Fondazione Eni Enrico Mattei

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NOTA DI LAVORO 58.2000

Corso Magenta, 63, 20123 Milano, tel. +39/02/52036934 – fax +39/02/52036946
E-mail: letter@feem.it
C.F. 97080600154

Conspicuous consumption, social status and clubs

Fernando Jaramillo[¤], Hubert Kempf[j] and Fabien Moizeau[k]

[¤]Universidad de los Andes, Bogota,

and EUREQua, Université Paris-1 Panthéon-Sorbonne.

[j]EUREQua, Université Paris-1 Panthéon-Sorbonne.

[k]EUREQua, Université Paris-1 Panthéon-Sorbonne.

Abstract

The paper develops a signalling theory of conspicuous consumption where the drive toward spending on an otherwise unuseful good comes from the desire to enter clubs and benefit from the provision of club good financed by members of a club and from a social status effect. Individual incomes are unobserved and admission to a club is based on the inference of an individual's capacity to contribute to the public good. By entering in a club, individual also gain a certain social status. This inference in turn is based on the signal emitted by spending on a conspicuous good. Because of the joint incentives of club good and social status, people may be induced to over-spend in the conspicuous good. We characterize both the pooling equilibria and the separating equilibria of the signalling game played by individuals. We then ask whether taxation can be Pareto-improving and which tax scheme would be chosen by the median voter in this society.

1 Introduction

It has been known and observed for a long time that individuals care about their social status, and in market economies tend to over-consume so as to "impress" their neighbours (see Bagwell and Bernheim, 1996, Frank, 1985, Veblen, 1910). The existing literature on conspicuous consumption as a signal for status-oriented individuals aims at explaining this behaviour. Ireland [1994, 1998] in particular has shown how individuals may be tempted to over-consume so as to reach a higher social status than the one they would get in case of perfect information and how this can be doomed to failure when people are able to correctly infer from the signals the true types of people. This leads to a suboptimal consumption in a positional good without intrinsic utility. Then the issue arises of whether some taxation scheme could redress this over-consumption and lead to a Pareto improvement.

However there are other means to be socialized than just gaining some social status and social recognition. Agents tends to form groups so as to perform some activities or obtain some bene¯ts. Indeed individuals get involved into "clubs" which by de¯nition are social groups. These groupings in turn tend to a®ect the social status of individuals. Examples that membership to a club is related to social status are countless. Think about the British clubs of the gentry in Victorian and post-Victorian Britain, or the various charities organized in New-York high society, or the ability to join certain "good" school districts in the US. We may think that individuals tend to spend some of their income on positional goods or conspicuous goods so as to be admitted in a club from which they expect some bene¯ts: driving a Porsche or living in luxurious oversized mansions makes it easier for you to be accepted in the Jockey Club or to the New York Yacht Club.

The aim of the present paper is to explore this line of reasoning and to mix these two ways of socialization: membership to a club producing some valuable public good and the concern for social status.[1] We do so by using the same signalling theory already used to justify conspicuous consumption. The simple argument is the following. Suppose that agents's incomes are not observed but that only individual spending on a conspicuous good is observable by any member of a given society. Suppose that clubs provide a "useful" public good to their members, but that this good is ¯nanced through member fees or individual contributions by members. Then it is in the interest of any member of a club to admit the

[1] The case without social status has been studied by Jaramillo and Moizeau [1999].

richest possible individuals. Similarly it is in the interest of every individual to be accepted in the richest club so as to bene¯t from a high level of public good. This mechanism is reinforced when individual care about their social status because being accepted in a "rich" club increases one's social standing. Hence, using a simple rule for club formation, namely that two individuals making the same signal are to be accepted in the same club, we provide a rational link between conspicuous consumption and the segmentation of society into clubs.

We shall explore the various signalling equilibria which can arise in this setting. The equilibria lead to di®erent patterns of club formation, and henceforth to di®erent types of social segmentation. Namely we shall characterize the pooling and the separating equilibria of the game, where conspicuous consumption is used as a signal from which the income of an agent is inferred. A pooling equilibrium corresponds to a unique club where there is no possibility to distinguish between people whereas a separating equilibrium leads to the formation of multiple clubs because individual incomes are correctly perceived and thus contributing capacities in a club perfectly anticipated.

Then we can address the issue of taxation. If individuals tend to over-spend in conspicuous consumption, isn't it the case that taxing or even forbidding conspicuous consumption is Pareto-improving? Studying this issue using the "best" separating equilibrium, the answer we get is ambiguous: even though positive taxation is Pareto-improving, an in¯nite taxation is not necessarily Pareto-optimal. This depends on the relative concern for social status in individual preferences and the distribution of income. If people care enough about their social standing and if the income gap between "rich" and "poor" is large enough, then some social segmentation through clubs is appreciated by the rich even at the expense of waste on conspicuous consumption whereas in¯nite taxation tends to amalgamate people in the same society-wide club.

Section 2 presents the model of economy-society we shall study. Pooling and separating equilibria are studied in section 3, whereas the taxation issues is tackled with in section 4. Section 5 concludes.

2 The model

2.1 The economy

We shall consider a society S formed of N individuals, indexed by z; living over two periods. Each individual earns the same income in each period. There are two classes of income. Individuals may be "rich", earning a level of income y^r; or "poor", earning a level of income y^p. There are n^r rich individuals and n^p poor ones (with $n^p = N - n^r$). Both N and n^r are assumed to be sufficiently large. Individuals are ranked so that, denoting y^z the level of income of the z-th agent, we can write:

$$y^z = \begin{cases} y^p & \text{if} \quad z \leq n^p \\ y^r & \text{otherwise} \end{cases} \tag{1}$$

There may be J clubs segmenting society into groups. We denote C_j the j-th club, with $j = 1, ...J$. J will be determined endogenously. Given the formation rule for clubs to be developed below, it will be apparent that:

$$C_j \bigcap C_{j'} = \emptyset \qquad \forall j, j' \qquad \bigcup_{j=1} C_j = S$$

The two periods differ in the following way: in the second period, individuals may be admitted in clubs, providing a club good. Each member of a club voluntarily contributes to the provision of the club good, once she is accepted in a given club.[2] There are also two other goods: a purely private consumption good, which can be consumed in both periods, and a conspicuous consumption good which can be purchased in the first period only. These goods are non-durable. The income is readily spent in either consumption goods; similarly, the level of club good is equal to the sum of voluntary provisions by members of the club. Namely the budget constraints for individual z belonging to club j for the two periods are:

$$y^z = c_j^z + f_j^z \tag{2}$$

$$y^z = x_j^z + g_j^z \tag{3}$$

where c_j^z denotes the consumption of the purely private good in the first period, f_j^z denotes the level of consumption of the conspicuous consumption good in the first period, x_j^z the

[2] By so doing, we follow the tradition initiated by Bergstrom, Blume and Varian (1986). However, the use of this assumption in relation to the formation of club theory has rarely been done.

level of consumption of the purely private good in the second period and finally g_j^z the voluntary amount given by z for the provision of the club good, when z belongs to club C_j. The individual contribution of z, g_j^z, will turn out to be a function of z's income y^z: $g_j^z = \gamma(y^z)$. The provision of club good in club C_j, which we denote by G_j is equal to:

$$G_j = \sum_{z \in C_j} g_j^z = \sum_{z \in C_j} \gamma(y^z) \tag{4}$$

2.2 Clubs

Let us now turn to the issue of club formation. We make the two following assumptions.

