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Self-Defeating Antitrust Laws How Leniency Programs Solve Bertrand's Paradox and Enforce Collusion in Auctions

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Self-Defeating Antitrust Laws

How Leniency Programs Solve Bertrand's Paradox and Enforce Collusion in Auctions*

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Abstract

I find that current US's and EU's Antitrust laws – in particular their "moderate" *leniency programs* that only reduce or at best cancel sanctions for price-fixing firms that self-report – may make collusion enforceable even in one-shot competitive interactions, like Bertrand oligopolies and first-price auctions, where no collusion would be supportable otherwise. The reduced sanctions for firms that self-report provide the otherwise missing credible threat necessary to discipline collusive agreements: they ensure that if a firm unilaterally deviates from collusive strategies, other firms find it convenient to punish it by reporting information to the Antitrust Authority.

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1 Introduction

This paper addresses some possibly counterproductive features of modern Antitrust legislations. In particular, it questions the current design of *leniency programs*, the reduced-sanctions schemes that encourage price-fixing firms to self-report and cooperate with Antitrust Authorities.

The main finding of the paper is that Antitrust Law against cartels, as currently designed and enforced in the United States and in the European Union, may make perfect collusion enforceable even in occasional market interactions, like one-shot (or infrequently repeated) Bertrand oligopolies and first-price auctions, where little or no collusion would be supportable if there were no Antitrust legislation or, of course, if this legislation was better designed.

The way current Antitrust Law may achieve the remarkable result of solving one of the oldest puzzles in economics is by providing firms with the otherwise missing credible threat necessary to enforce promises of cooperation (collusion) among rivals. In fact, two are the features of Antitrust Law that, together, lead to this paradoxical result, and possibly to deadweight welfare losses in society. The first one is the size of expected sanctions against colluding firms, way too low to deter cartels given the investigation resources available to Antitrust Authorities.[1] The second feature is the current design of leniency programs, the modifications of Antitrust Law that allow – both in the US and in the EU – to reduce sanctions against firms that choose to report information to the Antitrust Authority. The crucial aspect here is *the size of discounts in sanctions awarded to firms who spontaneously report information on their cartel when the Antitrust Authority has no information on it, that is, before any investigation is opened.*[2]

The first feature, too low overall sanctions, makes sure that firms are willing to collude if they find a way to discipline each other's tendency to cheat. The second fea-

[1] In the US, the Sherman Act allows to charge monetary fines against price-fixing firms up to 10 millions US dollars, and to charge monetary fines not exceeding 350.000 US dollars and to imprison up to three years individuals responsible of a cartel. The current EU's Antitrust law has no criminal sanctions and monetary fines up to 10% of a firm's yearly turnover. The fact that many cartels have recently been uncovered in the US and the EC reveals that the deterrence effect of current expected sanctions is still low. For the US, McCutcheon (1997) showed with a calibration exercise that expected sanctions are far below the level that would deter firms from colluding. In Europe expected sanctions are even lower, since fines are not much larger than in the US but no criminal sanctions are available.

[2] US's Antitrust Law includes both a *corporate* and an *individual* leniency policy that grant automatic amnesty from criminal charges to firms and/or individuals that report information on a cartel before an investigation is opened. The EU has also recently introduced a leniency program that, among other things, allows for the total exemption from fines, or for a very substantial reduction in their amounts (at least 75%), to firms that reveal information on a cartel about which the Community had no previous knowledge.

More limited forms of leniency also exist, both in the US and the EU, for firms that start co-operating with the Antitrust Authority only after an investigation on them has been opened. The present paper does not address these forms of leniency, directed more to facilitate prosecution than to deter cartels. For a thorough analysis of these forms of leniency see Motta and Polo (1999).

ture, the low but weakly positive reduced sanctions for firms that spontaneously self-report, provides the threat necessary to discipline the collusive agreement in one-shot interactions: it ensures that if a firm unilaterally deviates from the agreed collusive strategies, the other firms will find it convenient to punish it by reporting information on the initial agreement to the Antitrust Authority.

By incorporating these features of reality, results similar to "folk theorems" obtain for a number of models of one-shot market interaction. Collusion becomes enforceable in the homogeneous Bertrand oligopoly, in the Bertrand-Edgeworth (capacity-constrained) oligopoly, in split-award low-price (pay-your-bid) procurement auctions, in discriminatory auctions of shares, in other multi-unit first-price auctions, and even – under some additional conditions – in single-object first-price auctions. In all these one-shot games, the current leniency programs are shown to make any sufficiently profitable collusive agreement supportable in subgame perfect equilibrium.

Such general results do not obtain for games like the Cournot oligopoly, where the payoff of a player that sticks to the agreed collusive strategies is a smooth function of the strategic variable set by a player that unilaterally deviates. This is because the deviating player can then choose to restrain the "size" of its deviation to leave sufficient profits to non-deviating firms to induce them not to report. Analogously, the folk theorems do not obtain when players can costlessly administer side payments, these are so secret that they do not affect the probability of being detected by the Antitrust Authority, non-deviating players have zero bargaining power in the renegotiation stage that follows a deviation, and side-payments are recognized as shares of collusive profits and requested back from reporting firms with probability one. When all these (quite unrealistic) conditions are simultaneously satisfied, a deviating player can successfully "bribe" non-deviating ones to induce them not to report. But even in these last situations, leniency programs make profitable collusion supportable as long as there are moderate negative externalities.[3] Negative externalities may reinforce the pro-collusive effect of leniency programs by increasing non-deviating players' incentives to report if a player deviates unilaterally from the agreed collusive strategies.

The paper is most closely related to Buccirossi and Spagnolo's (2000) analysis of leniency programs in criminal Law, and complements Spagnolo's (2000) characterization of optimal leniency programs for the deterrence of organized crime. Buccirossi and Spagnolo show that moderate discounts in sanctions for self-reporting agents involved in sequential illegal exchanges, like trade in bads or corrupt deals, may enforce occasional deals that would otherwise be unfeasible. Spagnolo (2000) shows, among other things, that "moderate" leniency programs that – like the US's and EU's ones – only reduce or cancel sanctions for firms that spontaneously self-report to the Antitrust Authority when no investigation is open, cannot have any deterrence effect on long-term collusive arrangements. Here I find that, besides being valueless for deterring long-term collusive arrangements, these moderate leniency programs may also have the counterproductive effect of enforcing collusion in one-shot (or infrequently re-

[3]Externalities analogous to those considered by Jehiel and Moldovanu (1996), Jeihel *et al.* (1996), and Caillaud and Jehiel (1998).

peated) simultaneous competitive situations, where no collusion would be enforceable otherwise.

The paper's results are also very close in spirit to Marshall *et al.* (1994) and McCutcheon (1997). Marshall *et al.* showed that the possibility of private settlement between bidders in procurement auctions allows them to share any extra profits from preferential treatment, thereby allowing bidders to reach profitable quasi-collusive (and inefficient) outcomes in one-shot procurement auctions. Apparently, their voice has been heard and possibilities for private settlements have been reduced in the US and subjected to the control/approval of the General Services Administration Board of Contract Appeals, an independent and hard to capture body. McCutcheon showed that the current American Antitrust Law may have counterproductive effects with respect to cartels because expected sanctions are too low to deter collusion, but sufficiently high to deter renegotiation of finite-length punishments strategies. By enhancing the threats' credibility the low expected sanctions may end up stabilizing collusive agreements (in infinitely repeated Bertrand oligopolies). McCutcheon's work is a strong call for higher fines or stronger non monetary sanctions against cartels. I am not aware of whether her call has yet reached any legislator, but the call seems to have at least reached the US's Department of Justice, who has turned tougher with cartels in recent times.

At the end of the paper several policy options are discussed that easily eliminate the counterproductive effect of leniency programs highlighted below. I hope to be as lucky as the above mentioned authors in being heard: the design of leniency programs in Antitrust needs to be carefully re-examined.

2 How current Leniency Programs solve Bertrand's paradox

Consider a classical Bertrand duopoly where two symmetric firms produce an homogeneous good at constant marginal cost and compete in price once for a pool of perfectly informed consumers. I adopt the standard assumption of bounded, continuous, downward sloping demand with finite choke-price, so that monopoly profits are bounded.[4] Let $\pi(p)$ denote industry profits, the profits a monopolist would earn in the same market if he were to charge the price p, and $p^M \equiv \arg\sup\{\pi(p)\}$ the finite price that maximizes industry (or the monopolist's) profits.

As usual, I assume that after firms simultaneously quote prices, all consumers buy from the firm that quoted the lowest price, and that if firms quote identical prices consumers divide themselves equally between the two firms.

[4]When one allows for unbounded (infinite) monopoly profits, the Bertrand paradox needs not occurr (Baye and Morgan, 1999).

2.1 No Antitrust Law

Suppose first that there exist no Antitrust Law, as implicitly done in previous analyses of Bertrand competition. Then the "Bertrand paradox" where both firms set prices equal to marginal cost and earn zero profits is the only equilibrium outcome (e.g. Harrington, 1989). As is well known, this is the case because whatever price above marginal cost a firm chooses, the other firm has always an incentive to slightly undercut that price and capture the whole market. However, this is also the case because there are no threats firms can use to discipline each other's tendency to undercut and enforce a more cooperative outcome.

For the purposes of this paper, the timing of the Bertrand duopoly can be described as in Figure 1.

– Figure 1 About Here –

Absent Antitrust Laws, firms would always like to enter a collusive agreement at the first stage, if they could thereby sustain a price higher than marginal cost. However, the no-Antitrust-Law Bertrand game is a "pure" one-shot game in the sense that after prices are quoted there is no future action that firms can use as threat to enforce the agreement.

2.2 A simple Antitrust legislation

Suppose now that a simple Antitrust legislation is introduced, one that outlaws collusive agreements between firms and sets the (monetary) fine $F_i > 0$ that each firm i must pay if detected colluding. To simplify exposition I let F_i include also the monetary value of any non monetary sanctions firms caught colluding would incur. Let $\alpha > 0$ denote the probability by which a cartel is discovered by the Antitrust Authority (Antitrust Authority) and successfully prosecuted, which I take as given, and let $\pi_i^M = \frac{\pi(p^M)}{2}$ denote a firm i's share of industry profits at the symmetric joint monopoly collusive agreement. Firms are then willing to try collude and sustain a price above marginal cost, possibly the monopoly price, only if colluding is not too risky; that is, only if the expected sanctions for price-fixing αF_i are not too large relative to collusive profits. Assume that when a collusive agreement is detected and successfully prosecuted, firms pay their fines and, if sales already took place, they pay back collusive profits as damages; if not, the pricing game is nullified and re-started from the beginning. Moreover, assume that when a pricing game is nullified and replicated because a cartel has been discovered, the fines for a new collusive agreement between the same firms and/or the probability of detection of such an agreement increase up to F' and α', with $\alpha' F' > (1-\alpha')\pi_i^M$ (so that firms that have been caught once colluding will find it too risky to collude again in the same, re-started pricing game).[5] Under these assumptions, firms would be willing to collude in our one-shot

[5]This assumption is analogous to Marshall *et al.*'s (1994) assumption that no more distortions are present when a procurement auction is re-run after a distortion was detected. The (quite realis-

Bertrand game only if $\alpha F_i \leq (1-\alpha)\pi_i^M$, that is, when *ex ante* there are positive expected profits from the joint monopoly collusive agreement

$$E_{t=1}[\pi_i(p_i = p_j = p^M)] = (1-\alpha)\pi_i^M - \alpha F_i \geq 0.$$

To make things interesting, I assume that this condition is satisfied (that is, I focus on oligopolies that satisfy this condition).

