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THE VALUE OF LICENCES FOR RECREATIONAL RESOURCES USE*

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Abstract

Tools for recreational resources management are topics of great theoretical and practical interest. The most commonly used tool undoubtedly is the licence or permit. The implementation of a system of licences to regulate the use of a natural resource is far from simple as it presupposes the answer to a number of questions. This paper focuses on the point of view of the purchaser and, in particular, models his behaviour when considering the purchase of a licence that authorises him to benefit from a natural resource in accordance with certain rules and procedures. The model assumes that the behaviour can be compared to that of an investor faced with a *Call option*, i.e. the right (but not the obligation) to make an investment at any time in a given financial unit at a pre-established price. The model was used to study the effect on purchase timing and licence duration of certain factors such as the uncertainty of the benefits and the irreversibility of the purchase. The results show that under uncertainty the consumer tends to delay the purchasing time and, then, to purchase a licence of longer duration than the one that would be purchased considering only the present value deriving from the expected flow of benefits.

Key words: Option value, Irreversibility, Natural resources, Licences.

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1. Introduction

The management of natural resources recreational use, such as fishing and hunting, is a topic of great theoretical and practical interest. This interest is due, basically to: a continuously increasing demand for recreation and a rapidly diminishing of the stocks. The growth in demand is linked to improvement of the social and economic conditions of consumers in many countries accompanied by increasing pressure from the economic interest groups operating in the recreation sector (tour operators, equipment manufacturers and retailers, television entertainment, etc.). Resources stocks, on the other hand, are rapidly diminishing due to exploitation for production purposes, environmental pollution or reduction of reproduction habitat. At the same time, various tools to regulate recreation have been conceived, used and studied. The most common is the licence. The licence is a right to exercise, in accordance with pre-established rules and procedures, a certain activity for a certain period of time in a given area. This right can be acquired in various ways - it can be inherited and/or related to citizenship, residence and land ownership - but in the majority of cases it has to be purchased from a public agency. The contents of the right can be varied and are closely linked to the objectives of the regulation. Generally speaking, the aim of the regulation is to guarantee a sustainable use of the resource in the long term, in other words to ensure an extraction rate compatible with the reproduction capacity of the resource. In addition, the regulation system should boost the efficiency of the activities that use the resource. The licence is usually nominal, i.e. it can be used by one person only, and it cannot be transferred or refunded. Licences can be granted for different periods (daily, weekly, seasonal, annual etc.) and can permit the user to keep a certain amount of fish, mushrooms, game etc. It is evident, therefore, that setting up a licences system to regulate recreational natural resources use is far from simple as it presupposes the answer to a number of questions. In this regard, extensive literature exists concerning the analysis of professional licence systems (Anderson, 1958; Campbell and Linder, 1990; Karpoff, 1989) but the consequences of a licence system for recreational resources use is still relatively unexplored.

The paper models the behaviour of a consumer when considering the purchase of a licence that allows him to benefit from a natural resource in a certain place, for a certain period of time and according to pre-established rules and procedures. As the purchase of a licence has some characteristics similar to an investment decision, i.e. it is irreversible, the future benefits are uncertain and, to a certain extent, it can be postponed, the problem is to establish if and when the is optimal to purchase the licence[1]. Due to the above characteristics, it is reasonable to assume that the optimal purchase decision must satisfy a slightly more restrictive rule than the one suggested by the usual Net Present Value. The uncertainty about the availability of natural resources implies that in the future the consumer may wish to abandon the recreational activity. Irreversibility means that if the consumer purchases a licence now, he cannot re-sell it in the future. Finally, the expediency of postponing the purchase gives him the chance to gather further more information before committing himself to an irreversible decision. So, the possibility of purchasing a licence is similar to a perpetual *Call option* or a right, not an obligation, to make an investment at any time in a certain

[1] This aspect is of particular interest as it coincides with starting of the recreational activity. It is reasonable to assume that the consumer can decide to purchase the licence at any time even though the seasonal climatic trends, variability of the quality of the resource and the characteristics of the licence can considerably limit the possibility of choice. For example, in many countries annual fishing or hunting licences must be purchased before the beginning of the hunting or fishing season. Daily or weekly permits, on the other hand, can be purchased at any time except, of course, during the reproduction period.

financial activity at a pre-established operating price. As a financial option, the uncertainty of future benefits generates a value related to the opportunity to invest[2].

The plan of the paper is as follow. Section 2 sets out the basic model and examines the consumer's value of the option to buy the licence and the optimal purchasing time. The optimal duration of the licence and the trade-off between cost of the licence and its duration are discussed in section 3. Section 4 gives the conclusions.

2. The model

The expediency of purchasing a licence is assessed by weighing its cost against the satisfaction expected from the future recreational activity. This assessment can be made at any time and therefore it is not sufficient for the value of the expected satisfaction to be higher than the expenditure - it must also be higher than the satisfaction value that can be obtained, currently, by exercising the purchase at any time in the future. This aspect is very important when dealing with resources, like stocks of biological resources, the level of which varies due to natural causes like seasonal variation, or due to management such as restocking, environmental rehabilitation operations or over-exploitation of the resource by professional activities.

In this paper the basic assumptions are the following: a) the licence cannot be transferred and therefore the decision is irreversible and generates a sunk cost; b) the licence can be purchased at any time, i.e. the purchase can be postponed; c) the actions authorised by the licence can be undertaken immediately after purchase; d) the duration of the licence can vary: a day, a whole season (annual), several seasons or even for life. Having said this, let's consider a consumer who, at one time, assesses the expediency of purchasing a licence. He will obviously assess the expenditure with respect to the expected utility of the recreation. The latter will reasonably depend on the quantity of natural resources he expects to acquire. Therefore, the instantaneous net benefit expected by the consumer from the recreational activity is given by the following function $B(x, n)$ ($0 < x < X \leq \infty, 0 \leq n < N \leq \infty$) where x is the stock of resources and n indicates the number of rival consumers drawing from the same stock. The marginal willingness to pay for the recreational activity is positive and decreasing with respect to the amount of stock, i.e. $B_x > 0$, $B_{xx} \leq 0$.

The number of rival consumers is included in the benefit funtion as we assume that the instantaneous quantity of resources available for the consumer is negatively correlated with the number of consumers simultaneously present due to a competition effect[3]. An increase in rival agents produces a decline in the benefit and a reduction in the marginal willingness to pay, i.e. $B_n < 0$, $B_{nn} > 0$ and $B_{xn} < 0$.

The amount of the resource is measured by a variable of the biomass x_t which is assumed to satisfy the following stochastic differential equation[4]:

$$dx_t = \mu^x (x_t)\, dt + \sigma^x x_t\, dW_t^1 + x_t\, dQ_t, \text{ with } x_0 = x << X,\ \sigma^x > 0 \tag{1}$$

[2] Dixit (1992), Pindyck (1991) and a recent manual by Dixit and Pindyck (1994) provide a thorough review of this approach applied to various economic questions. For example, as regards fishing, the only work that uses this approach, as far as we know, is that of Karpoff (1989), in which the author highlights how the option component of each licence triggers a process of regulation of the number of licences.

