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Growth Maximising
Patent Lifetime

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1. Introduction

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Since Schumpeter (1942), the need to provide innovators with some form of market power (i.e. patent protection) in order to stimulate investment in R&D has been widely recognized. As patent protection helps innovators to benefit from their research efforts, it creates or increases the incentives to innovate. It also seeks to avoid duplication of research efforts and to promote technological progress.

The analysis of optimal patent lifetime is founded on two branches of literature. In the industrial organization literature, the analysis is built on the hypothesis that a society must balance the gains accruing from rapid technological progress against the welfare losses associated with the presence of temporary monopolies in the use of new technologies. Nordhaus (1969), Loury (1979) and recently Denicolò (1999), among others, all study the incentives and the distortions induced by a patent system within a static, partial equilibrium framework. Judd (1985) represents the first attempt to study the optimal patent lifetime through maximizing social welfare function in a dynamic, general equilibrium set-up. He builds an exogenous growth model where innovation, though endogenous, is not sustainable when there is no exogenous increase in the labor force.

Within the endogenous growth literature, innovation-based economic growth has become an important field of research due to the works of Segerstrom et al. (1990), Romer (1990), Grossman and Helpman (1991), Ahion and Howitt (1992) and Barro and Sala-i-Martin (1995). Considering innovations as to create a market for differentiated products by increasing the degree of product variety or quality, in these models no attenction is paid to the patent system as a government policy tool. Indeed Romer (1990) and Grossman and Helpman (1991, Chapter 3) assume that a successful innovator accrues an infinite patent protection. In a recent work, Michel and Nussen (1998) analyse patent lifetime in an endogenous growth model with horizontal product differentiation. However, their analysis is limited in the sense that they consider a deterministic innovative activity. Since Loury (1979), the limits of modelling innovative activity as a deterministic process, instead of a stochastic one, have been widely recognized.

Given all this, it is surprising that the impact of patent lifetime as a government policy tool on economic growth is almost entirely absent from theoretical work. Our motivation in the present paper is to fill this gap in the literature. In this sense, we focus on the optimal patent lifetime the government would set in order to maximize economic growth in an endogenous technological change model with vertical differentiation and stochastic R&D

activity. Within this framework we also study how this optimal patent lifetime would change in response to changes in the level of competition in R&D, the interest rate, the productivity parameter of research technology and the monopoly profit.

The paper is organized as follows. Section 2 presents the model. Section 3 introduces economic growth. Section 4 characterizes, through numerical analysis, the growth maximizing patent lifetime and performs comparative statics. Finally, section 5 concludes.

2. The Model

We consider an economy composed of three sectors. Final output sector produces an homogenous consumption good, intermediate sector produces capital goods whereas research and development sector produces innovations. Final output is obtained by combining labour and capital goods through a Cobb-Douglas technology and is used as an input in the intermediate goods and R&D sectors. Firms which operate in the research and development sector race to produce innovations in terms of quality improvements in the existing capital goods' product lines. The successful innovator wins a patent which is licensed to capital goods' producers.

2.1. The Final Output Sector

Final output (the numeraire) is produced in a competitive industry according to the following constant returns to scale production function:

$$(1) \qquad Y_t = L^{1-\alpha} \int_0^1 \left(\widetilde{X}_{jt}\right)^{\alpha} dj\,, \qquad \widetilde{X}_{jt} \equiv \sum_{m=0}^{M_j} \left(q^m X_{m_j t}\right), \qquad 0 < \alpha < 1\,, \qquad q > 1$$

where Y, L and q denote output, fixed supply labor and a quality-ladder index, respectively. The degree of horizontal differentiation is fixed over time, $j \in [0;1]$. $X_{m_j t}$ is the quantity used of the j^{th} type of intermediate good with quality level m_j. With M_j being the highest quality level available in sector *j*, $\widetilde{X}_{jt}$ is the quality-adjusted amount of the variety *j* employed at time *t*.

Supposing that for each variety *j*, only the top-quality generation $\left(q^{M_j}\right)$ is produced in equilibrium, the final output production function can be recast as follows:

(2) $$Y_t = L^{1-\alpha}\int_0^1 \left(q^{M_j} X_{M_j t}\right)^{\alpha} dj .$$

The representative firm producing the homogeneous final output maximizes its instantaneous profit with respect to X_{M_j}:

(3) $$\underset{X_{M_j}}{Max}\left[Y - wL - \int_0^1 \left(P_{M_j} X_{M_j}\right) dj\right].$$

In equation (3), the price of the final good has been normalized to unity and for ease of notation the subscript *t* has been omitted. The solution to this problem yields the set of intermediate inputs demand schedules:

(4) $$X_{M_j} = \left(\frac{\alpha q^{\alpha M_j}}{P_{M_j}}\right)^{\frac{1}{1-\alpha}} L .$$

The implied price-demand elasticity (μ) faced by each intermediate monopolist is given by $\dfrac{1}{1-\alpha}$.