When an Antitrust legislation is present and a collusive agreement is reached, besides setting their prices firms can also choose, *anytime* after the agreement is reached, to report information about the collusive agreement to the Antitrust Authority. To take into account this possibility in a discrete time environment, the timing of the Bertrand game must change as in Figure 2.

– Figure 2 About Here –

Is there any credible threat firms may use to enforce collusion after the introduction of this simple Antitrust legislation? Again, the answer is no. Now a possible threat does exist: firms may threaten to report information about the collusive agreement to the Antitrust Authority in case the other firm undercuts the agreed collusive price. However, such a threat is empty, is not credible. It is straightforward to verify that the choice of reporting information to the Antitrust Authority is dominated at all stages, also at Stage 4 after a firm deviated at Stage 3, since it leads to negative payoffs $-F_i$. It follows that under the simple Antitrust legislation above, even though collusion would be profitable (I assumed $\alpha F_i \leq (1-\alpha)\pi_i^M$), the unique subgame perfect Nash equilibrium outcome of the game remains the Bertrand paradox, with prices equal to marginal costs and zero profits for both firms.

2.3 Current Antitrust legislations: a Folk Theorem

Consider now more sophisticated Antitrust legislations that – like current EU's and US's legislations – include leniency programs that encourage firms to report information to the Antitrust Authority. Suppose an Antitrust legislation is introduced that, besides defining the fine F_i that a colluding firm must pay if its cartel is detected by the Antitrust Authority, also defines a reduced fine $RF_i < F_i$ the colluding firm must pay if it spontaneously self-reports, allowing its cartel to be prosecuted, when the Antitrust Authority has no information on the cartel. Since, as above, firms can report information to the Antitrust Authority anytime after the collusive agreement is reached, the game still follows Timing 2.

As mentioned in the introduction, both the US's and the EU's leniency programs are "moderate" in the sense of not allowing reporting firms to be rewarded (so that $0 \leq RF_i < F_i$). The EU's leniency program emphasizes the case of *non-imposition of*

tic) idea behind the assumption is that the new auction/oligopolistic interaction takes place under much closer supervision, both from the Antitrust Authority and from buyers (in oligopolies and procurement auctions) or the seller (in other auctions).

fines against firms that spontaneously report hard information on cartels of which the Commission had no knowledge. Analogously, when publicizing its leniency program, the US Department of Justice is keen to focus on cases like the "marine construction investigation" and the "graphite electrodes investigation," where the applicants both received amnesty from criminal prosecution and total cancellation of fines (e.g. Spratling, 1998). So let us begin by setting $RF_i = 0$.

It is easy to verify that when $RF_i = 0$ marginal cost pricing is *not* anymore the unique subgame perfect Nash equilibrium outcome of the one-shot Bertrand game. Consider Timing 2 backwards. If firms agreed at Stage 1 to both quote the monopoly price, respected the agreement at Stage 3, did not report information to the Antitrust Authority before, and were not caught by the Antitrust Authority at Stage 7, then a firm cannot gain by reporting information at Stage 8, since this would lead to payoff $-RF_i = 0$, instead of $\pi_i^M \geq 0$. Analogously, a firm cannot gain by reporting information at Stage 6, since reporting it would get $-RF_i = 0$, instead of $(1-\alpha)\pi_i^M - \alpha F_i \geq 0$. The same reasoning applies to the choice of reporting at Stage 4. If, instead, a firm has undercut the agreed monopoly price at Stage 3, then at Stage 4 (and at Stage 6, if it were reached) it becomes a dominant strategy for the other firm to report information to the Antitrust Authority, since this firm loses nothing by reporting ($-RF_i = 0$) but it avoids the risk of paying the fine F_i if the Antitrust Authority finds out by itself about the initial agreement (at Stage 7, with probability α). It follows that at Stage 3 a firm cannot gain by undercutting the agreed monopoly price, since doing it leads to a payoff of at best $-RF_i = 0$ (if the deviating firm itself reports information to the Antitrust Authority), and at worst $-F_i$ (if it does not), and $-F_i < 0 \leq (1-\alpha)\pi_i^M - \alpha F_i$. Given that the collusive agreement would be respected from Stage 3 onwards, firms could not gain by reporting the agreement to the Antitrust Authority at Stage 2, thereby getting $-RF_i = 0$ instead of $(1-\alpha)\pi_i^M - \alpha F_i \geq 0$. In summary, a Stage-1 agreement to quote the monopoly price would be respected by both firms, and no firm would ever report information to the Antitrust Authority. That is, when $F_i > 0$, $(1-\alpha)\pi_i^M - \alpha F_i \geq 0$ and $RF_i = 0$, (joint) monopoly pricing is a subgame perfect Nash equilibrium outcome of the one-shot Bertrand oligopoly game.

It is easy to verify that the reasoning above applies identically to any other collusive price $p^C \neq p^M$ that satisfies $-F_i < 0 \leq (1-\alpha)\pi_i^C(p^C) - \alpha F_i$, therefore any such price is also a subgame perfect Nash equilibrium of the Bertrand game.

As mentioned in the introduction, the US's leniency program cancels criminal sanctions but not always the monetary fines when firms spontaneously self-report. And although the EU's leniency program emphasizes the non-imposition of fines, it *guarantees* only "a very substantial reduction" of them (between 75 and 100%) to firms that spontaneously report hard information on their cartel when this is not already under investigation. Also, the EU's leniency program establishes that only after a firm has reported its information and this has been evaluated, the Antitrust Authority will judge whether the firm is eligible to the benefits of the leniency program and set the fine discount. Analogously, in the US only amnesty from criminal prosecution is automatic for firms that spontaneously self-report. The reduction in fine is instead decided case by case, after firms have reported information. This means that at the time a firm decides to report, it only has an expectation of the reduced fine it will eventually face.

What happens to our Bertrand game when $RF_i > 0$, or when the exact value of RF_i is uncertain at the time a firm reveals, with $0 \leq RF_i \leq F_i$? It is straightforward to verify that as long as $0 < RF_i \leq \alpha F_i$ or, if RF_i is uncertain, as long as $0 < E[RF_i] \leq \alpha F_i$, where E is the expectation operator, the same reasoning as for the case $RF_i = 0$ applies. Therefore, this section's findings can be summarized with the following (almost) Folk Theorem for one-shot Bertrand oligopolies played under modern Antitrust Law.

Theorem 1 *Suppose Antitrust Law establishes (i) fines F_i against price-fixing firms such that not all collusion is deterred ($(1-\alpha)\pi_i^M \geq \alpha F_i$), and (ii) reduced fines RF_i for firms that spontaneously report information on their cartel, with $0 \leq E[RF_i] \leq \alpha F_i$. Then any price $p^C > c$ that satisfies $(1-\alpha)\pi_i^C(p^C) \geq \alpha F_i$ can be supported in subgame perfect Nash equilibrium in the one-shot Bertrand oligopoly.*

Theorem 1 is not a "complete" Folk Theorem in the sense that prices in the interval $c < p < \underline{p}$, with $\underline{p} = (1-\alpha)\pi_i(\underline{p}) - \alpha F_i$, are not supportable in equilibrium. This restriction of the equilibrium set, however, cuts out only the *less* profitable collusive agreements, therefore it facilitates coordination on most collusive ones. One can state the following corollary without proof.

Corollary 1 *As long as $(1-\alpha)\pi_i^M \leq \alpha F_i$ remains satisfied, increases in α and in F_i*

(i) increase the range of reduced fines that enforce collusion in the one-shot Bertrand game (i.e. that satisfy $E[RF_i] \leq \alpha F_i$), and

(ii) facilitate coordination by restricting the set of agreements supportable in equilibrium to the most collusive ones.

Increases in the size of fines or in the intensity of investigation will increase collusion deterrence, reducing the number of oligopolies for which $(1-\alpha)\pi_i^M \geq \alpha F_i$ does not hold. However, they will also simultaneously worsen the counterproductive effect of Theorem 1 in oligopolies where, after the increase in fines, it still holds $(1-\alpha)\pi_i^M \geq \alpha F_i$.

3 Discussion and Extensions

3.1 Robustness

3.1.1 Timing, damages, and restitution

For the pro-collusive effect to obtain, it is crucial that a firm that deviates and simultaneously self-report cannot realize or retain the gains from the unilateral deviation.

In our model this cannot happen because the deviating firm must self-report immediately when setting its price (at Stage 3) otherwise the other firms do it (at Stage 4). Such early reporting allows the Antitrust Authority to nullify the pricing game before any gain from deviation realizes. Timing 2, in particular its Stage 4, follows naturally from Bertrand's (1883) original framework where sellers could immediately observe each other's price and react (several times) before the game is over.[6] However, previous section's results (as the ones in the forthcoming sections) are remarkably robust: we can fully dispose of Stage 4, and still get the Folk Theorem(s). Absent Stage 4, if a deviation occurs at Stage 3 non-deviating firms would still find it convenient to report at Stage 6. And even if a deviating player would self-report just before Stage 6, that is, after sales took place and gains from deviation realized, he would anyway lose these gains by the assumption that when a cartel is successfully prosecuted all supracompetitive profits are lost in damage suits. This assumption is close to reality. Under the current US's and EU's legislations firms that self-report and obtain leniency remain fully exposed to buyers' damage suits, as any other member of the prosecuted cartel. Being the only firm who profited from the collusive agreement, the deviating firm will be the first to be forced to pay back supracompetitive profits as damages (plus legal costs). But even if we were to fully abstract from damage suits – assuming, say, that free riding problems undermine buyers' ability to suit cartel members – the correct assumption would still be that a deviating firm that self-reports loses the realized supracompetitive profits, so the Folk Theorem(s) would remain. This is because, at least in the US, leniency programs normally require "restitution." That is, in order to obtain amnesty self-reporting firms are *required* to pay back to buyers all extra profits linked to the collusive agreement (unless this would drive the firm bankrupt; see Section A5 and B of US's Corporate Leniency Policy, and Spratling, 1998).