[3] It should be noted that the problem faced by a single consumer sharing a resource with others differs from that of the policy maker who considers the aggregate benefit of all the consumer. In this context, a rational consumer will include an assessment of the stock of resources in its benefit function and this assessment will vary inversely to the number of rivals present (Arnason, 1990). This assumption is obviously slightly restrictive as the effect of congestion concerns not only competition between consumers in use of the stocks but, as we are considering recreational activities in the open air, also disturbance caused by the intrusion of outsiders in addition to the need for *wilderness*. However, it is assumed that the main effect is that of competition.

[4] For a discussion of differential stochastic equations and Brownian motions, see Cox and Miller (1965) and Harrison (1985).

$$\mu^x(x_t) = \gamma(x_t)x_t - h(x_t, n_t)\, n_t$$

In (1) $\gamma(x_t)$ represents the expected growth rate of stock and is decreasing in the amount of the stock x_t[5]; $h(x_t, n_t)\, n_t$ is the expected individual harvest rate multiplied by the number of consumers present at time t and represents the reduction of the stock produced by recreation activity. This reduction is generally positively related to the stock level. The expected individual harvest rate is positively related to the stock and negatively related with the number of rivals[6]. In addition it is assumed that certain random factors such as weather affect the stock and the growth rate of the biomass. The term $\sigma^x x_t$ measures the instantaneous volatility and dW_t^1 is the increment of a Brownian motion with mean $E(dW_t^1) = 0$ and variance $E[(dW_t^1)^2] = dt$. Finally, dQ_t is the positive or negative variation of a Poisson process, independent of W_t, with mean arrival time λ. This process represents sudden variations in x due to "exceptional" events such as pollution, poaching or illegal harvesting, restocking. This variation of the stock is assumed to be a fixed percentage ϕ ($-1 \le \phi \le 1$). The process dQ_t has the following probability distribution: $dQ_t = \phi$ with probability λdt; $dQ_t = 0$ with probability $1 - \lambda dt$. Therefore, (1) establishes that the stock of resources x fluctuates over time according to a Brownian motion but that in each interval of time (t, $t + dt$) there is a probability λdt of instantaneous variation occurring (positive or negative), bringing the stock to level $(1 + \phi)$ times the initial level, subsequently beginning to fluctuate again until the next variation[7].
As the number of rivals is concerned, it is driven by the following stochastic differential equation:

$$dn_t = \mu^n(n_t)\, dt + \sigma^n n_t\, dW_t^2 \text{ with } n_0 = n << N,\ \sigma^n > 0 \tag{2}$$

$$\mu^n(n_t) = n_t u(x_t) p(n_t) - b n_t$$

Equation (2) models the so-called "social dimension" of the recreational activity. We assume that the number of consumers depends basically on the knowledge they have and/or think they have of the stock of resources. This knowledge derives essentially from the circulation of information by people who consider themselves "informed" as they have recently benefited (or attempted to) from the resource[8]. By indicating $u_t = u(x_t)$ as the contact rate between people who discuss stock levels x_t, at time t the "informed" consumers communicate with $n_t u_t$ people, only a fraction of whom $p(n_t)$ will be newly informed. In addition the number of consumers drops in time at a constant rate b for reasons not connected with the recreational activity such as saturation of the need to go out fishing or hunting. Finally, a level of uncertainty is considered with regard to the number and trend of the rivals: this is expressed by the term $\sigma^n nt$, where dW_t^2 is the usual increment of a Brownian motion with mean, $E(dW_t^2) = 0$, and variance $E[(dW_t^2)^2] = dt$, and possibly correlated with the process W_t^1, i.e. $E(dW_t^1 dW_t^2) = \rho dt$, where $-1 < \rho < 1$ indicates the instantaneous correlation coefficient between the two processes.

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5 The biological growth processes most widely used in models to optimise exploitation of sustainable resources (fish, wild animals etc.) incorporate the fact that when stocks increase, self-limitation processes are triggered with consequent reduction in the growth rate (see Clark, 1990).

6 This assumption is in line with the classic approach to modelling of the use of sustainable resources, where the effort spent in harvesting (E), understood as the aggregate of capital energy, and work expended in a certain interval of time (Schaefer, 1954), necessary for the collection of a certain quantity (h) of resources, is inversely proportional to the stock level (x), i.e. $E = h/x$, (Pearce and Turner, 1989, p. 242). It can therefore be assumed, for the sake of simplicity, that collection is proportional to the stock level, $h(x)n = (\vartheta E x)\, n$, $0 \le \vartheta \le 1$, where $E = 1$.

7 Without altering the results we can assume that the size of the jump is uncertain.

8 Here it should be remembered that this information is often misleading. This possibility has not been included in the model.

2.1. The value of the licence

Assuming, in this section, that the licence will run indefinitely, its value is given by the discounted flow of benefits:

$$V(x_T, n_T) = E_T \left\{ \int_T^{\infty} H(t) B(x_t, n_t) e^{-r(t-T)} dt \right\} \tag{3}$$

where T is the time at which the licence is purchased, r is the discount rate and $H(t)$ represents the probability of instantaneously benefiting from the resource at time t[9].

The consumer will purchase the licence when its expected present value exceeds the cost K. In reality, this is a reasonable policy only if the purchase cannot be postponed; in fact, although the net present value $V(x_T, n_T) - K$ is positive, uncertainty of benefits, the seasonal trend and the sunkness of the cost may induce the purchaser to be cautious before committing to an irreversible decision. We therefore need to assess the value of waiting in order to model the expediency of postponing the purchase, in other words assess the option value to purchase the licence.

2.2. The value of the option to purchase the licence

As the net benefit deriving from purchase of the licence at time T is $V(x_T, n_T) - K$, the opportunity value at time zero, indicated by F, can be calculated by maximising the following expression:

$$F(x,n) = \max_T E_0 \left[(V(x_T, n_T) - K)\, e^{-rT} \,\middle|\, x_0 = x, n_0 = n \right] \tag{4}$$

The maximisation problem involves two states variables: x and n. However, as the option will probably be withheld if x is low or if n is high and, conversely, will be exercised when n is low enough for a given value of x, the rule that guides purchase can be represented by a curve $x = x^*(n)$, where x^* is always increasing with respect to n.

