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Stratified or Comprehensive? The Economic Efficiency of School Design

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Abstract

We study the efficiency of secondary school design by focusing on the degree of differentiation between vocational and general education. Using a simple model of endogenous job composition, we analyze the interaction between relative demand and relative supply of skills and characterize efficient school design when the government runs schools and cares only about total net output. We show that neither a comprehensive nor a stratified system unambiguously dominates the other system for all possibile values of the underlying parameters. Therefore, the relationship between efficiency and equity in secondary education is not necessarily a trade off. We also show that net output maximizing government policy is not always supported by majority voting. When schools are stratified, majority voting could increase the elitist nature of general schools by rising the admission standard above efficient levels. At the same time, and depending on the values of the underlying parameters, efficient stratified schools could be voted down in favor of less efficient comprehensive schools.

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1 Introduction

An adequate supply of educated workers is important because it facilitates the adoption of new and highly productive technologies (Acemoglu (2000)). First, educated labor is more flexible. Second, when a new product or process is introduced, its superior signal extraction capability reaps a greater premium and leads to faster learning (Bartel and Lichtenberg (1987)). In these occasions, educated workers can also substitute for expensive company training (Bartel and Sicherman (1998)). Last but not least, because of the increased uncertainty at times of rapid technical change, better educated workers are better at coping with this uncertainty (Aghion, Howitt and Violante (2000)).

The supply of educated labor depends not only on the average number of years spent at school by the active population but also on school quality and on the way education is organized. Interestingly, educational systems differ greatly across countries. Consider secondary education. As shown by Shavit and Muller (1998) and Green, Wolf and Leney (1999), whilst some systems emphasize the development of specific vocational skills, other systems focus on the provision of general knowledge. In some countries (USA, Japan, Britain), general and vocational education are combined in comprehensive schools. In other countries (Germany, the Netherlands, Switzerland and Austria), there are different tracks and students are selected early on and streamed into tracks on the basis of their academic talents (see OECD (1998)).

As discussed by Muller, Ringer and Simon (1977) and Bertocchi and Spagat (2000), the observed institutional variety in the design of schooling systems is the outcome of complex national developments. Institutions have also changed over time. In some countries, most notably in Britain and Italy, equality of opportunity and a negative view of the selection criteria have been among the driving forces behind the shift from a stratified system, with schools organized in different tracks, to a comprehensive system, considered to be more suitable for the provision of better social opportunities to students from working class families[1].

An important question that, to our knowledge, has not attracted much attention in the literature so far is whether differences in (secondary) school design matter for the production of human capital and for national (net) output. Suppose that a stratified secondary school system yields higher efficiency than a comprehensive system. In this case, transitions from the former to the latter system, motivated by equality of opportunity and by distributional concerns, carry an efficiency loss, and there is a trade off between equity and efficiency.

A closely related question is how the relative efficiency of stratified and comprehensive systems is affected by the recent wave of organizational and technical change. On the one hand, the development of flexible organization and "just in time" practices in manufacturing has created demands for people who respond rapidly and have good communications skills. On the other hand, information technologies have placed greater emphasis on analytical and conceptual skills, and employees are increasingly required to work with symbolic systems and to develop holistic understanding of organizations and processes [2].

A final question is whether, in modern democracies, efficient school design is supported by the majority of voters. When efficiency and majority voting yield different policies, governments could prefer to the efficient design the design that maximizes consensus.

The paper addresses these questions by focusing on a key aspect of secondary school design, the degree of differentiation between vocational and general education. Following the definitions by Bertocchi and Spagat (2000) and Shavit and Muller (1998), vocational education is directly related to specific occupations, with a curriculum devoted to learning practical skills. General education, instead, provides basic knowledge that can be used in many occupations.

Given the complexity of national schooling systems, we are aware that our focus on secondary education overlooks important differences in the design of tertiary education. Lower and upper secondary education, however, still repre-

[1] See Reynolds (1987) and Weeks (1986) for a discussion of the British reform, and Barbagli (1974) for a study of the Italian reform.

[2] See Green, Wolf and Leney (1999) and Oakes (1994).

sents the most frequent highest completed degree in developed countries. According to the OECD (1998b), the percentage of the population with 25 to 64 years of age who in 1995 had an upper secondary degree as its highest completed degree was 53% in the US, 54% in Britain and 61% in Germany. Therefore, secondary schools remain of paramount importance when discussing the economic implications of school design.

We start by characterizing the main features of stratified and comprehensive schools. In so doing, we assume that schools provide both human capital and signals to firms, that cannot observe individual talents. Next, we use a version of the simple endogenous job composition model discussed by Acemoglu (1999) to study the interactions between school design and the demand for high school graduates. To sharpen our focus on these interactions, we ignore the issues related to school finance and assume that all secondary schools are public, a reasonable approximation of reality.

A key difference stressed in the paper is that, whilst comprehensive schools admit all individuals, independently of their ability, stratified schools stream them into different tracks, depending on measured academic ability. When secondary schools are public, the government decides both school design and the allocation of students into different tracks. We establish the efficient admission criteria in a stratified system and show that whether a stratified system is more efficient than a comprehensive system cannot be established in general but depends on the values taken by the underlying parameters. Since stratified systems can be less efficient than comprehensive systems, that generate by definition more equal outcomes, the relationship between equity and efficiency needs not be a trade off.

In the long run, government decisions in democratic countries need to be sustained by the approval of the majority of voters. We compare net output maximizing government policy with a policy dictated by majority voting, where each household in the economy decides sequentially both on the schooling system and on selection in stratified schools. We show that majority voting can lead to substantially different outcomes. First, the relative size of vocational schools in a stratified system can be higher than the efficient size, a result found also by Bertocchi and Spagat (2000). Therefore, majority voting imparts an "elitist" bias to stratified schools, because it reduces the relative size of general schools. Second, comprehensive systems can be preferred by voters even though stratified systems are more efficient.

The paper is organized as follows. Section 2 sets up the model. Sections 3 and 4 study stratified and comprehensive schools. Efficiency is examined in section 5. The next section illustrates the main properties of the model by presenting some numerical examples. Section 7 discusses majority voting and section 8 looks briefly at the implications of having endogenous growth. Conclusions follow.

2 The Model

2.1 Setup

Consider an economy with a mass of risk neutral individuals and a mass of risk neutral firms. Each individual lives for two periods. In the preliminary period, she attends school. In the second and last period, she matches with a firm, produces, gives birth to an offspring and retires. Following Acemoglu (1999), firms in each period choose which job type to offer and search for suitable workers. The choice of job type is costless but the posted vacancy carries a sunk cost, that varies both with the type of job and among firms. To focus on the design of secondary education, we assume that there are only secondary schools and ignore both tertiary education and dropouts. Secondary schools are attended by all individuals, a reasonable assumption for developed economies, either because these schools are partly compulsory or because the expected net return from post-compulsory education is assumed to be positive[3].

Secondary schools can be either stratified or comprehensive. Stratified systems are composed of vocational and academic schools, and pupils are allocated

[3] With this assumption, our analysis is partial equilibrium.

to school type either by an admission test or on the basis of their performance in primary schools[4]. In both cases, allocation to academic tracks depends on measures of academic talent. Define vocational schools as V schools and general (academic) schools as G schools. While vocational schools specialize in technical education, academic schools specialize in the development of general education. Comprehensive systems (C schools), on the other hand, do not stream individuals into separate tracks and provide pupils with both technical and general education[5].

In the simple economy discussed in this paper, schools have two purposes, to increase the human capital of pupils and to provide signals to the labor market. These signals are useful because individuals differ in their innate talents, that affect production but are not observed by firms. Since admission to G schools in stratified systems is based on a test, stratified schools provide better information about the average ability of admitted pupils than comprehensive schools, where admission is independent of individual ability.