3.1.2 More than two firms

It is easy to verify that when $N > 2$ everything works as in the duopoly case, again, as long as $E[RF_i] \leq \alpha F_i$. A difference that one must take into account when $N > 2$ is that, according to both the US's and the EU's leniency programs, only the first firm that reports information on a cartel gets the reduction in sanctions. Then, if at Stage 3 a firm undercuts the collusive price, none of the other firms can be sure to get the reduced fine by self-reporting at Stage 4. Though, everything goes through

[6] Bertrand writes: *"...one of the proprietors will reduce price to attract buyers to him, and [the] other will in turn reduce his price even more to attract buyers back to him. They will stop undercutting each other in this way, when either proprietor, even if the other abandoned the struggle, has nothing more to gain from reducing his price"* (as quoted in Baye and Morgan, 1999). In the case of procurement and other public auctions it will be even more natural to assume that bidders participating to a "ring" can self-report and nullify the auction's outcome before any profits realize. There a minimum time span *must* usually elapse after the auction before sales or procurement contracts can be formalized, exactly to leave bidders the time to observe the outcome of the contest and file any complaint they deem appropriate (see Section 4.1) .

unchanged because, given that $E[RF_i] \leq \alpha F_i < F_i$, for each of these firms it is a dominant strategy to report information to the Antitrust Authority. To verify this suppose that if, after a firm deviated at Stage 3, M firms simultaneously report at Stage 4, then each of these firms gets the reduction in fines with equal probability $\frac{1}{M}$ (other probability distributions would lead to the same conclusion). Let $\widetilde{M}_i$ denote firm i's belief about the number of other firms (or the actual number of other firms) reporting at Stage 4, with $0 \leq \widetilde{M}_i \leq N-1$. Then if it reports information firm i's expected payoff is $\frac{F_i - E[RF_i]}{\widetilde{M}_i+1} - F_i$, if it does not its expected payoff is $-\alpha F_i$ if $\widetilde{M}_i = 0$, and $-F_i$ when $\widetilde{M}_i > 0$. Since $E[RF_i] \leq \alpha F_i$, the payoff from reporting is never smaller than that from not reporting, and it is strictly larger for any $\widetilde{M}_i > 0$. That is, after a deviation has taken place the choice of reporting dominates that of not reporting for each of the non-deviating firms, independent of their number.[7]

3.2 Bertrand-Edgeworth oligopolies

Price competition leaving firms with zero profits is not the most realistic outcome for a model of oligopolistic interaction. This is why many have considered Bertrand's model as a puzzle, or as a benchmark from which to start the analysis of features of the real world that soften price competition. Edgeworth (1897) was the first to note that if firms have capacity constraints, price competition would lead to positive profits. What is the effect of leniency programs if the price-setting oligopoly is *á la* Bertrand-Edgeworth, if firms face capacity constraints? The main feature that distinguishes Bertrand-Edgeworth oligopolies from pure Bertrand ones, that in the formers profits both at firms' minimax and at the Nash equilibrium (typically in mixed-strategies) are strictly positive, turns out to reinforce the results of the previous section.

Let π_i^N denote firm i's profits at the standard Nash equilibrium, $\pi_i^{\min}$ denote firms' minimax profits (obtained by setting the monopoly price on the firm's residual demand, i.e. on total demand minus the capacity of the other firms), and $\underline{\pi}_i(p^C)$ the firm's profit when it sticks to an agreed collusive price $c < p^C \leq p^M$ but another firm undercuts it, where $\pi_i^N > \pi_i^{\min} \geq \underline{\pi}_i(p^M) > 0$.[8] Under the previous section's assumptions amended for $\pi_i^N > 0$ (e.g., assuming $\alpha' F' > (1-\alpha')\pi_i^M - \pi_i^N$, and so on) firms will be willing to collude only if

$$E_{t=1}[\pi_i(p_i = p_j = p^M)] - \pi_i^N \geq 0, \;\; \Rightarrow \;\; (1-\alpha)\pi_i^M - \alpha F_i \geq \pi_i^N.$$

Suppose this condition is satisfied. Then I can state a result analogous to Theorem 1.

[7]Moreover, if one introduces risk consideration, the procollusive effect leniency programs is reinforced by an increase in the number of colluding firms. This is because when $N > 2$, after a deviation has taken place each non-deviating firm may prefer to report even when $E[RF_i] > \alpha F_i$, to avoid the risk of getting $-F_i$ because another of the non-deviating firms reports. Then the Folk Theorem of the previous section applies even when $E[RF_i] > \alpha F_i$.

[8]It is $\pi_i^{\min} \geq \underline{\pi}_i^M$ because, depending on the shape of demand, the monopoly price on residual demand may differ from the joint monopoly price.

Theorem 2 *Suppose Antitrust Law establishes (i) finite fines F_i against price-fixing firms such that not all collusion is deterred ($(1-\alpha)\pi_i^M - \alpha F_i \geq \pi_i^N$), and (ii) reduced fines RF_i for firms that spontaneously report information on their cartel, with $E[RF_i] \leq \alpha F_i + \pi_i^N - \underline{\pi}_i(p^M)$. Then any price $p^C > c$ that satisfies $(1-\alpha)\pi_i^C(p^C) - \alpha F_i \geq \pi_i^N$ can be supported in subgame perfect Nash equilibrium in the one-shot Bertrand-Edgeworth oligopoly.*

Proof. Please see the Appendix.

Theorem 2 differs from Theorem 1 in two respects. The first one is that now firms are willing to sustain a collusive price p^C only if $(1-\alpha)\pi_i^C(p^C) - \pi_i^N \geq \alpha F_i$, instead of $(1-\alpha)\pi_i^C(p^C) \geq \alpha F_i$. In other words, the positive non-cooperative profits obtainable with Bertrand-Edgeworth competition increase the deterrence power of expected fines αF_i, and facilitate coordination by further restricting the set of supportable agreements to the most collusive ones in oligopolies that satisfy $(1-\alpha)\pi_i^M - \pi_i^N \geq \alpha F_i$.

The second, more important difference is that where $(1-\alpha)\pi_i^M - \alpha F_i \geq \pi_i^N$ is satisfied, leniency programs make collusion supportable in one-shot oligopolies when $0 \leq E[RF_i] \leq \alpha F_i + \pi_i^N - \underline{\pi}_i(p^M)$ instead of $0 \leq E[RF_i] \leq \alpha F_i$. So that, since $\pi_i^N - \underline{\pi}_i(p^M) > 0$, the following corollary follows.

Corollary 2 *The positive non-cooperative profits obtainable with Bertrand-Edgeworth competition widen the range of moderate leniency programs that enforce collusion.*

The positive expected gains from participating to a new, fully non-cooperative pricing game reinforce the pro-collusive effect of moderate leniency programs because they are an additional incentive to self-report at in Stage 4, when the Law enforcing agency can require the distorted price-setting stage to be replicated. Then, even leniency programs with higher expected reduced sanctions $\alpha F_i < E[RF_i] \leq \alpha F_i + \pi_i^N - \underline{\pi}_i(p^M)$ enforce collusion in one-shot interactions.[9]

Finally, although this section focused on capacity constraints, I speculate that the same results should hold for pricing games with decreasing returns.

3.3 "Smooth-deviation games"

A crucial feature behind previous results, present both in the Bertrand and Bertrand-Edgeworth models of oligopoly, is the non linearity of a colluding firm's profits as a function of other firms' strategies. A firm that in one of these models sticks to a collusive price $p^C > c$ receives payoffs

$$\pi_i(p_i^C, p_k) = \begin{cases} \pi_i^C(p_i^C) & \text{if } p_k = p^C, \ \forall k \\ \underline{\pi}_i^C(p_i^C) & \text{if } p_k < p^C, \text{ for some } k \end{cases}$$

[9]In many real world Auctions agents have positive ex ante expected profits from participating, even if bidding is fully non cooperative. As we will see, these positive expected profits will play the same role that π_i^N plays here in ensuring that agents have incentives to punish deviations from agreed collusive strategies by revealing information to the AA even when $\alpha F_i < E[RF_i]$.

with $\pi_i^C(p_i^C) > \underline{\pi}_i^C(p_i^C)$, $\underline{\pi}_i^C(p_i^C) = 0$ with Bertrand competition, and $\underline{\pi}_i^C(p_i^C) > 0$ with Bertrand-Edgeworth competition. This bang-bang property, which as we will see is also a feature of more commonly used auctions, facilitates the pro-collusive effect of leniency programs by limiting a deviating firm's ability to fine tune the loss it induces on other firms.

Other models of strategic interaction, for example those of Cournot's output competition and of differentiated-good price competition, do not share this bang-bang property. In these models, under standard assumptions, if N firms agree on a collusive strategy profile $s^C = \left\{s_1^C, ..., s_N^C\right\}$ and a firm i unilaterally deviates by choosing some $s_i \neq s_i^C$, other (non-deviating) firms' payoffs $\underline{\pi}_j^C(s_{-i}^C, s_i)$, with $j \neq i$, are smooth functions of s_i. This means that the deviating player has the ability, when it deviates at Stage 3 of Timing 2, to determine the exact amount of losses incurred by non-deviating firms with an appropriate choice of s_i. This characteristic of smooth-deviation games has the consequence – in the absence of externalities (see the next section) – of undoing the pro-collusive effect of leniency programs. To see why this is the case, consider first the joint monopoly agreement $s^M = \left\{s_1^M, ..., s_N^M\right\}$, again within Timing 2 (where the word "price" can be replaced by the word "strategic variable"). A firm i that unilaterally deviates from the collusive agreement knows that at least one of the other firms will report information to the Antitrust Authority at Stage 4 as long as $E[RF_j] \leq \alpha F_j + \pi_j^N - \underline{\pi}_j(s_{-i}^M, s_i)$. The deviating firm i can then avoid being punished by choosing an s_i that satisfies a no-reporting constraint, that is, by choosing

$$\begin{aligned} \widehat{s}_i &= \arg\max_{s_i} \; \pi_i(s_{-i}^M, s_i) \\ \text{s.t.} \quad E[RF_j] &\leq \alpha F_j + \pi_j^N - \underline{\pi}_j(s_{-i}^M, s_i) + \epsilon, \end{aligned}$$

where ϵ denotes the smallest monetary unit. The same reasoning applies to any other collusive agreement s^C for which there exist some $s_i \neq s_i^C$ that satisfies the system of inequalities

$$\begin{cases} \pi_i(s_{-i}^C, s_i) > \pi_i^C(s^C) \\ E[RF_j] \leq \alpha F_j + \pi_j^N - \underline{\pi}_j(s_{-i}^M, s_i) + \epsilon \end{cases}.$$

It follows that there exist only one collusive agreement s^{C*} supportable in equilibrium, the one that delivers expected profits

$$E[\pi_i(s^{C*})] = (1-\alpha)\pi_i^{C*}(s^{C*}) - \alpha F_i = \pi_i^N - E[RF_i] + \epsilon,$$

at which any (however small) deviation from the collusive strategies violates the no-reporting condition inducing other firms to punish by reporting. However, as long as $E[RF_i] \geq \epsilon$ this collusive equilibrium implies $E[\pi_i(s^{C*})] \leq \pi_i^N$, while when $E[RF_i] = 0$ it implies $E[\pi_i(s^{C*})] = \pi_i^N + \epsilon$. Since moderate leniency programs have $E[RF_i] \geq 0$, we have proved the following (negative) result.

Theorem 3 *Suppose the market game is such that if a firm i unilaterally deviates from an agreed collusive strategy profile, the other (non-deviating) firms' payoffs*

$\underline{\pi}_j^C(s_{-i}^C, s_i)$, *with* $j \neq i$, *are smooth functions of the deviating firm's strategic variable* s_i. *Then even if Antitrust Law establishes (i) finite fines* F_i *against price-fixing firms such that not all collusion is deterred* ($(1-\alpha)\pi_i^M - \alpha F_i \geq \pi_i^N$), *and (ii) reduced fines* RF_i *for firms that spontaneously report information on their cartel, with* $0 \leq E[RF_i] < \alpha F_i$, *no collusive agreement that improves on the fully non cooperative outcome of more than the smallest monetary unit* ϵ *is supportable in subgame perfect equilibrium in the one-shot market game.*

3.4 Externalities

Suppose there are externalities between firms, in the sense that even if a firm i's profits do not change, firm i looses if a competing firm gains higher profits. In an oligopoly framework this may be due, for example, to top managers being under an incentive contract with relative performance evaluation (in an auction framework many other interpretations are available; see e.g. Jehiel and Moldovanu, 1996; Jehiel *et al.*, 1996). Then, a firm i's objective function would be some function $U_i = U(\pi_i, \pi_{-i})$, with $U_i^1 > 0$, $U_i^2 < 0$, and $U_i^1 > -U_i^2$, where the superscript k denotes the partial derivative with respect to argument k. How would this kind of externalities affect previous results?