By a non-arbitrage argument the *Bellman equation* of (4) simply requires the capital gain $E[dF(x,n)]$ to be equal at all times to the return $rF(x, n)dt$, i.e.:

$$rF(x, n)dt = E[dF(x, n)] \tag{5}$$

which can be rewritten as a partial differential equation[10]:

$$\begin{aligned} &\tfrac{1}{2} [(\sigma^x)^2 x^2 F_{xx}(x,n) + (\sigma^n)^2 n^2 F_{nn}(x,n) + 2\rho\sigma^x\sigma^n xnF_{xn}(x,n)] + \\ &+ \gamma(x)x - h(x,n)n)F_x(x,n) + (nu(x)p(n) - bn)F_n(x,n) + \\ &+ \lambda\ F((1+\phi)x,n) - (r+\lambda)F(x,n) = 0 \end{aligned} \tag{6}$$

Equation (6) must also satisfy the following boundary conditions:

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9 Generally, this probability varies according to the seasonal trend; it is zero during periods when the activity is prohibited and maximum when the availability of the resource and/or the availability of time on the part of the consumer is maximum.

10 Expanding dF and applying Itô's lemma for combined Brownian and Poisson processes, we get the following partial differential equation (Dixit and Pindyck, 1994):

$$\begin{aligned} &\tfrac{1}{2} [(\sigma^x)^2 x^2 F_{xx}(x,n) + (\sigma^n)^2 n^2 F_{nn}(x,n) + 2\rho\sigma^x\sigma^n xnF_{xn}(x,n)] + \\ &+ (\gamma(x)x - h(x,n)n)F_x(x,n) + (nu(x)p(n) - bn)F_n(x,n) + \\ &+ \lambda \left[F((1+\phi)x,n) - F(x,n)\right] - rF(x,n) = 0 \end{aligned}$$

which can be reduced as in the text.

$$\lim_{x \to 0} F(x,n) = 0 \quad (7)$$

$$F(x^*(n),n) = V(x^*(n),n) - K \quad (8)$$

$$F_x(x^*(n),n) = V_x(x^*(n),n) \quad (9)$$

Condition (7) establishes that when the resource stocks x tend to zero, the option value also tends to zero. Condition (8) states that, within the purchase rule *(x*(n))*, the option value must equal the current value of the licence net of the outlay for the purchase (*matching value condition*). As long as $x_t < x^*(n)$, we get $F(x(n), n) > V(x(n), n) - K$, and the consumer considers it convenient to postpone the purchase. In other words, $V(x(n), n) < K + F(x(n), n)$ or the value of the licence is lower than its total cost obtained by adding its direct cost K to the opportunity cost of the option expressed by $F(x(n), n)$. Finally, the condition (9) excludes the possibility of the investment being made, for reasons of speculation, at a moment other than the one identified by (8) (*smooth pasting condition*). That is, the tangency point between the curve that represents the option value $F(x(n), n)$ and the net present value $V(x(n), n) - K$ identifies the critical value $x^*(n)$. Consequently, it is expedient to purchase the licence if, for every given value of n, stock levels at that time are higher than the critical value, i.e. $x_t \geq x^*(n)$[11].

2.3. Optimal purchase time

Although the model illustrated in the paragraph 2.2 fully represents the strategy of the licence purchaser, it is of little practical use as the complexity of the functions make a close solution impossible. To highlight the effect of context factors (like uncertainty) on consumer choices, a number of simplifying assumptions have been adopted to make the model solvable. The assumptions are the following:

Assumption 1: the instantaneous benefit function is $B(x,n) = x^{\alpha} n^{-\beta}$, with $0 < \alpha \leq 1$, $\beta > 0$.

Assumption 2: the weighted sum of the growth rates of x and n is constant, i.e. $\mu(x,n) \equiv \frac{\mu^x(x)}{x} - \frac{\mu^n(x)}{n} = \hat{\mu}$[12].

Assumption 3: the probability of the consumer to benefit from the resource at time t has an exponential distribution $H(t) = e^{-ht}$[13].

Making use of the above assumptions the optimisation problem reduces to one dimension. In particular, by expanding dB_t and applying Itô's lemma it can be easily shown that B varies according to the following function[14]:

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11 In this case we have $\frac{dx^*}{dn} > 0$, i.e. if the number of rivals increases, the stock must increase more than proportionally for the purchase to remain viable.

12 This assumption implies that: 1) the growth rate of the resource is constant, $\gamma(x) = \gamma$; 2) the harvesting rate is linear, $h(x,n) = a\frac{x}{n}$, or $h(x,n) = ax$; 3) the contact rate is constant, $u(x) = u$; 4) the fraction of possible people contacted by the consumer is constant, $p(n) = p$. This set of conditions gives $\hat{\mu} = \alpha(\gamma - a) - \beta(up - b)$.

13 This implies that the possibility of taking a trip out between time t and time $t + dt$ can be described by a constant rate h.

14 This derives from the fact that the log-linear function of a log-normal random variable is distributed log-normally. The first and second moment of the process B_t are given by $E(dB/B) = (\mu_B + \lambda\phi)dt$ and $V(dB/B) = (\sigma^2_B + \lambda\phi^2)dt$ (see Dixit and Pindyck, 1994, p.169).

$$dB_t = \mu_B B_t dt + \sigma_B B_t dW_t + \alpha B_t dQ_t \text{ with } B_0 = B \tag{10}$$

where:

$$\mu_B = \hat{\mu} + \frac{1}{2}(\sigma^x)^2 \alpha(\alpha - 1) + \frac{1}{2}(\sigma^n)^2 \beta(\beta + 1) - \rho\sigma^x\sigma^n\alpha\beta$$

$$\sigma_B = \sqrt{(\sigma^x)^2\alpha^2 + (\sigma^n)^2\beta^2 - 2\rho\sigma^x\sigma^n\alpha\beta}$$

In short, we assume that the stock of resources x_t and the expected number of rivals n_t increase (or decrease) at a certain constant mean rate but that the actual growth is stochastic, described by a normal distribution and independent over time[15]. The model has now been sufficiently simplified to provide a closed form solution for the purchasing rule.

Proposition 1

(i) The value of the licence is:

$$V(B_T) = E_T \{ \int_T^{\infty} B_t e^{-(r+\lambda+h)(t-T)} dt \mid B_0 = B \} = \frac{B_T}{\delta} \tag{11}$$

with:

$$\delta = r + \lambda + h - \mu_B > 0$$

(ii) The option value is an increasing convex function of the instantaneous benefit B:

$$F(B) = AB^{\xi} \quad \text{for} \quad x \in (0, B^*] \tag{12}$$

where:

$$A = \frac{(\xi - 1)^{\xi - 1} K^{1-\xi}}{(\delta\xi)^{\xi}} > 0$$

and $\xi > 1$ is the positive root of the following non-linear expression:

$$\Phi(\xi) \equiv \frac{1}{2}\sigma^2{}_B\xi(\xi - 1) + \mu_B\xi - (r + \lambda) + \lambda(1 + \phi\alpha)^{\xi} = 0$$

(iii) The purchasing rule is given by:

$$B^* = \frac{\xi}{\xi - 1}\delta K > 0 \tag{13}$$

Proof: see Appendix.

