement in the product (D/Y) is equal to 1.8 %. Then, we supposed that to obtain an optimal level of D/Y we should double the environmental tax rate. The sensitivity analysis conduced for some parameters confirms the main result of our numerical exercise : the environmental tax has a positive effect on long-run growth.

The next step of our work was to simulate and comment the transitional dynamics associated to an environmental policy change. Along the transitional dynamics, agents face different trade-offs. On the one hand, firms initiate a factorial substitution process that progressively neutralizes the crowding-out effect caused by pollution abatement. On the other hand, households substitute education time for leisure. These effects finally combine into a higher long-run growth rate.

Our third and final step was an assessment of the welfare cost induced by variations of the environmental tax rate. We have made a clear difference between the total welfare cost and that

[14] Note that our computational method concerns a transition from an optimal situation to a non optimal one, which induces a welfare cost. Conversely, it will induce a welfare gain when we start from the distorted economy and reach the optimal steady state.

[15] Since the transitional welfare cost is negative, it's synonymous to a welfare gain.

associated with the transitional dynamics. Measuring these di¤erent welfare costs caused by the environmental tax impact on growth con…rms the results found elsewhere in the literature, and emphasize both the sizable welfare cost in the short-run and the overall welfare bene…t in the long-run. The magnitude of the short term cost is an increasing function of the weights of leisure in utility.

If the agents' labor supply is elastic, an environmental tax increase can stimulate growth, improve welfare in the long-run, but causes an important welfare decline in the short-run.

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Appendix

A The Loglinear System

Once the system has been stationarized, we can linearize it in the neighborhood of its rest point. Let any lower-case letter without time subscript denote the steady state value of the associated stationary variable, and in the same manner, let a lower-case letter with a hat denote the log-deviation of the associated stationary variable. We resort to the method proposed by Uhlig (1995), and the approximate log-linear system writes:

$$\hat{l}_t - \hat{o}_t - \hat{c}_t + \hat{y}_t \simeq 0 \qquad \text{(E1)}$$

$$r\hat{r}_t - \alpha y\hat{y}_t + d\hat{d}_t = 0 \qquad \text{(E2)}$$

$$w\hat{w}_t - \hat{y}_t + \hat{o}_t + \hat{n}_{t-1} = 0 \qquad \text{(E3)}$$

$$\hat{e}^P_t - (1+\chi)\, d_t = 0 \qquad \text{(E4)}$$

$$\hat{p}_t + \chi\hat{d}_t = 0 \qquad \text{(E5)}$$

$$\hat{y}_t - (1-\alpha)\,\hat{o}_t - (1-\alpha)\,\hat{n}_{t-1} = 0 \qquad \text{(E6)}$$

$$g\hat{g}_t - y\hat{y}_t + d\hat{d}_t + c\hat{c}_t = 0 \qquad \text{(E7)}$$

$$\hat{g}_t + \hat{n}_t - \hat{n}_{t-1} + \frac{Bu}{g}\hat{u}_t + \frac{Bl}{g}\hat{l}_t = 0 \qquad \text{(E8)}$$

$$\hat{g}_t + \hat{c}_{t+1} - \hat{c}_t - \frac{\beta r}{g}\hat{r}_{t+1} = 0 \qquad \text{(E9)}$$

$$\hat{g}_t + \hat{c}_{t+1} - \hat{c}_t + \frac{\beta Bl}{g}\hat{l}_{t+1} = 0 \qquad \text{(E10)}$$

One can write this log-linearized system in the following matrix form:

$$\begin{aligned} 0 &= AX_t + BX_{t-1} + CY_t \\ 0 &= FX_{t+1} + GX_t + HX_{t-1} + JY_{t+1} + KY_t \end{aligned}$$

Then we use Matlab routines to compute the matrix P and Q from the recursive equilibrium law of motion.