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1. Introduction

Human capital accumulation and R&D are certainly two of the most important engines of growth and, strangely enough, in the last few years only few attempts have been made in order to integrate them within a unified and homogeneous framework. Stokey (1988) and Young (1993), for example, build theoretical models in which the interaction between endogenous technological change and human capital formation is explicitely taken into account. However, in these papers skills accumulation happens through learning-by-doing and on-the-job-training in the production activity rather than a separate formation (or education) sector. Grossman and Helpman (1991, Ch. 5.2) also endogenize both human capital and technical change. In their set-up, though, education sector does require no skilled labour and incentives to accumulate human capital depend, in equilibrium, only on an exogeneously specified schooling technology and agents' time preference. Eicher (1996) addresses a very interesting puzzle one faces when analysing the technology-skills relationship. Indeed, from a theoretical point of view, although technical change may increase the demand for human capital, and consequently the wage premium of skilled over unskilled workers, a higher level of education should lead to a higher supply of skilled labour and then to a fall in the wage differential. In Eicher's 1996 paper, skilled labour is assumed to be an input in education, research, and in the *absorption* of innovations into production. As a consequence, the absorption of new technologies requires the withdrawal of human capital from research and education, so that, at the end, higher rates of technological change lower relative supply of skilled labour and increase the relative wage.

Unlike the literature cited so far, we suppose that in the economy there exist two different inputs (human capital and ideas) that can be accumulated over time in two separate and skilled labour intensive sectors. In the long-run equilibrium these two inputs are complements to each other. In addition, and unlike Eicher (1996), in our model final output production does not employ directly human capital, so that this input can be used exclusively to form new skilled workers and to produce horizontally differentiated intermediates and ideas. We analyse in detail under which conditions the inter-sectoral competition for the acquisition of human capital may hamper economic growth.

The paper is mainly motivated by the attempt to integrate different research lines of the *Endogenous Growth Theory* and places emphasis on three aspects of economic growth (scale effects, complementarities and multiple equilibria and the relationship between product market competition and growth) that, for the most part, have been analysed separately in the recent past.

In Jones' view (1995a; 1998) the scale effects prediction stemming from a vast majority of ideas-based growth models (e.g. Romer (1990) and Grossman and Helpman (1991)) seems to be a natural consequence of the fact that *"...new ideas are discovered by individuals so that the number of innovations is inherently tied to the number of persons engaged in R&D"* (C. Jones, 1995a, p.763). Indeed, in these models the existing stock of ideas is an input to the invention of new varieties of capital goods. We remove this hypothesis and show that in this way the equilibrium growth rate turns out to depend on the fraction (rather than the stock) of human capital being devoted to research

As for the second aspect we will emphasize in the paper, an extensive and well established research line (both theoretical and empirical) suggests that technology and skills are, in a sense, complements. The most immediate consequence of the existence of such complementarities is represented (as Redding (1996) has recently pointed out) by the rise of poverty traps and multiple steady states. In the model that follows, we find, indeed, two growth equilibria. However, unlike Redding (1996), who introduces pecuniary externalities in the accumulation of human and knowledge capital, we show that, in the presence of complementarities, multiple equilibria may well exist even when any kind of positive spillover in the accumulation of skilled work and R&D is explicitly ruled out.

Finally, the structure of our model allows a deep analysis of the possible connections between competition and growth[1]. The most important result we get within this particular research area is that, under specific conditions, even in an horizontal product differentiation framework, we may restore the well known Aghion and Howitt's (1992) result that the relationship between monopoly power and economic development is unambiguously positive.

The organisation of the paper is as follows: in the second section we present the basic model, compute the long-run equilibria (paragraph 2.1.) and discuss the results obtained in the light of the literature on the scale effects (paragraph 2.2.) and the existence of complementarities between endogenous technological progress and skills (paragraph 2.3.). In the third paragraph we study the relationship between product market competition and growth when both knowledge and human capital can be accumulated over time. We analyse separately the general case (paragraph 3.1.) and the one in which the two forms of capital are perfect complements (paragraph 3.2.).

[1] Recent works that study this link include Smulders and van de Klundert (1995) and Aghion and Howitt (1998, Chp. 7).

Finally, in section 4 we present a summary of the main conclusions of the model and suggestions for possible extensions. All the results we will get throughout the paper will be stated in the form of proposition.

2. The Model

Consider an economy with four different sectors. There exists an undifferentiated consumption good which is produced with unskilled labour and technologically advanced capital goods (these are available, at time t, in N_t different varieties). In order to produce such inputs, intermediate firms employ only human capital. Technical progress takes place as a continuous expansion in the set of available horizontally differentiated intermediates. Unlike the traditional *R&D-based growth models*, we assume that the total supply of human capital may grow over time and postulate the existence of a separate education sector in charge of forming new skilled workers. Additionally, we suppose that specialised workers can be put into use to produce (not only capital goods and ideas, but also) new human capital in a separate education sector. In the model the total quantity of unskilled labour (L) is exogeneously given and is used only by final output producers.

• ***The final output and human capital formation sectors.***

The homogeneous, undifferentiated consumer good is produced in a competitive industry according to the following constant returns to scale technology:

$$Y_t = L^{1-\alpha} \cdot \int_0^{N_t} \left(x_{jt}\right)^{\alpha} dj\,, \qquad 0 < \alpha < 1. \tag{1}$$

Therefore, output at time t (Y_t) is obtained combining (fixed supply) unskilled work, L, and N different varieties of technologically advanced goods, each of which is employed in the quantity x_j.