The optimal threshold value B^* identifies the net benefit which makes for the user expedients to purchase the licence. The consumer will purchase the licence the first time B_t exceeds the threshold defined by B^*[16]. It should also be noted that B^* is greater than the "common" opportunity cost of

[15] In this specific, the exogenous variables x and n are not stationary and do not possess long period distributions (non-conditioned). According to the information at time $t = 0$, the future values of the exogenous variables x and n possess a joint log-normal distribution, with variance proportional to the time horizon of the forecast. In addition, since it is a log-linear function (with constant elasticity) of a Brownian motion, the benefit B_t follows a Brownian motion as described by (10). The trend and the standard variation of the function B_t are a result of the linear combination of the corresponding parameters contained in the primitive variables x_t and n_t, with weights given by the elasticity α and β and by the correlation coefficient ρ.

[16] As $B(x,n) = x^{\alpha}n^{-\beta}$, the purchase rule (13) can be easily represented by a strictly increasing function:

the licence expressed by δK. In fact, in the absence of uncertainty and/or the licence cost were recoupable, it would be expedient to purchase it when the current value of the benefits exceeds the cost of the licence, i.e. when B assumes a value such that:

$$\frac{B}{\delta} = K \tag{15}$$

In the presence of uncertainty, however, the critical level B^* is increased by the option factor $\frac{\xi}{\xi - 1} > 1$ which measures the additional benefit required by the consumer before making an irreversible outlay that provides uncertain benefits.

Using (11), (12) and (15) we can deal with the purchasing time of the licence, comparing the opportunity cost of immediate purchase of the licence with the corresponding benefit that can be obtained by purchasing at the optimal time. This can be assessed via the difference $F(B) - F_0(B)$, where we indicate with $F_0(B) = V(B) - K$ the value of the licence when purchased immediately (t = 0). Since $F(B) = AB^{\xi}$, if the current value of B is below B^* thus inducing the consumer to postpone the purchase, we get:

$$F(B) - F_0(B) = K + AB^{\xi} - \frac{B}{\delta} \tag{16}$$

The first term of the right-hand side of the equation (16) is the actual cost of the licence. The second term is the value of the purchase option. Since purchasing implies killing this option, in the formula, it represents a cost. The third term is the present value of the future flow of benefits and therefore measures the value of immediate purchase of the licence. In short, as long as $B < B^*$ and $F(B) - F_0(B) > 0$, the total cost of the licence (given by the direct cost and the option value) will be higher than the present value of the benefits and then the decision should be postponed[17].

Via equations (10) and (13) it is possible to perform some comparative statics. For example, an increase in the discount rate r increases the value of the purchase option and therefore also increases B^*. In fact, as it has been assumed that the cost K is sustained when the licence is purchased, an increase in r implies a greater reduction in the present value of the cost and therefore the option value increases but will tend to be exercised later (Dixit and Pindyck, 1994, p. 152-161).

Finally, since $\frac{\delta\xi}{\delta\sigma_B} < 0$, an increase in uncertainty of the future trend of B determines an increase of $\frac{\xi}{\xi - 1}$ and therefore a greater gap between B^* and δK. Although the purchase option value increases when uncertainty as to future benefits increases, the expediency of postponing the purchase decision also increases, i.e. for purchase to be expedient, the expected benefit flow must increase.

3. The optimal duration of the licence

The model presented is also useful for solving the problem of identify the optimal duration of the licence. If we assume that the consumer can purchase only one type of licence allowing him to use the resource forever (or for such a long period that it can be considered indefinite), the expected value is equal to the discounted integral of the net benefit as expressed in (3). In reality the

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$$x^* = \left(\frac{\xi}{\xi - 1} \delta K \right)^{-\alpha} n^{\beta/\alpha} \tag{14}$$

with $\frac{dx^*}{dn} \geq 0$, and $\frac{d^2x^*}{dn^2} \geq 0$ if $\alpha \geq \beta$ and $\frac{d^2x^*}{dn^2} \leq 0$ if $\alpha \leq \beta$.

17 Similar considerations apply also in the context of tax incentives for the adoption of eco-compatible environmental processes, see for example Dosi and Moretto (1996a,b) and Dosi and Moretto (1997, 1998).

consumer can choose from a number of alternatives and can purchase licences entitling him to benefit from the resource for one day, one week, one year etc. Obviously, the shorter the licence period, the lower the total cost and the higher the mean cost per trip out. In this case the consumer's problem is not only when to exercise the purchase option but also what duration to choose. Since the purchase of a licence is always an irreversible decision, the possibility of choosing among licences with different durations generates a further opportunity for postponing the purchase. In other words, we have an option value for every type (length) of licence that can be purchased[18].

In this paragraph it is assumed that the consumer can choose from a menu of N licences of different duration identified by i, cost K_i and expected present value, ranked in increasing licence duration, given by:

$$V_i(B_T) = E_T\{ \int_T^{T+\tau i} B_t e^{-(r+\lambda+h)(t-T)} dt \mid B_0 = B \} = \frac{B_T}{\sigma(\tau_i)} \tag{17}$$

where T is the time at which the licence is purchased, τ_i its duration, $\frac{1}{\sigma(\tau_i)} = \frac{1-e^{-\delta\tau i}}{\delta}$ is the "real" discount factor that depends on the duration of the licence. The shortest duration is the case in which the consumer purchases at time T a licence for one single trip out between T and $T + dt$, so that $V_I(B_T) = B_T$[19]. The longest duration is given by $\tau_N = \infty$ so that $V_N(B_T)$ is equal to (11).

A consumer who assesses the purchase with respect to the benefit B_T will choose the licence that appears preferable at time T. The net purchase value is $\max_i [V_i(B_T) - K_i]$, for $i = 1, 2, \ldots, N$. In practice we have to calculate when (T) and for what type of licence (i) the option value is maximised.

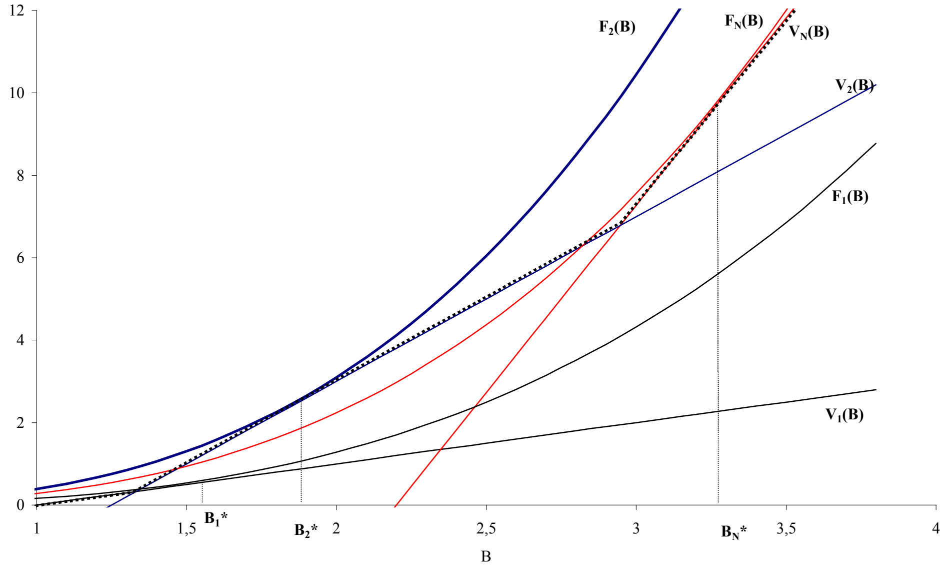
The solution to the problem is shown in Figure 1 where three types of licence are illustrated: V_1, V_2, and V_N. The first one permits only one trip out, the second is of intermediate duration $1 < \tau_2 < \infty$ and the third is "for life" $\tau_N = \infty$.