1. Clubs are formed at the beginning of the second period, or "between periods", according to the amount of conspicuous consumption spent by individuals. More precisely, the following rule is imposed on society:

$$\text{if} \qquad f_j^z = f_{j'}^{z'}, \qquad \text{then} \qquad j = j', \forall z, z' \tag{5}$$

2. The level of income y^z is a private information, known only by z. Similarly, we assume that only the level of conspicuous consumption made by any individual is observed by all agents.

According to (5), if two individuals make the same signal, they must belong to the same club. The admission to a club depends only on the amount of income spent on conspicuous consumption. Remark that the functioning of a club is a two-step procedure: the first step corresponds to the formation of clubs, and the second step corresponds to the voluntary financing by members of the club good, from which they benefit. Once accepted in a club, an individual has no obligation whatsoever. There is no prior sum given for the financing of the club good.

2.3 Beliefs and signals

Individuals will form beliefs about the level of income of any other individual. This in turn will affect the "social status" of an individual, and as such, will affect both her financing of the club good and her displaying of conspicuous consumption. We shall denote

- $\mu^z\left(f_j^{z'}\right)$ is the probability (belief) given by z to the claim " z' is rich" when $f_j^{z'}$ is observed. This probability is revised according to Bayes's rule.
- $\varphi^z\left(f_j^{z'}\right)$: the subjective expectation formed by individual z on z''s income and derived from the observation of the signal emitted by z', $f_j^{z'}$:

$$\varphi^z\left(f_j^{z'}\right) = \mu^z\left(f_j^{z'}\right) \cdot y^r + \left(1 - \mu^z\left(f_j^{z'}\right)\right) \cdot y^p \tag{6}$$

- $\bar{\varphi}\left(f_j^{z'}\right)$: the average expectation formed by "society" of individual on z''s income derived from the observation of the signal emitted by z', $f_j^{z'}$, defined as follows:

$$\bar{\varphi}\left(f_j^{z'}\right) = \frac{1}{N-1} \sum_{z \neq z} \varphi^z\left(f_j^{z'}\right) \tag{7}$$

It is assumed that N is large so that each individual has a negligible weight and does not take into account his own type when forming beliefs.

2.4 Individual utilities and social status

Let us now turn to the individual utility function. All individuals are characterized by the same utility function. We shall follow the framework developed by Ireland [1994, 1998]. Following Ireland, "social status" to be defined later matters for any individual and is an argument of the utility function. Consequently, this function W has two components: a "private component" U and a "social status component" S.[3] We shall assume that it is a simple linear combination of these two components:

$$W = (1 - \alpha) U + \alpha S \tag{8}$$

In turn, the arguments of the "private component" are the levels of pure private consumptions and the level of club good provision enjoyed by a given individual: conspicuous consumption generates no direct utility whatsoever. The private component enjoyed by individual z when she belongs to club C_j has a simple form:

$$U\left(c_j^z, x_j^z, G_j\right) = \ln\left(c_j^z\right) + \ln\left(x_j^z\right) + \beta G_j \tag{9}$$

[3] This represents a generalization of a model with social status studied by Jaramillo and Moizeau[1999].

β represents the relative desirability of public consumtion compared to private consumption. As in Ireland, we simply define the social status enjoyed by z as the direct inference of her private utility, based on the signal she emits, that is her level of conspicuous consumption:

$$S\left(\bar{\psi}\left(f_j^z\right);\left\{f_j^z\right\}\right) = \ln\left(\bar{\psi}\left(f_j^z\right) - f_j^z\right) + \ln\left(\bar{\psi}\left(f_j^z\right) - \gamma\left(\bar{\psi}\left(f_j^z\right)\right)\right) + \beta\left(\sum_{z' \in C_j} \gamma\left(\bar{\psi}\left(f_j^z\right)\right)\right) \tag{10}$$

where we use the average expectation formed by society about z's income and the $\gamma(\cdot)$ function. We now introduce two additional assumptions on income distribution which help us to solve the signalling game in a non-degenerate manner. We assume:

$$y^r > \frac{1}{=} > y^p \tag{11}$$

$$\bar{y} \equiv \frac{n^r y^r + n^p y^p}{N} > \frac{1}{=} \tag{12}$$

2.5 Individual optimization

An individual has two decisions to make: the first one is about her level of conspicuous consumption f_j^z, as it will simultaneously determine her first period private consumption and her access to a club, which in turn will determine her voluntary contribution to the financing of the club good. The second one is taken in the second period, once she is accepted, in a club and is about her voluntary contribution. Remember the two-step procedure about clubs. It means that the decision about g_j^z is logically dependent on the decision about f_j^z: Therefore the individual behaviour of z consists in solving the following programme:

$$\max_{f_j^z; g_j^z} W = (1 - \alpha)\, U + \alpha S$$

and that it is solved by bakward induction. On the whole, the decision on f_j^z will condition the level of voluntary contribution and is used both as a way to signal her possibility to contribute to the club good in a given club and her social status.

