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Option to Revoke and Regulation of Local Utilities.*

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Abstract

We study a long-term relationship between a risk-neutral firm that has been delegated to manage a local utility project and a regulator that has always the option-to-revoke the delegation. We show that when the threat of revocation is credible and the cost of exercising it is not too high, the "cooperative" equilibrium is an efficient solution which guarantees the utility with an appropriate level of return. The regulation timing consists of a endogenous regulatory lag where the regulation has a fixed-price nature followed by a period of rate-of-return regulation in which the firm is motivated to adjust its output price downward to avoid revocation. We also show that excessive revocation costs make the firm an unregulated monopolist with an infinite regulatory lag.

JEL: C73, L33, L51

Key words: Public utilities, Option-to-revoke, Stochastic games.

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1 Introduction

This paper investigates a long-term contract between a private firm and a regulator that has the option-to-revoke the contract. The problem we address stems from the recent trend to delegation in managing public utilities such as sewage and fresh water management, urban waste infrastructures and provision of public transport. These markets are usually supplied by local authorities which can decide to delegate their provision to a private company as alternative to direct management or to full privatization. The delegation is done under specific institutional features and determines the partial and temporary management of the service on the ground of a contract between the local authority and the private firm; this contract is usually long-term and provides the ongoing opportunity to renegotiate prices and terms (Spulber and Sabbaghi, 1994). Broadly speaking, the ownership of the assets remains in the hand of the collectivity, while "the right to use it" becomes private.

In France - the country with the widespread experience in this field - delegation in local utilities has been fostered by the national legal code: in the water resource sector, for istance, the municipality is in charge of managment of the service and the mayor is personally liable for any damage due to negligence on its part. Moreover, the mayor has been prohibited by law from insuring himself against the risk of damage and against the uncertainty regarding how his negligence will be defined by courts. By delegating the service to a private firm, however, the mayor can offset his personal liability, which is transferred to the delegated firm along with the management of the service. In this framework, the adoption of new technologies is a further incentive for delegation when local authorities cannot easily speed up the required technological change. In fact, technology determines the basic limits on quality of the service and the potential for accidents; consequently, the adoption of new technology represents a relevant element for the mayor's liability in the court *ex-post* decision when an accident occurs (Clark and Mondello, 2000). Hence, delegation becomes the most effective means of restricting the mayor's personal liability in the aim to reduce the full risk of direct management of the local utility.

In other countries, however, delegation is popular with local authorities as an intrument to promote efficiency in the allocation and the managment of the service. Referring again to the water resource sector, delegation finds its main theoretical justification in the aim to get better and/or cheaper services: the delegated firm is potentially capable of injecting technological, financial and managerial resources which the local authority may be unable to come up with because of fiscal and bureaucratic constraint and the lack of adequate incentives (Dosi and Easter, 2000). These features showed to be particularly relevant in developing countries, and recently in Spain and Portugal (OECD, 1999).

In the model we present we consider a risk-neutral firm that has been delegated to manage an indivisible public project and a regulator that has always the right to revoke delegation and return to direct management if the project is a positive net present value investment. The long-term contract signed by the two parties provides for an "allowed" rate of return as a maximum ceiling for the firm and the regulator threatens the firm with revocation of the assignment if that rate is crossed. Revocation is, then, analogous to a perpetual call option where the local authority has the right - but not the obligation - to purchase at any time an asset (the utility) of uncertain value for a present exercise price. The option-to-revoke will be exercised optimally when the rate of return of the project exceeds a critical value (i.e. the allowed rate of return). The trigger value is determined endogenously in the model but the optimal exercise time is stochastic.[1]

We, then, offer an optimal regulation mechanism where the commitment by the regulator to end the contract if the allowed rate of return is exceeded ensures that the private firm will behave consistently with the contract itself: once the firm's costs or production conditions improve, it adjusts prices to keep its rate of return below the allowed one and therefore to prevent revocation. However, as the termination threat is costly, a stochastic regulatory lag may follow over which prices are fixed and its not optimal for the regulator to recall the contract. Based on the above argument, these costs may refer to the sum necessary to overcome the obstacles to renewing direct management of the service such as contractual indemnities on the value of the investment, technological costs, recruiting and training costs, loss of fiscal advantage and, no less important, legal costs if the firm decides to sue the regulator for breaking the contract. Excessive revocation costs make the firm an unregulated monopolist with a infinite regulatory lag.

Our model is closest in spirit to the theory of monopoly regulation in a dynamic setting, in which mechanisms such as rate-of-return (ROR) and

[1] Reffering to the French municipalitis' negotiating disadvantage in the face of some cartelized water management, Clark and Mondello (1997) model the municipality's right to revoke delegation as a call option.

price cap arise endogenously as a self-enforcing and mutually beneficial co-operative equilibrium.[2] However, in this literature both the regulator and the regulated firm share the same bargaining power (i.e. either player has incentive to violate the contract) and they are not affected by regulatory lags. Although playing a crucial role in determining the incentive property of the regulation mechanism, these lags are of fixed time and exogenous. On the contrary, in our setup, the different bargaining positions of the two parties coupled with the regulator's option-to-revoke determine these lags *endogenously* as it is in the essence of ROR regulation (Laffont and Tirole, 1994, p.15). Regulatory reviews are initiated by the local authority when the option-to-revoke is worth to be exercised.[3] The result of an *endogenous* regulatory lag may thus explain the empirical evidence that, although contracts between local authorities and private operators are of limited duration, their renewals are often signed without any variations of contractual terms (Joskow and Schmalensee, 1986, p.7). Finally, we look at the option-to-revoke from the perspective of collective welfare maximization and discuss the specific characteristics of the dynamic regulatory rule stemming from the continuous rate of hearing between the regulated firm and the regulator, as a tool for obtaining a long-term efficient equilibrium.

On a formal level, our paper builds upon two distinct streams of literature. The first one relates to the stochastic control techniques recently developed to identify optimal timing rules and optimal barrier regulations.[4] These techniques have been widely used in the literature of irreversible investments[5], and emphasize the role of the option value of delaying investment decision, i.e. the value of waiting for better (although never complete) information on

[2] See for example Salant and Woroch (1991, 1992) and Gilbert and Newbery (1989).

[3] Our option-to-revoke is similar in spirit to the option-to-own studied by Nöldeke and Schmidt (1998). In a hold up problem in which two parties have to make relationship-specific investments, Nöldeke and Schmidt show that an option-to-own contract where one party owns the firm initially while the other has the option to buy it at a price specified (in the contract) at a later date, induces both parties to invest efficiently. They also show that this result is robust to renegotiation and uncertainty, and that it permits specification of side payments for the joint surplus between the parties.

[4] In particular, we refer to the works of Harrison and Taksar (1983) and Harrison (1985). Applications of this methodology to economic problems can be found in Bentolila and Bertola (1990), Dosi and Moretto (1994) and in a strategic context by Moretto and Rossini (1999), and Moretto (2000).

[5] Much of this literature was recently surveyed by Pindyck (1991), Dixit (1992) and Dixit and Pindyck (1994).

the stochastic evolution of a basic asset.

The second one considers the existence of efficient sub-game perfect equilibria for infinite-horizon-threat-games where, in the absence of a binding commitment, for the threatener it is an equilibrium for the victim to make a stream of payment over time[6]. The expectation of future payment keeps the threatener from exercising its threat. Indeed, we formulate a time-dependent supergame in continuous time, where optimal revocation for the regulator requires identification of the time at which to pay a sunk cost in return for a public project whose value is stochastic. The regulator does not revoke the contract until the revenue that it expects to earn from managing the investment by itself is equal to the expected present value of the rates regulation that the firm announces.

The plan of the paper is as follows: Section 2 sets out the basic assumptions of the model. Section 3 presents the regulator's option-to-revoke. Section 4 examines the regulation that belongs to this scheme. Section 5 discusses the regulatory rule, while the Appendixes collects all the proofs.

2 Basic model and assumptions

We consider the simple relationship between a self-interested-risk-neutral regulator and a risk-neutral firm that was delegated, at $t = 0$, to manage a one-time sunk indivisible public project[7]. Here, what is called "delegation" is the temporary (generally long-term) supply of a public service by a local authority (i.e. a municipality) to some private operator under contractual relationship[8].