Since the inverse of the discount factor is a decreasing convex function with respect to the duration, i.e.: $\delta'(\tau_i) < 0$ and $\delta''(\tau_i) > 0$, with $\delta(0) = +\infty$ and $\delta(\infty) = \delta$, the net value of each licence is a straight line, growing in B, with gradient that increases as the duration increases, while the greater value of the three determines the upper envelope curve.

[18] Dixit (1993b) highlights that the choice between investment projects of different scale creates an option extravalue to wait. Moretto (1999) extends this result to the case of abandonment of a multiplant firm.

[19] By expanding in series the right-hand side of (17), we get $e^{-\delta\tau} = \sum_{n=0}^{\infty} \frac{1}{n!}(-\delta)^n \tau^n$. By truncating the series expansion at the first term and placing $\tau_1 = 1$ we get $\frac{B}{\delta}\{1 - (-\delta)^0 \tau_1{}^0 - (-\delta)^1 \tau_1{}^1\} = B$.

Figure 1 - The case of the three licences



Unlike (4), by the above arguments, the investment option value at the present time can be obtained from the following equation:

$$F(B) = \max E_0 \{ [\max_i (V_i(B_T) - K_i] e^{-rT} \mid B_0 = B \} \tag{18}$$

As the differential equations (6) and (12) still apply, the solution to (18) is fairly easy. The set of option values, with respect to B and τ, is represented by a sheaf of exponential curves. The optimal solution can be identified by calculating for each hypothetical τ the tangent between the exponential curve (12) with the respective straight line indicating the net value $Vi(B) - K_i$, and choosing, from all the solutions, the one that provides the highest $F(B)$, or equally the one with the highest value of the constant A_i.

Proposition 2

(i) A simple application of the procedure performed for a single licence shows that:

$$A_i = \frac{(\xi - 1)^{\xi - 1} (1 - e^{-\delta \tau i})^{\xi} K_i^{\,1-\xi}}{(\delta \xi)^{\xi}} > 0 \tag{19}$$

(ii) and the investment rule is given by:

$$B_i^* = \frac{\xi}{\xi - 1} \delta(\tau_i) K_i > 0 \tag{20}$$

Proof: straightforward from proposition 1.

Choosing the highest value of the constant A_i is equivalent to choosing the licence with the highest term $(1-e^{-\delta\tau i})^{\xi} K_i^{1-\xi}$. For example, from Figure 1, this condition occurs for a licence with intermediate duration $1<\tau_2<\infty$. If the present value of the benefit B is below B^*_2 the consumer will wait until it exceeds B^*_2 and will then purchase the licence with duration τ_2 at cost K_2. It can also be easily seen that is not optimal to purchase the licence with the longest duration (infinite) $\tau_N=\infty$.[20] In other words, in the numerical example given, the intermediate licence represents the best compromise between duration and cost (K, τ), and it is not expedient for the consumer to wait and purchase the longest licence; he will prefer to take advantage of the possibility offered by a shorter licence which costs less. Neither is the daily licence $\tau_1=1$ an optimal choice, as can be seen again from Figure 1. In this case the consumer will by-pass this opportunity and wait for the expected benefit to grow sufficiently to make the intermediate licence worthwhile.

3.1. The trade-off between cost and duration of the licence

The trade-off between cost and duration may be analysed generalising the maximisation of the function $(1-e^{-\delta\tau i})^{\xi} K_i^{1-\xi}$ and then exploring the possibility of purchasing licences with continuum of durations.

Assumption 4: the inverse supply function of the agency's with respect to the duration is given by $K=K(\tau)$ with $K'>0$, $K''\leq 0$, and $K(0)=K_{\min}\geq 0$, $K(\infty)=K_{\max}\leq\infty$[21].
Choosing the licence implying the highest value of the constant A is equivalent to solving the following problem:

$$\max_{\tau}(1-e^{-\delta\tau})^{\xi} K^{1-\xi} \quad , \quad \text{subject to } K=K(\tau) \tag{21}$$

For an internal solution, $0<\tau<\infty$, the first order condition is the following:

$$\frac{K'(\tau)\tau}{K(\tau)}=\frac{\xi}{\xi-1}\frac{\delta e^{-\delta\tau}\tau}{1-e^{-\delta\tau}} \tag{22}$$

and given that $\left|\varepsilon_{\tau,K(\tau)}\right|\equiv\dfrac{K(\tau)}{K'(\tau)\tau}$ and $\left|\varepsilon_{\delta(\tau),\tau}\right|\equiv\dfrac{\delta e^{-\delta\tau}\tau}{1-e^{-\delta\tau}}$ equation (22) can be expressed as: $|\varepsilon_{\tau,K(\tau)}||\varepsilon_{\delta(\tau),\tau}|=\dfrac{\xi-1}{\xi}<1$ from which we get that the necessary condition implies that the product of the elasticity of the supply function and the elasticity of the discount factor must be below one. In other words, this means that as the discount rate grows, the duration for which the previous condition is valid increases and this is reasonable because as the rate increases, shorter and therefore intermediate licence terms (< ∞) are favoured. In addition, the fact that the product of the two elasticities is lower than one also depends on the form assumed for the cost function. With a linear

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20 This licence will be chosen when a higher minimum present benefit threshold is exceeded, identified by the tangent between $F_N(B)$ (upper curve) and $V_N(B)-K_N$.

21 Alternatively we can consider $K(\tau)$ as a function that expresses the cost sustained by the consumer to obtain a licence of duration τ.

cost function $K(\tau)$, condition (22) reduces to $|\varepsilon_{\delta(\tau),\tau}| = \frac{\xi - 1}{\xi}$, and the most expedient licence will always have an intermediate duration. If, on the other hand, $K(\tau)$ is constant, the most expedient licence will always have unlimited duration. Finally, if $K''(\tau) < 0$, then it will depend on how $K(\tau)$ grows with respect to τ : the quicker it reaches $\overline{K}$ the greater the weight of the constant cost conditions with respect to the duration. In other words, for the consumer the intermediate solutions are expedient when the discount rate is high and the elasticity of the cost function is high (i.. the elasticity of the supply function is low).