It is an open question whether stratified schools are better than comprehensive schools in the provision of human capital. According to Bedard (1997), stratified schools have the advantage of specialization and can increase vocational and general skills at a higher pace than comprehensive schools, that teach both types of skills at a lesser pace. While stratified schools separate ability groups, comprehensive schools mix them. The value of grouping (peer effects) versus mixing ability types is very much debated and the empirical evidence is not conclusive (see Hoxby (2000) and Betts and Shkolnik (2000) for a review). If mixing types yields better aggregate results than grouping, comprehensive schools could turn out to be relatively more efficient in the production of human capital than stratified schools.

Assume that individuals differ both in academic ability $\alpha \in (0,1)$, where $\int_0^1 d\alpha = 1$, and in vocational / technical ability $\beta \in (0,1)$, where $\int_0^1 d\beta = 1$.

[4] Green, Wolf and Leney (1999) describe the main differences in admission rules among European countries.

[5] Tracking by ability does occur in American comprehensive schools, but it is limited to a restricted subset of subjects (see Epple, Newton and Romano (2000)).

These abilities are not necessarily independently distributed among individuals. A characterization of correlation between abilities is

$$\beta_i = (1-\lambda)\gamma_i + \lambda\alpha_i \tag{1}$$

where $\int_0^1 d\gamma = 1$ and $\lambda \in (0,1)$. The component $\gamma \in (0,1)$ is orthogonal to academic ability. With uniform distributions, $Cov(\alpha, \beta) = \frac{\lambda}{12}$.

As mentioned above, allocation to G schools in a stratified system is based on an academic test. The test needs not be a formal exam, but could be orientation and screening of students by primary school teachers. Let the pass standard be θ. Individuals admitted to G schools have a test performance θ_i at least as good as the standard. Performance in the test depends both on academic ability and luck. Therefore, the test is noisy and the outcome reflects this noise[6]. Luck has the following simple structure. Individual performance is $\theta_i = \alpha_i + \xi$ with probability χ and $\theta_i = \alpha_i - \xi$ with probability $1-\chi$.

The less precise the test, the higher the probability of assigning individuals with low academic ability to G schools and individuals with high academic ability to V schools. The higher the positive correlation of technical and general abilities, the more likely the assignment of individuals with relatively high vocational talents to general rather than to vocational schools. Clearly, both these shortcomings of the selection process reduce the relative efficiency of stratified schools over comprehensive schools.

Schools are public and are run by the government, that can use non distortionary taxation to finance the costs of education. We ignore the important issues associated to school finance and income distribution by assuming that these costs are equal to zero[7]. The government has two choices to make. First, it must choose between a stratified and a comprehensive system. Second, and conditional on the choice of a stratified system, it selects the appropriate admission standard to G schools in order to maximize total net output in the

[6] The relevance and size of errors in admission tests are discussed in detail by Allen and Barnsley (1993) and are considered by Reynolds (1986) as one of the reasons for the demise of the secondary modern - grammar school stratified British system.

[7] Nothing of substance changes in the rest of the paper if we introduce positive tuition fees that do not vary with the type of school.

economy. In so doing, the social planner takes into account the fact that the design of the schooling system will affect human capital in the economy and the composition of jobs offered by firms.

In this simple setup, all the decisions about schooling are taken by the government. We allow individuals to have a voice, however, by asking in a later section whether they would support government choices in a democratic setting based on majority voting. Individual voting is assumed to involve both the education system and, when the system is stratified, the admission standard. The higher the standard, the more "elitist" the G school sector.

Turning to the demand side of the model, firms are allowed to choose between two types of job, the V and the G type. This choice occurs at the beginning of every period, given the existing supply of school graduates and the current vintage of technologies. While technical progress in each period updates this vintage, school curricula can only be based on the previous vintage. The sluggishness in the adjustment of school curricula implies that technical progress has two effects (see Galor and Moav (2000) and Gould, Moav and Weinberg (2000)): a productivity effect, that increases the productivity of each job; and an erosion effect, that depreciates the human capital produced by schools. Depreciation is faster the higher the rate of technical progress and the more specialized is the production of human capital at school.

We posit that skill erosion is faster when human capital is produced by vocational schools in a stratified system than when this capital is generated either by general (G) or by comprehensive (C) schools. One reason is that..."where the occupation - specific component of vocational education is large, graduates have few transferrable skills, and can only cash in on them by transforming them into the corresponding occupations in the labor market.." (Shavit and Muller (1998), p. 5). General skills, instead, endow individuals with higher flexibility and versatility. Another reason is that recent waves of technical progress have increased the relative demand of general skills at the expense of specific vocational skills (Lindbeck and Snower (2000)). Let the rate of erosion be equal to g, the rate of technical progress. Since it is relative erosion that matters, we assume this rate to be positive for accumulated vocational skills and equal to zero for the human capital accumulated in general or in comprehensive schools.

Let the number of firms be given and normalized to 1. The endogenous number of V jobs is $F \in (0, 1)$. While jobs of type V require vocational skills and talents, jobs of type G require general (academic) skills. Output in each job depends on human capital. When schools are stratified, firms with a V job can employ either a V or a G graduate. With specialization, the former graduate is more productive in a V job, but the latter graduate has the advantage of being more flexible and to be effective in both types of job. The difference in productivity between the two types of graduates in a V job can be interpreted in terms of training costs born by the firm: hiring a G graduate for a V job requires that the firm invests in additional training to compensate for this difference. On the other hand, firms offering a G job can only employ a G graduate, because the productivity of a V graduate in the job is equal to zero. This characterization captures in a sharp way the lack of flexibility associated to vocational training[8]. When schools are comprehensive, firms offering either type of job are indifferent between graduates, who come all from the same type of school.

With frictions, job offers are not immediately satisfied and real resources must be spent in the matching process. The simplest way to capture this is to assume that firms offering jobs in each period have to sink a nonzero fixed cost. This cost could be justified either with the choice of productive capacity (see Acemoglu (1999)) or with the cost needed to open and advertise a vacancy (see Pissarides (1990)). Since jobs must be created before a match occurs, employers must choose ex-ante the type of job they want to offer.

Because matching is time consuming and workers only live a single period, there are no opportunities of a second match during a worker's lifetime. Moreover, there is no learning by firms about individual abilities. When schools are stratified, firms can use school type to infer the expected academic and tech-

[8] As argued by Bedard (1997), there is a trade off between faster learning associated to streaming and the rigidity imposed by early streaming. Our results would not change in a qualitative way if we were to assume that the productivity of V graduates in G jobs is positive but lower than the productivity of G graduates.

nical ability of graduates. This is not possible, however, when schools pool all abilities in a comprehensive setup. Wage setting occurs after the match has been struck. Since fixed costs have already been sunk and bygones are bygones, workers and firms share rents and wages are gross output shares.

The sequence of events in this economy is as follows: first, either the government or a democratic voting mechanism choose between stratified and comprehensive schools. When streaming is chosen, the admission standard is selected. Next, individuals in a stratified system sort into school types or join the same school in a comprehensive system. After schooling is completed and technical progress has occurred, firms choose job types and sink setup costs by taking into account both the latest vintage of techniques and the composition of graduates by school type. Therefore, the composition of jobs into "vocational" (type V) and "general" (type G) is endogenous and depends on the relative supply of graduates, that varies with the design of the school system[9]. It also depends on the rate of technical progress g. Finally, wages are set and production occurs. As usual, we characterize the equilibrium and its properties by starting from the last decision and by working our way backwards.

2.2 Wages

Consider first stratified schools and define the human capital of V graduates either as H_{VV} or as H_{VG}, depending on whether they are employed in a V or in a G job. Similarly, the human capital of G graduates also varies with the type of job and is H_{GG} in a G job and H_{GV} in a V job. Output per head at time t is the product of the productivity index A_t and of individual human capital.