2.2. The Intermediate Goods Sector

At any point in time, there is a continuum of firms indexed by $j \in [0;1]$ operating in this sector. Each firm produces a differentiated intermediate good that is used in the production of final output as an input. One unit of foregone consumption allows each firm to produce one unit of intermediate good irrespective of its variety. Accordingly, the marginal cost of each firm is equal to one. In order to obtain the optimal price of the highest available quality of each intermediate good, firms equate their marginal cost to their marginal revenue leading to $P_{M_j} = \frac{1}{\alpha}$. Thus, price is a fixed mark-up on the marginal cost of production. A firm that has already incurred the fixed-cost investment in a patent will obtain the following instantaneous profit[1]:

(5) $$\Pi_{M_j} = (P_{M_j} - 1)X_{M_j} = (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} q^{\frac{\alpha}{1-\alpha}M_j} L, \qquad \forall j \in [0;1].$$

2.3. The R&D Sector

In this section, we postulate an R&D technology where the date of innovation is uncertain. Following Loury (1979), Dasgupta and Stiglitz (1980) and Denicolò (1999), we assume a Poisson process for innovations which will bring discrete quality improvements for the intermediate goods. The timing of innovations stochastically depends on the R&D investment, namely the amount of final output devoted to R&D. A firm introducing an innovation will maintain a legal patent whose lifetime is set optimally by government in order to maximize economic growth.

Concerning the structure of the sector, at time *t* each firm determines the amount of resources devoted to R&D through maximizing its expected profit:

(6) $$E[\pi] = \lambda N_i E[V] - N_i - F,$$

where

[1] At any point in time, the only sunk cost faced by each firm operating in the intermediate goods sector is the initial expenditure on the patent. This assumption is harmless in the present context as the intermediate input

(7) $$E[V]=\Pi_{M_j}\lambda\left(\sum_i N_i\right)\int_0^T e^{-\left(r+\lambda\sum_i N_i\right)\tau}d\tau+\frac{\Pi_{M_j}}{r}\left(1-e^{-rT}\right)e^{-\left(r+\lambda T\sum_i N_i\right)}.$$

In equation (6), λN_i is the Poisson arrival rate of innovation for firm i devoting to R&D an amount of resources equal to N_i, with $\lambda > 0$ being a parameter reflecting the productivity of the research technology. Apart from foregone consumption, there is a fixed cost, *F*, that firms pay in order to engage in R&D. At any point in time, firm *i* innovates with probability λN_i and accrues the value of the patent, *V*.

A firm that obtains a certain quality improvement keeps the private property of this innovation only during the legal patent lifetime, *T*. However, if the next innovation arrives before *T* elapses, then the actual lifetime of the patent will be limited by the duration between these two consecutive innovations. As a Poisson process is assumed for innovations, the duration between two consecutive innovations follows an exponential distribution with an arrival rate of $\left(\lambda\sum_i N_i\right)$. Accordingly, the expected value of a patent given in equation (7) is composed of two terms that capture the probability of having the next innovation before or after *T* elapses, with *r* denoting the interest rate.

Concentrating on a symmetric equilibrium, we analyse the case where all firms devote the same amount of resources to R&D in order to improve the quality of the variety which would induce the highest level of monopoly profit. Thus, without loss of generality, assuming that $N_i = N$ for all *i* and max $\{\Pi_{M_j},\ j\in[0;1]\}=\Pi$, the first order condition for a maximum becomes:

(8) $$\frac{\Pi\lambda^2 NS}{r+\lambda NS}\left(1-e^{-(r+\lambda NS)T}\right)+\frac{\Pi\lambda\left(1-e^{-rT}\right)e^{-\lambda NST}}{r}+\frac{\Pi\lambda^2 Nr}{(r+\lambda NS)^2}\left(1-e^{-(r+\lambda NS)T}\right)+\frac{\Pi\lambda^3 N^2 TSe^{-(r+\lambda NS)T}}{r+\lambda NS}-$$
$$-\frac{\Pi\lambda^2 NT\left(1-e^{-rT}\right)e^{-\lambda NST}}{r}=1,$$

where *S* is the number of symmetric firms engaged in R&D. It is evident that S is bounded as it is a decreasing function of the fixed R&D cost, *F*.