We assume that, once set up, the single project allows some flexibility in its operation at each time $t \geq 0$, by varying certain inputs according to the following production function:

$$q_t = a_t l_t^{\varphi} \qquad \text{with } 0 < \varphi < 1 \tag{1}$$

[6]We refer to the works of Klein and O'Flaherty (1993) and Shavell and Spier (1996).

[7]To simplify matters we assume that in doing delegation the local authority becomes *de facto* the regulator for the firm.

[8]In principle, our analysis could be applied to utilities of global range (national utilities), but given our assumption on revocation of the contract by the regulatory body, the local dimension is more realistic (see below). In fact, the management of a contract at national level can affect the regulated firm's bargaining power which, in turn, can affect the regulator's decision to revoke (*regulatory capture*).

where q_t denotes the production at time t, l_t is the operating input such as labor (or some intermediate input) and a_t is a technology-efficiency parameter whose value is determined stochastically. The firm can sell the output at price p_t which is bounded by a ceiling $\hat{p}$, and the operating input is a perfectly flexible factor which can be rented at the instantaneous price w_t whose value is also stochastic. In addition, the firm is faced by a (constant elasticity) demand function:

$$D(p_t) = d_t p_t^{-\mu} \qquad \text{with } \mu \geq 0 \tag{2}$$

where the parameter d_t is an index of the position of the demand curve: it may be a function of the consumers' income or of a price index for substitutes which the firm takes as given in its optimization. The operating cash flow function is defined as:

$$\pi(p_t; a_t, w_t) = \max_{p_t, l_t} p_t q_t - w_t l_t \tag{3}$$

subject to equation (1), (2), $D(p_t) \leq q_t$ and $p_t \leq \hat{p}$. We abstract from production decision as well as from market uncertainty by considering the case of a steep demand function (i.e. $\mu \to 0$).[9] This leads to the following expression for the operating cash flow:

$$\pi(\hat{p}; \theta_t) = \Pi(\hat{p})\theta_t \tag{4}$$

where:

$$\Pi(\hat{p}) = (1-\varphi)\varphi^{1-\xi}\hat{p}^{\xi}$$

and:

$$\theta_t = \theta(a_t, w_t) \equiv a_t^{\xi} w_t^{1-\xi} \qquad \text{with } \xi = \frac{1}{1-\varphi} > 1 \tag{5}$$

We adopt this additional assumption for the rest of the paper, since it appears to do little violence to the general substantive properties of the results. The new variable θ_t summarizes at every instant the business conditions for the project, and satisfies the conditions $\frac{\partial \theta_t}{\partial a_t} > 0$ and $\frac{\partial \theta_t}{\partial w_t} < 0$: it is higher the higher is the productivity indicator a_t and the lower is the flexible-factor rental cost w_t.

[9] Joskow and Schmalensee (1986, p.3) underline that the demand for utilities such as electricity, water and gas by most industrial customers and all residential custormers is very inelastic especially in the short run before stock of plant, equipment, appliances, and housing can be replaced in response to higher electricity prices.

Uncertainty is introduced in the model by assuming that both a_t and w_t evolve over time according to geometric Brownian motions, with instantaneous rates of growth $\alpha_a \geq 0$, $\alpha_w \geq 0$ and instantaneous volatilities $\sigma_a > 0$, $\sigma_w > 0$. That is:

$$da_t = \alpha^a a_t dt + \sigma^a a_t dz_t^a, \quad a_0 = a$$

$$dw_t = \alpha^w w_t dt + \sigma^w w_t dz_t^w, \quad w_0 = w$$

where dz_t^a and dz_t^w are the standard increments of two Wiener processes (possibly correlated), uncorrelated over time and satisfying the conditions that $E(dz_t^a) = E(dz_t^w) = 0$ and $E[(dz_t^a)^2] = E[(dz_t^w)^2] = dt$. In other words, we assume that the input's price and the factor's productivity are expected to grow at a constant mean rate, but the realized growth rates are stochastic, normally distributed and independent over time.

These assumptions allow us to reduce the model to one dimension. By expanding $d\theta_t$ and applying Itô's lemma for Brownian process it is easy to show that θ_t is driven by:

$$d\theta_t = \alpha\theta_t dt + \sigma\theta_t dz_t \quad \text{with } \theta_0 = \theta, \tag{6}$$

with:

$$\alpha = \xi\alpha^a - (\xi - 1)\alpha^w + \xi(\xi - 1)[\frac{1}{2}(\sigma^a)^2 + \frac{1}{2}(\sigma^w)^2 - \gamma\sigma^a\sigma^w],$$

and:

$$\sigma = \sqrt{(\sigma^a)^2\xi^2 + (\sigma^w)^2(\xi - 1)^2 - 2\gamma\sigma^a\sigma^w\xi(\xi - 1)}.$$

The drift and the standard deviation parameters of the process θ_t are linear combinations of the corresponding parameters of the primitive processes a_t and w_t, with weights given by the exponents of (5) and $\gamma = E(dz_t^a dz_t^w)/dt$. Hence, making use of (4) and (6), and provided that $\rho - \alpha > 0$, the expected value at time t of discounted cash flows from an infinite-lived project can be expressed as $V(\theta_t) = \frac{\Pi(\hat{p})\theta_t}{\rho-\alpha}$, resulting in dV_t being given simply by:

$$dV_t = \alpha V_t dt + \sigma V_t dz_t, \quad V_0 = V \tag{7}$$

In this respect V_0 can be interpreted as the "reasonable" rate of return at the delegation time to induce the firm to manage the utility. However, as any reasonable rate of return on an investment could be imbedded directly through a contractual (fixed) price for the service, this formulation sacrifices no generality.[10] [11]

In the remainder of this paper the value of the utility V_t, which evolves according to (7) with starting state V_0, is taken as the primitive exogenous variable for the regulator's delegation-revocation process.

We conclude by assuming that the regulator is theoretically always in the position of being able to revoke the contract with the firm and manage the utility by itself. However, to manage the utility it has to pay a sunk cost I. If revocation is carried out the firm suffers a loss V, while the regulator derives a gain $V - I$. As $V > V - I$, the revocation is inefficient given that the firm's loss exceeds the local authority's gain.[12]

3 The regulator's option-to-revoke

For the regulator, optimal revocation implies finding the point at which to pay the sunk cost I in return for a project whose value V evolves according to (7). If we denote the value of the regulator's investment opportunity at $t = 0$ by $F_m(V)$ then:

$$F_m(V) = \max E_0 \left[(V_T - I) e^{-\rho T} \mid V_0 = V \right] \tag{8}$$

where $T(V^*) = \inf (t \geq 0 \mid V_t - V^* = 0^+)$ is the unknown future time when the revocation is made and V^* is the value that triggers it. The maximization is subject to equation (7), ρ is the constant discount rate and V_0 is the value of the utility at time zero. To simplify discussion, we assume, if no otherwise indicated, that $V_0 < V^*$ so that $T^* > 0$ (see Appendix A for the general case).

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[10] In terms of cash flow, the regulator may set at time zero the price of service $\hat{p}$ so that the firm breaks even:

$$\Pi(\hat{p})\theta_0 \leq (\rho - \alpha) s_0$$

where s_0 is a "reasonable" rate of return (Joskow, 1973).

[11] It is easy, at this level, to relax the assumption that the ceiling for the output price is fixed. We can introduce an automatic adjustment clause such as $p_t = \hat{p} e^{\omega t}$, where the price is indexed, for example, to the planned inflation or to the price of the input, i.e. $\omega = \alpha^w$.

[12] Without any loss of generality we can consider the case in which revocation requires some contractual indemnities K (i.e. the underpreciated value of investment in technology and infrastructures) from the municipality to the firm, with $-V + K < 0$, $I = K + I'$ and $I' \geq 0$.