Consider first the case of Bertrand competition. It is straightforward to verify that these externalities do not change the equilibrium outcome in the absence of Antitrust Law or of leniency program. And it is almost as straightforward to see that when a leniency programs with $0 \leq E[RF_i] < F_i$ is present, externalities do affect the one-shot game's equilibrium set. In fact, the combination of moderate externalities and moderate leniency programs turn out to make collusion enforceable even in "smooth-deviation games." One can state the following result.

Theorem 4 *Low or moderate externalities enlarge the parameters configuration under which leniency programs enforce collusion in the one-shot Bertrand and Bertrand-Edgeworth games, and allow collusive agreements that improve on non-cooperative play to be supportable in equilibrium in one-shot smooth-deviation games. Sufficiently strong externalities unmake the pro-collusive effect of leniency programs by inducing firms to always report collusive agreements in the attempt to damage rivals.*

Proof. Please see the Appendix.

The intuition behind the first part of the result is that moderate externalities raise non-deviating firms' incentives to report information to the Antitrust Authority after a deviation takes place. This raises a deviating player's cost of preventing reports from non-deviating ones, which makes deviations less profitable and collusion easier to support.

4 Multi-Unit Auctions

4.1 Preliminaries

Previous sections' results on the counterproductive effects of current leniency programs in oligopolies have counterparts in multi-unit auctions. This is important for several reasons. One is that oligopolistic interaction is most often dynamic. Even though what shown in previous sections for one-shot oligopolies also applies to more or less frequently repeated ones, the possibility of sustaining collusion by standard price-war threats may somewhat reduce the practical relevance of the results. Instead, one finds easily examples of (almost) one-shot auctions, where no threat is available to players in the near future that might enforce collusion in alternative to leniency programs. Another reason is that the assumption of homogeneous good and perfectly informed and reactive consumers in Bertrand's and Bertrand-Edgeworth oligopoly models are often violated in reality, while the correspondent assumptions are easily met by real world auctions.

Of course, not all auctions are, or should be subject to Antitrust Law. When a private, monopolistic seller auctions goods to some potential buyers, collusion among the lasts need not always be a concern for Antitrust Authorities, since transfers of monopoly rents from the seller to the buyers can often be regarded as welfare neutral. Still, "bid rigging" is considered a *per se* illegal business practices by both the US's and EU's Antitrust legislations.[10] Bid rigging cases, both in government and private auctions, constitute a major share of total investigations on price-fixing agreements (see e.g. Porter and Zona, 1993).

I focus mainly on multi-unit or divisible goods' auctions because these are becoming more and more common for the sale of public assets (e.g. the Spectrum), and because it is this kind of auctions that is most affected by the design of leniency programs in antitrust (single-unit auctions are discussed is Section 5.2). Also, I restrict attention to first-price (i.e. pay-your-bid) sealed-bid auctions since, as is well known, ascending and second-price seal-bid auctions do not lack means of enforcing collusion.[11]

The literature on collusion in multi-unit auctions begins with Wilson's (1979) famous analysis of auctions of shares, later extended by Back and Zender (1993), which showed that in a one-shot uniform-price auction of a divisible good, bidders

[10] In particular, both in the EC and in the US, government's procurement contests and auctions of state assets are always subject to standard competition law against cartels. The logic behind this is presumably that when governments use auctions to procure public goods, and the auction's competitive mechanism is distorted by collusion, the resulting higher price would induce a suboptimally low provision of the public good, that is, an efficiency loss. Analogously, if governments must finance themselves through distortionary taxation, a reduction in government revenue from auctioning public assets (e.g. in privatizations) due to bid rigging implies relatively higher distortionary taxation.

[11] In ascending auctions deviations can be immediately detected and punished; in second-price sealed-bid auctions deviations are unprofitable, since the collusively agreed winner can bid a very high price. This was first noted by Robinson (1985) and Graham and Marshall (1987) for the case of single-unit auctions, but the same argument applies also to multi-unit auctions.

can sustain highly profitable collusive outcomes by submitting very steep demand schedules. The literature has been driven by the debate on how the US Treasury should auction its securities, so it has mainly focused on the comparative efficiency of the two auction formats the US Treasury experimented: the discriminatory first-price auction and the uniform-price auctions proposed by Milton Friedman (1960) (see e.g. Chari and Weber 1992; Goswami, Noe and Rebello 1996). A generally accepted conclusion is that one-shot uniform price auctions are subject to collusion – although the practical significance of the problem is debated – while first-price discriminatory auctions are not (see e.g. Nyborg, 1997).

Regarding procurement auctions, Anton and Yao (1989, 1992) showed that in split-award low-price (pay-your-bid) auctions, where two bidders can submit prices both for the whole and for one or more predetermined shares of an award (contract), a form of coordination must take place in split-award equilibria (equilibria at which firms split the contract), and that these equilibria only exist when there is "dual source efficiency," that is, when a split award (dual sourcing) implies strictly lower costs than a unit award (sole sourcing). Moreover, these authors find that rents from coordination are greatly reduced when firms have private information on their production costs.

The remainder of the section shows that all these standard conclusions do *not* apply when auctions are run under the current US's or EU's Antitrust legislations. Then, highly profitable collusive split-award equilibria are sustainable in split-award procurement auctions both with asymmetric cost information and when dual sourcing is highly inefficient, and collusion is easily enforced in first-price multi-objects auctions and discriminatory auctions of shares.

To show this, in what follows I will refer to a new timing, Timing 3, which follows naturally from the US's and EU's public procurement regulations.

– Figure 3 About Here –

Note that in the case of procurement and other public auctions, the *only* realistic assumption is the one incorporated in Timing 3, that when a deviation from collusive bids takes place, non-deviating bidders can report the bid rigging agreement and thereby have the auction nullified and re-run before any real transaction takes place (before any gain from deviation realizes).[12] In public auctions the information on submitted bids is released as early as possible, and some time must elapse before any contract can be formalized. This is typically so for "transparency" reasons, to avoid abuses. It is exactly to allow competitors to observe the outcome of the auction and

[12]In the EU Antitrust Law has horizontal validity, and Article 2 of Council Directive 89/665/EEC (21/12/89) requires Member States to establish review procedures for violations of EU law in awarding public procurement contracts, including measures to "suspend or ensure the suspension for the award of a public contract" (1a) and "set aside or ensure the setting aside of decision taken unlawfully..." (1b). In the US, the Federal Acquisition Regulation (FAR) explicitly requires the "rejection of offers suspected of being collusive" (3.103-2b).

make any complain, or undertake any other admissible legal action (such as reporting about the bid-rigging agreement), before the game is over.[13]

4.2 Split-award procurement auctions

Anton and Yao (1989, 1992) model split-award procurement auctions where a buyer can divide production between two suppliers or award all production to a single one, and where suppliers submit bids (prices) both for the whole production and for split awards.[14] They show that split award equilibria – σ equilibria in their terminology – where a split award obtains for all cost types, exist only when diseconomies of scale determine "dual source efficiency" (DSE), that is, when the joint firms' costs in dual sourcing is lower than a firm's cost when it produces alone (for all cost types, when costs are private information). In their words, "in equilibrium a split award must be the efficient award, and if it is efficient, then a σ equilibrium exists" (1992, p. 692), and "a sole-source outcome is the unique equilibrium outcome whenever sole-source production is efficient" (1989, p. 543). They also show that in split award equilibria prices must be implicitly coordinated, since each firm can unilaterally veto a split by submitting a very high split price (relative to its whole award price). Firms can then support in equilibrium split awards with coordinated prices above costs by setting the whole award prices low enough to make any deviation from the agreed strategies unprofitable. Though, the rents firms obtain from this form of coordination are limited to the cost differential between sole and dual sourcing, and are further reduced by information asymmetries on costs, since with private cost information firms must reduce prices to ensure that deviations are deterred for all cost types.

Suppose that, as in Anton and Yao (1992), two firms can bid for the whole or for a split σ of a procurement contract. Let C_i and $C_{i\sigma}$ denote firm i's cost of producing the whole and the split award respectively, which may or may not be private information for firms. At Stage 1 of Timing 3 bidders may agree to split the award by bidding some high collusive split prices, which will be denoted by $p^c_{i\sigma}$, and some whole award prices, which will be denoted by p^c_i. Let $V_i \geq 0$ denote a firm i's expected gains from participating to the split award auction if this is re-run under close supervision (and played non-cooperatively, by the assumption that expected fines go up to $\alpha' F_i^{'}$) after having been run once and then nullified because a firm reported evidence of a bid rigging agreement.[15] I can now state the next result.

[13] Still, all what we wrote in Section 3.1.1 regarding the robustness of Theorem 1 applies equally to the results in this section.

[14] This auction format has been used extensively by governments in public procurement, particularly in defence and high-tech telecommunications systems, but also in the private sector procurement of computer chips, commercial aircrafts, and other items (see Anton and Yao, 1992).

[15] It is $V_i \geq 0$ because there are no negative externalities and firms can always choose not to bid. The exact value of V_i will depend on the specification of the auction. For example, if one assumes that the form of coordination described by Anton and Yao (1989, 1992) cannot be detected by the Antitrust Authority, whenever there is dual source efficiency it will be $V_i > 0$. If, instead, one assumes that also the form of coordination described by Anton and Yao is subject to detection by the AA

Theorem 5 *Suppose Antitrust Law establishes (i) finite fines F_i against bid rigging firms and (ii) reduced fines RF_i for firms that spontaneously report information on their ring, with $0 \leq RF_i < F_i$. Then, in any split award auction (with full or asymmetric cost information) as long as $V_i - E[RF_i] \geq -\alpha F_i$:*

i) Split award collusive equilibria exist, independent of efficiency considerations;

ii) Any sufficiently high collusive split price can be supported in equilibrium.

Proof. Please see the Appendix.

The intuition behind the result, another "almost" Folk Theorem, is that a split award auction is isomorphic to a Bertrand game from the point of view of this paper. The only way a firm can profitably deviate from a collusive agreement to quote a high split award price and a whole award price equal to the sum of split award ones, is by reducing the whole price and winning the whole award. As in Bertrand oligopoly, this leaves the other firm with zero profits and under the risk of being fined by the Antitrust Authority if this independently detects the (violated) bid rigging agreement. As long as the reduced fines RF_i (or the expected reduced fines) are not too large, the undercut firm will then prefer to immediately self-report, thereby avoiding the risk of being fully fined and getting the chance to bid again when the auction is re-run. This makes deviations from the agreed bids unprofitable and highly collusive split award equilibria supportable, independent of efficiency or informational considerations.[16]

Until now I have disregarded the possibility that the buyer set a reserve price. By setting a reserve price, the buyer is of course able to reduce bidders' ability to collude. Let r denote the buyer's reserve price, so that for a whole award bid to be accepted it must be $p_i \leq r$, and for a split award bid to be accepted it must hold $p_{i\sigma} + p_{j\sigma} \leq r$. Let $V_i^0 \geq 0$ denote firm i's expected payoff from the auction when in Stage 1 it chooses not to rig bids. For simplicity, suppose that the form of collusion studied by Anton and Yao is subject to detection and prosecution exactly as the form of collusion considered by Theorem 5 is. Then one can state the following corollary.