The choice over g_j^z is made in the second period. Remark that the actual value of the voluntary contribution of z has no impact on her social status, since it depends on the inference of her income by society ("Is she rich? Is she poor?") which is based on f_j^z and secure by the end of first period. The second-period problem, once society has been segmented into clubs and z has been accepted in a club C_j; given the set of conspicuous consumption levels $\left\{f_j^z\right\}_{z\in S}$; therefore consists in solving the following problem:

$$\max_{g_j^z} \ln\left(x_j^z\right) + \beta G_j$$

$$\begin{aligned} s.t.: \quad y^z &= x_j^z + g_j^z \\ G_j &= \sum_{z'\neq z;\, z'\in C_j} g_j^{z'} + g_j^z \\ & \quad g_j^{z'} \text{ given} \\ g_j^z &\geq 0 \end{aligned}$$

The first-order condition implies that:

$$g_j^z = \begin{cases} y^z - \mu & \text{if } \quad y^z = y^r \\ 0 & \text{if not} \end{cases} \tag{13}$$

Two remarks are worth to be made about (13). First, individual contributions never depend on contributions made by other individuals neither on the characteristics of the club to which z belongs, nor more generally on the partition of society into clubs. It solely depends on the individual income. Second, poor people never contribute and rich people always contribute to the financing of the club good, in whatever club to which they belong. This is why, ceteris paribus, the more numerous rich people are in a club, the better it is for any member of this club. Consequently, any individual always wants to join a club with rich people only, and her club to be joined by another rich individual. In particular this generates a desire to spend on conspicuous consumption even though no direct utility is provided by such consumption. If you are poor, and spend as much as a rich, you will belong to the same club as he does and therefore benefit from his contribution to the club good; moreover, your social status will be the "rich" one, which is preferrable to the "poor" one. If you are rich, you prefer your clubmates to be rich people and not poor ones, so as to get a larger provision of the club good. Moreover without ambiguity, your social status is the "rich" one.

We are now able to rewrite the maximisation programme to be solved in the first period by individual z: Given (4) and (6), the expected utility function for individual z can be rewritten as:

$$
\begin{aligned}
E_z W = {} & (1-\alpha)\left[\ln\left(y^z - f_j^z\right) + \ln\left(y^z - \gamma\left(y^z\right)\right) + \beta \sum_{z \in C_j} \gamma\left(\phi^z\left(f_j^z\right)\right)\right] \\
& + \alpha\left[\ln\left(\bar{\phi}\left(f_j^z\right) - f_j^z\right) + \ln\left(\bar{\phi}\left(f_j^z\right) - \gamma\left(\bar{\phi}\left(f_j^z\right)\right)\right) + \beta \sum_{z' \in C_j} \gamma\left(\bar{\phi}\left(f_j^{z'}\right)\right)\right]
\end{aligned}
\tag{14}
$$

Each individual has to choose her level of conspicuous consumption, given the choice of all other individuals, only as a signal about her income and capacity to contribute once she belongs to a club. Of course, this is true for all individuals in this society and clearly gives rise to signalling games within this setting. We shall denote σ^z the strategy played by z; with $\sigma^z = \left(g^z; f_j^z\right)$ depending on the partition of society and the characteristics of the club to which z belongs.

3 Complete information

As a benchmark, it is useful to develop the case of an economy with perfect information. That is, it is assumed that each agents's income is perfect knowledge in this economy. In that case, it is obvious that it unnecessary for every agent to care about her social status and there is no conspicuous consumption, since signalling is purposeless. Because there is no congestion effect linked to the public good and applying the club formation rule, a unique club is formed which includes all individuals. Only the rich contribute and this is to the advantage of the poor. But by being joined by the poor in a single society-wide club, the rich do not lose any social status. They keep the "rich" status. Altogether, this is clearly the Pareto-optimal situation.

4 Incomplete information

In this section, we assume imperfect information and we shall characterize the pooling and separating equilibria of this signalling game. In each game, the incentive-compatibility constraints are built under the following assumption about defection

Assumption a defection from a club by one individual implies that she does not join any other club or coalition, remains isolated. As a consequence, there is no need for her to make any conspicuous consumption and she is therefore immediately given the social status of a poor.

4.1 Pooling equilibria

Given our notations, a pooling equilibrium is characterized by $J = 1$: The entire society belongs to the same club. This comes from the fact that everybody makes the same signal in a pooling equilibrium and therefore, applying our formation rule, is put in the same club as everybody else. In a pooling equilibrium, it is impossible to differentiate agents and to distinguish between poor and rich people.

Definition 1 A pooling equilibrium is an equilibrium where every individual has the same level of conspicuous consumption:

$$f^z = f^{z'} = f^{pool} \qquad \forall z, z' \in S$$

Let us now prove that there exist pooling equilibria, that is non-zero levels of conspicuous consumption that all agents can afford. We shall prove the following

Proposition 2 There exist pooling equilibria, characterized by $f^{pool} \in \left[0; \hat{f}^p\right]$; such that the set of strategies $\{\sigma^z\}_{z \in S}$ played in a pooling equilibrium and the set of beliefs $\left\{\mu^z\left(f^{pool}\right)\right\}_{z \in S}$ held by individuals define a perfect Bayesian equilibrium and are defined by:

1. For any $z > n^p$; $\sigma^z = \left(y^r - \frac{1}{\,\cdot\,}; f^{pool}\right)$; $\mu^z\left(f^{pool}\right) = \frac{n^r - 1}{N - 1}$
2. For any $z \leq n^p$; $\sigma^z = \left(0; f^{pool}\right)$; $\mu^z\left(f^{pool}\right) = \frac{n^r}{N - 1}$

Proof. See appendix A. ■

Several remarks are worth to be made about pooling equilibria.

First, it is shown in the proof that the threshold level of conspicuous consumption $\hat{f}^p$ of the poor individual, derived from her incentive-compatible constraint is increasing in α the more important is the social status component in the preferences of individuals, the more poor people are willing to ostentatiously consume. This is easily understandable. A higher α means that the poor are caring more about their social status and are more willing to be considered as rich eventhough they indeed are poor; hence they are willing to make more sacri¯ce in terms of \directly useful" private consumption in the ¯rst period. Once accepted in the society-large club, they will nevertheless not contribute.

Second, in a pooling equilibrium, there is maximum free-riding from the poor. This comes from the fact noted above that the poor never contribute and the rich always contribute, and from the assumption that there is no congestion e®ect or negative e®ect linked to the club size. Henceforth, in a pooling equilibrium, the maximum possible level of public good is produced, hence generating the maximum bene¯t for the poor, at stricly no cost for them, except the \wasted" level of conspicuous consumption. They bene¯t from no social discrimination (they bene¯t from the \average" social status / income) and they enjoy the largest possible level of public good. The rich on the other hand su®er from not being distinguished from the poor and be granted an "average" social status / income, and not being recognized for what they are, rich people, even though they too bene¯t from the largest possible level of public good. On other words, a pooling equilibrium is always best for the poor, but not necessarily so for the rich.