After clarifying the effect of the discount rate and the cost function on the optimal solution, we shall now consider the effect of irreversibility and uncertainty. The first point to note is that if the purchase of the licence is a reversible decision or there were no uncertainty about future benefits of the resource, it would be sufficient $|\varepsilon_{\tau,K(\tau)}||\varepsilon_{\delta(\tau),\tau}| = 1$ to have an intermediate solution.

Let us consider the option value implicit in (22) and assume a supply function with constant elasticity:

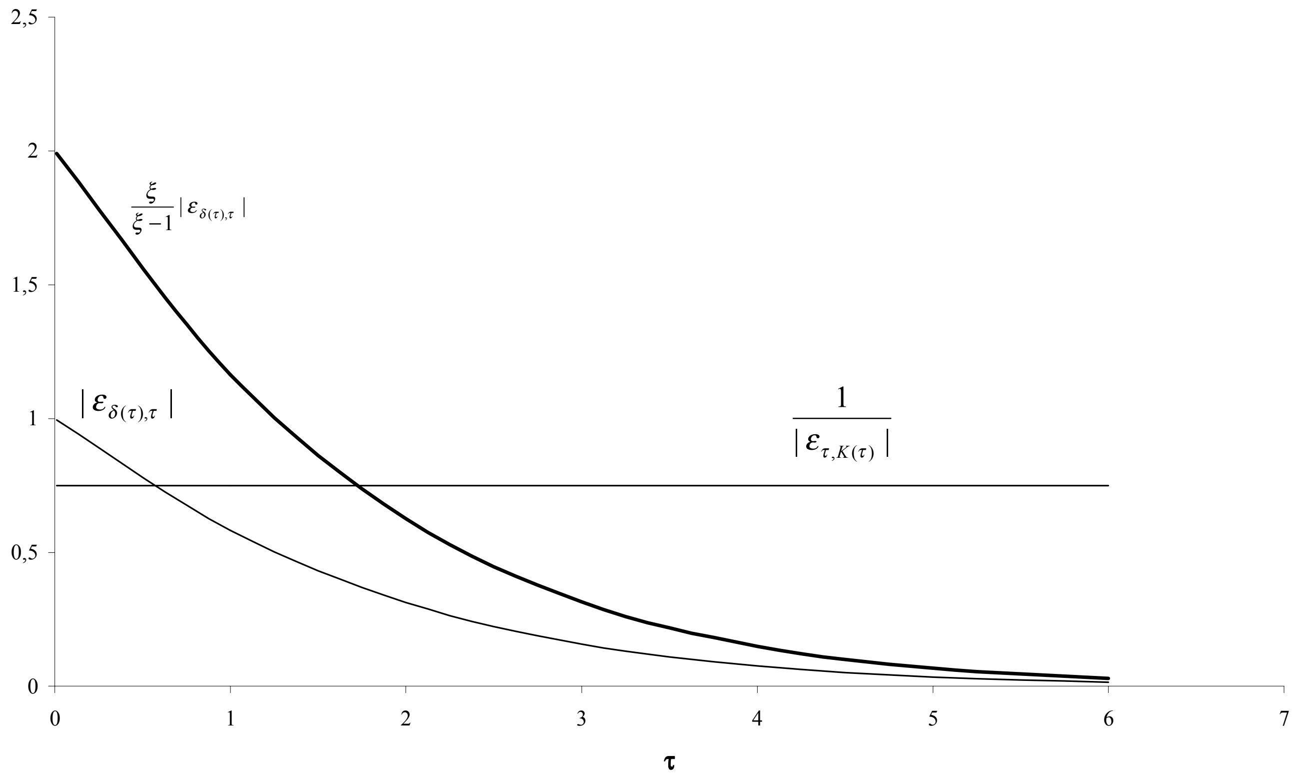
Assumption 4bis: the supply function of the agency in terms of duration with respect to the cost of the licence is given by $\tau = K^{1/a}$ with $0 < a \leq 1$.

From the properties of $\delta(\tau)$, we get $|\varepsilon_{\delta(\tau),\tau}| \rightarrow 1$ for $\tau \rightarrow 0$ and $|\varepsilon_{\delta(\tau),\tau}| \rightarrow 0^{+}$ for $\tau \rightarrow +\infty$, with:

$$\frac{d|\varepsilon_{\delta(\tau),\tau}|}{d\tau} = \begin{cases} -\tfrac{1}{2}\delta & con \;\; \tau \rightarrow 0 \\ 0^{-} & con \;\; \tau \rightarrow +\infty \end{cases}$$

which guarantee that if an intermediate solution exists this is unique. Having said that, the implications of the uncertainty on the choice of purchasing the licence can be assessed with reference to the following Figure 2.

Figure 2 - Optimal duration of the licence with and without uncertainty



The thin lower curve represents the elasticity of the discount factor with respect to the duration τ, i.e. $|\varepsilon_{\delta(\tau),\tau}|$. The horizontal straight line is the inverse of the supply function elasticity, i.e. $\frac{1}{|\varepsilon_{\tau,K(\tau)}|} = a$. Finally, the upper curve represents $\frac{\xi}{\xi-1}|\varepsilon_{\delta(\tau),\tau}|$. From condition (22) we get that the optimal duration under uncertainty is obtained when the straight line intersects the upper curve. If there were no uncertainty the optimal condition would be represented by the intersection of the straight line with the lower curve.

The distinguishing element is therefore given by the coefficient $\frac{\xi}{\xi-1} > 1$. The consequence of introducing the factor $\frac{\xi}{\xi-1}$ is that the optimal duration of the licence increases. By increasing the duration of the licence the consumer reduces the risk of not achieving the expected benefits. This first consideration has a very important implication because, from (20), as τ increases, the threshold benefit that triggers the decision to purchase the licence also increases. This can be stated in the following proposition:

Proposition 3

(i) The consumer is willing to purchase a licence with longer duration only if there is an adequate expected benefit.

(ii) Furthermore, if uncertainty grows, the consumer will tend to postpone the decision to purchase, while awaiting an increase in the benefit due to a reduction in the number of rivals and/or an increase in the stock of resources, and then to purchase a licence of longer duration.

Now, expanding the discount factor $\delta(\tau)$ in a Taylor series and ignoring the terms of second orders, the condition (22) can be solved for the optimal duration τ as follows:

$$\tau = \frac{2}{\delta}(\frac{1-a}{a}) + \frac{2}{\delta a}(\frac{\xi}{\xi - 1} - 1) > 0 \tag{23}$$

By replacing the linear approximation of $\delta(\tau)$ in (20) we get:

$$B^* = \frac{\xi}{\xi - 1}(\frac{\delta}{2} + \frac{1}{\tau})\tau^a \tag{24}$$

The first term on the right-hand side of (23) represents the optimal duration under certainty, whereas the second term incorporates the option value of postponing the purchase[22]. From (23), it can easily be seen that the optimal duration decreases as a increases, i.e. $\frac{\partial \tau}{\partial a} < 0$. It is also confirmed that an increase in the uncertainty of future benefits lengthens the optimal duration, i.e. $\frac{\partial \tau}{\partial \sigma^2_B} > 0$, and that an increase in the discount rate leads to a reduction in the optimal duration, i.e. $\frac{\partial \tau}{\partial \delta} < 0$.