As the industry is competitive, in equilibrium each variety of intermediates receives its own marginal product (in terms of the only final good, the numeraire):

$$p_{jt} = \alpha \cdot (x_{jt})^{\alpha-1}, \qquad \forall j \in (0; N_t). \tag{2}$$

In (2), p_{jt} is the inverse demand function faced, at time t, by the generic *j-th* intermediate producer, after normalizing L to one. From (2), the direct demand function for the *j-th* type of intermediates is:

$$x_{jt} = \left(\frac{\alpha}{p_{jt}}\right)^{\frac{1}{1-\alpha}}, \qquad \forall j \in (0; N_t). \tag{3}$$

As already mentioned, the total amount of human capital (or specialised work, H), unlike L, is not fixed, but is allowed to grow over time according to the following law of motion:

$$\dot{H}_t = H_{Ht} \equiv s_{Ht} \cdot H_t, \qquad 0 \leq s_{Ht} \leq 1, \quad \forall t. \tag{4}$$

In other words, we assume the existence of a separate competitive education sector (think of universities, for example) which employs a constant returns to scale technology in order to form new human capital (graduate students). This technology is such that, using H_H units of input (teachers) at time t, dH new skilled people are formed in the time interval of length dt. We denote with s_{Ht} the fraction of the total stock of human capital (available at time t) which is devoted (at the same point in time) to the formation of new skilled workers. This variable will be endogenously determined in the model. Hence, unlike Lucas (1988), where each agent chooses optimally at each date how to allocate her time between production and schooling, we suppose that skilled people are *"produced"* by a separate sector whose activity is financed through a lump-sum tax (T_t) on the labor income of each member of the population (composed of skilled and unskilled workers).

• ***The Research Sector.***

Producing the generic *j-th* variety of capital goods entails the purchase of a specific blueprint (the *j-th* one) from the competitive research sector, characterised by the following technology:

$$\dot{N}_t = H_{Nt} \equiv s_{Nt} \cdot H_t, \qquad 0 \le s_{Nt} \le 1, \quad \forall t, \tag{5}$$

where s_{Nt} is the fraction of the total stock of human capital (existing at t) devoted (at the same point in time) to this sector. The share s_{Nt} is a technological parameter and is considered as exogeneous here.

The production function of new ideas displays two peculiar features that are worth pointing out. First of all, it is a deterministic function of H_N (notice, in addition, that human capital is employed in the sector with a marginal productivity that is constant and equal to one). Secondly, it does not depend on N_t (the stock of knowledge accumulated up to t). This is an alternative to the canonical assumption one may find in the literature. In fact, unlike the P. Romer's (1990) and Grossman and Helpman's (1991, Chap.3) models (where the total cost, in terms of skilled work, to be borne in order to invent a new variety of capital goods declines monotonically with the number of intermediates already existing), we explicitly assume that no (positive) externality effect is attached to N_t in discovering a new product variety.

As the research sector is competitive, new firms will enter it till when all profit opportunities will be completely exhausted. The static zero profit condition amounts, in this case, to set:

$$w_{Nt} = P_{Nt}, \tag{6}$$

with P_N being the price of one unit of research output (the generic *j-th* idea allowing to produce the *j-th* variety of technologically advanced good). It is equal to the discounted present value of the profit flow a local monopolist can potentially earn from t to infinity:

$$P_{Nt} = \int_t^{\infty} e^{-r(\tau - t)} \cdot \pi_{j\tau} d\tau, \qquad \tau > t. \tag{7}$$

The symbols used in (6) and (7) have the following meaning: w_N is the wage paid to one unit of human capital devoted to research; *r* denotes the interest rate (it will turn out to be constant in equilibrium) and π_j is the profit accruing to the *j-th* intermediate producer (once the sunk cost for the acquisition of the *j-th* infinitely-lived patent has already been borne).

• ***The production of technologically advanced goods.***

The capital goods industry is monopolistically competitive and in it each firm produces with the following one-for-one technology:

$$x_{jt} = h_{jt}, \qquad \forall j \in (0; N_t). \tag{8}$$

This production function is characterised by constant returns to scale in the only input employed (human capital) and, according to it, one unit of skilled work is able to produce (at each time) exactly one unit of whatever variety. Since the number of varieties invented up to t is equal to N_t, from (8) it follows that the total stock of human capital allocated (at time t) to this sector (H_{jt}) is:

$$\int_0^{N_t} x_{jt} dj = \int_0^{N_t} h_{jt} dj \equiv H_{jt} = s_{jt} \cdot H_t = (1 - s_{Nt} - s_{Ht}) \cdot H_t, \qquad 0 \le s_{jt} \le 1, \qquad \forall t. \tag{8a}$$

The generic *j-th* firm maximises (with respect to x_t) its own instantaneous profit, under the demand constraint (given by (2)). From the first order conditions:

$$w_t = \alpha^2 \cdot (x_{jt})^{\alpha-1}, \tag{9}$$

where w_t is the wage (paid at time t) to one unit of human capital employed in the sector.[2] Plugging (3) into (9) yields:

$$p_{jt} = \frac{1}{\alpha} \cdot w_t = p_t \quad , \qquad \forall j \in (0; N_t). \tag{9a}$$

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[2] As we will state more clearly later, this wage rate must coincide with the one accruing to one unit of human capital devoted both to the formation of new skilled workers and to research. This derives from the hypothesis that this input (skilled labor) is homogeneous and is equally productive irrespective of the sector in which it is used.

Therefore, each local monopolist sells its own output at a price equal to a constant mark-up over the marginal cost (w_t). From (3) above, it is possible to show that the *mark-up rate* $(1/\alpha)$ turns out to be (as we would expect) decreasing in the price elasticity of the demand faced by each capital goods firm (and equal to $1/1-\alpha$).

As the price (p) is equal for each variety, from (3) it follows that the output produced by a generic local monopolist (x) is the same for each j as well. Under this hypothesis (all the capital goods producers are perfectly symmetric), from (8a):

$$x_{jt} = \frac{H_{jt}}{N_t} = x_t, \qquad \forall j \in (0; N_t). \tag{8b}$$

Finally, the profit function is given by:

$$\pi_{jt} = (p_t - w_t) \cdot x_t = (1-\alpha) \cdot p_t \cdot x_t = \alpha \cdot (1-\alpha) \cdot \left(\frac{H_{jt}}{N_t} \right)^{\alpha} = \pi_t, \qquad \forall j \in (0; N_t). \tag{10}$$

Just as p and x, so too the profit is equal for every variety (such a conclusion derives from the symmetry with which intermediates are assumed to enter the consumer good technology).