By an arbitrage argument and applying the Ito's lemma, the value of the opportunity to invest held by the regulator is given by solution of the following Bellman equation (Dixit and Pindyck, 1994, p. 147-152, Clark and Mondello, 1997):

$$\frac{1}{2}\sigma^2 V^2 F_m{}'' + \alpha V F_m' - \rho F_m = 0 \qquad \text{for } V \in (0, V^*], \tag{9}$$

where $F_m(V)$ must satisfy the following boundary conditions:

$$\lim_{x \to 0} F_m(V) = 0 \tag{10}$$

$$F_m(V^*) = V^* - I \tag{11}$$

$$F_m'(V^*) = 1 \tag{12}$$

If the value of the utility goes to zero, the value of the option to invest should also go to zero. Efficient operation conditions (11) and (12) respectively imply that, at the trigger V^*, the value of the option is equal to its liabilities where I indicates the sunk cost for revoking the contract (*matching value condition*) and suboptimal exercise of the option to invest is ruled out (*smooth pasting condition).* By the linearity of (9) and using (10), the general solution is:

$$F_m(V) = AV^{\beta_1}, \tag{13}$$

A is a constant to be determined and $\beta_1 > 1$ is the positive root of the quadratic equation:

$$\Phi(\beta) = \frac{1}{2}\sigma^2 \beta(\beta - 1) + \alpha\beta - \rho = 0 \tag{14}$$

Furthermore, as (13) represents the option value to optimally revoke the contract, the constant A must be positive and the solution is valid over the range of V for which it is optimal for the regulator to keep the option alive (0, V^*]. By substituting (13) for (11) and (12) we get:

$$V^* = \frac{\beta_1}{\beta_1 - 1} I, \quad \text{with} \quad \frac{\beta_1}{\beta_1 - 1} > 1 \tag{15}$$

and:

$$A(V^*) = \frac{1}{\beta_1}(V^*)^{1-\beta_1} > 0,$$

Putting together (8), (11), (12) and (15), we can write the regulator's investment opportunity at time t as:

$$F_m(V_t) = \begin{cases} AV_t^{\beta_1} & \text{for all } V_t < V^* \\ \\ V_t - I & \text{for all } V_t \geq V^* \end{cases} \tag{16}$$

The optimal trigger value V^* indicates the utility's value for which the regulator will find it profitable to revoke the contract. Or, in other words, the local authority will find it expedient to manage the public service by itself the first time V_t, randomly fluctuating, hits the upper threshold level V^*. We are able to highlight the revocation timing by comparing the opportunity costs of currently revoking the contract and the corresponding benefits of optimally postponing the decision. This can be done by evaluating the difference $F_m(V_t) - V_m^0(V_t)$, where, by (16), $V_m^0(V_t) = V_t - I$ is the net value of the public utility when it is acquired at time t, and $F_m(V_t) = AV_t^{\beta_1}$. If we assume $V_t < V^*$ so that the regulator finds it optimal to wait before revoking, we get:

$$F_m(V_t) - V_m^0(V_t) = I + AV_t^{\beta_1} - V_t \tag{17}$$

The first term on the r.h.s. of (17) is the direct cost of revocation. The second term is the value of the option-to-revoke, and since revocation implies "killing" this option, in (17) it appears as an opportunity cost of current revocation. The third term is the current value of the project and is thus an opportunity benefit. Since $V_t < V^*$ and $F_m(V_t) - V_m^0(V_t) > 0$, the direct cost plus the opportunity cost are greater than the opportunity benefit, and the revocation decision should be delayed.

4 Revocation and firm regulation

From the previous section, once the delegation is in place, the regulator does not have any incentive to revoke the contract as long as V_t is below the revocation level V^*. Indeed, as $V_t - I - AV_t^{\beta_1} < 0$ for all $V_t < V^*$, recalling the contract implies a cost to the local authority which makes the (threat of) revocation not credible. On the contrary, for $V_t > V^*$ the local authority's gain from managing the utility is strictly positive, $V_t - I > 0$. Here, the threat is credible. This reveals the simple stationary nature that this extreme threat possesses: the first time V hits V^* revocation is carried out, the firm suffers the loss V^* and the regulator's gain is $V^* - I$. This extreme equilibrium represents the minimax point of the game.

To avoid revocation, the firm may be willing to reduce profits to keep V_t below V^* and then to guarantee its contract. However, without a binding commitment any *lump sum regulation,* evaluated from the difference $V_t - V^*$, will be inefficient (Klein and O'Flaherty, 1993; Shavell and Spier, 1996). The firm knows that the regulator has an incentive to carry out the threat of revocation as soon as V^* is hit. In this respect, the regulator can set the length of the relationship whereas the firm cannot. If the firm makes a once-for-all reduction of the utility's value the first time V_t hits V^*, the regulator will revoke *immediately after* regardless of the level of the regulation unless $V_t < V^*$. Furthermore, by backward induction, the same happens for any finite number of controls. The firm does not have any incentive to regulate the utility's level to delay the revocation decision. The regulator does not expect to see regulations. It, then, optimally carries out the threat as soon as V^* is hit. The *unique* sub-game perfect equilibrium is inefficient, since the revocation is carried out regardless of the firm's gain staying in the market.[13]

To avoid this inefficiency the firm must *regulate in continuum the utility's value.* As decisions are taken in continuous time, for $t \geq T^*$ the firm elects V^* as its ceiling and chooses to reduce expected profits via a downward adjustment of the output price just enough to keep V_t from crossing the ceiling V^*, so that continuing the contract or revoking it makes no difference to the regulator.[14]

[13] For $V > V^*$, "...the threatener's problem is that he will have an incentive to carry out his threat even if he is paid.........Because this means that the victim will not prevent the threatened act by paying, he will not pay. The threatener cannot overcome this problem in a single (or finite) period setting, and his threat will therefore fail in this version of the model" (Shavell and Spier, 1996, p. 3-4).

[14] In continuous time repeated games there is no notion of *last time before t*. The real line is not well ordered and then induction cannot be applied. Continuous time can be seen as discrete-time with a length of reaction (or information lag) that becomes infinitely negligible to allow the threateners to respond immediately to the firm's actions. In Simon and Stinchcombe (1989), for example, a class of continuous strategies is defined so that any increasingly narrow sequence of discrete-time grids generates a convergent sequence of game outcomes whose limit is independent of the grid sequence. In Bergin and MacLeod

Letting the firm start with the initial value V_0, the optimal policy from here on is a simple one: for $V_t < V^*$, it allows the value of the utility to evolve over time according to the geometric Brownian motion (7); at V^* a costless regulator dr_t is applied to stop the process V_t from going above V^*.[15] The overall process can be described as:

$$dV_t = \alpha V_t dt + \sigma V_t dz_t - dr_t, \quad V_0 = V, \text{ for } V \in (0, V^*] \tag{18}$$

where r_t represents the upside value of the project cut by regulation. The increment dr_t gives the sum the firm is willing to pay (i.e. the loss that the firm is able to bear) between t and $t + dt$ to keep the contract alive. In technical terms, V^* is no longer an absorbing barrier but is a (reflecting) barrier control (Harrison and Taksar, 1983; Harrison, 1985), while the optimal control r_t is a right-continuous, non-decreasing and non-negative adapted process that takes the form (see Appendix A and figure 3):

$$r_t = [1 - \inf_{T^* \leq v \leq t} \left(\frac{V^*}{V_v}\right)]V_t \qquad \text{if } V_t \geq V^* \tag{19}$$

This control has several interesting features. Firstly, it represents the cumulative amount of the project's value that the firm abandons up to time t. The firm must increase r_t fast enough to keep $V_t - r_t$ below V^* but wishes to exert as little regulation as possible subject to this constraint. Secondly, the regulation r_t is parametrized by the initial condition V^* which, in turns, depends on the revocation cost I. Thirdly, as r_t depends only on the primitive exogenous process V_t, the regulated process $V_t - r_t$ is also a Markov process in levels (Harrison, 1985, Proposition 7, p. 80-81).

The first and the second property make the regulation related to past realizations and then to the history of the game. Since V fluctuates stochastically over time, although the intervention is continuous, its rate of change is discontinuous. Furthermore, the last property is important as it effectively makes the regulated process (18) a function solely of the starting state. At the beginning of each period both the firm and the regulator can predict the evolution of the utility value referring only to its current state.

(1993) a class of *inertia strategies* represents a delay in response: an action at time t must also be chosen for a small period of time after t, with this small period of time tending to zero.

[15] The assumption that the regulation is cost-free is not technically necessary for the analysis. We can alternatively assume that the firm faces a cost $C_t = cdr(V_t)$ without this altering the results.