With steady growth, $A_t = (1+g)^t A_i$, $i = V, G$. It is reasonable to assume that G graduates are more productive in a G than in a V job ($A_G H_{GG} > A_V H_{GV}$). On the other hand, V graduates are less versatile than G graduates, because their skills are geared to a specific occupation and/or machine. Lack of versatility is modeled here by assuming $H_{VG} = 0$. This implies that G graduates

[9] Models where job composition is a function of the supply of skilled labor are surveyed by Acemoglu (2000) and Aghion, Caroli and Penalosa (1999).

are always better at G jobs than V graduates. On the other hand, V jobs can be filled by both types of graduates and either V graduates are better at V jobs ($H_{VV} > H_{GV}$) or G graduates are better at both jobs (absolute advantage). In the former case, specialization implies that firms offering a G job will search for a G graduate and firms offering a V job will search for a V graduate. In the latter case, all firms prefer G graduates. Since these graduates are more productive in G jobs, however, because $A_G H_{GG} > A_V H_{GV}$, firms offering V jobs have to make do with V graduates unless there is an excess supply of G graduates with respect to G jobs[10].

Profits gross of sunk costs in job V are

$$\pi_{GVt} = (1+g)^t A_V H_{GV} - w_{GVt} \tag{2}$$

when the job is filled by a G graduate and

$$\pi_{VVt} = (1+g)^t A_V H_{VV} - w_{VVt} \tag{3}$$

when the job is filled by a V graduate. Here, w is the individual (real) wage. Profits gross of sunk costs in job G are

$$\pi_{GGt} = (1+g)^t A_G H_{GG} - w_{GGt} \tag{4}$$

when the job is filled by a G graduate and 0 otherwise.

Wage bargains takes place after a job match between graduates and jobs has occurred and the fixed cost has been sunk by firms. With only one period, there are no economic separations. With no other opportunity to match, the outside option is the value of leisure, that we normalize to zero. Outside options for firms are also normalized to zero. Letting the bargaining power be $\frac{1}{2}$, wages are respectively $w_{GGt} = \frac{(1+g)^t A_G H_{GGt}}{2}$, $w_{GVt} = \frac{(1+g)^t A_V H_{GVt}}{2}$ and $w_{VVt} = \frac{(1+g)^t A_V H_{VVt}}{2}$.

[10] In principle, firms with V jobs could try to attract G graduates by offering higher wages. They realize, however, that firms with G jobs can offer even higher wages, because $A_G H_{GG} > A_V H_{GV}$.

Next consider comprehensive schools. With a comprehensive system, graduates have the same human capital in V and G jobs and can be employed indifferently in either type of job. Profits in these jobs are

$$\begin{aligned} \pi_{CVt} &= (1+g)^t A_V H_C - w_{CVt} \\ \pi_{CGt} &= (1+g)^t A_G H_C - w_{CGt} \end{aligned} \tag{5}$$

and wages are $w_{\mathrm{Ci}\,t} = \frac{(1+g)^t A_i H_{\mathrm{Ci}}}{2}$, $i = V, G$.

2.3 Firms

Each period, firms choose job types and match these jobs with new school graduates. When schools are comprehensive, each firm can match with certainty with a graduate in each period. When schools are stratified, however, the probability that a firm choosing job type V matches with a V graduate depends both on the share of V graduates θ and on the share of firms offering V jobs, F. With random matching, the probability of filling the vacancy with a V graduate is $p = \frac{\theta}{F}$ when $\theta < F$ and $p = 1$ when $\theta \geq F$. On the other hand, the probability of filling the same vacancy with a G graduate is equal respectively to $1 - p$ and to zero. Turning to G jobs, the probability of matching a G job with a G graduate is $q = \frac{1-\theta}{1-F}$when $\theta > F$ and $q = 1$ when $\theta \leq F$. When the firm offering a G job cannot find a match with a G graduate, the job remains unfilled.

Suppose for a moment that the government sets θ, the share of V graduates, to exactly match the share of V firms, F (perfect sorting). This happens if the government correctly anticipates both the decision taken by firms and the bargaining game between firms and employees. In these circumstances, each firm selects to offer a V or a G job depending on relative (expected) profits.

The expected (net) profit of setting up a job of type V is

$$E\pi_{Vit} = (1+g)^t \left(A_V \frac{H_{VV}}{2} - c_{Vi} \right) \tag{6}$$

where c_{Vi} is the fixed cost of a type V job for firm i, that must be sunk before the matching takes place. Notice that we have multiplied the setup costs by the growth factor $(1+g)^t$, to ensure that these costs are not wiped away by economic growth, a necessary condition for the existence of a steady state solution (see Pissarides (1990)).

Similarly, the expected profit of setting up a job of type G is

$$E\pi_{Git} = (1+g)^t \left(A_G \frac{H_{GG}}{2} - c_{Gi} \right) \tag{7}$$

where c_{Gi} is the fixed cost of type G jobs for firm i.

Indifference between the two jobs is given by

$$\sigma_s = c_{Gi} - c_{Vi} = \frac{A_G H_{GG} - A_V H_{VV}}{2} \tag{8}$$

Firms with relative costs lower than or equal to σ_s will choose to offer G jobs and firms with relative costs higher than σ_s will choose to offer V jobs. Let $c_{Vi} = c_V$, so that firms share the same setup costs for type V jobs, and assume that the distribution of c_{Gi} be uniform

$$\int_0^\eta \frac{1}{\eta} dc_{Gi} = 1 \tag{9}$$

In this case the share of firms offering V jobs is

$$F = \int_{\sigma_s}^{\eta - c_V} \frac{1}{\eta} dc_{Gi} = \frac{\eta - c_V - \sigma_s}{\eta} \tag{10}$$

It follows that the relative demand of V and G jobs depends on the human capital accumulated at school by new labor market entrants. This capital depends on school design, to which we now turn.

3 Stratified Schools

Individuals are allocated to academic tracks in a stratified system if they pass an admission test. When the test is not very precise, individuals with relatively low ability can pass the test while individuals with relatively high ability can fail. Expected academic ability for those who pass the test and are allocated to G schools is

$$E[\alpha \mid \theta_i \geq \theta] = \frac{1+\theta}{2} + \frac{\xi(1-2\chi)}{2} \tag{11}$$

On the other hand, the expected ability of those who fail is

$$E\left[\alpha \mid \theta_i < \theta\right] = \frac{\theta}{2} + \frac{\xi\left(1-2\chi\right)}{2} \tag{12}$$

Since $E\left[\alpha \mid \theta_i \geq \theta\right] > E\left[\alpha \mid \theta_i < \theta\right]$, the admission test is informative. Notice that an increase in the standard θ rises the expected academic ability of either group (see Betts (1998)).

When technical ability β is positively correlated to academic ability α, an admission test based upon academic ability introduces a further distortion by allocating individuals with relatively high β to G schools (see Bedard (1997)). In particular, expected technical ability is

$$E\left[\beta \mid \theta_i \geq \theta\right] = \frac{1+\lambda\theta}{2} + \frac{\lambda\xi\left(1-2\chi\right)}{2} \tag{13}$$

for those who pass and

$$E\left[\beta \mid \theta_i < \theta\right] = \frac{1-\lambda(1-\theta)}{2} + \frac{\lambda\xi\left(1-2\chi\right)}{2} \tag{14}$$

for those who fail the test.

Expected ability is independent of luck ξ if $\chi = \frac{1}{2}$. In the rest of the paper, we shall assume this to be the case. This is equivalent to assuming that luck affects only individual allocation to school, with no consequence for the average ability signalled by stratified schools[11].

The larger the positive correlation between tested academic and untested technical abilities, the lower the expected technical ability of graduates from V schools. The reason is that individuals endowed with both relatively high α and β are allocated to G schools[12]. Notice that $E\left[\beta \mid \theta_i \geq \theta\right] > E\left[\beta \mid \theta_i < \theta\right]$ if the two abilities are positively correlated.

[11] Removing this assumption would complicate the algebra and add an additional distortion to the working of stratified schools.

[12] See Brunello and Giannini (1999) for a discussion of the problems associated to testing in the presence of multiple abilities.