Corollary 3 *Suppose in the above split award auction the buyer sets a reservation price r. Define $\underline{r}$ as*

$$\underline{r} = C_{i\sigma} + C_{j\sigma} + \frac{V_i^0 + V_j^0}{1-\alpha} + + \frac{\alpha(F_i + F_j)}{1-\alpha}.$$

Then:

with probability α, then it may well be $V_i = 0$, since we assumed that then the expected fines $\alpha' F_i'$ are high enough to deter any detectable form of collusion. It is easy to verify that Theorem 5 holds under any of these alternative specifications of the model.

[16]Theorem 5 holds independent of the number of feasible splits defined by the buyer. If there are more possible splits, then a firm could try to deviate only "partially," by taking a larger split but still leaving enough collusive rents to the non-deviating firm to refrain it from reporting. However, firms can exclude this possibility by bidding very high (very low, when bidders are buyers) or not bidding at all for splits different from the collusively agreed one, much like in Wilson's (1977) collusive equilibria in uniform-price auctions of shares (see Section 4.3).

(i) When $r < \underline{r}$ no collusive split award equilibrium exists.

(ii) When $r \geq \underline{r}$ any collusive split award with prices $\underline{r} \leq p_{i\sigma} + p_{j\sigma} \leq r$ can be sustained in equilibrium.

Proof. Please see the Appendix.

4.3 Discriminatory auctions of shares

Consider a discriminatory (first-price, pay-your-bid) sealed bid auction of a finitely or infinitely divisible good G. Let each of N bidders be allowed to submit a bid function (or inverse demand function) $\mathbf{p}_i(q_i)$ indicating the price $p_i(q_i')$ bidder i offers to pay for each quantity/share q_i' of G. No constrains are put on bid functions. The functions $\mathbf{p}_i(q_i)$ can be defined on the superset of the infinite possible shares, when the good is perfectly divisible and bidders have no restrictions on the number of bids they can submit, as in Wilson's (1977) original model. Alternatively, the bid functions can consist of a finite number of price/quantity combinations, either because the seller restricts the number of bids a bidder can submit (as in the Italian and British Treasury auctions) and/or predetermines a finite set of shares on which bids are admitted; or because the good itself can only be divided in a finite number of ways. Let again $V_i \geq 0$ denote a bidder i's expected gains from participating to the auction if this is re-run under close supervision (and played non-cooperatively by the assumption that expected fines go up to $\alpha' F_i^{'}$) after having been run once and then nullified because a bidder reported evidence of a bid rigging agreement.[17] At Stage 1 of Timing 3 bidders may agree on a collusive allocation of the objects. Let $\Theta \equiv \{(q_1^c, p_1^c), ..., (q_N^c, p_N^c)\}$ denote a collusive allocation, where $q_i^c > 0$ denotes the share of the object allocated to bidder i, and $p_i^c > 0$ denotes the collusive price to be submitted by i for q_i^c. Also, let $\upsilon_i(q_i)$ denote bidder i's (nondecreasing) valuation of a share q_i of G. An allocation Θ will then be defined *exhaustive* when $\sum_i q_i = G$, and will be defined *sufficiently profitable* when it satisfies $(1-\alpha)(\upsilon_i(q_i^c) - p_i^c) - \alpha F_i \geq V_i - E[RF_i]$, $\forall_i$.[18] I can now state the next result.

Theorem 6 *Suppose Antitrust Law establishes (i) finite fines F_i against bid rigging bidders and (ii) reduced fines RF_i for bidders that spontaneously report information on their ring, with $0 \leq RF_i < F_i$. Then, as long as $V_i - E[RF_i] \geq -\alpha F_i$, any exhaustive and sufficiently profitable collusive allocation can be supported in equilibrium in a discriminatory sealed-bid auction of shares.*

Proof: Please see the Appendix.

The intuition for this theorem is analogous to that behind the previous one. Bidders can agree on a collusive division of the object, and submit bidding schedules such

[17] As in the previous section, it is $V_i \geq 0$ because bidders can always choose not to bid, and the exact value of V_i will depend on the specification of the auction.

[18] Note that the condition can always be satisfyed by choosing sufficiently low p_i^c.

that any profitable deviation causes some player to report to the Antitrust Authority. Of course, also in this case the seller may limit losses from collusion by setting a reservation price r for G. In fact, we could state a corollary similar to Corollary 2 since it is easy to verify, along the lines of the proof of Theorem 6, that as long as $(1-\alpha)(\upsilon_i(q_i^c) - rq_i^c) - \alpha F_i \geq V_i - E[RF_i]$, splitting the object at the reservation price can be supported as a collusive equilibrium.

4.4 Other multi-unit auctions

The previous result does not immediately extend to auctions where a limited number of distinguishable goods are on sale (different goods, or homogeneous but individually identifiable goods). This is because in this last case bids are linked to a specific set of the objects on sale, not only to a generic fraction of them. Consider, for example, the Spectrum Auctions. Typically, firms can submit bids on one or more specific element of a set of pre-defined (and more or less homogeneous) frequencies, but not on "any share q" of the auctioned spectrum. To extend our results to these multi-unit auctions, let there be a set Ω of countably many objects, $\Omega \equiv \{1, 2, ..., n\}$ to be sold simultaneously by a seal-bid first-price auction, and let ω denote a subset of Ω, so that $\omega \subseteq \Omega$. Assume N buyers can submit a schedule of bids $\mathbf{p}_i(\omega\ )$ defined on the set of subsets of Ω on which bids are allowed (again, the partitions of Ω on which bids are allowed may or may not restricted by the seller). As usual, $V_i \geq 0$ will denote buyer i's expected gains from participating to the auction if this is re-run under close supervision (and played non-cooperatively by the assumption that expected fines go up to $\alpha' F_i^{'}$) because a bidder reported evidence on a bid rigging agreement. Let again $\Theta \equiv \{(\omega_1^c, p_1^c), ..., (\omega_N^c, p_N^c)\}$ denote a collusive allocation, where $\omega_i \neq \varnothing$ denotes the subset of objects allocated to buyer i, and $p_i^c > 0$ denotes the collusive price to be submitted by bidder i for ω_i^c. Let Θ be defined as *exhaustive* when $\bigcup_i \omega_i^c = \Omega$ and $\bigcap_i \omega_i^c = \varnothing$. Finally, let $\upsilon_i(\omega_i)$ denote bidder i's valuation of the subset ω_i, and let a collusive allocation Θ be defined as *sufficiently profitable* when it satisfies $(1-\alpha)(\upsilon_i(\omega_i^c) - p_i^c) - \alpha F_i \geq V_i - E[RF_i]$, $\forall_i$.[19] Once more, an "almost" Folk Theorem obtains.

Theorem 7 *Suppose Antitrust Law establishes (i) finite fines F_i against colluding bidders and (ii) reduced fines RF_i for bidders that spontaneously report information on their ring, with $0 \leq RF_i < F_i$. Then, as long as $V_i - E[RF_i] \geq -\alpha F_i$, any exhaustive and sufficiently profitable collusive allocation can be supported in equilibrium in a multi-unit first-price sealed-bid auction.*

Proof. Please see the Appendix.

Again, by bidding so that for any deviation non-deviant players find it convenient to report, bidders can support any sufficiently profitable collusive allocation in

[19]Note that the last requirement can always be satisfyed by choosing sufficiently low $p_i^{\omega i}$.

equilibrium. And again, if the seller sets reservation prices $r(\omega)$ to limit losses from collusion, bidders can sustain collusive prices $p_i^c = r(\omega_i^c)$ in equilibrium as long as $(1-\alpha)(\upsilon_i(\omega_i^c) - r(\omega_i^c)) - \alpha F_i \geq V_i - E[RF_i]$, $\forall_i$. A remark is in order here.

Remark 1 *For Theorem 7 to hold all active bidders must receive some rent ($\omega_i \neq \varnothing$ $\forall i$), otherwise some of them would not stick to collusive strategies.*

Note that this does not imply that Theorem 7 is only relevant when the number of bidders is smaller than the number of units for sale. It is the number of *colluding* bidders that must be smaller or equal to the number of auctioned objects. It is easy to envisage a situation where there are $M > n$ bidders, and where the $N \leq n$ bidders with highest valuations agree to split the objects at some collusive prices bounded below by the valuation of the remaining $M - N$ bidders.

5 Renegotiation, Side Payments, and Single-unit Auctions

5.1 More on robustness

5.1.1 Renegotiation

An agent who unilaterally deviates from a collusive agreement disciplined by the threat of reporting to the Antitrust Authority will have incentives, right after the deviation, to renegotiate the agreement to try induce non-deviating agents not to carry out the punishment. In the previous sections we disregarded the possibility that after a deviation agents may renegotiate the agreement before information is revealed to the Antitrust Authority. This was in the spirit of McCutcheon (1997), who convincingly argued that any new meeting to renegotiate a collusive agreement – and even more any side payment between cartel members – would increase the probability of being caught and fined by the Antitrust Authority. When this increase is substantial, as it is reasonable to think (the non-competitive outcome should have alerted the Antitrust Authority at the time renegotiation occurs), renegotiation is deterred by the fear of being caught renegotiating and fined.

However, as a theoretical issue it is interesting to consider what happens to our folk theorems when renegotiation is feasible since, as is well known, costless renegotiation undermines the credibility of threats and may destroy all (pure-strategy) collusive agreements in standard supergame-theoretic models (at least in the repeated Bertrand game; see e.g. Farrell and Maskin, 1989). It is easy to realize that costless renegotiation has no bite in our model. To see why, suppose a renegotiation stage is added between Stage 3 and Stage 4, and consider the extreme case where renegotiation is both costless and so secret that it does not raise the probability of been detected by the Antitrust Authority. Still, the threat of reporting to the Antitrust Authority if a deviation takes place remains credible, it is carried out if a deviation takes place.

This is because – in contrast to what happens in more dynamic frameworks – in a one-shot game there is no future choice of actions through which gains from renegotiation could be shared. In our model there is nothing the deviating player can offer at the renegotiation stage to induce non-deviating firms not to report.

This conclusion holds, of course, also for any less extreme assumption on the cost of renegotiation and on its effects on the probability to be detected. On the other hand, being due to the deviating agent's lack of means of payment at the renegotiation stage, this conclusion might change if, besides allowing for renegotiation, we also allow agents to exchange side payments.