The above strategies and the regulation mechanism (19) can improve upon static non-cooperative outcomes. These strategies imply an instantaneous response by the regulator when the firm departs from the regulation rule (19) with the minimax threat: revocation. In addition, since the project is infinitely lived, the present value of foregone profits will ensure participation by the firm and the expectation of future regulations keeps the regulator from exercising the threat.

Proposition Part I (Threat equilibria). For any $V^* > V_0 > 0$, if the firm regulates the utility's value with the non-decreasing proportional rule (19), then the following regulator's strategy is a sub-game perfect equilibrium:

$$\phi(V_t, r_t) = \begin{cases} \textbf{Do not revoke} \\ \text{at } t = T^* \text{ if the firm has followed the rule } r_t \\ \text{to keep } V_t < V^* \text{ for } t' < t \\ \\ \textbf{Revoke} \\ \text{if the firm has deviated from } r_t \\ \text{at any } t' < t \end{cases}$$

Proof. see Appendix A.

According to the decision rule strategy the firm observes V_t, chooses an action (19) and the regulator stays ($\phi(V_t, r_t) =$ "Not Revoke" for all $t \geq T^*$) or, equivalently, at T^*, sets a continuous time control rule for each realization of V_t for any $t \geq T^*$.[16] The regulated utility's value is obtained from V_t by imposition of an upper control barrier at V^*. Regulation increases to keep V_t lower than V^* and it is given by the cumulative amount of control exerted on the sample path of V_t up to t. Regulation is related to the history of the game and past value realizations, this makes $\phi(V_t, r_t)$ a time-dependent strategy. The regulator's "threat" strategy is adopted if the firm deviates from the regulation rule (19). The regulator believes that this mechanism, from initial date and state (T^*, V^*), is kept in use for the whole (stochastic) planning horizon. If the firm deviates, the regulator expects a fresh rule. The punishment for the firm deviating from the announced rule is revocation.[17]

[16] In our continuous time setting we can assume, without any loss of generality, that when the regulator is indifferent it does not exercise the threat; see footnote n.13

[17] The firm cannot commit itself to changing the rule without losing its credibility. In

However, although the utility lives forever, the regulation takes place within a finite (stochastic) time span. Owing to uncertainty, neither player can perfectly predict V_t each time. As V_t follows a random walk there is, for each time interval dt, a constant probability of moving up or down, i.e. of the game continuing one more period. The game ends in finite (stochastic) time with probability one, but everything is as if the horizon were infinite.[18]

Proposition Part II (Regulation timing). As long as $V_t < V^*$ nothing is done. The first time V_t *crosses from below* V^*, at $T^* = \inf(t \geq 0 \mid V_t - V^* = 0^+)$, the firm regulates using (19) to keep the regulator indifferent to revoking. Regulation goes on up to the point where the unregulated utiltiy's value V_t *crosses from above* the trigger V^* and the regulator becomes (again) indifferent, i.e. $T^{*\prime} = \inf(t \geq T^* \mid V_t - V^* = 0^-)$.

Proof. see Appendix A.

Since the regulator's strategy is time-dependent, the firm cannot decide whether to continue or stop the regulation referring only to the current realization of V_t. If the regulated value $V_t - r_t$ goes below V^*, in the interval $[T^*, T^{*\prime})$ the firm may be willing to stop regulation to increase the utility's value. However, for the sake of perfectness, earlier interruption is not allowed before $T^{*\prime}$. Earlier interruptions are not feasible as long as the threat of closure by the public authority is credible. The credibility relies on the fact that the regulator's option-to-revoke if the firm deviates from r_t is always worth exercising at $V_t \geq V^*$, i.e. $F_m(V_t) \geq F_m(V^*)$. At $T^{*\prime}$, however, the firm is able to restore the process V_t and the game can start afresh. The timing of the game is shown in figure 1 below.

Figure 1 about here

---

this respect, a change in the regulation policy is perceived by the regulator as a stoppage of regulation.

[18] In a discrete-time and constant-payoffs game, Shavell and Spier (1996) propose a similar scheme, where the threatener uses a *threat strategy* with maximal punishments. Our continuous time framework calls for a refinement of the *threat strategy* as in footnote n.13.

5 Analysis of the results and policy implications

Although our regulation mechanism may lack some features of real schemes, several novel implications follow from our analysis. We summarize the discussions of our results in the following six items.

- **The regulatory rule**

The regulatory rule (19) is *endogenous.* It rises as optimal response from a continuous rate of hearing between a regulated firm and a regulator that cannot sign binding contracts with the firm it regulates. The rule is dynamic in nature: such a repetition of the relationship substitutes long-term contracts and guarantees the firm with an "allowed" level of return. Moreover, recalling the non-decreasing property, this rule may appear as an "insurance premium" based on the value of the utility V_t, paid in continuous time and in advance by the firm to avoid revocation. The firm starts paying the first time V_t goes above V^* (the first occurrence time) and cannot stop or reduce it since this would cancel its coverage. When the utility's value goes again above V^* (the second occurrence time), the firm will be asked to increase its premium to maintain the coverage. In other words, it continues paying even when "things get better" (profits decrease as well as the local authority's value of revoking the contract) in order to have the option of being active next time the value goes above V^*. It follows that the new regulation is higher, since the firm pays the premium due after the "second occurrence" (see figure 3 in the Appendix A).

- **Option-to-revoke as a tool for collective welfare**

The higher the cost of revoking the contract, the higher the option-to-revoke which increases the value of waiting for better information on the stochastic evolution of the utility before the local authority commits itself to the investment decision. With this in mind, we can look at the regulatory rule from the perspective of collective welfare maximization. In this specific instance the expected value of cumulative future premiums (equations (27) and (40) in Appendix) can be expressed at time t as :

$$
\begin{aligned}
R(V_t;V^*) &= E_t\left\{\int_t^{\infty} e^{-\rho(s-t)}dr(V_s) \mid V_t^r = V_t\right\} \\
&= (\rho-\alpha)E_t\left\{\int_t^{\infty} e^{-\rho(s-t)}r(V_s)ds \mid V_t^r = V_t\right\} \\
&= B(V^*)V_t^{\beta_1},
\end{aligned}
\tag{20}
$$

with $B(V^*) = \frac{1}{\beta_1}(V^*)^{1-\beta_1} > 0$ and V_t^r is the regulated value as in (18). Equation (20) is the firm's expected cumulative controls in terms of lower output price. The adoption of the policy rule (19) means that it makes no difference to a "local community" whether it receives benefits from the firm's regulations or from the local authority's maximization of the discounted consumer surplus (see Appendix B), i.e.[19]:

$$
A(V^*)V_t^{\beta_1} - B(V^*)V_t^{\beta_1} = 0, \qquad \text{for } t \geq T^*.
$$

- **Price adjustment**

Once the numerical value for V^* is known, by using (4) and (5), the optimal policy (15) can be written as:

$$
(1-\varphi)\varphi^{1-\xi}p^{\xi}\theta = \frac{\beta_1}{\beta_1 - 1}(\rho-\alpha)I \tag{21}
$$

with $\theta = \theta^*$ for $p = \hat{p}$. By inverting (21) we can obtain the optimal boundary function $p(\theta)$ which determines the optimal price regulation as a function of the sole state variable θ and the parameter of the problem ξ :

$$
p = \hat{p}\left(\frac{\theta^*}{\theta}\right)^{1/\xi} \qquad \text{with} \quad \frac{dp}{d\theta} < 0 \tag{22}
$$

The boundary function is shown in Figure 2. For any given value of contractual price $\hat{p}$, random fluctuations of θ move the point (θ, p) horizontally on the left or on the right. If the point goes on the right of the boundary, then

[19] Formal proof (see Appendix A) shows that the regulator in revoking the contract does so in rational expectation of subsequent (marginal) price regulations by the firm. It turns out that for the regulator this makes no difference to the trade-off between revoking now and waiting another instant, i.e. the regulator's option value is identically zero. See Leahy (1993) for the same result in the context of a competitive industry.

a price reduction is immediately undertaken so that the point shift down to the boundary. If θ stays on the left of the boundary, no new price regulation is undertaken. Price reduction proceeds gradually to maintain (21) as an equality.