The analysis of preferences closes the model.

• ***Preferences.***

Final output (Y) can be consumed only and there is full employment. An infinitely-lived representative agent supplies inelastically one unit of (skilled or unskilled) labour services per unit of time and gets a total income which is equal to the sum of labour income $(\overline{w}_t)$ and interest income $(r_t \cdot a_t)$. He/she maximises, under constraint, his/her own intertemporal utility function and solves the following dynamic problem:

$$\begin{cases} \underset{\{Y_t\}_{t=0}^{\infty}}{Max} U_0 \equiv \int_0^{\infty} e^{-\rho t} \log(Y_t) \\ s.t.: \\ \dot{a}_t = \overline{w}_t + r_t \cdot a_t - Y_t - T_t \\ \lim_{t \to +\infty} \lambda_t \cdot a_t = 0 \end{cases}$$

where Y denotes consumption (of the homogeneous final good); log (Y) is the agent's instantaneous utility function; ρ is her subjective discount rate; λ is the *co-state* variable and, finally, *a* and T are, respectively, the asset (measured in terms of Y) she holds and the lump-sum tax (on her labor income, $\overline{w}_t$) which is used to finance the education sector. The solution to this problem gives the *Euler Equation*:

$$\gamma_Y = \frac{\dot{Y}_t}{Y_t} = r_t - \rho \; . \tag{11}$$

2.1. Steady State Analysis

We concentrate on a long-run equilibrium where N_t and H_t grow at the same positive (and constant) rate. Therefore:

a) $\dfrac{\dot{H}_t}{H_t} = s_H$;

b) $\dfrac{\dot{N}_t}{N_t} = s_N \cdot \dfrac{H_t}{N_t} = s_H$.

From (b), given the way we define the equilibrium, s_N turns out to be constant. Thus, on the balanced growth path, a constant share of human capital is allocated both to the formation sector (s_H) and to research (s_N). Consequently, in equilibrium, s_j will be constant as well (since the condition

$s_j + s_N + s_H = 1$ must always be checked). Moreover, from (b), it is possible to conclude that in the symmetric case:

c) $\frac{H_{jt}}{N_t} \equiv s_j \cdot \frac{H_t}{N_t} = s_j \cdot \frac{s_H}{s_N} = Constant.$

Such a result allows us to infer that the profit accruing in equilibrium to each local monopolist (π_j) does not vary over time (see (10)) and, then, under the hypothesis that *r* is constant, equation (7) can be recast as:

$$P_{Nt} = \int_t^{\infty} e^{-r(\tau - t)} \cdot \pi_j d\tau = \frac{1}{r} \cdot \pi_j = \frac{1}{r} \cdot \pi = P_N . \tag{7a}$$

Equation (7a) suggests that in the steady state, P_N (the market value of each new idea or patent), and w_N (the wage rate earned by one unit of human capital devoted to research), are constant (see (6)).
In order to determine the optimal allocation of human capital among the sectors using this input (the intermediate, education and research sectors, respectively), we impose the following three conditions:

d) $w_N = w_t$;

e) $w_t = w_H$;

f) $s_H = 1 - s_N - s_j, \quad \forall t$.

From (d) and (e), jointly considered, we also obtain, implicitly, the condition $w_H = w_N$.
Basically, since human capital is a perfectly homogeneous input, we impose that it be paid the same wage rate across all sectors (this is stated by the *no-arbitrage conditions* (d) and (e)) and that the sum of the shares of skilled work (allocated to each market) be equal to one for each *t* (condition (f)).

From (d):[3]

$$\frac{H_{jt}}{N_t}=\left(\frac{\alpha}{1-\alpha}\right)\cdot r\,. \tag{12}$$

As the left-hand side in (12) is constant, the interest rate (r) will be constant as well.

To find w_H, let us express P_{Ht} (the price, at time t, of one unit of human capital devoted to formation) as the present discounted value of all the future earnings (wages) accruing to it:[4]

$$P_{Ht}=\int_t^{\infty} e^{-r(\tau-t)} w_{H\tau}\, d\tau=\frac{1}{r} w_H=P_H \Rightarrow rP_H=w_H=w_N=P_N=\frac{1}{r}\alpha(1-\alpha)\left(\frac{H_{jt}}{N_t}\right)^{\alpha}. \tag{13}$$

Given w_H, it is possible to use condition (e) and get:

$$\frac{1}{r}\cdot\alpha\cdot(1-\alpha)\cdot\left(\frac{H_{jt}}{N_t}\right)^{\alpha}=\alpha^2\cdot\left(\frac{H_{jt}}{N_t}\right)^{\alpha-1} \quad\Rightarrow\quad \frac{H_{jt}}{N_t}=\frac{\alpha}{1-\alpha}r\,. \tag{14}$$

Notice that (14) is perfectly consistent with (12). Finally, from (f):[5]

$$s_H=1-s_N-s_j=(1-s_N)-\left(\frac{\alpha}{1-\alpha}\cdot\frac{s_N}{s_H}\cdot r\right)\Rightarrow s_H=\frac{1}{2}\left[(1-s_N)\pm\sqrt{\frac{(1-\alpha)(1-s_N)^2-4\alpha s_N r}{(1-\alpha)}}\right] \tag{15}$$

To compute the output growth rate of this economy, let us rewrite (1) as:

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[3] In order to obtain this result, we use the fact that $x_{jt}=x_t=H_{jt}/N_t$ and $w_N=(1/r)\cdot\pi_j$.

[4] In (13) the result follows from the observation that r and w_H $(=w_N)$ are constant and $w_N=P_N$.