Schools not only select students by academic ability but also provide skills. Education augments innate talents. School V increases the vocational skills (talents) of admitted students by a factor $\delta > 1$. Similarly, school G increases the general skills (talents) of admitted students by a factor $\delta > 1$. These upgrades of human capital, however, can only be used productively in the right jobs. Moreover, the human capital accumulated in V schools depreciates at the rate g, the rate of technological progress. Faster growth means faster development and introduction of new machines and techniques, and more rapid depreciation of existing vocational skills, that need further upgrading and additional on the job training[13].

If a V graduate works in a type V job, that requires vocational abilities, her human capital is the product of her expected technical ability, $E[\beta \mid \theta_i \geq \theta]$, and the factor $\delta(1-g)$, that measures the accumulation of human capital at school. Therefore we have

$$H_{VV} = \delta(1-g)\, E[\beta \mid \theta_i < \theta] = \delta \frac{1-\lambda(1-\theta)}{2}(1-g) \tag{15}$$

If the same graduate works in a G job, depreciation is so large that her human capital is equal to zero. The human capital of G graduates is given by

$$H_{GV} = \delta(1-\nu)E[\beta \mid \theta_i \geq \theta] = \delta(1-\nu)\frac{1+\lambda\theta}{2} \tag{16}$$

$$H_{GG} = \delta E[\alpha \mid \theta_i \geq \theta] = \delta \frac{1+\theta}{2} \tag{17}$$

where $\nu \in (0,1)$ measures the fact that the human capital of G graduates cannot be fully exploited in V jobs ($\nu = 1$ for V graduates in G jobs). Assuming that the government sets the size of the V school sector to exactly match the percentage of V jobs in the economy, F, the critical value σ is given by

$$\sigma_s = \frac{1}{2}\left[\delta A_G \frac{1+\theta}{2} - \delta(1-g)A_V \frac{1-\lambda(1-\theta)}{2}\right] \tag{18}$$

Using the fact that

$$F = \theta = \frac{\eta - c_V - \sigma}{\eta}$$

[13] See Aghion, Howitt and Violante (1998) and Caselli (1999).

we obtain the admission standard θ that guarantees equality of relative demand and relative supply. Straightforward differentiation yields

$$\theta = \theta(\underset{-}{g}, \underset{-}{\lambda}, \underset{-}{\delta}, \underset{-}{A_G}, \underset{+}{A_V}) \tag{19}$$

Admission to G schools in a stratified system is tighter the slower is technical progress, the lower the correlation between ability types and the lower the rate of accumulation of human capital at school. One implication is that an acceleration of technical progress reduces the admission standard and the relative importance of vocational schools relative to more general education[14]. A second implication is that a relative productivity shift that favors V jobs ($\frac{A_V}{A_G}$ increases) increases both the expected share F of firms offering V jobs and the percentage of slots available in V schools.

4 Comprehensive schools

Compared to stratified schools, comprehensive schools admit all candidates and do not differentiate by (academic) ability. Since human capital formation involves both academic and vocational skills, a disadvantage of comprehensive schools is that the rate of accumulation of these skills is slower than in stratified schools (see Bedard (1997)). Therefore, these schools provide more flexible training but lack the relative advantage of specialized instruction. This disadvantage could be compensated, however, if mixing ability types turns out to be more effective than grouping types in the production of human capital. Let the differential in the rate of accumulation between school types be captured by the parameter ρ. This rate is less than 1 when stratified schools are more effective and higher than 1 when they are less effective than comprehensive schools in the production of human capital.

Compared to stratified schools, an advantage of comprehensive schools is that the skills learned in these schools can be used both in type V and in type

[14] Lindbeck and Snower (2000) discuss the relationship between technical progress and the importance of general education in the context of organizational change.

G jobs (higher versatility). Furthermore, these skills are subject to less erosion than vocational skills learned at V schools when technical progress occurs. A disadvantage is that they do not provide firms with an informative signal about individual abilities. The human capital of graduates of C schools who are employed in V and G jobs is respectively

$$H_{CV} = \rho\delta E(\beta) = \frac{\rho\delta}{2} \tag{20}$$

$$H_{CG} = \rho\delta E(\alpha) = \frac{\rho\delta}{2} \tag{21}$$

The share of firms offering type V jobs when schools are comprehensive, F_c, is

$$F_c = \frac{\eta - c_V - \sigma_c}{\eta} \tag{22}$$

where

$$\sigma_c = c_{Gi} - c_{Vi} = \frac{(A_G - A_V)\,\rho\delta}{2} \tag{23}$$

Relative wages when schools are comprehensive are equal to $\frac{A_G}{A_V}$. When schools are stratified and there is perfect sorting of graduates to jobs, relative wages are higher and equal to $\frac{A_G(1+F)}{A_V(1-g)[1-\lambda(1-F)]}$. Therefore, equality is higher in the former type of schools.

Finally, net output in an economy where schools are comprehensive is

$$\frac{NY_{ct}}{(1+g)^t} = F_c A_V \frac{\rho\delta}{2} + (1-F_c)A_G \frac{\rho\delta}{2} - F_c c_V - \frac{\eta\,(1-F_c)^2}{2} \tag{24}$$

where the first two elements on the right hand side make up total output and the last two elements are the total cost of setting up the two types of job[15].

5 Efficiency

An equilibrium in this model economy is defined by the following sequence of events: given expected job composition and labor demand, the government selects the schooling system and the admission standard. Expectations are based

[15] Recall that we have normalized to zero the cost of education.

upon the decision rules used by firms, that are known to the government. Individuals are allocated to schools and graduate; firms select the jobs to offer after observing the average ability of graduates and the current menu of technologies; matching occurs and wages are bargained over. Finally, production takes place with a job composition that is equal to the composition expected by the government. In equilibrium, expectations are fulfilled.

Assume that, when schools are stratified, the government sets the admission standard θ to equalize the demand with the supply of jobs. An important question is whether perfect sorting is efficient and maximizes net output in the economy. To answer this question, we first define net output when schools are stratified (NY_s) and $\theta = F$ as[16]

$$\frac{NY_{st|\theta=F}}{(1+g)^t} = \delta(1-F)A_G\frac{1+F}{2} + FA_V\delta\frac{(1-g)}{2}\left[1-\lambda(1-F)\right] - Fc_V - \frac{\eta\,(1-F)^2}{2} \tag{25}$$

Next we ask whether net output can be improved upon by increasing or decreasing θ with respect to F. We consider the two possibilities in order, starting from $\theta < F$.

A reduction in θ has the following two effects: a) the average ability of both V and G graduates declines. Hence, aggregate human capital decreases; b) the share F of V jobs is also affected. Intuitively, a decline in θ reduces the relative supply of V graduates in favor of G graduates. The main consequence is that firms offering V jobs are forced to face the possibility of having a G graduate fill a V job. This affects the relative profitability of V jobs and the equilibrium share of V jobs, that is now determined by the following condition

$$\sigma_{s|\theta<F} = c_{Gi} - c_{Vi} = \frac{A_G H_{GG} - \frac{\theta}{F}A_V H_{VV} - \frac{(F-\theta)}{F}A_V H_{GV}}{2} \tag{26}$$

because each firm offering a V job can fill this job with a V graduate with probability $\frac{\theta}{F}$. It is shown in the Appendix that the share F varies with the academic standard θ as follows

$$\frac{\partial F}{\partial \theta} = \frac{A_V H_{GV} - A_V H_{VV} + F\left[A_G - A_V(1-g)\lambda\right]\frac{\delta}{2}}{A_V H_{GV} - A_V H_{VV} - 2\eta F} \tag{27}$$

[16] From this section onwards, we ignore time subscripts when unnecessary.