Figure 2 about here

- **The regulation as a sliding scale**

Our result establishes a connection between ROR regulation and price cap regulation. In fact, simple algebra allows us to write the policy rule (19) as a *sliding scale* over an allowed rate of return (Joskow and Schmalensee, 1986, p. 29):[20]

$$s_t^r = s_t + h_t\left(s^* - s_t\right), \quad \text{with } h_t = \begin{cases} 0 & , \quad \text{for } s_0 \leq s_t < s^* \\ \frac{1-\inf\limits_{T^* \leq v \leq t}(V^*/V_v)}{1-(V^*/V_t)}, & \text{for } s_t \geq s^* \end{cases} \tag{23}$$

where $s_t^r = \frac{V_t^r}{I}$, $s_t = \frac{V_t}{I}$ and $s^* = \frac{V^*}{I}$. By (23), the actual rate of return under regulation s_t^r is given by the actual rate of return without regulation s_t, i.e. at the output price that prevails in time t, plus the adjustment $s^* - s_t$, where the *revocation rate* s^* plays the role of the upper "allowed" rate of return. Thus, if at time t the earned rate of return goes above s^*, the output price is adjusted according to (22) by the firm to decrease the rate of return by the fraction $h_t \geq 1$ of the difference between the earned rate of return and the allowed rate of return.

Contrasting with the formula proposed by Joskow and Schmalensee (1986), in (23) h_t is time-dependent and not-decreasing. That is, h_t is the optimal adjustment rate that keeps the regulator indifferent between revoking the contract and leaving the project to the firm. To obtain consistent behavior by the firm, h_t cannot decrease when the difference between the earned rate

[20] The formula proposed by Joskow and Schmalensee (1986, p. 29) would adjust prices so that the actual rate of return s_t^r at new prices would be given by: $s_t^r = s_t + h\left(s^* - s_t\right)$, where s_t is the rate of return at the prices in the year t (old prices), h is a constant between zero and one and s^* is the ROR target.

of return and the allowed rate of return drops. It follows that in the period $0 \leq t < T^*$ where $s_t < s^*$, we will have $h_t = 0$ and $s_t^r = s_t$. During this regulatory lag the firm is allowed to earn the actual rate of return at the rates fixed at time $t = 0$, i.e. $p_t = \hat{p}$ (a period of price-cap or fixed price regulation period).[21] When $s_t \geq s^*$, in period $t \geq T^*$, the adjustment rate h_t jumps to 1 and it will remain at that value until $dV_t > 0$ so that $s_t^r = s^*$. The firm is allowed to earn a rate of return no greater than the upper rate $s^* = \frac{\beta_1}{\beta_1 - 1} > 1$ and $p_t < \hat{p}$ (a period of ROR regulation). However, in periods where $dV_t < 0$ we will have $h_t > 1$ in order to keep the difference $s_t^r - s_t$ constant at the highest level reached up to t.

- **Regulatory information**

The rule (23) is parametrized by the *revocation rate* s^*. So, in the specific, the key variable for the value of the option-to-revoke and thus for the regulator's position during the delegation period is the direct cost I which - in turn- depends, excluding indemnities on the value of the investment, on training and hiring costs for qualified personnel, costs of procuring new technology and legal costs if the firm decides to sue the local authority for breaking the contract. Thus, the initial set of information and the demand/cost data the regulator uses are fundamental in determining the length of the regulatory lag. This effect could be weighted with respect to the well-known tradeoff in ROR literature between a short regulatory lag that promotes allocative efficiency but is bad for productive efficiency, and a long regulatory lag that produces the opposite effect on allocative and productive efficiency.

Furthermore, the *revocation rate* s^* is related to the credibility of the regulator, given the firm's information on the regulator's revocation costs. This credibility is relevant for the renegotiation process since it determines the regulator's bargaining power with the delegated firm and - in turn - the timing of contract renewal. Indeed, if the revocation costs, on the one hand, measure the "inefficiencies" the local authority incurs by direct management and are, therefore, used to positively evaluate the decision to delegate the public service to a private operator, on the other hand, they raise the problem of the irreversibility of the delegation once it is taken. In the case of local provision of utilities we refer to, after the delegation has taken place

[21] Since the difference between a price-cap and a fixed price regime seems to be relevant when the regulated firm faces competition, where the price-cap provides only a ceiling rather than both a ceiling and a floor, we do not adopt this distinction here.

the municipal authority plays the role of a regulator with respect to the private firm: the inexperience of the municipal authority in this role can affect negatively its credibility and thus determine a negotiating disadvantage.

- **Market expectations**

Finally, as long as public utilities are not traded in limited supply for investment purposes by a large number of investors, their growth rate α may actually fall below the equilibrium total expected rate of return $\hat{\alpha}$ required in the market by investors from an equivalent-risk traded financial security, i.e. $\delta \equiv \hat{\alpha} - \alpha > 0$ (McDonald and Siegel, 1986). However, by the asset price equilibrium relationship $\hat{\alpha} - r = \lambda\sigma$, we are able to evaluate the regulator's value of the option-to-revoke, replacing α with the risk-adjusted rate of growth $\alpha - \lambda\sigma = r - \delta$ and behaving as if the word were risk neutral: where r is the risk-free rate of interest, δ is the below-equilibrium return shortfall and λ is the utility's market price of risk. The allowed rate of return becomes:

$$s^* = s^*(r, \lambda, \sigma)$$

Although, it seems reasonable to assume that utilities with higher "capital costs" will be allowed to earn higher rates of return, i.e. $\frac{\partial s^*}{\partial r} > 0$, it is also confirmed the empirical evidence that a higher systematic risk, as measured through the market price of risk λ, results in a higher allowed rate-of-return, i.e. $\frac{\partial s^*}{\partial \lambda} > 0$ (Fan and Cowing, 1994). Furthermore, a higher volatility also increases the allowed rate of return, i.e. $\frac{\partial s^*}{\partial \sigma} > 0$, but for reasons other than those related to interest rates and systematic risk. From section 3 we know that an increase in the instantaneous variance, σ^2, of the revenue process reduces β_1 and then increases the *option multiply* $\frac{\beta_1}{\beta_1 - 1}$. As a result, when the utility's market or economic environment becomes more volatile, the market value of the public project can go up, but it also increases the regulator's value of keeping the revocation opportunity alive. Thus, the allowed rate of return s^* is higher since the regulator optimal policy is to lag behind in revoking the contract with the firm.

6 Conclusions

In this paper we have modelled the regulation of a local public utility as a long-term relationship between a firm and its regulator. After formulating a time-dependent supergame in continuous time, we have shown that when the regulator has a credible threat of revoking the contract and the cost of revocation is not too high, the cooperative equilibrium is an efficient solution. Such a repetition of the relationship may substitute long-term contracts and guarantee utilities with an appropriate level of profits. Furthermore, since the output price is contractually fixed at time zero and the firm is the residual claimant for its profits, we have a stochastic regulatory lag where the regulation has a fixed-price nature. Excessive revocation cost makes the firm an unregulated monopolist with an infinite regulatory lag. This fixed-price regulation is followed by a period of rate-of-return regulation in which the firm is induced to adjust its output price downward to keep its rate of return below the allowed one set by the regulator and avoid revocation.

A Appendix: The threat game

We prove that the regulatory scheme proposed is a perfect equilibrium belonging to the class of efficient perfect equilibria (which may be very large) for the continuous time threat-game described in the text.

1) Regulation mechanism

We define the regulation as the negative increment dV_t to let V_t stay at V^*,that is, a policy control is a process $Z = \{Z_t, t \geq 0\}$ and a regulated process $V^r = \{V_t^r, t \geq 0\}$ such that

$$V_t^r \equiv V_t Z_t, \quad \text{for} \quad V_t^r \in (0, V^*], \tag{24}$$

where:

- $i)$ V_t is a geometric Brownian motion, with stochastic differential as in (7);
- $ii)$ Z_t is a decreasing and continuous process with respect to V_t ;
- $iii)$ $Z_0 = 1$ if $V_0 \leq V^*$, and $Z_0 = V^*/V_0$ if $V_0 > V^*$ so that $V_0^r = V^*$;
- $iv)$ Z_t decreases only when $V_t^r = V^*$.