[5] In (15), s_j is derived combining (c) and (12).

$$Y_t = L^{1-\alpha} \cdot N_t \cdot x_t^{\alpha} = L^{1-\alpha} \cdot N_t \cdot \left(\frac{H_{jt}}{N_t} \right)^{\alpha} = \Psi N_t \,, \qquad \Psi \equiv L^{1-\alpha} \left(\frac{H_{jt}}{N_t} \right)^{\alpha}$$

Taking logs of both sides of this expression and totally differentiating with respect to time, we get:

$$\frac{\dot{Y}_t}{Y_t} \equiv \gamma_Y = \frac{\dot{N}_t}{N_t} = s_H \,, \tag{16}$$

since Ψ is constant in the long-run equilibrium and $\frac{\dot{H}_t}{H_t} = \frac{\dot{N}_t}{N_t} = s_H$.

Thus, growth is entirely determined in this economy by the innovation rate, being equal (in steady state) to the human capital accumulation rate (s_H). To find out s_H, we plug in (15) the value of r deriving from the resolution of the *consumer side* of the model (see (11)) and obtain:

$$\frac{\dot{Y}_t}{Y_t} = \gamma_Y = s_H = \frac{[\beta(1-s_N)-1] \pm \sqrt{\beta^2 s_N^2 - 2[(\beta-1)(\beta+2\rho)]s_N + (\beta-1)^2}}{2(\beta-1)} \,, \beta \equiv \frac{1}{\alpha} > 1 \,. \tag{17}$$

In brief, along a balanced growth path, the model just outlined exhibits the following properties:

- s_H, s_N and s_j are constant (each sector using human capital receives, at each point in time, a constant share of such an input);
- the interest rate (r), prices (P_N e P_H) and wages (w_t, w_N and w_H) are constant as well;[6]
- each local intermediate monopolist produces a constant quantity of output (x) and sells it at a constant price (p), accruing a constant profit (π);

[6] $P_Y = 1$ (Y is the numeraire) and $w_t = w_H = w_N = P_N = r \cdot P_H = \alpha^{1+\alpha} \cdot (1-\alpha)^{1-\alpha} \cdot r^{\alpha-1}$.

- finally, from (9) and (8b), $\lim_{\alpha \to 0} w_t = \lim_{\alpha \to 0} \alpha^2 \cdot (H_{jt} / N_t)^{\alpha-1} = 0$. As $w_t = w_N = w_H = w$, this means that the more uncompetitive the intermediate sector is (α goes to zero), the more the equilibrium wage (paid to human capital throughout the all economy) falls below its marginal productivity, equal to one in each industry (we will come back later on the interpretation of α as a measure of the level of competition present in this economy). At the limit, the wage rate approaches zero. In this case, indeed, the return from human capital investment $(= r \cdot P_H)$ declines and so does the incentive to accumulate this resource over time, too.[7]

In the next section, we analyse in greater detail the main implications stemming from (17).

2.2. Discussion

From (17), it is possible to show that:

- the two roots (s_{H1} and s_{H2}) are real and distinct for:
  1. $s_N < \frac{\beta - 1}{\beta^2}\left[(\beta + 2\rho) - 2\sqrt{\rho(\beta + \rho)}\right]$,
  2. $s_N > \frac{\beta - 1}{\beta^2}\left[(\beta + 2\rho) + 2\sqrt{\rho(\beta + \rho)}\right]$;
- when $0 < s_N < \frac{\beta - 1}{\beta} < 1$, both growth rates are positive.

In addition, since $\frac{\beta - 1}{\beta^2}\left[(\beta + 2\rho) + 2\sqrt{\rho(\beta + \rho)}\right]$ is greater than $\frac{\beta - 1}{\beta}$, we can easily neglect those values of s_N falling in the interval:

$$s_N > \frac{\beta - 1}{\beta^2}\left[(\beta + 2\rho) + 2\sqrt{\rho(\beta + \rho)}\right]$$

[7] This observation will be useful when we will analyse the relationship between markup rate and aggregate growth.

(whose elements are such that s_{H1} and s_{H2} are always negative) and concentrate only on the range:

$$0 < s_N < \frac{\beta-1}{\beta^2}\left[(\beta+2\rho)-2\sqrt{\rho(\beta+\rho)}\right] < \frac{\beta-1}{\beta}.$$

For example, if we set β =1.5 and ρ =0.03, then γ_Y behaves, as a function of s_N, as follows (the function $\gamma_Y(s_N)$ displays qualitatively the same behaviour even for different values of β):

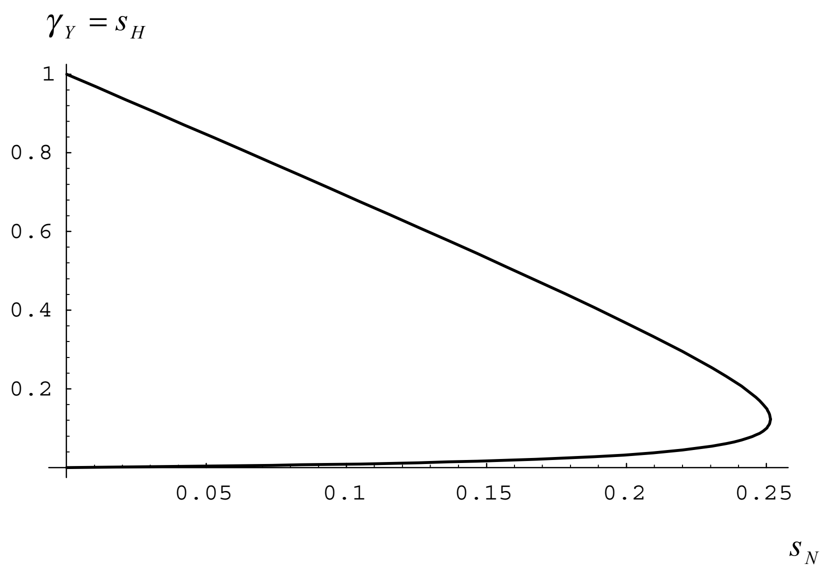

Figure 1

The Relationship between Aggregate Growth and the Share of Resources devoted to R&D

With these parameters ($\beta = 1.5$ and $\rho = 0.03$), when $s_N = 0.25144762$, there exists a unique aggregate growth rate equal to: $s_H = \gamma_Y \cong 0.12$. The figure reported above must be interpreted in terms of a simple comparative statics exercise. Indeed, in this model the share of human capital devoted at each *t* to R&D is an exogenous technological parameter. In addition we know that in the long-run equilibrium it is constant. Figure 1 tells us what happens to the aggregate growth rate of this economy (γ_Y) when s_N is allowed to assume different values within a given interval in which $\gamma_Y \geq 0$.