This derivative can take either sign. It is positive when the difference between the human capital of G and V graduates is large in absolute value and negative when this difference is relatively small. Using the subscript E to denote net output when $\theta < F$, an equilibrium with an excessive supply of G graduates improves on an equilibrium with perfect sorting when

$$\frac{\Delta NY}{(1+g)^t} = (1-F)A_G(H_{GGE} - H_{GG}) + [A_G H_{GGE} - A_V H_{GVE}](F - F_E)$$

$$+A_V(H_{GVE} - H_{VV})F - A_V\theta_E(H_{GVE} - H_{VVE})$$

$$+(F - F_E)c_V - \frac{\eta}{2}(1-F_E)^2 + \frac{\eta}{2}(1-F)^2 > 0 \qquad (28)$$

The first element in the right hand side of (28) is negative and represents the efficiency loss associated to the reduction in the average academic ability of G graduates. The second element can take either sign, depending on whether the share of firms offering V jobs increase or decrease; the next two elements are both positive if the human capital of G graduates in V jobs, evaluated at a lower θ, is higher than the human capital of V graduates in the same jobs. The remaining terms are the changes in total setup costs associated to changes in the number of firms offering V jobs. Depending on the values of the underlying parameters, the overall sign can be positive or negative. A necessary but not sufficient condition for a net output gain when $\theta < F$ is that G graduates have an absolute advantage over V graduates in both types of job. In this case, the social planner can find it efficient to increase the relative supply of G graduates above the share of G jobs.

Next, consider the case $\theta > F$. Compared to the previous case, when signalling is informative an increase in θ raises the average ability of both V and G graduates. Hence, aggregate human capital increases. The share of V jobs is also affected. With random matching, an increase in θ reduces the relative supply of G graduates in favor of V graduates and firms offering G jobs must consider the positive risk of having their posted vacancy unfilled, because the lack of flexibility and adaptability of V graduates implies that these graduates cannot productively be employed in G jobs. As before, this affects the relative profitability of G jobs and the equilibrium share of firms offering these jobs, that is determined by the following condition

$$\sigma_{s|\theta>F} = c_{Gi} - c_{Vi} = \frac{\frac{1-\theta}{1-F}A_G H_{GG} - A_V H_{VV}}{2} \tag{29}$$

because each firm offering a G job can fill it with a G graduate with probability $\frac{1-\theta}{1-F}$. We show in the Appendix that in this case the share of firms offering V jobs, F, is an increasing function of the academic standard θ. Using again the subscript E to denote net output when $\theta > F$, an equilibrium with an excessive supply of V graduates improves on an equilibrium where the demand and supply of types exactly match when

$$\frac{\Delta NY}{(1+g)^t} = (1-F)A_G(H_{GGE} - H_{GG}) + A_G H_{GGE}(F - \theta_E)$$

$$+A_V(H_{VVE} - H_{VV})F + A_V(F_E - F)H_{VVE}$$

$$+(F - F_E)c_V - \frac{\eta}{2}(1-F_E)^2 + \frac{\eta}{2}(1-F)^2 > 0 \tag{30}$$

The first element in the right hand side of (30) is positive and is the efficiency gain associated to the increase in the average academic ability of G graduates. The second element is negative and is the output loss associated to unfilled G jobs. The next two elements are both positive and measure the relative gain of having graduates with higher average academic quality. As before, the remaining terms are the changes in total setup costs associated to changes in the number of firms offering V jobs. Once again, and depending on the values of the underlying parameters, the overall sign can be positive or negative. A positive sign requires that the gain associated to higher academic ability be larger than the cost of having unfilled job slots. In this case, it is efficient for the government to restrict access to G schools and to have $\theta - F$ unfilled vacancies and unemployed V graduates.

In the next section, we present three examples that illustrate the possibility of having efficient equilibria with $\theta = F$, $\theta < F$ and $\theta > F$. We also compare these equilibria with equilibria with comprehensive schooling.

Notice that, when perfect sorting is not efficient, relative wages are affected by the mismatch between graduate and job types. In particular, relative wages increase when $\theta > F$ if income from unemployment is lower than income from employment, because the expected wage of V graduates declines. When $\theta < F$, relative wages decline because $A_G H_{GG} > A_V H_{GV}$ and $(F - \theta)$ G graduates are employed in V jobs.

6 Numerical examples

We illustrate the outcomes of the model by assigning numerical values to the parameters and by computing numerical solutions. In the first exercise, we set parameters as follows

$\delta = 1.2$	$\lambda = 0.1$	$c_V = 0.2$	$\eta = 1$	$\rho = 0.95$	$\delta(1-\nu) = 1$
$A_G = A_V = 1$	$g = 0.05$				

Therefore, we assume that mixing abilities is less productive that separating them ($\rho < 1$). Moreover, productivity levels do not differ across jobs and the distortion induced by admission tests is small ($\lambda = 0.1$). Assuming that the government sets $\theta = F$, we obtain the following equilibrium

$H_{GG} = .957$	$H_{GV} = .529$	$H_{VV} = .547$
$F_s = .595$	$NY_s = .537$	
$F_c = 0.8$	$NY_c = .410$	

We find that a stratified system yields higher net output than a comprehensive system. Moreover, either reducing or increasing θ with respect to F in a stratified system does not improve net output. When $\theta = 0.5$, we find that $F = 0.571$ and $NY_{s|\theta<F} = .453$. When $\theta = 0.6$, $F = 0.597$ and $NY_{s|\theta>F} = .510$. Larger increases or decreases in θ do not change the ranking of equilibria. In this example, efficiency requires that the central government selects a stratified system with perfect sorting.

Next, we modify the parameters by increasing both the distortion induced by the admission test in a stratified system and the relative productivity of comprehensive schools:

$\lambda = 0.50$	$\rho = 1.06$

Not surprisingly, we obtain

$H_{GG} = .933$	$H_{GV} = .638$	$H_{VV} = .443$
$F_s = .555$	$NY_s = .474$	
$F_c = 0.8$	$NY_c = .479$	

In this example, net output is higher in a comprehensive system than in a stratified system. As in the previous case, the government cannot improve the outcome of the stratified system either by increasing or by reducing θ with respect to F. It turns out that, when $\theta = 0.5$, net output is equal to 0.459; when $\theta = 0.6$, it is equal to 0.463. Both values are lower than the value of net output when $\theta = F$. Again, larger changes in θ do not modify the ranking of equilibria in terms of efficiency.

Interestingly, a deceleration in the rate of exogenous growth g from 5 to 1% is sufficient in this example to increase net output in stratified schools above net output in comprehensive schools. Hence, the relative efficiency of alternative school designs can vary with changes in the rate of technical progress.

In the third example, we alter the values of the parameters to obtain an efficient equilibrium with stratified schools and $\theta < F$. In order to do so, we need to increase the gap between human capitals H_{GV} and H_{VV}. This is done by assuming that the erosion of V skills is much faster, at the rate $\phi g = 0.35$, while the rate of technical progress remains at $g = 0.05$ ($\phi = 7$). We also set

$\lambda = 0.7$. We find that, when the government sets $\theta = F$, the share of F firms is 0.48 and net output is 0.367. The government can do better, however, by choosing $\theta = 0.47$, which implies $F = 0.49$ and $NY = 0.372$.

In the final example, we use the same values assigned to the parameters in the second example but increase the relative productivity of V jobs with respect to G jobs by setting $A_V = 1.1$ and $A_G = 0.7$. When the government chooses perfect sorting, we find that $F = 0.709$ and $NY_s = 0.425$. Again, the government could do better than this by pushing θ towards unity[17]. When $\theta = 0.95$, the equilibrium share of V firms increases to 0.90 and net output turns out to be equal to 0.427, higher than with perfect sorting. By so doing, average human capital of V and G graduates increase respectively from 0.555 to 0.611 and from 0.718 to 0.819. The real cost of this increase is a 5% unemployment rate, that affects V graduates, who cannot be gainfully employed in G jobs[18].

These examples suggest two things. First, neither the stratified nor the comprehensive schooling system discussed in this paper unambiguously dominate the other system. Dominance is restricted to particular subsets in the space of parameters, and both systems can be superior in efficiency terms. Since the comprehensive system delivers more equality in the distribution of wages, the fact that the stratified system does not dominate in terms of efficiency suggests that the choice between alternative school designs cannot easily be characterized in terms of a trade off between equality and efficiency.

Second, when schools are stratified, the most efficient policy is not necessarily to match the relative demand of jobs with the relative supply of graduates.