Applying Ito's lemma to (24), we get:

$$d\,V_t^r = \alpha\, V_t^r dt + \sigma V_t^r dz_t + V_t^r \frac{dZ_t}{Z_t}, \qquad V_0^r \in (0, V^*]$$

where $V_t^r \frac{dZ_t}{Z_t} \equiv V_t dZ_t = -dr_t$ is the infinitesimally small level of value given up by the firm. In terms of the regulated process V_t^r, we can write:

$$r_t \equiv r(V_t) = V_t - V_t^r \equiv (1 - Z_t)V_t, \tag{25}$$

Although the process Z_t may have a jump at time $t = 0$ it is continuous and maintains V_t below the barrier using the minimum amount of control, in that control takes places only when V_t crosses V^* from below with probability one in the absence of regulation. Therefore, in the case of $V_0 < V^*$, we get $V_t^r \equiv V_t$, with initial condition $V_0^r \equiv V_0 = V$, and $Z_t = 1$. At $T^* \equiv T(V^*) = \inf(t \geq 0 \mid V_t - V^* = 0^+)$ the regulation starts so as to maintain $V_t^r = V^*$.

The firm regulates the utility's value by the amount $r_t = V_t - V_t^r \geq 0$ every time V^* is hit.
Finally, the same conditions $(i) - (iv)$ uniquely determine Z_t with the representation form (Harrison,1985; proposition 3, p. 19-20):[22]

$$Z_t \equiv \begin{cases} \min(1, V^*/V_0) & \text{for } t = 0 \\ \inf\limits_{0 \leq v \leq t} (V^*/V_v) & \text{for } t \geq 0 \end{cases} \tag{26}$$

Figure 3 about here

2) Cost of regulation

Let's now indicate with $R(V^r; V^*)$ the expected value of future cumulative losses in terms of the firm's value due to the regulation. As neither player can predict the end of the game, the rational player evaluates R considering the infinite life of the project:

$$\begin{aligned} R(V_0^r; V^*) &= E_0 \left\{ \int_0^\infty e^{-\rho t} dr(V_t) \mid V_0^r \in (0, V^*] \right\} \\ &= -E_0 \left\{ \int_0^\infty e^{-\rho t} V_t dZ_t] \mid V_0^r \in (0, V^*] \right\} \end{aligned} \tag{27}$$

Since V_t^r is a Markov process in levels (Harrison, 1985, proposition 7, p.80-81), we know that the above conditional expectation is in fact a function solely of the starting state.[23] Keeping the dependence of R on V_t^r active

[22] This is an application of a well-known result by Levy (1948), for which the process:

$$\ln V_t^r \equiv \ln V_t + \ln Z_t \equiv \ln V_t - \inf_{0 \leq v \leq t} (\ln V_v - \ln V^*)$$

has the same distribution as the "reflected Brownian process" $\mid \ln V_t - \ln V^* \mid$.

[23] For $V_0 = V > V^*$ optimal control would require Z to have a jump at zero so as to ensure $V_0^r = V^*$. In this case the integral on the right of (27) is defined to include the control cost r_0 incurred at $t = 0$, that is (see Harrison 1985, p.102-103):

$$\int_0^\infty e^{-\rho t} dr_t \equiv r_0 + \int_{(0,\infty)} e^{-\rho t} dr_t$$

where $r_0 = V - V_0^r$.

and assuming that it is twice continuously differentiable, by Ito's lemma we get:

$$
\begin{aligned}
dR &= R' dV_t^r + \frac{1}{2} R'' (dV_t^r)^2 \qquad (28)\\
&= R'(Z_t dV_t + V_t dZ_t) + \frac{1}{2} R'' Z_t^2 (dV_t)^2 \\
&= R'(\alpha\, V_t^r dt + \sigma V_t^r dz_t + V_t^r \frac{dZ_t}{Z_t}) + \frac{1}{2} R'' Z_t^2 \sigma^2 dt \\
&= \frac{1}{2} R'' \sigma^2 V_t^{r2} dt + R' \alpha\, V_t^r dt + R' \sigma V_t^r dz_t + R' V_t^r \frac{dZ_t}{Z_t}
\end{aligned}
$$

where it has been taken into account that for a finite-variation process like Z_t,$(dZ_t)^2 = 0$. As $dZ_t = 0$ except when $V_t^r = V^*$ we are able to rewrite (28) as:

$$
dR(V_t^r; V^*) = [\frac{1}{2} \sigma^2 V_t^{r2} R''(V_t^r; V^*) + \alpha\, V_t^r R'(V_t^r; V^*)] dt \qquad (29)
$$

$$
+\sigma V_t^r R'(V_t^r; V^*)\, dz_t - R'(V^*; V^*) dr(V_t)
$$

This is a stochastic differential equation in R. Integrating by part the process Re^{-rt} we get (Harrison, 1985, p.73):

$$
e^{-\rho t} R(V_t^r; V^*) = R(V_0^r; V^*) + \qquad (30)
$$

$$
+ \int_0^t e^{-\rho s} \left[\frac{1}{2} \sigma^2 V_s^{r2} R''(V_s^r; V^*) + \alpha\, V_s^r R'(V_s^r; V^*) - \rho R(V_s^r; V^*) \right] ds
$$

$$
+\sigma \int_0^t e^{-\rho s} V_s^r R'(V_s^r; V^*)\, dz_s - R'(V^*; V^*) \int_0^t e^{-\rho s} dr(V_s)
$$

Taking the expectation of (30) and letting $t \to \infty$, if the following conditions apply:

(a) $\lim_{l \to 0} \Pr[T(l) < T(V^*) \mid V_0^r \in (0, V^*]] = 0$ for $l \leq V_t^r < V^* < \infty$, where $T(l) = \inf(t \geq 0 \mid V_t^r = l)$ and $T(V^*) = \inf(t \geq 0 \mid V_t^r = V^*)$;

(b) $R(V_t^r; V^*))$ is bounded within $(0, V^*]$;

(c) $e^{-\rho t} V_t^r R'(V_t^r; V^*)$ is bounded within $(0, V^*]$;

(d) $R'(V^*; V^*) = 1$;

(e) $\frac{1}{2}\sigma^2 V_t^{r2} R''(V_t^r; V^*) + \alpha V_t^r R'(V_t^r; V^*) - \rho R(V_t^r; V^*) = 0$,

we obtain $R(V^r; V^*)$ as indicated in (27). Condition (a) says that the probability that the regulated process V_t^r reaches zero before reaching another point within the set $(0, V^*]$ is zero. As V_t^r is a geometric type of process this condition is, in general, always satisfied (Karlin and Taylor, 1981, p. 228-230). Furthermore, if condition (a) holds and $R(V^r; V^*)$ is bounded then conditions (b) and (c) also hold. According to the linearity of (e) and using (d), the general solution has the form:

$$R(V_0^r; V^*) = B(V^*)(V_0^r)^{\beta_1}, \tag{31}$$

with:

$$B(V^*) = \frac{1}{\beta_1}(V^*)^{1-\beta_1} > 0. \tag{32}$$

As for $V_0 \leq V^*$, $Z_0 = 1$ and $V_0^r = V_0 = V$, then $R(V_0^r; V^*) = R(V; V^*)$. On the other hand, if $V_0 > V^*$, we get $Z_0 = V^*/V_0$, so that $V_0^r = V^*$ and $R(V_0^r; V^*) = R(V^*; V^*)$.