Before commenting on Figure 1, only few observations about (17):

- **Endogenous Growth**: the aggregate growth rate (γ_Y) of this economy is endogenous in the sense that it depends exclusively on the structural parameters of the model. In particular, for given s_N, it is a function of ρ (the subjective discount rate) and β (the mark-up rate charged over the marginal cost by each intermediate local monopolist). In addition, γ_Y coincides with the accumulation rate of human capital (s_H). In this sense, the model supports the main conclusion reached by that branch of the Endogenous Growth literature pioneered by Uzawa (1965) and Lucas (1988);[8]

- **Scale Effects**: our model does not display any scale effect, since γ_Y depends neither on the absolute dimension of the economy (its total human capital stock), nor on the population growth rate. On the contrary, as already mentioned, the development rate comes out to be a function of β, ρ and s_N (the fraction of skilled work devoted to research) only. The *scale effect prediction* is present in almost all the innovation driven growth models (included those by P. Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992)). In these approaches, indeed, an increase in the total size of population determines, ceteris paribus, a proportional increase in the number of researchers and, consequently, in the real per-capita growth rate. As Jones himself (1995a) points out, such a result is not corroborated by the data and, therefore, it should be rejected on empirical grounds (Jones, 1995b). Recently, many works (Kortum, 1997; Aghion and Howitt, 1998, Chap.12; Dinopoulos and Thompson, 1998; Howitt, 1999; Peretto, 1998; Peretto and Smulders, 1998; Segerstrom, 1998; Young, 1998) attempted to find a key to this apparent riddle. But, unfortunately, despite the title of some of their papers, the result remains that a scale effect continues to show itself either in the real per-capita income level, or in the per-capita growth rate;
- **Multiple Equilibria**: the model also features multiple (two) steady states. A possible economic interpretation of this result has to do with the existence of complementarities between human and technological capital. In the next paragraph we investigate more deeply the mechanics of these links.

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[8] Benhabib and Spiegel (1994), Islam (1995) and Pritchett (1996) all suggest that, unlike Lucas (1988), international differences in per-capita growth rates depend exclusively on the differences in the respective human capital stocks each country is endowed with. Klenow (1998), also argues that the models based on the endogenous accumulation of embodied technical change explain better (in comparison with the human-capital-based ones) the inter-industry differences in productivity growth. He finds, indeed, that the TFP growth is fastest in those industries being more capital goods intensive and that the sectors where technologically advanced goods prices decline rapidly exhibit above average TFP growth rates.

2.3. Complementarities, Multiple Equilibria and Growth

The idea that there may exist a relationship of complementarity among human, physical and technological capital dates back to the sixties, and in particular to the seminal works by Nelson and Phelps (1966) and Griliches (1969), among the others (see J. Mincer, 1995). However, in recent years the strong investment in information technologies and the dramatic change in the wage structure (both in Europe and the U.S.) have jointly contributed to renew the interest in the possible relationships between human and technological capital. On the theoretical side, such an interest is justified by the fact that, as Redding (1996) has pointed out, the existence of complementarities between skilled work and R&D represents an important element in explaining both multiple equilibria and underdevelopment traps in the growth process of a country. Arnold (1998), following Uzawa (1965) and Lucas (1988), introduces human capital accumulation in a standard model of endogenous innovation (Grossman and Helpman, 1991, Chap.3). However, unlike Redding (1996) and the model we have just outlined, his analysis is mainly focused on showing a particular way through which another prediction of the *ideas-based-growth models*[9] can be falsified. In a word, he is not interested in studying how the (rational and profit-seeking) decisions to accumulate both forms of capital may interact with each other, giving potentially rise to a multiplicity of equilibria. Recently, Blackburn et al. (2000) have extended Arnold's model (1998) in the direction of a more complete micro-foundation of the R&D process.

On the empirical side, instead, many contributions claim the relevance of the skill-technology connections even at the sectoral level (Goldin and Katz, 1998), whereas De la Fuente and da Rocha (1996) also find evidence of strong complementarities between human capital stock and investment in R&D for the OECD countries. Finally, the *skilled-biased technological progress hypothesis* has been analysed by many scholars in the last few years[10] and empirically tested by J. Mincer (1989).

Turning back to our model, in what follows we first show in which sense human and technological capital are complements. According to Matsuyama (1995, p.702): *"...Complementarities are said to exist*

[9] Namely, that the equilibrium growth rate is very much sensitive to policy changes.

[10] Bartel and Lichtenberg (1987); Berman, Bound and Griliches (1994); Doms, Dunne and Troske (1997); Autor, Katz and Krueger (1997); Bell (1996); Machin, Ryan and Van Reenen (1996); Berndt, Morrison and Rosenblum (1992); Bartel and Sicherman (1995); Krueger (1993), among the others.

when two phenomena (or two actions, or two activities) reinforce each other. For example, if expansion of industry A leads to expansion of industry B, which in turn leads to further expansion of A, then the two industries are complementary to each other. ...Such complementarity introduces some circularity in the economic system, which has profound implications for the stability of the system. If a change in a certain activity is initiated by an exogenous shock, this leads to a similar change in complementary activities and starts a cumulative process of mutual interaction in which the change in one activity is continuously supported by the reaction of the others in a circular manner".

Applying such a definition, we will say that technological (N) and human (H) capital are complements if an exogenous increase in N (caused, for example, by a proportional increase in s_N) implies an increase in H as well, and this effect, in turn, reinforces the initial expansion of N. In other words:

Remark

In the economy under analysis, N_t and H_t are complements in the long run in the sense that $\partial H_t / \partial N_t$ and $\partial N_t / \partial H_t$ are both positive when $t \to \infty$.