Although the negative effect of the discount rate, it deserves further attention. First of all, as it is given by $\delta = r + \lambda + h - \mu_B$, it responds negatively to an increase of the growth rate of the expected benefits μ_B increases, thus increasing the optimal duration. In fact, with positive forecasts on the growth of stocks and negative forecasts on the trend of the number of rivals, the consumer will tend to purchase longer-lasting licences. Second, an increase in the probability λ of a sudden change in the stock of resources, has an ambiguous effect on the optimal duration. In the first place the discount factor δ increases, with the effects already illustrated; in addition λ affects the option multiplier $\frac{\xi}{\xi - 1}$ according to the sign of ϕ which acts both on the expected rate of increase of B and on its variance. If ϕ is positive, then μ_B will increase to $\mu_B + \lambda\phi$ and the variance will increase from σ^2_B to $(\sigma^2_B + \lambda\phi^2)$ involving an increase of $\frac{\xi}{\xi - 1}$. The combined effect therefore depends on the prevalence of the "rate" effect, which reduces the duration of the licence, or of the "growing stock" effect which, conversely, increases it.

If ϕ is negative, although the variance will still increase, there will be a drop in μ_B. The overall effect is negative, with a reduction of $\frac{\xi}{\xi - 1}$ which, combined with the increase in δ, will reduce the optimal duration of the licence.

4. Conclusions

The paper has illustrated the consumer behaviour with respect to the purchase of a licence for recreational use of a natural resource. We developed a model starting from the assumption that the opportunity of purchasing a licence can be likened to a *call option* given that the uncertainty

[22] It should be noted that an optimal duration can exist also in the presence of a concave supply function ($a > 1$), which would never be possible if $\sigma^2_B = 0$.

concerning the future benefits and the irreversibility of the expenditure may make it expedient to wait before purchasing. Furthermore, the model includes the main factors conditioning the perceived utility of the recreational activity, i.e. the stock of resources, the number of rivals and operations designed to enrich/deplete stock levels, with particular attention to the uncertainty that characterises activities pertaining to biological resources.

The first result concerns the effect of uncertainty. It always acts as a deterrent to purchase of the licence and it lengthens the optimal duration. Generally speaking, therefore, the first tool available to an agency appointed to regulate the purchase of licences for recreational activities consists in the quality of the information to be circulated. From this it follows that operations designed to "programme" future conditions by reducing uncertainty can accelerate purchase of the licences.

A subsequent analysis enabled us to highlight the effect produced by an uncertain sharp change in the stock of resource on the optimal duration of the licence. In the first place it emerged that, as the discount rate increases, shorter durations are not always favoured as might be expected with respect to the criterion of the net present value. This is due to the fact that the various components of δ do not act univocally. As they are conditioned by the probability λ of sudden shocks ϕ in the stock and by a variance in benefits, they can produce effects contrary to those expected.

Optimal licence duration also depends on the elasticity of the supply function: as the elasticity decreases, optimal duration decreases. This highlights a second possibility of intervention for the agency: by modulating the supply function with respect to the duration, it can directly condition consumers' choices. Finally, irreversibility and uncertainty lengthen the optimal duration of the licence as longer durations reduce the effect of the variability of future benefits.

In conclusion, this study has highlighted that any policy for the management of recreational activities via licence must take account of the fact that uncertainty of the benefits and irreversibility of the expenditure generate an opportunity cost linked to the option value of the licence, causing the consumer to behave very differently from what we would expect if we considered only the present value of the benefits.

Appendix

Firstly, from Fubini's theorem and from (10), it is possible to bring the factor $E_T(.)$ within the integral, thus obtaining, by simple integration (Harrison, 1985, p. 44):

$$V(B_T) = E_T\left\{\int_T^{\infty} B_t e^{-(r+\lambda+h)(t-T)} dt \mid B_0 = B\right\} = \frac{B_T}{\delta} \tag{25}$$

where $\delta = r + \lambda + h - \mu_B$. We assume that $r - \mu_B > 0$, otherwise the maximisation would be indefinite as an increasingly bigger moment T would be chosen. It should be noted that $r - \mu_B > 0$ also implies $\delta > 0$. Yet, assuming $r - \mu_B > 0$ the option value can be estimated by maximising $\max E_0\left[(V(B_T) - K)e^{-rT} \mid B_0 = B\right]$, which permits reduction of the differential equation (6) in:

$$\frac{1}{2}\sigma^2{}_B B^2 F''(B) + \mu_B F'(B) - (r+\lambda)F(B) + \lambda F((1+\phi\alpha)B) = 0 \tag{26}$$

In addition, $F(B)$ must satisfy the boundary conditions (7), (8) and (9), purposely simplified.

$$\lim_{B\to 0} F(B) = 0 \tag{27}$$

$$F(B^*) = V(B^*) - K \tag{28}$$

$$F'(B^*) = V'(B^*) \tag{29}$$

At this point it is easy to verify that the solution of (26) can be expressed by a convex increasing function of B as:

$$F(B) = AB^{\xi} \quad \text{for } x \in (0, B^*] \tag{30}$$

The parameters A and ξ, together with the critical value B^* can be obtained from the previous boundary conditions. In particular, by replacing (30) in the equation (26), we obtain ξ as one of the roots of the following non-linear expression:

$$\Phi(\xi) \equiv \frac{1}{2}\sigma^2{}_B\xi(\xi-1) + \mu_B\xi - (r+\lambda) + \lambda(1+\phi\alpha)^{\xi} = 0 \tag{31}$$

From condition (27) it is known that ξ must be positive and from (29) that it must be greater than one. The solution ξ can be represented more intuitively by re-writing (31) as $\Phi(\xi) = \Phi_1(\xi) + \Phi_2(\xi)$, with $\Phi_1(\xi) \equiv \frac{1}{2}\sigma^2{}_B\xi(\xi-1) + \mu_B\xi - r$ and $\Phi_2(\xi) \equiv \lambda + \lambda(1+\phi\alpha)^{\xi}$ and drawing the two functions as shown in Figures A1 and A2, with $\phi > 0$ and $\phi < 0$ respectively.