5.1.2 Side payments

In the previous sections we disregarded the possibility that agents administer side payments because what we wrote about the effects of renegotiation on the probability of being detected by the Antitrust Authority hold even more for side payments. If it is hard to imagine that the communication required by renegotiation would not affect the probability of being detected, it is much harder to imagine a technology for side payments that would make these completely undetectable in an investigation (this is not only my view; see e.g. Stigler, 1964, p. 46). And even if such a secret side payment technology could be found, it would clearly not be costless.[20] However, as a theoretical benchmark it is useful to consider the most extreme assumption, that renegotiation and side payments are both costless and do not affect the probability of being detected by the Antitrust Authority. With costless side payments a player that deviates at Stage 3 may "bribe" non-deviating players not to report, e.g. leaving them the smallest share of the collusive rent sufficient to make them better-off by not reporting. One would then think that costless renegotiation and side payments would transform the one-shot games discussed in previous sections into "smooth-deviation games," so that Theorem 3 applies and the collusive effect of leniency programs disappears. Does the pro-collusive effect of leniency programs indeed disappear under the extreme assumptions above?

The answer is a qualified no. Even when renegotiation and side payments are both costless and do not raise the probability of being detected by the Antitrust Authority, the procollusive effect of leniency program may survive. This is because costless and riskless side payments transform the one-shot games discussed in previous sections into "smooth-deviation games" only if two additional assumptions also hold:

(a) that the side payments a deviating player secretly administers to non-deviating ones are recognized by the Antitrust Authority (and by the courts in the following damage suits) as shares of the collusive rent *with probability one*;

(b) that non-deviating players have no bargaining power whatsoever in the renegotiation stage that follows a deviation.

[20] In general, where the cost and risk of administering illegal side payments are not prohibitive, one would expect firms to invest in secrecy-enhancing technology up to where the cost of a marginal increase in secrecy outsets the gain from a marginal reduction in the probability of detection.

Only if assumption (a) holds a non-deviating player is sure to lose the side payment if it reports, and only if assumption (b) holds the deviating player can prevent non-deviating ones reporting by paying them only the minimum amount that makes them prefer not to report, as it is necessary for Theorem 3 to apply. These two assumption must simultaneously hold for costless and riskless renegotiation and side payments to transform "bang-bang" games like Bertrand competition or multi-unit auctions into "smooth-deviation games." However, both these assumption are quite unrealistic. Regarding assumption (b), there is no particular reason why the deviating player should have *all* bargaining power at the renegotiation stage. Considering that the deviating player lose more if renegotiation is unsuccessful, any more or less balanced bargaining game would leave non-deviating players with some of the surplus.[21] Regarding assumption (a), it is not clear that a secret side-payment administered after a deviation from collusive strategies occurred would be easily recognized as a share of the collusive profits and requested back by the Antitrust Authority or in damage suits. Remember that to be feasible these payments must be so secret that they do not affect the probability of the cartel to be detected. A non-deviating firm that receives such a secret side payment would then have a good chance to keep it even after it self-reports to the Antitrust Authority, and this would require higher "bribes" to prevent non-deviating firms from reporting.

To summarize, the possibility of renegotiation and side payments turns all one-shot games into "smooth-deviation games" if and only if the following four assumption hold simultaneously:

(a) *ex post* side payments are recognized as shares of the collusive rent with probability one;

(b) non-deviating players have no bargaining power in the renegotiation stage that follows a deviation;

(c) renegotiation and side payments are costless;

(d) renegotiation and side payments do not affect the probability of the cartel being detected by the Antitrust Authority.

If at least one of these condition is violated the pro-collusive effect of leniency programs discussed in previous sections applies.

5.2 Single-object auctions

Much previous theoretical work on bid rigging has focused on auctions of one object. However, the enforcement of collusion in one-shot single-object first-price auctions remains a problem. The authors who have studied bid rigging in first-price auctions have usually appealed to repeated play and similar arguments for the enforcement of collusive strategies (e.g. McAfee and McMillan, 1992; La Casse, 1996). Does the pro-collusive effect of leniency programs extend to single-unit first-price auction?

[21] And even if we let the renegotiation stage be an ultimatum game with the deviating player giving a take-it-or-leave-it offer to one party, we know that a real-world deviating player would leave some of the surplus to deviating ones.

Everything is more difficult for single-unit first-price auctions. The reason is what noted in Remark 1. In one-shot single-unit auctions it is always $N > n = 1$, so there is no way to split the collusive surplus. The only way of sharing collusive gains from a one-shot single-unit auction is therefore to arrange for a system of side payments. But as argued above, side payments are problematic. First of all, they may increase the probability of being caught at the point to make collusion too risky. Second, even if they would not, they should be arranged in such a way that the previous section's assumption (a) is satisfied, otherwise bidders who do not get the object might report even if a deviation did not take place. Third, side payments may destroy the collusive mechanism by turning the game into a smooth-deviation one. In particular, in the case of single-unit auctions also costly side-payments or costless side-payments that increase the probability of collusion being detected could turn the game into a smooth deviation one. This is because in these auctions the costs of side payments are not a deterrent for deviations. Here side payments must be administered – and the relative cost must be born – independently of whether a deviation occurs.

Let $\Pi > 0$ be the common and deterministic value of an object to be sold through a first-price sealed-bid auction to one among N identical potential buyers, and let p_i denote the price submitted by bidder i. Since the value Π is common knowledge, if played non-cooperatively this auction leads to a kind of Bertrand competition which drives up bids up to the Nash equilibrium level of $p_i^N = \Pi$ which implies $\pi_i^N = V_i = 0$ for all i, where π_i denote player i's payoff and π_i^N the payoff at the Nash equilibrium.

Consider now a collusive agreement to let player i win the object at the collusive price $p_i^C < \Pi$, and let T_j^C denote the monetary side transfer from player i to player j by which the collusive surplus is shared. Suppose administering these side transfers is costless and takes place between Stage 3 and Stage 4 of Timing 3 (as argued above, costly side transfers would lead to the same results here). Let us focus on symmetric collusive agreement, so that $T_j^C = T^C \; \forall j \neq i$. Since the collusive surplus is $\Pi - p_i^C$ and in a symmetric split each bidder must receive the same payoff, the monetary transfer will be $T^C = \frac{\Pi - p_i^C}{N}$. Collusion will be too risky unless

$$(1-\alpha)\left(\frac{\Pi - p_i^C}{N}\right) - \alpha F_i \geq 0, \quad \Rightarrow (1-\alpha)T^C \geq \alpha F_i,$$

where for notational convenience we are assuming that α already incorporates the effects of side payments on the probability of detection (or that there is no such effect).

Suppose this condition is satisfied, and let β denote the probability with which – if the collusive agreement is detected because a firm reported – the side payments the reporting firm received are recognized as shares of the collusive rent and required back by the Antitrust Authority (or by the court in damage suits).

Assuming $\beta < 1$, as is realistic to do, has two consequences on the model. First, even if no deviation occurred, a player $j \neq i$ would find it optimal to self report after having received the side payment as long as

$$(1-\beta)T^C - E[RF_i] > (1-\alpha)T^C + \alpha[(1-\beta)T^C - F_i],$$

$$\Rightarrow T^C < \frac{\alpha F_i - E[RF_i]}{(1-\alpha)\beta}.$$

Therefore a collusive agreement delivering per-bidder profits T^C can be sustained only as long this inequality is not satisfied, that is, as long as $T^C \geq \frac{\alpha F_i - E[RF_i]}{(1-\alpha)\beta}$.

Second, if bidder i, after having won the object at the collusive price p_i^c, deviates by administering lower than agreed side payments $T' < T^C$, at Stage 4 the other players find it convenient to punish the deviation by reporting as long as

$$(1-\beta)T' - E[RF_i] \geq (1-\alpha)T' + \alpha[(1-\beta)T' - F_i],$$

$$\Rightarrow T' \leq \frac{\alpha F_i - E[RF_i]}{(1-\alpha)\beta},$$

so the threat of reporting is credible as long as $T' \leq \frac{\alpha F_i - E[RF_i]}{(1-\alpha)\beta}$.

It is now possible to state the last result of this paper.

Theorem 8 *Suppose Antitrust Law establishes (i) finite fines F_i against colluding bidders and (ii) reduced fines RF_i for bidders that spontaneously report information on their ring, with $0 \leq RF_i < F_i$. Then:*

(i) As long as last section's condition (a) is not satisfied, so that $\beta < 1$, profitable collusive prices are supportable in subgame perfect Nash equilibrium in the above one-shot sealed-bid first-price auction, and the highest supportable collusive profits are decreasing in β;

(ii) When $\beta = 1$ the first-price single-object auction becomes a smooth-deviation game and Theorem 3 applies.

Proof. Please see the Appendix.

6 Concluding remarks

The models above showed that leniency programs for price-fixing firms that spontaneously report information to the Antitrust Authority, as they are currently designed and implemented in the US and in the EU, may be highly counterproductive. These results seem relevant for competition policy in general, but also for the current debate on the optimal design of Spectrum Auctions. For example, a standing proposal to limit collusion in the simultaneous ascending auction used in the US is to replace the current closing rule – a round where no bids are submitted – with one last round of first-price sealed bidding (see e.g. Cramton and Schwartz, 2000). The cost of this change would be a reduction in bidders' flexibility, which is important for an efficient assembling of groups of licences. Theorem 7 shows that there may be no benefit to balance this cost: under current Antitrust Law sealed-bid auctions need not solve the collusion problems.

The counterproductive effect of leniency programs discussed in this paper can be removed in a number of ways. A first, radical way is to cancel leniency programs for firms that spontaneously self-report. Since the current moderate leniency programs have practically no deterrence effects on long-run collusive transactions (Spagnolo 2000), this way of eliminating the counterproductive effect is costless.

However, there is a better way to eliminate the counterproductive effect, one which would also increase cartel deterrence: to *reward* (the first) firms that self-report, e.g. to allow them to cash (part or all of) the fines eventually paid by the other colluding firms. With sufficiently high rewards the counterproductive effect goes away, since firms would always prefer to report, so no collusion would sustainable in one-shot games. And letting a reporting firm cash the fines paid by other conspirators also undermines trust and steeply increases deterrence of long-term collusive arrangement (again, see Spagnolo, 2000).

An alternative and strictly related way of eliminating the counterproductive effect is that of protecting firms that receive leniency from their customers' damage suits and of not requiring restitution as a precondition to obtain leniency (as it is currently done in the US; see Spratling 1998). Then, deviating from a collusive agreement and simultaneously reporting to the Antitrust Authority becomes such a profitable strategy that collusion is no more sustainable in one-shot interactions.

Following Stigler (1964), one would think that postponing the revelation of information regarding firms' actions could also make collusion harder, by making deviations less observable.[22] However, this would not work here. In most of the auctions' settings discussed above it is sufficient to know whether one's own bid was successful to understand whether a deviation occurred. Moreover, reducing the level of "transparency" in the auctions could worsen other problems (corruption, abuses, etc.).

To conclude, there is a final obvious way to eliminate the counterproductive effects of leniency programs discussed in this paper and simultaneously increase cartel deterrence: to raise sanctions against colluding firms. Sufficiently high expected sanctions would make collusion too risky, independent of the shape of leniency programs.

[22]Regarding public procurement auctions, Stigler writes: "*The system of sealed bids, publicly opened with full identification of each bidder's price and specification, is the ideal instrument for the detection of price cutting.*" (1964, p. 48).