Proof: Suppose that both s_N and s_H are strictly between zero and one, so that on a balanced growth path both forms of capital are accumulated over time. In the long run, N_t and H_t grow at the same constant rate, s_H. From (4) and (5):

[A] $H_t = H_0 \cdot e^{s_H \cdot t}$, $H_0 > 0$, given;

[B] $N_t = \frac{s_N}{s_H} \cdot H_t = \frac{s_N}{s_H} \cdot H_0 \cdot e^{s_H \cdot t}$.

In turn, from [A] and [B], we get the following partial derivatives:

[A'] $\frac{\partial H_t}{\partial s_H} = H_0 \cdot e^{s_H \cdot t} \cdot t$;

[B'] $\frac{\partial N_t}{\partial s_H} = H_0 \cdot \frac{s_N}{s_H} \cdot e^{s_H \cdot t} \cdot \left(t - \frac{1}{s_H} \right)$.

Henceforth:

[C] $$\frac{\partial H_t}{\partial s_H}\cdot\frac{\partial s_H}{\partial N_t}=\frac{\partial H_t}{\partial N_t}=\frac{t}{\frac{s_N}{s_H}\cdot\left(t-\frac{1}{s_H}\right)}>0, \qquad \forall t>\frac{1}{s_H}>0.$$

This means that, for t sufficiently large, but finite, an increase in N_t produces a variation of the same sign in H_t. In addition, from [B] and [C] respectively:

[D] $$\frac{\partial N_t}{\partial H_t}=\frac{s_N}{s_H},$$

[C'] $$\lim_{t\to+\infty}\frac{\partial H_t}{\partial N_t}=\frac{s_H}{s_N}.$$

In other words, in the very long run ($t\to\infty$), an increase in N_t equal to ∂N_t (determined by an exogenous rise in s_N) produces an increase in H_t equal to $\partial H_t=\partial N_t\cdot s_H/s_N$. This variation in H_t reproduces, in turn, the same initial increase in N_t $(\partial N_t=\partial H_t\cdot s_N/s_H)$ and the process of reciprocal and mutual interaction between N_t and H_t remains unchanged over time. ■

We can state now the first two results of the model.

Proposition 1

In the very long run, if s_N and s_H are both close to zero, then the greater the increase in s_N, the higher the aggregate economic growth rate $(\gamma_Y=s_H)$.

Proof: Consider the lower branch of Figure 1 and notice that, along this branch, when s_N tends to zero, s_H tends to zero as well. It can be easily checked that, for *t* sufficiently larger than $1/s_H$, an exogenous increase in s_N unambiguously raises the stock of knowledge capital $\left(N_t=(s_N/s_H)e^{s_H t}\right)$. In the very long run, the increase in the human capital stock (H_t), caused by the positive variation of N_t, is equal to the ratio s_H/s_N and from Figure 1, it is clear that, such a ratio, for given β and ρ, is increasing in s_N along the lower branch. This explains why in this case γ_Y grows (even at increasing rates) with respect to s_N. ■

The economic intuition behind this result is straightforward: in the long run, when s_N increases both the number of intermediate firms and the total demand for human capital increase. Such an increase in the demand for skilled workers stimulates the supply of new human capital (s_H increases) which, in turn, boosts growth, the real engine of growth.

Proposition 2

In the very long run, if too many resources have already been devoted to the formation sector (s_H tends to one), then the greater s_N the lower the aggregate growth rate of the economy (γ_Y).

Proof: Consider now the upper branch of Figure 1. Along this branch, when s_N goes to zero, s_H tends to one. In this case, unlike the previous one, an increase in s_N is possible if and only if s_H decreases (since the sum of s_N, s_H and s_j cannot be greater than one). For *t* sufficiently larger than $1/s_H$, an increase in s_N may well reduce now the number of existing capital goods varieties (N_t).[11] When *t* becomes infinitely large, the reduction in H_t is much more sustained than that in N_t, since:

- $\lim_{t\to+\infty} \frac{\partial H_t}{\partial N_t} = \frac{s_H}{s_N}$;
- $\frac{s_H}{s_N}$ decreases quickly along the upper branch of Figure 1.

This explains why, in this circumstance, the greater the value of s_N, the lower the growth rate (γ_Y).■

[11] Indeed, it is possible to show that for a given (large enough) *t*, $N_t(s_N)$ takes an inverse-U shape. At first, an increase in s_N makes N_t rise, but after a threshold value a further increase in s_N unambiguously decreases the number of available varieties. For high values of s_N, N_t tends to zero. For example, when $t = [(1/s_H) + 50]$, then N_t reaches its maximum value for $s_N \cong 0.0066$. After this value, N_t declines monotonically towards zero. The higher *t*, the lower the value of s_N for which N_t reaches the maximum.

Again, the intuition behind this result is very simple: when almost all the available human capital is employed in formation (s_H is close to one), a rise in s_N cannot be supported anymore (as in the previous situation) by a simultaneous increase in s_H (which, on the contrary, goes down).

In the long run, as the number of specialised workers diminishes and the two forms of capital are complements, N_t (that depends both on s_N and s_H) goes down as well. The reduction in the human capital demand coming from the intermediate sector represents a further disincentive for the accumulation of new skilled workers and this hampers growth.

Putting together Propositions 1 and 2, we can conclude that the presence of s_N in the steady-state growth rate captures a *human capital inter-sectoral competition effect*. Indeed, on the lower branch of Figure 1, both s_N and s_H may increase at the same time, so that the allocation of higher shares of skilled workers to formation and R&D sectors is feasible. In this situation, to a rise in s_N it corresponds in the long run a monotonic increase in N_t, which stimulates, in turn, human capital accumulation and economic development. On the other hand, along the upper branch, the *inter-sectoral competition for human capital acquisition* is such that a contemporaneous increase in both s_N and s_H is not sustainable. Therefore, an exogenous rise in the share of resources devoted to research may cause, in the steady state, a sharp reduction in the number of available capital goods and, in turn, in the stock of human capital and in the rate of growth.