3) The value of the option-to-revoke

Although the firm prefers to regulate rather than close (i.e. the loss from closure is larger than the (expected) cost of regulation), it always prefers to stop regulation if the threat of revocation is not carried out, i.e. $r_t = V_t - V_t^r \geq 0$, for all $t \geq T^*$. To simplify discussion we assume that $V_0 < V^*$ so that $T^* > 0$. While regulation reduces the project's value but keeps the firm's contract alive, the regulator is not in the same condition. Indicating with $F_m^r(V; V^*)$ the regulator's option value when the firm controls itself, it can be expressed, at time zero, by:

$$F_m^r(V; V^*) = \max E_0 \left\{ (V_T^r - I)e^{-\rho T} \mid V_0 = V \right\} \tag{33}$$

or using $r_t = V_t - V_t^r = (1 - Z_t)V_t$:

$$F_m^r(V; V^*) = \max E_0[(V_T - I)e^{-\rho T} - (V_T - V_T^r)e^{-\rho T} \mid V_0 = V] \tag{34}$$

In (34) the regulator's option value, with a barrier control on V_t, takes account of two terms depending upon the joint evolution of V_t and V_t^r. The first $(V_T - I)$ is the net project's value without the barrier, while $(V_T - V_T^r)$ is the reduction in value due to the regulation. Again, keeping the dependence of F_m^r on V_t^r active and assuming it twice continuously differentiable, by Ito's lemma we obtain:

$$dF_m^r = \frac{1}{2} F_m^{r\prime\prime} V_t^{r2} \sigma^2 dt + F_m^{r\prime} \alpha \, V_t^r dt + F_m^{r\prime} \sigma V_t^r dz_t + F_m^{r\prime} V_t^r \frac{dZ_t}{Z_t} \tag{35}$$

As $dZ_t = 0$ except when $V_t^r = V^*$ the above differential equation becomes:

$$dF_m^r(V_t^r; V^*) = [\frac{1}{2}\sigma^2 V_t^{r2} F_m^{r\prime\prime}(V_t^r; V^*) + \alpha \, V_t^r F_m^{r\prime}(V_t^r; V^*)]dt \tag{36}$$

$$+\sigma V_t^r F_m^{r\prime}(V_t^r; V^*) \, dz_t - F_m^{r\prime}(V^*; V^*) dr(V_t)$$

Integrating by part the process $F_m^r e^{-\rho T^*}$ gives:

$$e^{-\rho T^*} F_m^r(V_T^r; V^*) = F_m^r(V; V^*) + \tag{37}$$

$$+ \int_0^{T^*} e^{-\rho s} \left[\frac{1}{2}\sigma^2 V_s^{r2} F_m^{r\prime\prime}(V_s^r; V^*) + \alpha \, V_s^r F_m^{r\prime}(V_s^r; V^*) - \rho F_m^r(V_s^r; V^*) \right] ds$$

$$+\sigma \int_0^{T^*} e^{-\rho s} V_s^r F_m^{r\prime}(V_s^r; V^*) \, dz_s - F_m^{r\prime}(V^*; V^*) \int_0^{T^*} e^{-\rho s} dr(V_s)$$

Taking the expected value of (37), if the following conditions apply*:*

(a) $e^{-\rho t} V_t^r F_m^{r\prime}(V_t^r; V^*)$ is bounded within $(0, V^*]$

(b) $F_m^r(V_{T^*}^r; V^*) = V_{T^*}^r - I$

(c) $F_m^{r\prime}(V^*; V^*) = 0$;

(d) $\frac{1}{2}\sigma^2 V_t^{r2} F_m^{r\prime\prime}(V_t^r; V^*) + \alpha V_t^r F_m^{r\prime}(V_t^r; V^*) - \rho F_m^r(V_t^r; V^*) = 0$

we obtain the expression for $F_m^r(V; V^*)$ as in (33). Now the two conditions (b) and (c) together with the fact that at T^* the regulation starts so as

to keep $V_t^r = V^*$ (i.e. compare condition (c) with condition (12)), give $F_m^r(V;V^*) = 0$.
From (34) and (31), a heuristic but direct way of looking at the same result is to see $F_m^r(V;V^*)$ as the difference between the regulator's option value to manage the utility, $F_m(V) = A(V^*)V^{\beta_1}$, and the firm's expected value of future cumulative controls due to the regulation, $R(V) = B(V^*)V^{\beta_1}$, that is:

$$F_m^r(V_t;V^*) = A(V^*)V_t^{\beta_1} - B(V^*)V_t^{\beta_1} = 0$$

In other words, it should make no difference whether the "community" receives the benefits in terms of the firm's regulation (lower output price) or by direct transfers from the regulator. For $0 < t < T^*$, $V_t^r \equiv V_t$ and then $F_m^r(V_t) \equiv F_m(V_t)$. At T^* regulation starts killing the option, i.e. $F_m^r(V) = 0$, for all $t \geq T^*$.

4) Optimal threat and perfect equilibrium

Since V_t follows a random walk there is, for each time interval of small length dt, a constant probability that the game will continue one more period. The game ends in finite (stochastic) time with probability one, but everything is as if the horizon were infinite. Neither player is able to perfectly predict V_t at each date and the regulation scheme described by (25) with the form (26) is viewed by both contenders as a rule for evaluating all future value reductions.[24] In the strategy space of the agency it appears as:

[24] It is well known that infinitely repeated games may be equivalent to repeated games that terminate in finite time. At each period there is a probability that the game continues one more period. The key is that the conditional probability of continuing must be positive (Fudenberg and Tirole, 1991, p.148). Integrating the differential form (7), the geometric Brownian motion can be expressed as:

$$V_{t+dt} = V_t e^{dY_t}$$

where $dY_t = \mu dt + \sigma dz_t$ and $\mu = \alpha - \frac{1}{2}\sigma^2$. The differential dY_t is derived as the continuous limit of a discrete-time random walk, where in each small time interval of length Δt the variable y either moves up or down by Δh with probabilities (Cox and Miller, 1965, p. 205-206):

$$\Pr(\Delta Y = +\Delta h) = \frac{1}{2}\left(1 + \frac{\mu\sqrt{\Delta t}}{\sigma}\right), \quad \Pr(\Delta Y = -\Delta h) = \frac{1}{2}\left(1 - \frac{\mu\sqrt{\Delta t}}{\sigma}\right)$$

or defining $\Delta h = \sigma\sqrt{\Delta t}$:

$$\phi(V_t, r_t) = \begin{cases} \text{Do not revoke at } t = T^* \text{ if the firm} \\ \text{plays the rule } r_t = (1 - Z_t)V_t \text{ for } t' < t \\ \\ \text{Revoke if the firm deviated from} \\ r_t = (1 - Z_t)V_t \;\; \text{at any } t' < t \end{cases}$$

where $\phi(V_t, r_t)$ is the action at t with history (V_t, Z_t). The regulator's "threat" strategy is chosen if the firm deviates by regulating V_t less than r_t or by abandoning $r_t = (1 - Z_t)V_t$ as a rule to evaluate future regulations. The regulator must believe that the regulation, from the initial date and state (T^*, V^*), will be kept in use for the whole (stochastic) planning horizon. If the firm deviates, the regulator believes that the firm switches to a different rule in the future and knows for sure that the regulator revokes immediately. The regulator does not revoke in t if $r_{t'} \geq V_{t'} - V_{t'}^r$ for all $t' \leq t$, because value controls are expected to continue with the same rule and $F_m^r(V) = 0$ for all $t \geq T^*$. If $r_{t'} < V_{t'} - V_{t'}^r$ for some $t' < t$ the regulator expects a different rule and carries out the threat, switching from $F_m^r(V_t) = 0$ to $F_m(V_t) \geq V^* - I$. The game is over.

To prove this, let's first consider R as in (27). For each $t' > T^*$, integration by parts gives:

$$\int_{t'}^{t} e^{-\rho(s-t')} V_s dZ_s = \tag{38}$$

$$e^{-\rho(t-t')} V_t Z_t - V_{t'} Z_{t'} + \rho \int_{t'}^{t} e^{-\rho(s-t')} V_s Z_s ds - \int_{t'}^{t} e^{-\rho(s-t')} Z_s dV_s$$

$$\Pr(\Delta Y = +\Delta h) = \frac{1}{2}\left(1 + \frac{\mu \Delta h}{\sigma^2}\right), \quad \Pr(\Delta Y = +\Delta h) = \frac{1}{2}\left(1 - \frac{\mu \Delta h}{\sigma^2}\right)$$

That is, for small Δt, Δh is of order of magnitude $O(\sqrt{\Delta t})$ and both probabilities become $\frac{1}{2} + O(\sqrt{\Delta t})$, i.e. not very different from $\frac{1}{2}$. Furthermore, considering again the discrete-time approximation of the process Y_t, starting at $V^* e^{+\Delta h}$, the conditional probability of reaching V^* is given by (Cox and Miller, 1965, ch.2):

$$\Pr(Y_t = 0 \mid Y_t = 0 + \Delta h) = \begin{cases} 1 & \text{if } \mu \leq 0 \\ e^{-2\mu \Delta h / \sigma^2} & \text{if } \mu > 0 \end{cases}$$

which converges to one as Δh tends to zero.