7 Appendix

Proof of Theorem 2. Consider again Timing 2, backwards. If firms agreed at Stage 1 to both quote the monopoly price, respected the agreement at Stage 3, did not report information to the Antitrust Authority before, and were not caught by the Antitrust Authority at Stage 7, they do not gain by reporting information at Stage 8 as long as $\pi_i^N - E[RF_i] \leq \pi_i^M$, which is always satisfied (by the assumption that $RF_i \geq 0$). Colluding firms cannot gain by reporting information at Stage 6 as long as $\pi_i^N - E[RF_i] \leq (1-\alpha)\pi_i^M - \alpha F_i$, which is also always satisfied (by the assumptions $E[RF_i] \geq 0$ and $(1-\alpha)\pi_i^M - \alpha F_i - \pi_i^N \geq 0$). The same reasoning applies to the choice of reporting at Stage 4. If at Stage 3 a firm undercuts the agreed monopoly price and does not simultaneously report, the other firm expects payoffs $\underline{\pi}_i(p^M) - \alpha F_i$ if it does not report in the following Stages (4 or 6), $\pi_i^N - E[RF_i]$ if it reports at Stage 4 (when the price setting stage can be replicated), and just $-E[RF_i]$ if it reports only at Stage 6. Therefore, if at Stage 3 a firm undercuts without reporting, at Stage 4 the other firm(s) finds it convenient to report information to the Antitrust Authority as long as $\pi_i^N - E[RF_i] \geq \underline{\pi}_i(p^M) - \alpha F_i$. Then a firm cannot gain by undercutting the agreed monopoly price at Stage 3, since doing it would lead to a payoff of at best $\pi_i^N - E[RF_i]$ (if the undercutting firm simultaneously reports) and at worst of $\pi_i^N - F_i$ (if it does not), and $\pi_i^N - F_i < \pi_i^N - E[RF_i] \leq (1-\alpha)\pi_i^M - \alpha F_i$ (by the assumptions $(1-\alpha)\pi_i^M - \alpha F_i - \pi_i^N \geq 0$ and $E[RF_i] \geq 0$). Given that the collusive agreement would be respected from Stage 3 on, firms could not gain by reporting the agreement to the Antitrust Authority at Stage 2, thereby getting $\pi_i^N - E[RF_i]$ instead of $(1-\alpha)\pi_i^M - \alpha F_i \geq \pi_i^N$. It follows that as long as $F_i > 0$, $(1-\alpha)\pi_i^M - \alpha F_i \geq \pi_i^N$ and $\pi_i^N - E[RF_i] \geq \underline{\pi}_i(p^M) - \alpha F_i$, (joint) monopoly pricing is a subgame perfect Nash equilibrium outcome of the one-shot Bertrand-Edgeworth oligopoly game.

Along the same lines one can verify that an analogous reasoning applies to any other collusive price $p^C \neq p^M$ that satisfies $-F_i < 0 \leq (1-\alpha)\pi_i^C - \alpha F_i - \pi_i^N$, therefore any such price is also a subgame perfect Nash equilibrium of the Bertrand game as long as $\pi_i^N - E[RF_i] \geq \underline{\pi}_i(p^C) - \alpha F_i$. ■

Proof of Theorem 3. Consider the Bertrand game of Section 2, and again Timing 2 backwards. If firms agreed at Stage 1 to both quote the monopoly price, respected the agreement at Stage 3, did not report information to the Antitrust Authority before, and were not caught by the Antitrust Authority at Stage 7, they do not gain by reporting information at Stage 8 as long as

$$U_i(-E[RF_i], -F_{-i}) \leq U_i(\pi_i^M, \pi_{-i}^M).$$

If the externality is so strong that this inequality is violated no collusive equilibrium is sustainable, since firms always prefer to report information on the agreement to the Antitrust Authority to get their competitors fined. If the externality is not that strong, so that the inequality above is satisfied, then colluding firms would not report at Stage 8. Consider now colluding firms' incentive to report at Stage 6. Firms loose by reporting information at Stage 6 as long as

$$U_i(-E[RF_i], -F_{-i}) \leq U_i((1-\alpha)\pi_i^M - \alpha F_i, (1-\alpha)\pi_{-i}^M - \alpha F_{-i}).$$

Therefore, again, if the externality is so strong that this condition is not satisfied, no collusive agreement is supportable in the one-shot Bertrand game, since firms always find it convenient to report information at Stage 6. Suppose again that the externality is not that strong. Then in Stage 6 colluding firms would not report in the absence of a deviation in previous stages. The same reasoning for Stage 6 applies to the choice of reporting at Stage 4 when no deviation occurred in Stage 3. However, if at Stage 3 a firm i undercuts the agreed monopoly price and does not simultaneously report to the Antitrust Authority, then a non-deviating firms j gets zero profits and utility $U_j(-E[RF_j], -F_{-j})$ if it reports information in the following Stages (4 or 6), $U_j(-\alpha F_{-i}, N\pi_i^M - \alpha F_i)$ if no firm reports, and $U_j(-E[RF_k], -F_{-k})$ with $k \neq j$ if another firm reports first, with

$$U_j(-\alpha F_{-i}, N\pi_i^M - \alpha F_i) > U_j(-E[RF_k], -F_{-k}).$$

It follows that if at Stage 3 a firm undercuts without reporting, at Stage 4 (or 6) non-deviating firms report information to the Antitrust Authority (as a dominant strategy) as long as

$$U_j(-E[RF_j], -F_{-j}) \geq U_j(-\alpha F_{-i}, N\pi_i^M - \alpha F_i).$$

As long as this condition is satisfied, the Folk Theorem of Section 2 applies. Absent externalities, the condition for the Folk Theorem to hold was, instead,

$$0 \leq E\left[RF_j\right] \leq \alpha F_j.$$

By inspection, the first condition is always satisfied when the second is, and that the first condition is also satisfied when the second is not (when $E[RF_i] > \alpha F_i$). I can therefore conclude that when the externality is so strong that

$$U_i(-E[RF_i], -F_{-i}) \geq U_i((1-\alpha)\pi_i^M - \alpha F_i, (1-\alpha)\pi_{-i}^M - \alpha F_{-i}),$$

it destroys all collusive agreements in the one-shot Bertrand game by inducing firms always to report in the attempt to damage competitors. When the externality is present but it is not that strong, then the Folk Theorem for the one-shot Bertrand game holds with a larger set of leniency programs than in the absence of externalities, with all leniency programs that satisfy

$$U_j(-E[RF_j], -F_{-j}) \geq U_j(-\alpha F_{-i}, N\pi_i^M - \alpha F_i).$$

Analogous reasoning applies to Bertrand-Edgeworth competition and to smooth-deviation games. In smooth-deviation games with externalities the unique collusive equilibrium supportable in smooth-deviation games, the one that guarantees that any deviation is punished by reporting information, entails

$$EU_i^{C*} = (1-\alpha)U_i\left(\pi_i^{C*}(s^{C*}), \pi_{-i}^{C*}(s^{C*})\right) + \alpha U_i(-F_i, -F_{-i}) = U_i(\pi_i^N - E[RF_i] + \epsilon,\ \pi_{-i}^N - F_{-i}),$$

and as long as $E[RF_i]$ is not too large (or externalities are sufficiently strong) it holds

$$EU_i^{C*} = U_i(\pi_i^N - E[RF_i] + \epsilon,\ \pi_{-i}^N - F_{-i}) > U_i^N(\pi_i^N + \epsilon, \pi_{-i}^N + \epsilon).$$

■

Proof of Theorem 5. Suppose that at Stage 1 of Timing 3 bidders agree to split the award and bid some high collusive split prices $p_{i\sigma}^c$, and whole award prices $p_i^c = p_j^c = p_{i\sigma}^c + p_{j\sigma}^c$ Consider Timing 3 backwards. If firms respected the agreement at Stage 3, did not report information to the Antitrust Authority before, and were not caught by the Antitrust Authority at Stage 5, they do not gain by reporting information at Stage 6 as long as $V_i - E[RF_i] \leq p_{i\sigma}^c - C_{i\sigma}$, which is always satisfied by a sufficiently high $p_{i\sigma}^c$. Colluding firms also cannot gain by reporting information at Stage 4 as long as $V_i - E[RF_i] \leq (1-\alpha)(p_{i\sigma}^c - C_{i\sigma}) - \alpha F_i$, which again is satisfied when agents choose a high enough $p_{i\sigma}^c$.

If at Stage 3 a firm j undercuts the agreed price by bidding $p_j < p_j^C$ (any other deviation is unprofitable), and does not simultaneously report, the other firm gets expected payoffs $-\alpha F_i$ if it does not report in the following Stage (4) and $V_i - E[RF_i]$ if it does. So as long as $V_i - E[RF_i] \geq -\alpha F_i$, if at Stage 3 a firm undercuts without reporting the other firm finds it convenient to report. Then a firm cannot gain by undercutting the agreed monopoly price at Stage 3, since doing it would lead to a payoff of at best $V_i - E[RF_i]$ (if the undercutting firm also simultaneously reports) and at worst of $V_i - F_i$ (if it does not), and $V_i - F_i < V_i - E[RF_i] \leq (1-\alpha)(p_{i\sigma}^c - C_{i\sigma}) - \alpha F_i$ for any sufficiently high $p_{i\sigma}^c$. Given that the collusive agreement would be respected from Stage 3 on, firms could not gain by reporting the agreement to the Antitrust Authority at Stage 2 either, thereby getting only $V_i - E[RF_i]$. The statement follows.
■

Proof of Corollary 3. For the two firms i and j to be willing to rig bids, and thereby risk being caught and fined by the Antitrust Authority, it must hold $(1-\alpha)(p_{h\sigma}^c - C_{h\sigma}) - \alpha F_h \geq V_h^0$ for each of them, so the following pooled incentive constrain must also hold

$$(1-\alpha)(p_{i\sigma}^c - C_{i\sigma}) - \alpha F_i + (1-\alpha)(p_{j\sigma}^c - C_{j\sigma}) - \alpha F_j \geq V_i^0 + V_j^0.$$

Substituting from the maximum collusive split prices condition $p_{i\sigma} + p_{j\sigma} = r$ and rearranging, for any level of collusion to be sustainable the following condition must hold

$$(1-\alpha)(r - C_{i\sigma} - C_{j\sigma}) - \alpha(F_i + F_j) \geq V_i^0 + V_j^0,$$

which leads to

$$r \geq \underline{r} = C_{i\sigma} + C_{j\sigma} + \frac{V_i^0 + V_j^0}{1-\alpha} + + \frac{\alpha(F_i + F_j)}{1-\alpha}.$$

(ii) Suppose that $r \geq \underline{r}$ and that at Stage 1 of Timing 3 the firms agreed to collude and bid prices $\underline{r} \leq p^c_{i\sigma} + p^c_{j\sigma} = p^c_i = p^c_j \leq r$. Then the reasoning in the Proof of Theorem 5 applies unchanged. ■

Proof of Theorem 6. Consider first the case of unconstrained bidding for a perfectly divisible good. Suppose that at Stage 1 of Timing 3, bidders agree on a collusive allocation Θ and to submit bid functions (inverse demand schedules) $\mathbf{p}^c_i(q_i)$ of the following shape:

$$\mathbf{p}_i(q_i) = \begin{cases} p'_i(q_i) \geq \upsilon_i(q_i) & \text{for any } q_i < q^c_i \\ p^c_i(< \upsilon_i(q^c_i)) & \text{for } q_i = q^c_i \\ p''_i(q_i) \leq p^c_i - (\upsilon_i(q_i) - \upsilon_i(q^c_i)) & \text{for any } q_i > q^c_i \end{cases}.$$

Now consider Timing 3 backwards. If bidders respected the agreement at Stage 3, did not report information to the Antitrust Authority before, and were not caught by the Antitrust Authority at Stage 5, they do not gain by reporting information at Stage 6 as long as $p^c_i \leq \upsilon_i(q^c_i) - V_i + E[RF_i]$ which is always satisfied when Θ is sufficiently profitable. Also, if Θ is sufficiently profitable, a bidder cannot gain by reporting information at Stage 4 (by definition $V_i - E[RF_i] \leq (1-\alpha)(p^c_i - \upsilon_i(q^c_i)) - \alpha F_i$).