[17] Stratified schools require that θ be strictly less than 1.

[18] An alternative set of values of the parameters that yield an internal solution to net output maximization with $\theta > F$ is

$\delta = 1.2$	$\lambda = 0.83$	$c_V = 0.45$	$\eta = 1$
$A_G = 0.7$	$g = 0.05$	$A_V = 1$	$\delta(1-\nu) = 1$

We obtain $\theta = 0.41 > F = 0.4$ and $NY = 0.11236$. Imposing the restriction $\theta = F$ reduces net output marginally to $NY = 0.11233$.

Depending on the values of the parameters, the government can do better either by increasing access to G schools or by pursuing an elitist policy, with a small share of G graduates and a nonzero rate of unemployment of V graduates.

7 Voting on admission tests and schooling systems

So far we have assumed that the government chooses both the standard for admission to general schools and the schooling system to maximize net output. In a democratic system, however, important decisions about educational policy need the support of voters. The question then arises whether the decisions taken by the government can be supported by the majority of voters, who vote on government policy in an uncoordinated fashion, by taking the behavior of the other voters and of the rest of the economy as given.

In this economy, there is a finite number of households, each consisting of a parent and an offspring. In each period, the parent is employed in a V or in a G job and the daughter goes to secondary school. Preferences are linear in income and the discount factor is normalized to 1. With innate abilities and no monetary costs of education, each parent votes on the schooling system by considering the expected income of her offspring[19]. Voting is not coordinated in the following sense: each parent knows the innate talents of her offspring and votes both on the admission standard and on school design (stratified versus comprehensive) by taking both the votes of the other households and the composition of jobs in the economy as given. By so doing, she ignores that both school design and the aggregate supply of graduates affect the job decisions taken by firms. This spillover is fully internalized by the social planner.

Households vote first on the design of schooling (stratified or comprehensive). Conditional on the choice of a stratified system, they vote on the admission standard. As usual, we start from the second choice. Consider the following

[19] Bertocchi and Spagat (2000) include also social status in the preferences of parents, that we ignore in the current setup. See DeFraja (1999) for a detailed discussion of preferences when households vote on educational policies.

hypothetical sequence of events: a) the government chooses both a stratified schooling system and perfect sorting, with $\theta = F < \alpha_m$, where $\alpha_m = \frac{1}{2}$ is median ability; b) the current generation goes to school with the selected system, graduates and in the next period is allocated to V and G jobs; c) the offspring is born and goes to school.

Suppose that parents belonging to the current generation are asked to vote on the schooling system before their offspring start school. The first question is whether there is a majority of households in favor of marginally increasing θ to θ^*, above the level set by the government in the previous period. We can distinguish voters in three groups. The first group is composed of households whose offspring has academic ability less than or equal to θ; the second group includes households whose offspring has ability higher than or equal to θ^* ; the last group includes the ability range $\theta^* - \theta$.

Recall that households vote before the schooling of their offspring takes place and by considering F as given. Starting from the former group, the offspring of these households is not affected in her schooling and is expected to go to V schools under both admission standards. Ignoring time subscripts, the expected utility of a V graduate belonging to this group, U_V, is

$$U_V = \frac{\theta}{\theta^*} W_{VV}(\theta^*) \tag{31}$$

For this group, an increase in θ has the advantage of raising the average expected ability of each graduate and the disadvantage of producing an excess supply of V graduates, who cannot be productively used in G jobs. Starting from $\theta = F$, a marginal increase in θ reduces U_V[20]. Therefore, the households in this group vote individually against a tightening of the standard.

Next consider the second group. Individuals in this group expect to be assigned to G schools even after the increase in the admission standard. Since their expected utility

$$U_G = W_{GG}(\theta^*) \tag{32}$$

[20] $\frac{\partial U_V}{\partial \theta^*} = \frac{\delta}{4}\frac{\theta}{\theta^*}\left[(1-g)\lambda - \frac{1}{\theta^*}(1-g)(1-\lambda(1-\theta^*)\right] < 0.$

is strictly increasing in the admission standard θ, their parents will vote in favor of a tighter standard.

Finally, the third group is composed of the marginal individual who is expected to shift school (from G to V) as a consequence of the tighter standard. This group is worse off if

$$\frac{\theta}{\theta^*} W_{VV}(\theta^*) < W_{GG}(\theta) \tag{33}$$

It follows that, when θ is set by the government at a value lower than α_m and equal to the share of V jobs, F, there is always a majority of voters (the second group) who vote in favor of marginally increasing the admission standard. When $\theta = \alpha_m$, further increases are turned down by the first group of households.

The next step is to consider whether there is a voting majority in favor of marginally reducing θ to θ^-, below the value $\theta = F > \alpha_m$ set by the government. As before, we classify households in three groups: a first group, of dimension θ^-, whose offspring is expected to remain in V schools independently of the standard; a second group, composed by individuals with academic ability higher than or equal to θ, who remains in G schools; a third group, composed of the marginal individual who shifts from V to G schools because of the lower standard. The expected utility of the former group is $W_{VV}(\theta^-)$, an increasing function of the standard θ. This group loses from a lower standard. The expected utility of the second group is

$$U_G = \frac{1-\theta}{1-\theta^-} W_{GG}(\theta^-) + \left[1 - \frac{1-\theta}{1-\theta^-}\right] W_{GV}(\theta^-) \tag{34}$$

because the lower standard, given the composition of jobs, implies that some G graduates have to take up V jobs. We show in the Appendix that this expression is increasing in θ^-. It follows that this group will also vote against a reduction in the standard. The last group, composed of the marginal individual who shifts from a V to a G school, will gain from the shift if

$$W_{VV}(\theta) < \frac{1-\theta}{1-\theta^-} W_{GG}(\theta^-) + \left[1 - \frac{1-\theta}{1-\theta^-}\right] W_{GV}(\theta^-) \tag{35}$$

We conclude that, when $\theta = F$ and $\theta > \alpha_m$, there are no voting majorities in favor of reducing the admission standard marginally below the level set by the government.

When majority voting leads to a change in θ above or below the value set by the government to maximize net output, efficiency falls. To illustrate, consider the first example in Section 6 and reduce η from 1 to 0.5. The efficient outcome is $\theta = F = 0.332$, with net output equal to 0.559. When θ is set to α_m by voters, F increases to reach 0.396 but net output falls to 0.519, a 7.2% decline.

The ex-post outcome of majority voting is $\theta > F$, with an excess supply of V graduates with respect to the share of V jobs. Is this a stable equilibrium? To answer is positive. To show this, we ask whether there would be a majority in favor of marginally reducing θ to θ^- and below α_m. Given F, a percentage θ^- of households would gain from such a reduction, because (33) is decreasing in θ. On the other hand, one half of the households, who send their offspring to G schools, would lose, because (34) is increasing in θ. Households whose offspring change from V to G schools as a result of a reduction in the admission standard gain if the expected return from switching to a G school with a lower standard $W_{GG}(\theta^-)$ is greater than the expected return from remaining in a V school with a higher standard, $\frac{F}{\alpha_m} W_{VV}(\alpha_m)$. Even when this group of households gain from a switch, there is no majority in favor of switching, because half of the households prefer the higher admission standard.

We conclude from this example that majority voting could prefer a less efficient equilibrium with unemployment of V graduates to an efficient equilibrium with no unemployment and perfect sorting of graduates to job types.

So far, we have assumed that it is efficient for the government to set $\theta = F$. We have seen above, however, that the maximization of net output could involve setting either $\theta < F$ or $\theta > F$. We have also shown that there is no voting majority in favor of reducing θ when $\theta \leq \alpha_m$ and $\theta > F$. In the Appendix we prove that, without perfect sorting, there is always a voting majority in favor of marginally increasing θ up to α_m when θ starts from below α_m. We also prove that, when $F < \theta$ and the initial value of θ is above α_m, there is a voting majority in favor of reducing θ and close the gap between demand F and supply θ.