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demand is stationary in equilibrium.

3. Economic Growth

Using (2) and (4), the level of aggregate output (Y) can be recast as:

(9) $$Y = L^{1-\alpha}\int_0^1 \left[q^{M_j}\left(\alpha^2 q^{\alpha M_j}\right)^{\frac{1}{1-\alpha}} L \right]^{\alpha} dj \Rightarrow Y = L\alpha^{\frac{2\alpha}{1-\alpha}}Q,$$

where $Q \equiv \int_0^1 q^{\frac{\alpha}{1-\alpha}M_j} dj$ is the aggregate quality index. Hence, the growth rate of output is equal to the growth rate of Q. For a variety j, the proportionate change in Q due to a successful innovation is $\left(q^{\frac{\alpha}{1-\alpha}} - 1\right)$ and, under the symmetric equilibrium hypothesis, the flow probability of a success due to a Poisson process is λNS. Accordingly, the expected growth rate of the economy can be stated as:

(10) $$g = E\left[\frac{\dot{Y}}{Y}\right] = \lambda NS\left(q^{\frac{\alpha}{1-\alpha}} - 1\right).$$

For having a positive economic growth, the amount of resources devoted to R&D by each firm should be positive as the number of firms engaged in R&D is bounded. Checking the first order condition given in (8) when $S \to 0$ and $S \to \infty$ leads to the conditions $\lambda\Pi > r$ and $\lambda\Pi = 1$, respectively, in order to have $N>0$. $\lambda\Pi$ can be interpreted as the expected rate of return from investing one unit of foregone consumption in R&D. Clearly, it should be bigger than the interest rate in order to give sufficient incentive to firms to innovate.

3.1. Growth-Maximizing Patent Lifetime

In a symmetric equilibrium, for given *S*, the government's problem to set the legal patent lifetime that maximizes economic growth reduces to maximizing *N* with respect to *T*. Implicit differentiation of (8) yields the following condition for a growth-maximizing patent lifetime:

$$(11) \quad \frac{\partial N}{\partial T} = 0 \Rightarrow$$

$$(1-\lambda NT)(1+\lambda NS)e^{-(r+\lambda NS)T} + \frac{\lambda N(\lambda NST - S - 1)(1-e^{-rT})e^{-\lambda NST}}{r} + \frac{\lambda Nre^{-(r+\lambda NS)T}}{r+\lambda NS} = 0$$

The solution to the government's problem can be obtained through solving (8) and (11) simultaneously. Unfortunately, it is not possible to conduct a complete algebraic analysis. Therefore we have proceeded with numerical simulations. These simulations show that, under the condition $r < \lambda\Pi < 1$, ***a finite growth-maximizing patent lifetime does exist and is unique***.

4. Numerical Analysis

The aim of our numerical analysis is to characterize the legal patent lifetime that maximizes economic growth and to see how it changes with respect to the level of competition in R&D, namely the number of firms engaged in research, the interest rate, the monopoly profit and the research productivity parameter.

Figure 1 can be used to illustrate the legal patent lifetime, *T**, that will be set by a government in order to maximize economic growth and the corresponding amount of resources devoted to R&D by each firm, *N**, under the parameter values reported in Table 1[2]:

[2] We have assigned the value of λ in the line of the empirical findings of Duguet and Kubla (1998). For sake of expositional simplicity, we have assigned arbitrary values to Π, being aware of the fact that, indeed, it depends on the parameters α, *q* and M_j. Finally, we have set an usual value for the interest rate (*r*).

Table 1: *Parameter Values and Corresponding T* and N**

$\lambda = 0.0005$	$\Pi = 500$	$r = 0.05$	$S = 100$	T*=10.354	N*=1.575

In Figure 1, according to the conditions given in (8) and (11), we have plotted two curves (with a continuous and dashed line, respectively) that give the amount of resources devoted to R&D by each firm as a function of legal patent lifetime. The intersection of the two gives the solution to the government's problem, *T**. As is clear from the Figure, for admissible parametrizations, a finite growth-maximizing patent lifetime exists and is unique.