In this sense, a policy implication stemming from the model has particularly to do with the less developed countries. They are generally characterised by a negligible fraction of skilled workers devoted both to human capital formation and research (low s_H and s_N). For these economies, our analysis seems to suggest that, in order to boost the equilibrium growth rate it would be beneficial to invest a greater amount of resources into R&D. The reason is simple. An increase in s_N (through, say, R&D subsidies) rises, *ceteris paribus*, the stock of technological capital available at time *t* (N_t), which, in turn, due to the complementarities between the two forms of capital, increases the number of skilled workers (H_t). This means that in the next period more human capital can be used to produce new *ideas* and new educated people, so fostering economic growth.

In the next section, we turn to the analysis of the relationship between mark-up and growth in a context where the supply of skilled workers is allowed to grow over time.

3. The Markup – Growth nexus

3.1. The General Case

In order to study the relationship between competition and growth within the present framework, first of all we should be particularly clear on what we mean by (imperfect) competition and where the mark-up measure (we are going to use in the remainder of the paper) comes from. Indeed, as already pointed out by Aghion and Howitt (1997, p.284), the natural measure of the degree of competition is, in this class of models, the parameter α. In fact, the higher α, the higher the elasticity of substitution between two generic intermediate inputs (equal to $1/(1-\alpha)$). This means that they become more and more alike when α grows and, accordingly, the price elasticity of the derived demand curve faced by a local monopolist (equal, again, to $1/(1-\alpha)$) tends to be infinitely large when α tends to one. In a word, the *"toughness"* of competition in the intermediate sector is strictly (and positively) depending on the level of α. Conversely, the inverse of α ($1/\alpha$), can be viewed as a proxy for how uncompetitive the sector is.[12]

In what follows $\beta \equiv 1/\alpha$ will represent the key variable in measuring the level of mark-up and (imperfect) competition in the intermediate goods production. From this premise it derives that, in the present context, studying the relationship between β and γ_Y amounts to analyse how (from (17)) $\gamma(\beta)$ behaves with given ρ and s_N.

In this respect, we obtain the following figure:[13]

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[12] van de Klundert and Smulders (1997) have compared, in an endogenous growth model, the "toughness" of Bertrand versus Cournot competition explicitely taking into account the perceived price-demand elasticity. They conclude that in an oligopolistic set-up: *"...price competition à la Bertrand is tougher than quantity competition à la Cournot because the former results…in higher elasticity e and lower profit margins set by firms"* (p.108). Sutton (1991) also points out that, for a given number of incumbents in the market, the lower the markup coefficient (in our case $1/\alpha$), the stronger the competition.

[13] In Figure 2, we concentrate exclusively on those values of β for which growth is positive. Additionally, we set ρ=0.03 and s_N=0.008. Therefore, we assume that only the 0.8% of the total stock of human capital is devoted in equilibrium to research. This number is consistent with the data recently proposed by P. Romer (2000, Figure 4) and C. Jones (1999, pag.17), who indeed notes: *"...In the United States and throughout the G-5 countries, less than one percent of the labor force is engaged in research according to the definition employed by the National Science Foundation"*. Even using a value less than 0.008 for s_N, the qualitative behaviour of γ doesn't change.

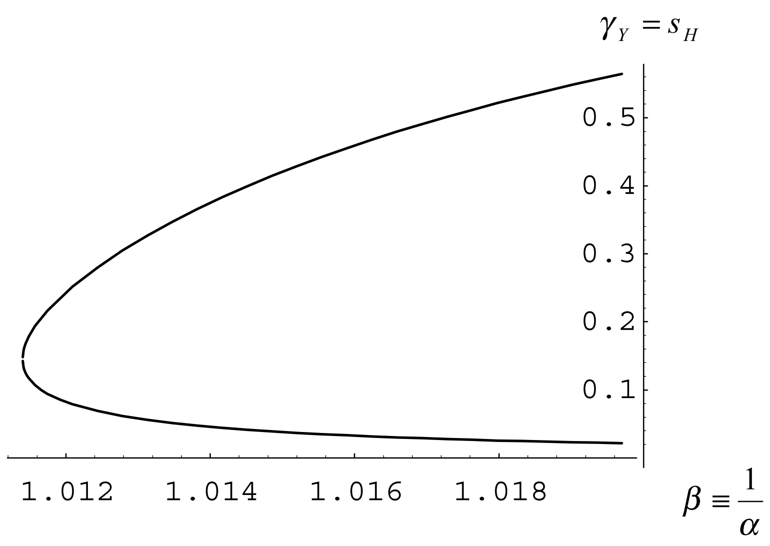

Figure 2

The Markup–Growth Nexus: the General Case

To explain economically the shape of $\gamma_Y(\beta)$, consider the value assumed by H_{jt} (from (14)):

$$H_{jt} = \left(\frac{\alpha}{1-\alpha}\right) \cdot rN_t .$$

With *r* positive and N_t greater than zero (this amounts to say that s_N and s_H are both strictly positive), we conclude that H_{jt} tends to zero when $\alpha \to 0$. Formally:

$$\lim_{\alpha \to 0} H_{jt} = \lim_{\alpha \to 0} (1 - s_N - s_H) \cdot H_t = 0 . \tag{18}$$

There are two ways for this condition to be checked. First of all, it can be:

- $H_t > 0$ and $\lim_{\alpha \to 0} s_j = 0$, which implies that $s_H \to (1 - s_N)$.

In this case, an increase in the monopoly power enjoyed by the local monopolists reduces the total output of the intermediate sector $(H_{jt} = N_t \cdot x_t)$ and, then, the human capital demand coming from it (s_j goes to zero). Since the constraint: $s_j + s_N + s_H = 1$ must always be met, s_H (and the aggregate growth rate) goes up. This is exactly what happens along the upper branch of Figure 2. Alternatively, (18) is checked when:

- $(1 - s_N - s_H)$>0 and $\lim_{\alpha \to 0} H_t = 0$.