Third, private consumptions of the rich are larger in both periods than private consumptions of the poor. In the ¯rst period private consumption for an individual is the di®erence between her income and f^{pool} and in the second period, the rich spend on the private consumption good a sum equal to 1=¯ whereas poor spend on this good all their income, as we assumed that 1=¯ is larger than y^p: Of course, all rich people enjoy the same level of utility, and all poor people enjoy the same level of utility, but a rich's utility di®ers from a poor's one. These two levels di®er because of the di®erences in private consumptions. The rich's welfare in a pooling equilibrium is higher than the welfare of a poor: their social status components are equal because of the pooling characteristics of the equilibrium but the personal component is higher for a rich than for a poor.

4.2 Separating equilibria

A separating equilibrium is such that every individual is recognized her exact level of income, inferred from the signal she emits.

De¯nition 3 A separating equilibrium is an equilibrium where every poor individual has the same level of conspicuous consumption, every rich individual has the same level of conspicuous consumption but the two levels di®er.

In our simple framework, it is then straightforward by applying the club formation rule, that in a separating equilibrium two clubs will form in society. One will encompass all rich people, whereas the other will be formed by all poor people. Hence the club-partition of society S is made of two clubs: C_1 and C_2 :

$$C_1 = \{1; ....; n^p\} \qquad C_2 = \{n^p + 1; ...; N\}$$

In a separating equilibrium, a rich is distinguished from a poor because their levels of conspicuous consumption differ:

$$f_1^z = f^p \quad \text{for } z \leq n^p; \qquad f_2^z = f^r \neq f^p \quad \text{for } z > n^p$$

We can then remark that in a separating equilibrium, because the inference made by everybody about everybody is perfect (the signal emitted by an individual perfectly reveals her type), beliefs are correct and therefore :

$$\mu^z\left(f^{z'}\right) = 1 \qquad \forall z; \text{ if } z' > n^p$$
$$\mu^z\left(f^{z'}\right) = 0 \qquad \forall z; \text{ if } z' \leq n^p$$

It immediately follows that the value of α has no influence on the utility levels obtained by individuals in a separating equilibrium. We can then prove the following

Proposition 4 There exist separating equilibria, characterized by $f^{sep} \in \left[\hat{f}^p; \hat{f}^r\right]$; where $\hat{f}^p$ and $\hat{f}^r$ are the threshold levels of conspicuous consumption of poor people and rich people respectively, such that the set of strategies $\{\hat{a}^z\}_{z \in S}$ played in a separating equilibrium and the set of beliefs $\{\hat{\mu}^z(f^{sep})\}_{z \in S}$ held by individuals define a perfect Bayesian equilibrium and are defined by:

1. For any $z > n^p$; $\hat{a}^z = \left(y^r - \frac{1}{\alpha}; f^{sep}\right)$; $\hat{\mu}^z(f^{sep}) = 1$
2. For any $z \leq n^p$; $\hat{a}^z = (0; 0)$; $\hat{\mu}^z(0) = 0$

Proof. See Appendix B. ■

A separating equilibrium is characterized by perfect discrimination between poor and rich, because of the partition of society in two clubs, one for the rich, the second for the poor. We can make several additional remarks concerning this partition, or alternatively a separating equilibrium.

First, again we find that the threshold levels of conspicuous consumption $\hat{f}^p$ (for the poor individual) and $\hat{f}^r$ (for the rich individual), derived from her incentive-compatible constraints, are increasing in α : the more important is the social status component in the preferences of individuals, the more individuals, poor and rich alike, are willing to ostentatiously consume. This is easily understandable. A higher α means that the poor are caring more about their social status and are more willing to be considered as rich eventhough they indeed are poor; hence they are willing to make more sacrifice in terms of "directly useful" private consumption in the first period. Once accepted in the society-large club, they will nevertheless not contribute. For the same reason, the higher α is the more important it is for rich people to be distinguished from poor people and be given the higher social status of "rich" compared to the much lower "poor" status. Hence they too are willing to spend more on the conspicuous good, even though it has no intrinsic value. It is proven in the appendix that $\hat{f}^p < \hat{f}^r$:

Second, because of the perfect discrimination between the poor and the rich, there is no free-riding of the poor at the expense of the rich. Rich people contribute the same amount of public good as in a pooling equilibrium and therefore enjoy the same amount of public good in their "rich-only" club. But poor cannot benefit from this provision because they do not belong to this club. In fact, in their club, no one contributes to the public good and the amount of public good enjoyed in the "poor-only" club is null.

Third, even though rich people "waste" more income on the conspicuous good in the first period and contribute to the public good, whereas the poor do not "waste money" and do not contribute, it can be shown that the level of welfare enjoyed by the rich in a separating equilibrium is higher than the level of welfare enjoyed by the poor.

Fourth, it can be checked that $\tilde{f}^p < \hat{f}^p$.[4] Therefore $f^{pool} < f^{sep}$ and there is no ambiguity about the characteristics of an equilibrium once the amount of conspicuous expenditures by the poor is observed.

5 Taxation

We now turn to the issue of taxation. We know that in this economy, individuals will engage into a race on conspicuous consumption with different objectives in mind: poor people will

[4] See the proof in the appendix B

attempt to obtain a higher social status than the one they deserve and be amalgamated with rich people; rich people will attempt to distinguish themselves from the poor so as to keep their higher social status. Because this conspicuous good generates no intrinsic utility, these expenditures can be considered as waste, generating negative externalities or inefficiencies. It immediately comes to mind that some taxation scheme could then remedy these inefficiencies and improve welfare in this economy. We shall consider a tax scheme based on conspicuous consumption. A tax is added to the price of one unit of the conspicuous good. The tax rate t is fixed. Finally the proceeds of taxation are shared equally among individuals, whatever their income. We then would like to know whether taxation is Pareto-improving and if an infinite taxation, amounting to a ban on conspicuous consumption is socially desirable. We then will turn to the issue of taxation in a political economy framework, assuming that the tax rate can be set by the median voter, which can be alternatively rich or poor.

Studying taxation requires the selection of a particular equilibrium since there is an infinite number of separating equilibria. To do so, we shall use the "intuitive criterion" proposed by Cho and Kreps. Applying this criterion to our signalling game, it immediately appears that

Proposition 5 The best separating equilibrium is characterized by $f^{sep} = \hat{f}^p$:

At the best separating equilibrium, rich people spend the minimum amount of income on the conspicuous good such that poor people prefer not to spend as much on this good.

5.1 Taxation and social welfare

First, we would like to consider taxation from the Paretian point of view. We shall first ask whether some taxation is Pareto-desirable, then whether infinite taxation is Pareto-optimal. On the first point, we shall make the following

Proposition 6 For any set of parameters $(\alpha; y^r; y^p; n^r; n^p)$ a non-zero taxation is Pareto-improving.