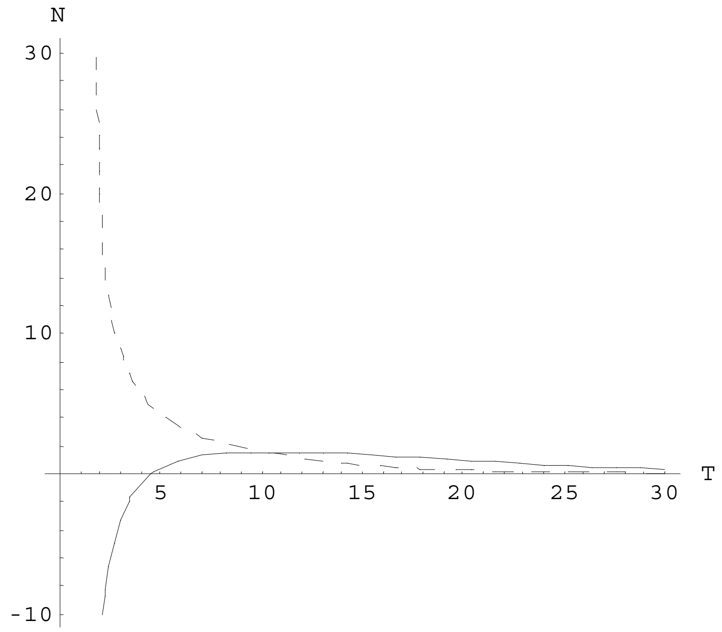

Figure 1: Characterizing the Growth-Maximizing Patent Lifetime

We observe that an increase in the number of firms, thus the level of competition in the R&D sector, leads to a decrease in the amount of resources devoted to research by each firm. However, it increases the total R&D investment, $N * S$.[3] This implies a decline in the expected duration that would elapse between two consecutive innovations, which in turn reduces the incentive of each firm to engage in R&D. To compensate for this, the

[3] Loury (1979) has also confirmed the same relationship between the number of firms, the total amount of R&D investment and the expected time to invention. However, he has not dealt with economic growth and thus growth-maximizing patent lifetime, which is the main concern of this paper.

governments sets a higher legal patent lifetime in order to maximize N, hence economic growth. In Table 2, for given parameter values (λ, Π and r), we have reported *T** and the corresponding *N** values with respect to different numbers of firms doing research.

Table 2: *The impact of the Level of Competition in R&D*

$\lambda = 0.0005$; $\Pi = 500$; $r = 0.05$	S=1	S=10	S=100	S=1000	$S = 10^6$
*T**	9.394	10.297	10.354	10.357	10.358
*N**	71.128	14.412	1.575	0.159	0.016

We also perform numerical exercises for analysing the impact of different *r*, λ and Π on *T**. The results are contained in the following Tables 3, 4 and 5.

Table 3: *The impact of the Interest Rate (r)*

$\lambda = 0.0005$; $\Pi = 500$; S=100	r=0.05	r=0.10	r=0.15	r=0.20
*T**	10.354	10.921	11.983	14.238
*N**	1.575	1.053	0.608	0.247

Table 4: *The impact of the Research Technology Productivity Parameter*

$r = 0.05$; $\Pi = 500$; S=100	$\lambda = 0.0002$	$\lambda = 0.0005$	$\lambda = 0.0010$	$\lambda = 0.0015$
*T**	29.175	10.354	4.669	2.709
*N**	0.761	1.575	2.320	3.455

Table 5: *The impact of Monopoly Profit*

$\lambda = 0.0005$; S=100; r=0.05	$\Pi = 200$	$\Pi = 500$	$\Pi = 1000$	$\Pi = 1500$

T^*	29.175	10.354	4.669	2.709
N^*	0.305	1.575	4.640	10.365

A higher interest rate implies a lower expected value for a patent as firms would discount future profits more. The incentive to innovate reduces due to a decrease in the expected net gain per unit of resources $(\lambda\Pi - r)$ devoted to R&D. Thus, the optimal legal patent lifetime maximizing economic growth increases. On the other hand, an increase in the expected net gain per unit of resources (due to an increase in either λ or Π) rises the incentive to innovate, so that a lower patent lifetime would suffice to reach the maximum level of economic growth.

5. Conclusions

In this paper we have characterized the patent lifetime that would be set by government (T^*) in order to maximize economic growth within an endogenous growth model with vertical differentiation and stochastic R&D activity. Through numerical simulation we have shown that T^* increases with the level of competition in the R&D sector and the interest rate, whereas it decreases with the monopoly profit and the productivity parameter of research technology. For future work, along with patent lifetime, the impact of other policy instruments (such as patent breadth) on economic growth would be analysed.

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