In the current discussion, we have focused on marginal changes with respect to the standard set by the government. While the majority of voters could be against marginal changes, it could favor large changes, that involve not a marginal voter but a substantial share of voters. Suppose for instance that households are asked to choose between a standard $\theta = F$ and $\theta^{+} = \theta + 0.5$. With the higher standard, half of the population is bound to change from a G to a V school. If changers are better off with this switch, a large increase in θ could in principle succeed in winning the necessary support. Since, however, the expected gain from the change, $\frac{\theta}{\theta^{+}} W_{VV}(\theta^{+}) - W_{GG}(\theta)$, is negative, this possibility can be ruled out[21]. Next consider the choice between $\theta = F$ and $\theta^{-} = \theta - 0.5$. In this case the shift from V to G schools generates a gain for school changers if $(1-\theta)\left(W_{GG}(\theta^{-}) + \frac{1}{2} W_{GV}(\theta^{-})\right) > [1-\theta+0.5]\, W_{VV}$, an unlikely occurrence given the sharp decline in θ[22].

We summarize results in the following

Proposition 1 *If households are allowed to vote individually on the admission standard in stratified schools and voting is about increases or decreases in the standard, the selected outcome is at least as high as α_m, the standard corresponding to median ability, depending on the initial value set by the government.*

Corollary 2 *When the admission standard in a stratified school needs the support of majority voting, stricter standards ($\theta \geq \alpha_m$) are likely to prevail over looser standards ($\theta < \alpha_m$), even when looser standards are more efficient.*

[21] The gain is positive if

$$\frac{\theta}{\theta^{+}}(1-g)[1-\lambda] + \theta\lambda(1-g) > 1 + \theta$$

While the left hand side is less than 1, the right hand side is greater than 1 and the inequality never holds.

[22] Consider for instance the first numerical example in the previous section. A reduction in θ from 0.595 to 0.095 implies that the right hand side and left hand side of the inequality in the text take on the following values: 0.195, 0.216. Therefore, the inequality is not satisfied.

Majority voting biases outcomes in favor of more "elitist" G schools. Intuitively, this occurs because, when θ is less than α_m and G schools are not elitist, there is a majority of households with offspring in G schools who would gain from a marginal increase in the standard, that raises average ability and human capital.

Households vote both on the admission standard, given that the schooling system is stratified, and on the schooling system itself. The key result in Proposition 1 is that, when schooling is stratified and government policy needs the support of majority voting, a percentage α_m of the young generation should go to vocational schools. It follows that the choice between stratified and comprehensive schools depends on the vote of the households whose offspring ends up in these schools. These households will choose a stratified system if the expected return from a V school is higher than the expected return from a C school. This occurs if the following conditions holds

$$A_V(1-g)\left[1-\lambda(1-\theta)\right] > \rho\left[A_G + F_c(A_V - A_G)\right] \tag{36}$$

when $\theta \leq F$, and

$$\frac{F}{\theta}(1-g)\left[1-\lambda(1-\theta)\right] > \rho\left[A_G + F_c(A_V - A_G)\right] \tag{37}$$

when $\theta > F$.

The above conditions are more likely to be satisfied the lower the rate of growth of technical progress g, the lower the relative efficiency of comprehensive education ρ, the more selective is admission to G schools and the lower the correlation between technical and academic abilities λ. To illustrate, consider the first numerical example in Section 6. In that case, $\theta = F = 0.595$, the left hand side of (36) is equal to 0.911 and the right hand side is equal to 0.95. In this example there is a majority of households who would vote in favor of the comprehensive system, even though the stratified system is more efficient.

Intuitively, this happens because the majority of households cares only about the expected income of their offspring in V schools, while the net output maximizing government cares also about the expected net output of G graduates. Assuming that the difference between total setup costs in a stratified and in a comprehensive system is small enough to be disregarded, majority voting is more likely than the net output maximizing government to select comprehensive schools whenever $H_{GG} > H_{VV}$.

We summarize as follows:

Remark 1 *When academic schools in an efficient stratified system are "elitist" ($\theta \geq \alpha_m$) and the human capital of G graduates is higher than the human capital of V graduates, majority voting could lead to the establishment of a less efficient comprehensive system.*

An interesting implication of majority voting is that reducing the admission standard is not necessarily the best way of supporting the existence of stratified systems: by so doing, the expected return from vocational schools also falls, and the majority of voters could be induced to choose a comprehensive system.

In this paper, we have assumed that talents are innate. If the development of academic talents depends also on family background (Heckman (1999)), students from working class families are more likely than students from wealthier families to end up in the V schools of a stratified system. In this case, less well off households could stand to gain from a transition to a more equal comprehensive system and vote in favor of it despite the fact that a stratified system is more efficient.

The shift in favor of comprehensive schools is more likely the higher the rate of technical progress, the higher the distortion induced by the admission test on the allocation of talents and the less efficient are stratified schools with respect to comprehensive education. Technological progress works against stratified systems and in favor of comprehensive education, that is more versatile and less specialized, by eroding the specialized skills acquired in vocational schools. This erosion process is faster with the introduction of information technologies, that use intensively general academic skills.

8 Growth

So far, we have assumed an exogenous rate of technical progress. Following Galor and Moav (2000), we can endogenize g by assuming that the rate at time $t+1$ be proportional to total accumulated human capital H at time t. Since total human capital depends on the rate of growth at time t, we obtain a nonlinear relationship between g_{t+1} and g_t. Let

$$g_{t+1} = \zeta H_t(g_t) \tag{38}$$

where $\zeta < 1$ is a positive parameter, and consider an economy with stratified schools characterized by perfect sorting of workers and job types. Total human capital at time t in this economy is:

$$H_t(g_t) = F(g_t)H_{VVt} + (1 - F(g_t))H_{GGt} =$$

$$= F(g_t)\left[\delta \frac{1 - \lambda(1 - F(g_t))}{2}(1 - g_t)\right] + (1 - F(g_t))\left[\delta \frac{1 + F(g_t)}{2}\right] \tag{39}$$

where

$$F(g_t) = \theta(g_t) = \frac{4(c_V - \eta) + \delta A_G + \delta A_V(g_t + \lambda(1 - g_t) - 1)}{A_V \delta \lambda(1 - g_t) - 4\eta - \delta A_G}$$

The strong non-linearity of the first order difference equation makes it difficult to study its behavior in a general way. As an alternative, we illustrate how endogenous growth affects our simple model by using the first numerical example described in section 6, with the additional assumption that $\zeta = 0.1$. In this specific case, the first order difference equation in the rate of technical progress g becomes

$$g_{t+1} = 0.1\left[1.2\frac{9797 - 3884g_t + 1287g_t^2}{(127 + 3g_t)^2}\right]$$

This relationship can be drawn as a downward sloping line in the phase plane (g_{t+1}, g_t), that crosses the 45 line once, and exhibits a unique steady state rate of growth, $g^*(\zeta)$, with $\partial g^*/\partial \zeta > 0$ (see Figure 1).

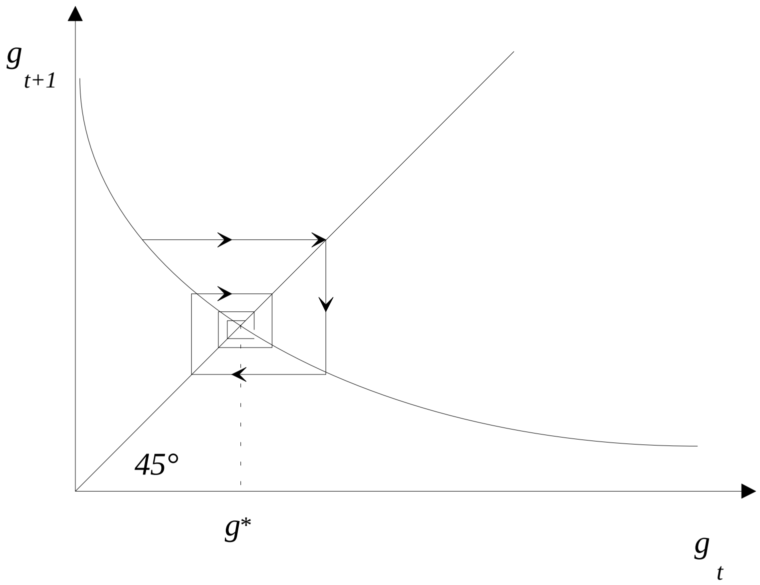

Figure 1: Cobweb Dynamics

Using a Taylor expansion in the neighborhood of g^* one can show that the steady state rate is locally stable for any $\zeta \in [0,1]$ and that the dynamical path is characterized by a convergent oscillatory trajectory. It is worth stressing that along the transition path the $F = \theta$ condition provides higher net output with respect to the alternatives $\theta < F$ and $\theta > F$. This example suggests that economies starting from a perfect sorting equilibrium can converge, under suitable values of the parameters, to a long run equilibrium still characterized by perfect sorting.