Proof. See appendix C. ■

This proposition states that it is always in the interest of every individual in this society to limit to a certain extent the conspicuous race by means of taxation and de facto tax the rich.

The poor are never taxed in the best separating equilibrium since they don't spend on the conspicuous good but they benefit from redistribution of the tax proceeds. The rich are taxed but they know that to a certain extent some taxation will still help them to remain separated from the poor, eventhough it makes the purchase of one unit of the conspicuous good more costly, because taxation will have discouraging effects on the poor. Indeed, it can be shown that $\partial \hat{b}^p/\partial t$ is negative: the rich partly compensate the introduction of a tax system by decreasing their consumption of the conspicuous good, while keeping their favorable status. Hence, some taxation can be Pareto-improving both because it decreases the "waste" on the conspicuous good, without altering the partition of society into two classes or clubs, and because it redistributes some income from the rich to the poor. This is exactly the scenario that Frank [1985] has in mind when he explains that status considerations explains why rich people not necessarily want to avoid taxes. A tax system has the benefit of cementing a segmented society and letting them enjoy the benefits of a favorable social status, which partially depends on the presence of the poor in society.

Then we may wonder whether taxation should not impede any conspicuous consumption since it is a waste, generating no direct pleasure. But remark that forbidding by taxation or by law any conpicuous consumption has mixed consequences. Clearly making conpicuous consumption unlawful or unaffordable implies that only a pooling equilibrium can exist: no one makes any signal and applying the club formation rule, every member of society is included in the society-large club which characterizes a pooling equilibrium. Indeed on the one hand, it allows individuals to devote more income in the first period to the intrinsicly "useful" good and gives a higher social status to the poor who cannot be "stigmatised" and cannot be distinguished from rich people since all belong to the society-club. But on the other hand, it inflicts some harm: to the poor it eliminates an extra source of income, and to the rich it represents a loss in social status, since the social status of a rich, ceteris paribus, is larger than the average social status. Hence a priori, we cannot say that infinite taxation is Pareto optimal. Indeed the following proposition makes this precise claim:

Proposition 7 When $\alpha = 0$, an infinite taxation is Pareto-optimal, but there exist values of parameters $(\alpha; y^r; y^p; n^r; n^p)$ such that an infinite taxation is not Pareto-optimal.

Proof. See Appendix C ■

Indeed it is shown in the appendix that for some positive and large enough ®; if the di®erence between the two levels of income is large enough, then the rich will loose when being moved because of in¯nite taxation from a separating equilibrium to a no-waste pooling equilibrium: the loss in social status is just too great for him and overcomes the gain from reduced wasteful conspicuous consumption. Remark that this is not true when ® equals zero. In this case, a rich always bene¯t from being forbidden to spend on the conspicuous good because it is a pure waste, without any bene¯cial e®ect. The rich will get the same provision of public good in the society at large or in the "rich-only" club, but will increase her consumption of private good in the ¯rst period. The rich always gains in not spending in the conspicuous good. On the whole, the situation is then equivalent to the perfect information case.

This is not true any more when ® is positive and large enough and the income gap is large enough, because social status is enjoyed per se by individuals and is the more valuable the larger is the income gap. This proposition indeed exempli¯es that some conspicuous consumption may be useful to rich individuals and even if quali¯ed as a waste, it cannot be claimed to be a \social" waste: it has the bene¯t of allowing social segmentation which the rich appreciate. Of course, the poor have an opposite view on the spending on conspicuous good: for them, it is a \bad" on any account in a separating equilibrium.

This suggests to make a link between social status and congestion e®ects. It is well known that congestion e®ects tend to limit the size of clubs. Viewed in this perspective, the pooling of agents of di®erent types tend to produce congestion e®ects. The adding of an additional low-type agent to a club produces adverse consequences on existing members, as it contributes to decrease their social status, and their utility, just as if there were congestion.

5.2 Taxation and the median voter

Of course, taxation is not usually decided by a benevolent planner but rather by political bodies which enjoy the "power of the purse". In unequal societies, we know that the political decision over taxation will depend on who has the political power to set the tax scheme and on redistributional considerations (cf. Meltzer and Richards 1986, on this perpective, and Tocqueville to name only a few authors who studied this problem). Suppose that the society we study is democratic and that given the tax scheme we presented above, the decision over the value of the parameter is taken applying a simple majority rule. Given the division of society in two income classes, the majority is either formed of poor people if $n^p > N=2$; or by rich people otherwise.[5] Obviously a "poor" majority will not choose the same tax rate as a "rich" majority as their interests are partially incompatible. On this problem of the choice of a tax rate by majority rule, we can state the following

Proposition 8 The median voter chooses a tax rate which

1. is always in¯nity if $n^p > N=2$
2. can be a ¯nite positive number if $n^p < N=2$; for some values of the vector parameter $(®; y^r; y^p; n^r; n^p)$:

Proof. 1 derives from the fact that a poor always prefers the pooling equilibrium with no expenditure on the conspicuous good because she bene¯ts from social status ambiguity, the provision of public good by the rich in society and no waste from demonstration e®ect; 2 comes from Proposition 6. ■

6 Conclusion

Conspicuous consumption matters for individuals both for social status reasons and for the purpose of entering closed social groups we call "clubs" and which provide some collective bene¯ts to their members. But these two e®ects cannot be thought as separate. To the contrary, these two e®ects are reinforcing: by entering a club, an individual bene¯ts both from the collective good supplied in the club and from the status associated by society to its members. Conspicuous consumption as a signal plays a dual role: its helps the sorting out of people and the making of teams and by so doing, it a®ects the status of individuals. As Frank wrote:

> " In societies in which economic and social interactions between people are important and pervasive..., information about people with whom we might interact ha obvious value. It determines for example, the people we consider as potential mates, the employees we hire, those whose company we seek, and so on".[6]

[5] Clearly, because of strict homogeneity among the poor, they all vote for the same tax rate if they represent more than half of society. The same is true for the rich.

[6] cf. Frank, p.137.

In the present paper these reinforcing incentives combine when individuals rationally choose their signal about themselves. This may lead to a segmentation of society into clubs where rich individuals are neatly separated from the poor. Taxing conspicuous consumption is beneficial to anybody, but an infinite taxation which would lead to forbidding any conspicuous consumption is not necessarily Pareto-optimal even though this consumption is a pure waste. This is due to the fact that the rich may prefer to be "distinguished" even at the cost of some loss in terms of private consumption.