Consider now a deviation at Stage 3. A bidder j can deviate either by setting $p_j(q^c_j) > p^c_j$ or by setting $p_j(q_j) > p^c_j$ for some $q_j > q^c_j$. The first deviation is unprofitable, since it increases the quantity won by other players while reducing quantity and markup of the deviating player. Suppose bidder j deviates by setting $p_j(q_j) > p^c_j$ for some $q_i > q^c_i$. Since $\sum_i q^c_i = G$ (Θ is exhaustive), there will be at least one bidder i who gets less than q^c_i, thereby earning zero from the auction (since $p'_i(q_i) \geq \upsilon_i(q_i)$ for any $q_i < q^c_i$). Then this bidder expects payoffs $-\alpha F_i$ if it does not report in the following Stage (4), and $V_i - E[RF_i]$ if it does. So as long as $V_i - E[RF_i] \geq -\alpha F_i$, if at Stage 3 a bidder j deviates without reporting at least one bidder finds it convenient to report. Therefore a bidder cannot gain by deviating at Stage 3, since any profitable deviation would lead to a payoff of at best $V_i - E[RF_i]$ (if the deviating bidder also simultaneously reports) and at worst of $V_i - F_i$ (if it does not), and $V_i - F_i < V_i - E[RF_i] \leq (1-\alpha)(p^c_i - \upsilon_i(q^c_i)) - \alpha F_i$ by the definition of a sufficiently profitable Θ. Given that the collusive agreement would be respected from Stage 3 on, firms could not gain by reporting the agreement to the Antitrust Authority at Stage 2 either, since doing it they get only $V_i - E[RF_i]$. The statement follows. ■

Proof of Theorem 7. Suppose that at Stage 1 of Timing 3, bidders agree on a collusive allocation Θ and to submit demand schedules $\mathbf{p}_i(\omega)$ such that $\mathbf{p}_i(\omega^c_i) = p^c_i$, $\mathbf{p}_i(\omega^c_j) = p^c_j + \epsilon$, $\forall j \neq i$ (where ϵ is the smallest monetary unit, or the "tick size" of bids in the auction), and either $\mathbf{p}_i(\omega) = 0$ or $\mathbf{p}_i(\omega) \geq \upsilon_i(\omega)$ $\forall \omega \notin \Theta$. Now consider Timing 3 backwards. If bidders respected the agreement at Stage 3, did not report information to the Antitrust Authority before, and were not caught by the Antitrust Authority at Stage 5, they do not gain by reporting information at Stage 6 as long as $p_i^c \leq \upsilon_i(\omega_i^c) - V_i + E[RF_i]$, which is always satisfied when Θ is sufficiently profitable. Also, if Θ is sufficiently profitable, a bidder cannot gain by reporting information at Stage 4, since by definition $V_i - E[RF_i] \leq (1-\alpha)(\upsilon_i(\omega_i^c) - p_i^c) - \alpha F_i)$.

Consider now a deviation at Stage 3. A bidder j can deviate by setting $\mathbf{p}_j(\omega_i^c) = p_i^c - \epsilon$ for some $i \neq j$, or by bidding a sufficiently attractive price for any other admissible ω' such that $\omega' \cap \omega_i^c \neq \varnothing$ for some $i \neq j$. For any profitable deviation, there will be at least one bidder who does not get the agreed allocation, nor wins positive payoffs. At Stage 4 this bidder expects payoffs $-\alpha F_i$ if he does not report, and $V_i - E[RF_i]$ if it does. So as long as $V_i - E[RF_i] \geq -\alpha F_i$, if at Stage 3 a bidder deviates without reporting at least one bidder will find it convenient to report at Stage 4. Then a bidder cannot gain by deviating at Stage 3, since a deviation would lead to a payoff of at best $V_i - E[RF_i]$ (if the deviating bidder also simultaneously reports) and at worst $V_i - F_i$ (if it does not), and $V_i - F_i < V_i - E[RF_i] \leq (1-\alpha)(\upsilon_i(\omega_i^c) - p_i^c) - \alpha F_i$ by the definition of a sufficiently profitable Θ. Given that the collusive agreement would be respected from Stage 3 on, bidders could not gain by reporting the agreement to the Antitrust Authority at Stage 2 either, since doing it they get only $V_i - E[RF_i]$. The statement follows. ■

Proof of Theorem 8. Since the threat of reporting is credible only if $\beta \leq \beta(T') = \frac{\alpha F_i - E[RF_i]}{(1-\alpha)T'} \Rightarrow T' \leq \frac{\alpha F_i - E[RF_i]}{(1-\alpha)\beta}$, as long as $\beta < 1$ bidders may agree on a collusive price such that the implied T^C satisfies $T^C = \frac{\alpha F_i - E[RF_i]}{(1-\alpha)\beta} + \varepsilon$. Then, for any $T' < T^C$ it is $\beta \leq \beta(T')$ and the punishment is credible for any deviation. To proof statement (i) it remains to be shown that a collusive agreement satisfying $T^C = \frac{\alpha F_i - E[RF_i]}{(1-\alpha)\beta} + \varepsilon$ can be *ex ante* profitable when $\beta < 1$. The non-cooperative payoff is 0. Expected the payoff from colluding is $(1-\alpha)T^C - \alpha F_i$. Profitable collusion is therefore enforceable when there is at least an agreement that satisfies the system

$$\begin{cases} (1-\alpha)T^C - \alpha F_i > 0 \\ T^C = \frac{\alpha F_i - E[RF_i]}{(1-\alpha)\beta} + \varepsilon \end{cases}.$$

Substituting T^C from the second to the first we obtain

$$(1-\alpha)\left(\frac{\alpha F_i - E[RF_i]}{(1-\alpha)\beta}\right) + (1-\alpha)\varepsilon - \alpha F_i > 0,$$

$$\Rightarrow (1-\beta)\alpha F_i - E[RF_i] + (1-\alpha)\beta\varepsilon > 0,$$

which, by inspection, is decreasing in increasing β ($\alpha F_i > (1-\alpha)\varepsilon$) and strictly positive as long as $E[RF_i]$ is not too large.

Statement (ii) follows straightfortwardly from substituting $\beta = 1$ in this last inequality. ■

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Timing 1

1. Firms choose whether or not to enter a collusive agreement
2. Firms simultaneously quote prices
3. Sales take place

FIGURE 1

Timing 2

1. Firms choose whether or not to enter a collusive agreement

   If they don't, the standard Bertrand game is played non-cooperatively

   If they do it, the game moves on to Stage 2

2. Firms choose whether or not to report about the agreement to the Antitrust Authority

   If at least a firm reports, fines are paid and the Bertrand game is played non-cooperatively

   If none does it, the game moves on to Stage 3

3. Firms simultaneously quote prices and choose whether to report to the Antitrust Authority

   If at least a firm reports, fines are paid and the Bertrand game is re-started and played non-cooperatively

   If none does it, the game moves on to Stage 4

4. Firms and consumers observe the quoted prices, then firms choose whether to report to the Antitrust Authority

   If a firm reports, fines are paid and the Bertrand game is re-started and played non-cooperatively

   If none does it, the game moves on to Stage 5

5. Sales take place and profits realize

6. Firms choose whether to report to the Antitrust Authority

   If at least a firm reports, fines are paid and collusive profits are paid as damages

   If none does it, the game moves on to Stage 7

7. If no firm reported before, the Antitrust Authority monitors the market

   With prob. α profits are paid back as damages and fines are paid

   With prob. $(1 - \alpha)$ no fines nor profits are paid, and the game moves on to Stage 8

8. Firms choose whether to report to the Antitrust Authority

   If at least a firm reports, fines are paid and collusive profits are paid as damages

   If none does it, firms earn collusive profits.

FIGURE 2

Timing 3

1. Bidders choose whether or not to enter a bid rigging agreement

   If they don't, the auction is played non-cooperatively

   If they do it, the game moves on to Stage 2

2. Bidders choose whether or not to report about the agreement to the Antitrust Authority

   If at least a bidder reports, fines are paid and the auction is played non-cooperatively

   If none does it, the game moves on to Stage 3

3. The auction takes place

   Bidders simultaneously submit bids and choose whether to report to the Antitrust Authority

   If at least a bidder reports, fines are paid and the auction is nullified, re-run, and played non-cooperatively

   If none does it, the game moves on to Stage 4

4. Bidders observe the outcome of the auction, and then choose whether to report to the Antitrust Authority

   If a bidder reports, fines are paid and the auction is nullified, re-run, and played non-cooperatively

   If none does it, the game moves on to Stage 5

5. If no bidder reported before, the Antitrust Authority monitors the auction outcome

   With prob. α bid rigging is detected, fines are paid, and the auction is nullified, re-run, and played non-cooperatively

   With prob. $(1-\alpha)$ bid rigging is not detected and the game moves on to Stage 6

6. Bidders choose whether to report to the Antitrust Authority

   If at least a bidder reports, fines are paid and the auction is nullified, re-run, and played non-cooperatively

   If none does it, collusive profits realize

FIGURE 3

Non-Technical Summary

This paper questions the current design of *leniency programs*, the reduced-sanctions schemes that encourage price-fixing firms to self-report and cooperate with Antitrust Authorities. Its main finding is that thanks to these programs Antitrust Law against cartels – as currently designed and enforced in the United States and in the European Union – may make perfect collusion enforceable even in occasional market interactions, like one-shot (or infrequently repeated) Bertrand oligopolies and first-price auctions, where little or no collusion would be supportable if there were no Antitrust legislation or, of course, if this legislation was better designed.

The way current Antitrust Law may achieve this paradoxical result is by providing firms with the otherwise missing credible threat necessary to enforce promises of cooperation (collusion) among rivals. The crucial aspect is the size of discounts in sanctions awarded to firms who spontaneously report information on their cartel when the Antitrust Authority has no information on it, before any investigation is opened. The low but weakly positive reduced sanctions for firms that spontaneously self-report ensure that if a firm unilaterally deviates from the agreed collusive strategies, the other firms will find it convenient to punish it by reporting information on the initial agreement to the Antitrust Authority.

By incorporating these features of reality, results similar to "folk theorems" obtain for a number of one-shot market interaction. Collusion becomes enforceable in the homogeneous Bertrand oligopoly, in the Bertrand-Edgeworth (capacity-constrained) oligopoly, in split-award procurement auctions, in discriminatory auctions of shares, in other multi-unit first-price auctions, and even – under some additional conditions – in single-object first-price auctions.