The numerical solution yields $g^* = 0.071$, $NY_s^* = 0.540$, $\theta^* = F^* = 0.59$, where the stars refer to the steady state solution with endogenous growth. Compared to the case of constant and exogenous growth, $g = 0.05$, the dynamical path with endogenous growth increases net output and reduces the selectivity of G schools. Net output increases because the productivity effect, that affects all jobs, prevails over the erosion effect, that affects only V graduates and their skills. Since the higher erosion reduces the relative efficiency of V schools, the relative importance of G schools increases and the admission standard to these schools falls. In a comprehensive system, the growth rate g also depends on total human capital. Human capital, however, is not affected by growth. In our example, the value of g when schools are comprehensive is 0.057, lower than the value obtained with stratified schools, and net output is $NY_c^* = 0.422$, higher than with exogenous growth.

9 Conclusions

When governments choose between comprehensive and stratified systems exclusively on the basis of economic efficiency, they must weight the relative advantages and disadvantages of both systems. Stratified systems trade the advantages of specialization and signalling against the disadvantages of producing skills with limited flexibility and versatility. The economic benefit of signalling is also reduced by the fact that academic tests are both incomplete and sensitive to random elements. Comprehensive systems, on the other hand, trade the relative advantage of providing higher versatility against the relative disadvantages of poor signalling and limited specialization. Government choice is affected by the rate of technical progress, because of the erosion effect on human capital, that is particularly significant for skills learned in V schools.

When the government chooses stratified schools and takes into account the effects of school design on the endogenous composition of jobs in the economy, it does not necessarily opt for perfect sorting, that matches the composition of graduates by school type with the composition of job types. Depending on the values of the parameters, it could do better either by having less elitist general schools, with graduates of these schools employed in jobs that require technical skills, or by having a very elitist system, with a percentage of graduates from vocational schools trapped into unemployment.

Government choices based on the maximization of net output are not always supported by majority voting. In an economy without liquidity constraints, the majority of voters could be in favor of a more elitist general school, when schools are stratified, and could at the same time prefer comprehensive to stratified schools, even when the latter are more efficient.

We do not find that one system clearly dominates the other in efficiency terms. This is a reasonable result, that squares well with the observed variation across different developed countries in the design of secondary schools. Our paper spells out a number of factors that could account for this variation. Importantly, only some of these factors are explicitly related to the organization of schools, and one should also consider differences both in the rate of technical progress and in the composition of job types (labor demand).

The classification of schooling systems into stratified and comprehensive stressed in this paper captures one important dimension of school design at the cost of drastically simplifying on other dimensions. First, comprehensive schools differ in their degree of standardization of curricula, examination and certification systems. While Germany and the Netherlands are examples of highly standardized and stratified systems (see Hannah et al (1999)), the United States has a comprehensive secondary school system with limited standardization (see Bishop (1996)). In between these two polar cases, there are countries with comprehensive and standardized schools (Japan and Scandinavian countries) and countries with an intermediate level of stratification (France, Italy and the UK).

Perhaps more importantly, comprehensive schools can also be selective. In Japan, for instance, secondary schools are comprehensive but clearly ranked in prestige and average achievement, and pupils compete very hard to be admitted to the top ranked schools. In the US, comprehensive schools often engage in tracking in a limited subset of subjects. In our simple theoretical framework, selection without differentiation clearly works in the direction of increasing the relative efficiency of comprehensive schools, because it provides firms with useful information about individual talents.

10 Appendix

Efficiency. We start by examining how F varies with respect to θ when $F > \theta$. Ignoring the time subscript, a reduction of θ below F implies that the new equilibrium share of V jobs, F, is given by

$$2F\left[\eta(1-F)-c_V\right] = F(A_G H_{GG} - A_V H_{GV}) - \theta(A_V H_{VV} - A_V H_{GV}) \quad \text{(A.1)}$$

Total differentiation with respect to F and θ and the use of (A.1) to eliminate $(A_G H_{GG} - A_V H_{GV})$ yields

$$\left[-2\eta F + \frac{\theta}{F}(A_V H_{GV} - A_V H_{VV})\right]\partial F =$$

$$\left[(A_V H_{GV} - A_V H_{VV}) + A_G F\frac{\delta}{2} - A_V\frac{F\delta(1-\nu)\lambda}{2}\right]\partial\theta$$

$$+\theta\left[A_V\frac{\delta(1-\nu)\lambda}{2} - A_V\frac{\delta(1-g)\lambda}{2}\right]\partial\theta$$

Evaluation of this derivative at $\theta = F$ gives the expression in the main text.

Next, we consider how F varies with respect to θ when $F < \theta$. When θ is set above F, the equilibrium value of F is

$$2F\left[\eta(1-F)-c_V\right] = \frac{1-\theta}{1-F}A_G H_{GG} - A_V H_{VV} \quad \text{(A.2)}$$

Total differentiation yields

$$\left[-2\eta F - \frac{1-\theta}{(1-F)^2}A_G H_{GG}\right]\partial F =$$

$$\left[\frac{A_G}{1-F}\frac{\delta}{2}\left[(1-\theta)-(1+\theta-\xi)\right] - A_V\frac{\delta(1-g)\lambda}{2}\right]\partial\theta$$

Since both elements within brackets are negative, the derivative $\frac{\partial F}{\partial\theta}$ is positive.

Preferences First, we show that the expression

$$U_G = \frac{1-\theta}{1-\theta^-} W_{GG}(\theta^-) + \left[1 - \frac{1-\theta}{1-\theta^-}\right] W_{GV}(\theta^-)$$

is increasing in θ^-. Differentiation yields

$$\frac{\partial U_G}{\partial \theta^-} = \frac{1-F}{(1-\theta^-)^2} \left[W_{GG}(\theta^-) - W_{GV}(\theta^-)\right] +$$

$$\frac{1-F}{(1-\theta^-)} \frac{\delta}{4} \left[A_G - A_V (1-\nu) \lambda\right] + \frac{\delta}{4} A_V (1-\nu) \lambda$$

The first element on the right hand side is positive by assumption (specialization of G graduates in G jobs). The sum of the second and third elements is also positive. It follows that $\frac{\partial U_G}{\partial \theta^-} > 0$.

Next, consider voting when $\theta < F$. Marginal increases of θ that leave $\theta < F$ are voted for both by the group who remains in V schools and by the group who remains in G schools. In the former case, expected utility is W_{VV}, an increasing function of θ. In the latter case, expected utility is given by $\frac{1-F}{1-\theta} W_{GG} + \left[1 - \frac{1-F}{1-\theta}\right] W_{GV}$, that is also increasing in θ. This voting behavior leads eventually to $\theta = F$. Further increases of θ depend on the initial value of θ: voting majority supports a higher θ and leads to $\theta = \alpha_m$ only if $\theta < \alpha_m$. When $F < \theta$, the group remaining in G schools gains from a higher θ, but the group remaining in V loses out. When $\theta > \alpha_m$, the latter group has the majority and θ declines towards F. When $\theta \leq \alpha_m$, θ increases to reach α_m.

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