Of course, the present paper can be extended in several directions. Two seem to be worthwhile. First, we could refine the treatment of beliefs on the one hand and the club formation. In particular we could use the theory of coalitions formation rather than the ad hoc rule we assume and be more in line with the microeconomic setting we investigate. However we doubt that these technical niceties would significantly modify our results. Second, we could see the signalling problem in a very different way: the observable item could be the individual contribution to the club, on which is based the society's inference over individual income and therefore on social status. In other words you contribute to the New York Yacht Club not because you like sailing but for snobbish reasons only: just to show off your fortune. It is then social segmentation into clubs which serves as the support of status discrimination or social segmentation into statuses. Again this exemplifies the complex relationships between both types of segmentation.

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A Pooling equilibria

Consider a level f^{pool} of conspicuous consumption corresponding to a pooling equlibirum. The incentive constraints in the case of a pooling equilibrium are the following:

- for a rich

$$
\begin{aligned}
& (1-\alpha)\left[\ln\left(y^r - f^{pool}\right) + \ln\frac{1}{=}^{\lambda}\right] + \\
& \alpha\left[\ln\left(\frac{n^r}{N}y^r + \frac{n^p}{N}y^p - f^{pool}\right) + \ln\left(\frac{n^r}{N}\frac{1}{=} + \frac{n^p}{N}y^p\right)^{\lambda}\right] + \beta n^r\left(y^r - \frac{1}{=}\right) \\
\geq\; & (1-\alpha)\left[\ln y^r + \ln\frac{1}{=} + \beta\left(y^r - \frac{1}{=}\right)^{\lambda}\right] + \alpha 2\ln y^p
\end{aligned}
\tag{15}
$$

- for a poor:

$$
\begin{aligned}
& (1-\alpha)\left[\ln\left(y^p - f^{pool}\right) + \ln y^p\right] + \\
& \alpha\left[\ln\left(\frac{n^r}{N}y^r + \frac{n^p}{N}y^p - f^{pool}\right) + \ln\left(\frac{n^r}{N}\frac{1}{=} + \frac{n^p}{N}y^p\right)^{\lambda}\right] + \beta n^r\left(y^r - \frac{1}{=}\right) \\
\geq\; & 2\ln y^p
\end{aligned}
\tag{16}
$$

The critical values $\hat{f}(y^r)$ and $\hat{f}(y^p)$ are such that (15) and (16) are true as equalities, respectively. (15) is equivalent to:

$$\Gamma_{y^r;\alpha}\left(f^{pool}\right) \geq -C_{y^r;\alpha}\left(f^{pool}\right)$$

where:

$$
\begin{aligned}
\Gamma_{y^r;\alpha}(f) &\equiv (1-\alpha)\ln\frac{(y^r - f)^{\lambda}}{y^r} + \alpha\ln\frac{(\bar{y} - f)^{\lambda}}{(y^p)^2} - (1-\alpha)\beta\left(y^r - \frac{1}{=}\right) \\
C_{y^r;\alpha}(f) &\equiv \beta n^r\left(y^r - \frac{1}{=}\right) + \alpha\ln\left(\frac{n^r}{N}\frac{1}{=} + \frac{n^p}{N}y^p\right)
\end{aligned}
$$

and (16) is equivalent to

$$\Pi_{y^p;\alpha}\left(f^{pool}\right) \geq -C_{y^r;\alpha}\left(f^{pool}\right)$$

where:

$$\Pi_{y^p;\alpha}(f) \equiv (1-\alpha)\ln\frac{(y^p - f)^{\lambda}}{y^p} + \alpha\ln\frac{(\bar{y} - f)^{\lambda}}{(y^p)^2}$$

1. Let us show that there exists a unique value f^*, independent from α, such that

$$\Gamma_{y^r;\alpha}(f^*) = \Pi_{y^r;\alpha}(f^*)$$

This equality is equivalent to:

$$\ln\frac{(y^p - f^*)}{y^p} = \ln\frac{(y^r - f^*)}{y^r} - \beta\left(y^r - \frac{1}{\beta}\right)$$

$$\iff f^* = y^r\frac{y^p\left(1 - e^{1-\beta y^r}\right)}{y^r - y^p e^{1-\beta y^r}} \tag{17}$$

2. We now prove:

$$\frac{\partial \Pi_{y^p;\alpha}(f)}{\partial f} = -\frac{1-\alpha}{y^p - f} - \frac{\alpha}{\overline{y} - f} < \frac{\partial \Gamma_{y^r;\alpha}(f)}{\partial f} = -\frac{1-\alpha}{y^r - f} - \frac{\alpha}{\overline{y} - f} < 0 \tag{18}$$

3. We now prove that $\tilde{f}(y^p)|_{\alpha=0} > f^*$. From (16), we get:

$$\tilde{f}(y^p)|_{\alpha=0} = y^p\left(1 - e^{n^r(1-\beta y^r)}\right)$$

Hence:

$$\begin{aligned}
\tilde{f}(y^p)|_{\alpha=0} &> f^* \\
&\iff \\
\left(1 - e^{1-\beta y^r}\right) &> y^r\frac{\left(1 - e^{1-\beta y^r}\right)}{y^r - y^p e^{1-\beta y^r}} \\
&\iff \\
y^r\left(e^{1-\beta y^r} - e^{n^r(1-\beta y^r)}\right) &> y^p\left(e^{1-\beta y^r} - e^{(n^r+1)(1-\beta y^r)}\right) \\
&\iff \\
\frac{y^r}{y^p} &> \frac{\left(e^{1-\beta y^r} - e^{(n^r+1)(1-\beta y^r)}\right)}{y^r\left(e^{1-\beta y^r} - e^{n^r(1-\beta y^r)}\right)} \simeq 1
\end{aligned} \tag{19}$$

Since $y^r >> y^p$, (19) is verified and

$$\tilde{f}(y^p)|_{\alpha=0} > f^* \tag{20}$$

4. We prove that:

$$\frac{\partial \tilde{f}(y^p)}{\partial \alpha} > 0 \tag{21}$$

From (16); we deduce:

$$d\alpha \left[-\ln\left[\frac{y^p - \hat{f}(y^p)}{y^p} \right] + \ln\left(\frac{\bar{y} - \hat{f}(y^p)}{(y^p)^2} \right) + \ln\left(\frac{n^r}{N}\frac{1}{=} + \frac{n^p}{N} y^p \right) \right]$$

$$= d\hat{f}(y^p) \left[(1-\alpha)\frac{1}{y^p - \hat{f}(y^p)} + \alpha\frac{1}{\bar{y} - \hat{f}(y^p)} \right]$$

( )

$$d\alpha \left[\ln\left(\frac{\bar{y} - \hat{f}(y^p)}{y^p - \hat{f}(y^p)} \right) + \ln\left(\frac{\frac{n^r}{N}\frac{1}{=} + \frac{n^p}{N} y^p}{y^p} \right) \right]$$

$$= d\hat{f}(y^p) \left[(1-\alpha)\frac{1}{y^p - \hat{f}(y^p)} + \alpha\frac{1}{\bar{y} - \hat{f}(y^p)} \right]$$

Because $\frac{1}{=} > y^p$; it is clear that $\frac{\partial \hat{f}(y^p)}{\partial \alpha}$ is positive.