Figure A1 - Solution for ξ. with $\phi > 0$

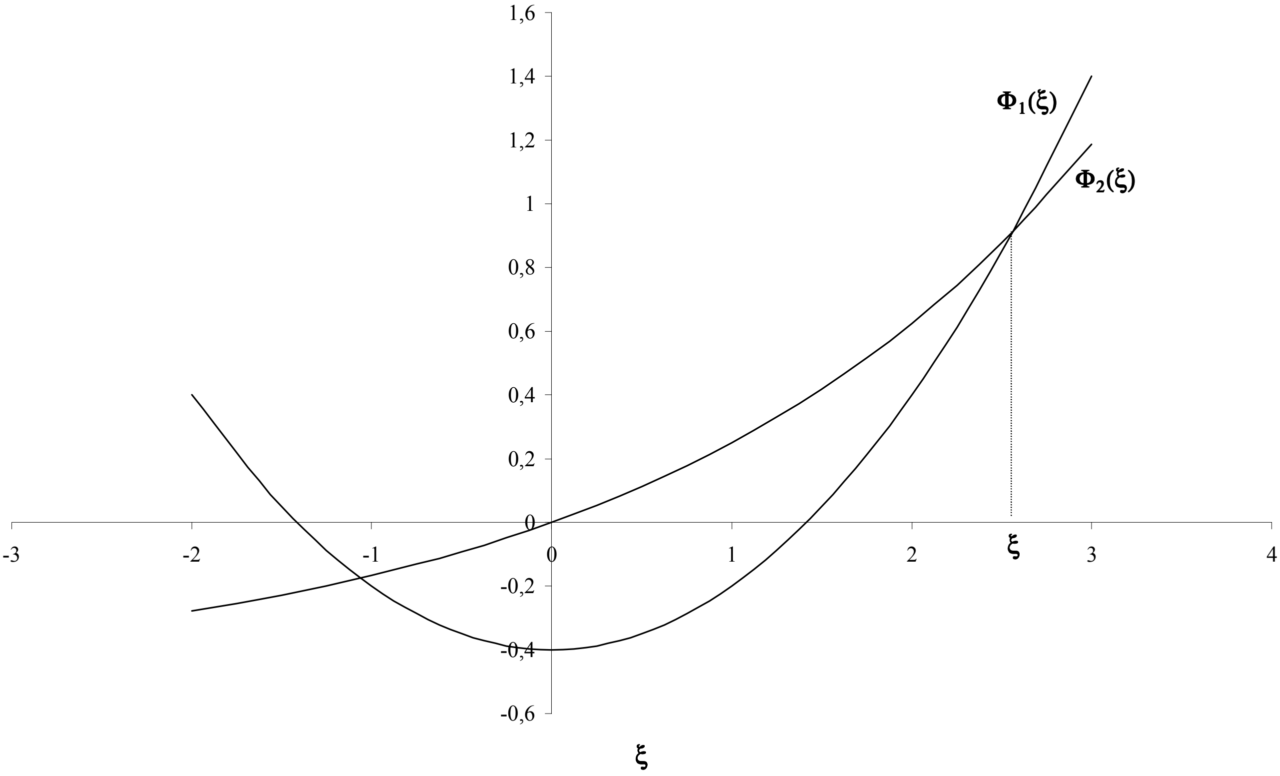
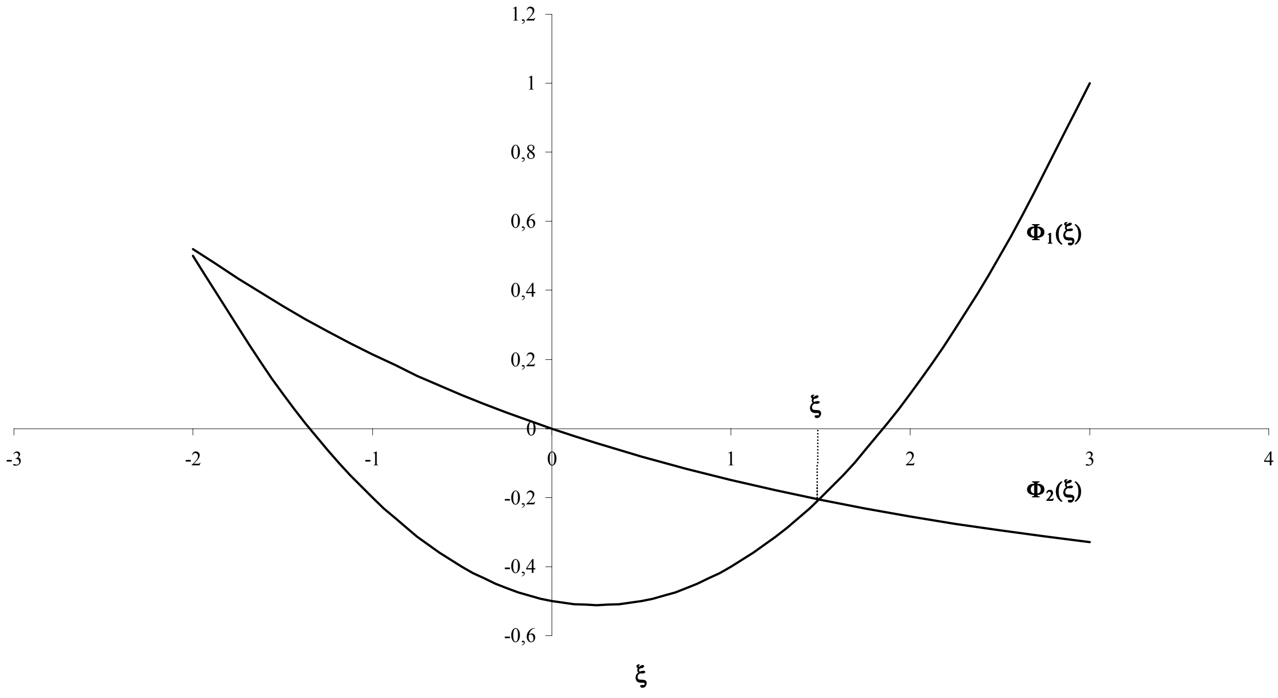


Figure A2 - Solution for ξ with $\phi < 0$ and $|\phi\alpha\lambda| < |\mu_B - r|$.



It should be noted that, if $\Phi_1(0) = -r < 0$, $\Phi_1(1) = \mu_B - r < 0$, $\Phi_2(0) = 0$ and $\Phi_2(1) = -\lambda\phi\alpha$ the solution of (31) is identified by the intersection of these two curves for $\xi > 1$[23].

In addition, since (30) represents the option value of purchasing the licence at optimal time, the constant A must be positive and the solution is valid in the interval of B within which it is preferable to keep the option alive $(0, B^*]$. By replacing (30) in (28) and (29), we get:

$$B^* = \frac{\xi}{\xi - 1}\delta K > 0 \qquad (32)$$

$$A^* = \frac{(\xi - 1)^{\xi - 1} K^{1-\xi}}{(\delta\xi)^{\xi}} > 0 \qquad (33)$$

as shown in the text and this completes the proof.

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