For positive *t* and H_0, H_t decreases over time when s_H is negative. However, under the constraint of a non-negative s_H (which, in our model represents the aggregate economic growth rate), if s_H tends to zero, then $s_j \to (1 - s_N)$, with s_N given. Unlike the previous case, in the present one an infinitely large monopoly power acts in the sense of reducing the return on human capital $\left(= r \cdot P_H = \alpha^{1+\alpha} \cdot (1-\alpha)^{1-\alpha} \cdot r^{\alpha-1}\right)$ and discouraging the decision to accumulate this input further. This is exactly what happens along the lower branch of Figure 2.
We can state, then, the following:

Proposition 3
In a context where technological progress takes the shape of a continuous horizontal expansion in the set of available capital goods, even when human capital supply is allowed to grow over time, the market power–growth nexus continues to be ambiguous (see also Bucci, 1998; 1999).

Indeed, a rise in the mark-up rate $(1/\alpha)$ may determine either an increase in the share of resources (human capital) devoted to growth-generating activities (formation), or an increase in the fraction of skilled workers allocated to non-growth-generating ones (capital goods production).

3.2. The mark-up/growth nexus when human and technological capital are perfect complements

One way to eliminate the above-mentioned ambiguity between product market competition and growth is to consider H_t and N_t as perfect complements.

Proposition 4

Suppose that $H_t / N_t = 1$ *for each t (this implies that* $s_N = s_H$ *on a balanced growth path). Under this assumption, human and technological capital are perfect complements and an increase in the mark-up rate unambiguously raises aggregate growth.*

Proof: In (17), set $s_N = s_H$ and solve for s_H. Two solutions are easily obtained:

g) $s_{H1} = 0$;

h) $s_{H2} = \dfrac{(\beta - 1)(\beta - 1 - \rho)}{2\beta^2 - 3\beta + 1}$.

We take into account exclusively solution (h), the one with potentially positive growth. In addition, as s_H represents at the same time the growth rate and the share of human capital devoted to the formation of new skilled workers, the following must be true:

i) $0 < s_{H2} = \dfrac{(\beta - 1)(\beta - 1 - \rho)}{2\beta^2 - 3\beta + 1} \leq 1$.

This condition is met when $\beta > (1 + \rho)$. Hence, in the case of perfect complementarity between N_t and H_t, and with ρ =0.03, $\gamma_Y(\beta)$ behaves in the following way:

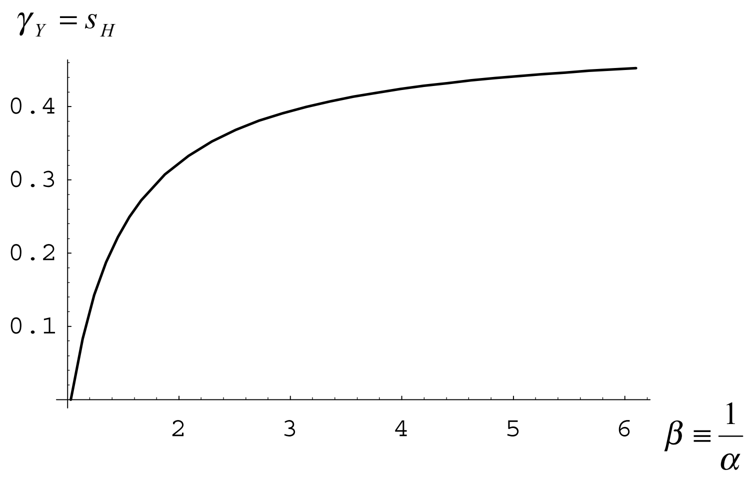

Figure 3

The Markup-Growth Nexus: the Case of Perfect Complementarity

Notice that when $\beta > (1+\rho)$, it turns out that: $s_H^{'}(\beta)$>0 and $s_H^{''}(\beta)$<0. In other words, γ_Y always increases (though at decreasing rates) with respect to β. When β becomes infinitely large, then γ_Y tends to 0.5. Therefore, we are able to restore (even in a deterministic, horizontal product differentiation framework) one of the main results one may find in the growth literature with *quality improvements* (namely, Aghion and Howitt, 1992; 1998). ■

The economic intuition behind this result is that when the mark-up rate goes up, the human capital demand coming from the intermediate producers drops so that a greater fraction of skilled labour can be put into use in the research and formation activities (complement to each other). When $\beta \to \infty$, then $s_j \to 0$ and s_N and s_H tend both to 0.5.

4. Concluding Remarks

In this paper, we have analysed the long-run equilibrium of an endogenous growth model in which the only two inputs that can be accumulated over time (ideas and human capital) are complements to each other. Many insights arise from this analysis. First of all, we find that, as in Lucas (1988), the real engine of growth is represented by human capital accumulation and, contrary to Redding (1996), we show that even without externalities (in the accumulation of human and technological capital), multiple steady-states may emerge. In particular, we find two steady state growth rates. These two equilibria (whose dynamic properties are not studied in the present paper) are such that neither of them exhibits scale effects. In our model, indeed, growth depends exclusively on the subjective discount rate, the mark-up rate (charged by each local monopolist producer of intermediates) and the fraction of skilled workers which is allocated to research. Finally, under the perfect complementarity assumption between human and technological capital, we conclude that product market competition is unambiguously bad for growth. This happens simply because an increase in the mark-up rate enjoyed by the local monopolist producers has the effect of attracting human capital away from the non-growth-generating activities (capital goods production), towards the growth-generating ones (skilled workers and ideas accumulation).

We think that the model could be extended to include more explicitly *regional convergence* considerations, as long as one thinks of the two branches of Figure 1 as belonging to two different regions: an advanced region that massively invests in human capital and grows fast, and a less developed one that invests less in formation and, because of this, always grows less than its counterpart, for each s_N. According to our approach, only for a sufficiently large value of s_N, the two regions are supposed to converge to each other in terms of growth rates (it is evident that, *ceteris paribus*, the two economies will continue to diverge in terms of per capita income levels). However, throughout the paper we have been considering the fraction of human capital devoted to research as an exogenous technological parameter. A fully endogenisation of this variable is left to future research.

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