5. Then, from (21), (20), and f^* independent from α imply:

$$\hat{f}(y^p)\,|_{\alpha>0} > \hat{f}(y^p)\,|_{\alpha=0} > f^* \tag{22}$$

(22), (18) and f^* independent from α then imply:

$$\hat{f}(y^r) > \hat{f}(y^p)$$

6. It is then immediate that f^{pool} must belong to the interval $\left[0; \hat{f}(y^p)\right]$:

7. Beliefs at a pooling equilibrium are obtained from the equilibrium strategies and are consistent with the Bayes rule.

B Separating equilibria

Assuming the simple belief that, if you display conspicuous consumption, then you are viewed as rich by the rest of society, the incentive constraints in the case of a separating equilibrium are the following:

- for a rich when she practices f^r

$$\ln(y^r - f^r) + \ln\frac{1}{=} + \beta n^r\left(y^r - \frac{1}{=}\right)$$
$$\geq (1-\alpha)\left[\ln y^r + \ln\frac{1}{=} + \beta\left(y^r - \frac{1}{=}\right)\right] + 2\alpha\ln y^p \tag{23}$$

- for a poor when she displays f^p:

$$2\ln y^p > (1-\alpha)\left[\ln(y^p - f^p) + \ln y^p\right] +$$
$$\alpha\left[\ln(y^r - f^p) + \ln\frac{1}{=}\right] + \beta n^r\left(y^r - \frac{1}{=}\right) \tag{24}$$

The critical values $\hat{f}(y^r)$ and $\hat{f}(y^p)$ are such that (23) and (24) are true as equalities, respectively. (23) is equivalent to:

$$\Theta_{y^r;\alpha}(f^r) \geq -D_{y^r;\alpha}$$

where:

$$\Theta_{y^r;\alpha}(f) \equiv \ln(y^r - f) - (1-\alpha)\ln y^r - (1-\alpha)\beta\left(y^r - \frac{1}{=}\right) - 2\alpha\ln y^p$$
$$D_{y^r;\alpha} \equiv \alpha\ln\frac{1}{=} + \beta n^r\left(y^r - \frac{1}{=}\right)$$

Similarly, (24) is equivalent to:

$$\Psi_{y^p;\alpha}(f^r) \geq -D_{y^r;\alpha}$$

where:

$$\Psi_{y^p;\alpha}(f) \equiv (1-\alpha)\ln(y^p - f) + \alpha\ln(y^r - f) - (1-\alpha)\ln y^p$$

1. Let us show that there exists a unique value f^*; independent from α; such that

$$\Theta_{y^r;\alpha}(f^*) = \Psi_{y^p;\alpha}(f^*)$$

This equality is equivalent to:

$$\ln\left(\frac{y^r - f^*}{y^r}\right) - \beta\left(y^r - \frac{1}{=}\right) = \ln\left(\frac{y^p - f^*}{y^p}\right)$$

and we remark that f^* is the same as the value computed in the pooling equilibrium part.

2. We now prove:

$$\frac{\partial \Psi_{y^p;\alpha}(f)}{\partial f} = -\frac{1-\alpha}{y^p - f} - \frac{\alpha}{y^r - f} < \frac{\partial \Theta_{y^r;\alpha}(f)}{\partial f} = -\frac{1}{y^r - f} < 0 \tag{25}$$

   Then we get from (24):

$$\hat{f}(y^p)\,|_{\alpha=0} = y^p\left(1 - e^{n^r(1-\beta y^r)}\right) = \tilde{f}(y^p)\,|_{\alpha=0}$$

   which immediately implies:

$$\hat{f}(y^p)\,|_{\alpha=0} > f^* \tag{26}$$

3. From (24) it is deduced that:

$$d\hat{f}(y^p)\left[-\frac{1-\alpha}{y^{rp} - \hat{f}(y^p)} - \frac{\alpha}{y^r - \hat{f}(y^p)}\right] = d\alpha\left[\ln\left(\frac{y^p - \hat{f}(y^p)}{y^r - \hat{f}(y^p)}\right) + \ln\frac{y^p}{1}\right]$$

   which implies:

$$\frac{d\hat{f}(y^p)}{d\alpha} > 0 \tag{27}$$

4. Then (27) and f^* independent from α imply:

$$\tilde{f}(y^p)\,|_{\alpha>0} > \tilde{f}(y^p)\,|_{\alpha=0} > f^* \tag{28}$$

   (28), (25) and f^* independent from α imply: $\hat{f}(y^r) > \hat{f}(y^p)$.

5. It is then immediate that f^{sep} must belong to the interval $\left[\hat{f}(y^p); \hat{f}(y^r)\right]$.

6. Beliefs at a separating equilibrium are obtained from the equilibrium strategies and are consistent with the Bayes rule.

7. Comparison between $\tilde{f}(y^p)$ and $\hat{f}(y^p)$: From the definitions of these two critical values, we can show that they imply:

$$X_1\left(\hat{f}(y^p)\right) \equiv (1-\alpha)\ln\left(y^p - \hat{f}(y^p)\right) + \alpha\ln\left(y^r - \hat{f}(y^p)\right) - (1+\alpha)\ln y^p + \alpha\ln\frac{1}{=}$$
$$-\beta(N - n^r)\left(y^r - \frac{1}{=}\right)$$

$$X_2\left(\tilde{f}(y^p)\right) \equiv (1-\alpha)\ln\left(y^p - \tilde{f}(y^p)\right) + \alpha\ln\left(\bar{y} - \tilde{f}(y^p)\right) - (1+\alpha)\ln y^p + \alpha\ln\bar{x}$$

   As $X_1(f)$ is bigger than $X_2(f)$, it immediately follows that:

$$\tilde{f}(y^p) < \hat{f}(y^p)$$

   and the two sets of equilibria have no common element.

C Taxation

C.1 Proof of Proposition 6

Given the taxation scheme, the incentive-compatible constraints for a separating equilibrium are modified. They become:

- for a rich when she practices f^r

$$\begin{aligned} &\ln\left(y^r + \frac{n^r f^r t}{N} - f^r(1+t)\right) + \ln\frac{1}{=} + \beta n^r\left(y^r - \frac{1}{=}\right) \\ \geq\ &(1-\alpha)\left[\ln\left(y^r + \frac{(n^r-1)f^r t}{N}\right) + \ln\frac{1}{=} + \beta\left(y^r - \frac{1}{=}\right)\right] \\ &+\alpha\left[\ln y^p + \ln\left(y^r + \frac{n^r f^r t}{N}\right)\right] \end{aligned} \tag{29}$$

- for a poor when she displays f^p:

$$\begin{aligned} \ln y^p + \ln\left(y^p + \frac{n^r f^r t}{N}\right) \geq\ &(1-\alpha)\left[\ln\left(y^p + \frac{(n^r+1)f^r t}{N} - f^p(1+t)\right) + \ln y^p\right] \\ &+\alpha\left[\ln\left(y^r + \frac{n^r f^r t}{N} - f^p(1+y)\right) + \ln\frac{1}{=} \right. \\ &\left. +\beta n^r\left(y^r - \frac{1}{=}\right)\right] \end{aligned} \tag{30}$$

The value of conspicuous consumption $f^{tax}_{\alpha;t}$ at the best separating equilibrium for given values of the two parameters t and α is such that (30) holds as an equality. It is easy to show that:

$$\frac{\partial f^{tax}_{\alpha;t}}{\partial t} < 0 \qquad \frac{\partial f^{tax}_{\alpha;t}}{\partial \alpha} > 0$$

For $\alpha = 0$; we get that:

$$f^{tax}_{0;t} = \frac{y^p\left(1 - e^{-\beta n^r(1-\beta y^r)}\right)}{1 + \frac{t}{N}(N - n^r(1 - e^{-\beta n^r(1-\beta y^r)}) - 1)} \tag{31}$$

In this case, to prove that a positive taxation is Pareto-improving, it suffices to show that the consumption of the rich in the first period is increased by taxation, since the poor always benefit from taxation in the best separating equilibrium: they do not consume the conspicuous good and they benefit from a net transfer. From the above definition for $f^{tax}_{0;t}$; this amounts to show that:

$$\ln\left(y^r + \frac{n^r f^{tax}_{0,t} t}{N} - f^{tax}_{0,t}(1+t)\right) > \ln\left(y^r - f^{tax}_{0,0}\right)$$
$$\iff$$
$$\frac{n^r f^{tax}_{0,t} t}{N} - f^{tax}_{0,t}(1+t) > -f^{tax}_{0,0}$$
$$\iff$$
$$\frac{n^r f^{tax}_{0,t} t}{N} e^{-\beta n^r (1-\beta y^r)} > 0.$$

which is true.

For α non zero, given the definition of the first-period consumption, we have to show that:

$$\frac{\partial c^r}{\partial t}\Big|_{t=0} > 0$$

We know that:

$$\frac{\partial c^r}{\partial t}\Big|_{t=0} = \left(\frac{n^r}{N} - 1\right) f^{tax}_{\alpha,0} - \frac{\partial f^{tax}_{\alpha,0}}{\partial t}$$

From (), we know that:

$$\frac{\partial f^{tax}_{\alpha,0}}{\partial t} = \frac{f\left[\frac{(1-\alpha)}{y^p - f}\left(\frac{n^r+1}{N} - 1\right) + \frac{\alpha}{y^r - f}\left(\frac{n^r}{N} - 1\right) - \frac{n^r}{N y^p}\right]}{\left[\frac{1-\alpha}{y^p - f} + \frac{\alpha}{y^r - f}\right]}$$

Hence, after some manipulation and using the approximation $\frac{n^r+1}{N} \approx \frac{n^r}{N}$, we get:

$$\text{sign}\left(\frac{\partial c^r}{\partial t}\Big|_{t=0}\right) = \text{sign}\left(\frac{n^r}{N}\frac{f}{y^p}\right)$$

This completes the proof.

C.2 Proof of Proposition 7

It suffices to prove that for some values of the parameters, an infinite taxation is not Pareto-optimal.

Again, we have to study the welfare of the rich, as the poor always benefit from taxation.

When taxation is infinite, then there is no conspicuous consumption. Hence we get:

$$W^r|_{t=\infty} = (1-\alpha)\left(\ln(y^r) + \ln\frac{1}{=}\right) + \alpha[\ln\bar{y} + \ln\bar{x}] + \beta n^r\left(y^r - \frac{1}{=}\right)$$

$$W^r|_{t\geq 0} = \ln\left(y^r + \frac{n^r f_{1,t}^{tax} t}{N} - f_{1,t}^{tax}(1-t)\right) + \ln\frac{1}{=} + \beta n^r\left(y^r - \frac{1}{=}\right)$$

Hence:

$$W^r|_{t\geq 0} > W^r|_{t=1}$$

( )

$$0 < \ln\left(y^r + \frac{n^r f_{1,t}^{tax} t}{N} - f_{1,t}^{tax}(1-t)\right) + \alpha\ln\frac{1}{=} - (1-\alpha)\ln(y^r) - \alpha\ln\bar{y} - \alpha\ln\bar{x}$$

When $\alpha + 1$; this inequality becomes:

$$0 < \ln\left(y^r + \frac{n^r f_{1,t}^{tax} t}{N} - f_{1,t}^{tax}(1-t)\right) + \ln\frac{1}{=} - \ln\bar{y} - \ln\bar{x}$$

Denote A the RHS term in the previous inequality. We know from the previous proposition that for t in the neighborhood of zero:

$$A > \ln\left[y^r - y^p\left[1 - e^{-\beta n^r(1-\beta y^r)}\right]\right] + \ln\frac{1}{=} - \ln\bar{y} - \ln\bar{x} \equiv B$$

Hence if B is positive, then A is positive. Clearly:

$$B > \ln(y^r - y^p) + \ln\frac{1}{=} - \ln\bar{y} - \ln\bar{x}$$

The RHS term fo this inequality can rewritten as:

$$\ln\left[(y^r - y^p)\frac{1}{=}\right] - \ln[\bar{y}\bar{x}]$$

It then suffices to prove that it is possible that:

$$\ln\left[(y^r - y^p)\frac{1}{=}\right] - \ln[\bar{y}\bar{x}]$$

Given the assumptions we made on β; it amounts to obtain as a sufficient condition:

$$y^r - y^p > \bar{y}$$

( )

$$\frac{y^r}{y^p} > 1 + \frac{N}{N - n^r} > 1$$