Taking expectation of both sides and using the zero expectation property of the Brownian motion (Harrison, 1985, p.62-63), we have:

$$E_{t'}\int_{t'}^{t} e^{-\rho(s-t')}V_s dZ_s = E_{t'}[V_t Z_t e^{-\rho(t-t')}] - V_{t'}Z_{t'} + (\rho-\alpha)E_{t'}\int_{t'}^{t} e^{-\rho(s-t')}V_s Z_s ds \tag{39}$$

By the Strong Markov property of V_t^{r}[25], it follows that $E_{t'}[V_t Z_t e^{-\rho(t-t')}] = E_{t'}[V_t Z_t]E_{t'}[e^{-\rho(t-t')}] = V^* E_{t'}[e^{-\rho(t-t')}] \to 0$ almost surely as $t \to \infty$, so that:

$$E_{t'}\int_{t'}^{\infty} e^{-\rho(s-t')}V_s dZ_s = -V_{t'}Z_{t'} + (\rho-\alpha)E_{t'}\int_{t'}^{\infty} e^{-\rho(s-t')}(V_s - r_s)ds$$

Since $-V_{t'}Z_{t'} + (\rho-\alpha)E_{t'}\int_{t'}^{\infty} e^{-\rho(s-t')}V_s ds = 0$, substituting in (27) and rearranging we get:

$$R(V_{t'};V^*) = (\rho-\alpha)E_{t'}\int_{t'}^{\infty} e^{-\rho(s-t')}r_s ds \tag{40}$$

Secondly, let us assume (t',t) be an interval in which r_s is flat so that $V_s^r \leq V^*$, and t as the first time in which $dZ_t > 0$. Considering the decomposition (39) we can write (40) as:

$$\begin{aligned} R(V_{t'};V^*) &= (\rho-\alpha)\left\{E_{t'}\int_{t'}^{t} e^{-\rho(s-t')}r_s ds + E_{t'}\left\{\int_{t}^{\infty} e^{-\rho(s-t')}r_s ds\right\}\right\} \\ &= (\rho-\alpha)\left\{E_{t'}\int_{t'}^{t} e^{-\rho(s-t')}r_s ds + E_{t'}\left\{e^{-\rho(t-t')}\int_{t'}^{\infty} e^{-\rho(s-t')}r_s^* ds\right\}\right\} \end{aligned}$$

where we have defined $V_s^{r*} = V_{t+s}^r$ and $r_s^* = r_{t+s} - r_t$ for $t' \leq t$. Applying, again, the Strong Markov Property of V_t^r we get:

$$\begin{aligned} R(V_{t'};V^*) &= E_{t'}\int_{t'}^{t} e^{-\rho(s-t')}r_s ds + E_{t'}\left\{e^{-\rho(t-t')}E_{t'}\int_{t'}^{\infty} e^{-\rho(s-t')\infty}r_s^* ds\right\} \\ &= (\rho-\alpha)E_{t'}\int_{t'}^{t} e^{-\rho(s-t')}r_s ds + E_{t'}\left\{e^{-\rho(t-t')}R(V_{t'};V^*)\right\} \\ &= (\rho-\alpha)E_{t'}\int_{t'}^{t} e^{-\rho(s-t')}r_s ds + R(V_{t'};V^*)E_{t'}\left\{e^{-\rho(t-t')}\right\} \end{aligned}$$

Since $r_s = r_{t'} \equiv V_{t'} - V_{t'}^r$ for all $s \in (t',t)$ we can simplify the above expression as:

[25] The Strong Markov Property of regulated Brownian motion processes stresses the fact that the stochastic first passage time t and the stochastic process V_t^r are independent (Harrison, 1985, proposition 7, p.80-81).

$$R(V_{t'};V^*) = \frac{(\rho-\alpha)}{\rho} r_{t'} = \frac{(\rho-\alpha)}{\rho}(V_{t'} - V_{t'}^r) \quad (41)$$

From (41), any application of controls $r_{t'} < V_{t'} - V_{t'}^r$, leads to a reduction of (40) for all $t \geq t'$ and then to $F_m^r(V_t;V^*) > 0$. Furthermore, the firm does not regulate more than r_t since, by doing so, it does not increase the probability of a delayed closure. It does not pay less, since $r_t < V_t - V_t^r$ induces closure making them worse off, i.e. $0 < V_t$. Finally, as V_t^r is a Markov process in levels, it is immediate by (40) that any sub-game beginning at a point at which revocation has not taken place is equivalent to the whole game. The strategy ϕ is efficient for any sub-game starting at an intermediate date and state (t, V_t). We have sub-game perfection.

6) Non-decreasing path of r_t within $[T^*, T'^*)$.

So far we have implicitly assumed that, once started at T^*, the regulation goes on forever. Earlier interruptions are not feasible as long as the threat of closure by the regulator is credible. Credibility relies on the fact that the agency's option-to-revoke the contract if the firm deviates from r_t is always worth exercising at $V_t > V^*$, i.e. $F_m(V_t) \geq F_m(V^*)$. As the decision rule strategy depends on the history of the game, the regulator expects regulation to continue according to the rule r_t and any premature stop could make it no longer subgame-perfect.

However, in an optimal Brownian path there is a positive probability that the primitive process V_t crosses V^* again starting at an interior point of the range (V^*, ∞). In this case, the firm may be willing to stop regulation. That is, the firm regulates its value until $V_t \geq V^*$, letting the agency expect the regulation to continue in the future according to the same rule $r_t = (1-Z_t)V_t$, but when V_t reaches, for the first time after T^*, a predetermined level, say $V' \leq V^*$, it stops the regulation. The regulator will face a jump from zero to $F_m(V') \leq F_m(V^*)$ making the threat of revocation no longer credible. To see this, consider the possibility of the firm's regulation terminating at time T' with $T^* < T' < \infty$, where $T' = \inf(t \geq T^* \mid V_t \geq V')$ is the first hitting time of $V' \leq V^*$ when regulation is on. The regulator's option value starting at any $t \in [T^*, \infty)$,can be expressed as:

$$\tilde{F}_m^r(V_t;V') = P(V';V_t)E_t[F_m^r(V_{T'})e^{-r(t-T')}] + \quad (42)$$

$$(1 - P(V'; V_t)) \max E_t[(V_T^r - I)e^{-r(t-T)}]$$

where $P(V'; V_t)$ is the probability of the unregulated process V_t reaching $V' \leq V^*$ starting at an interior point of the range (V^*, ∞), which is equal to (Cox and Miller, 1965, p. 232-234):

$$\Pr(T' < \infty \mid V_t) \equiv P(V'; V_t) = \left(\frac{V_t}{V'}\right)^{-2\mu/\sigma^2}$$

with $\mu = (\alpha - \frac{1}{2}\sigma^2)$.[26] As the starting point is now any $t \in (T^*, \infty)$, we can immediately see in (42) the dependence on both V_t^r and V_t. Recalling that the option value in the case of regulation is zero and that at time T' when the contract is revocated it is simply $F_m^r(V_{T'}) = F_m^r(V')$, we get:

$$\tilde{F}_m^r(V_t; V') = P(V'; V_t)E_t[F_m^r(V')e^{-r(T'-t)}]$$

According to the Strong Markov Property of V_t^r equation (42) becomes:

$$\tilde{F}_m^r(V_t; V') = P(V'; V_t)F_m^r(V')\left(\frac{V_t}{V'}\right)^{\beta_2} \tag{43}$$

where $\beta_2 < 0$ is the negative root of (14). Since at t the unregulated process V_t is greater than V' and $P(V'; V_t)\left(\frac{V_t}{V'}\right)^{\beta_2} = \left(\frac{V_t}{V'}\right)^{\beta_2 - 2\mu/\sigma^2} \leq 1$, we obtain $\tilde{F}_m^r(V_t; V') \leq F_m^r(V')$ for all $t \in [T^*, T')$, which implies that:

$$\tilde{F}_m^r(V_t; V') = F_m^r(V^*)\left(\frac{V'}{V^*}\right)^{\beta_1}\left(\frac{V_t}{V'}\right)^{\beta_2 - 2\mu/\sigma^2} \leq F_m^r(V^*) \tag{44}$$

Therefore, to avoid revocation the regulation continues until time $T'^* \equiv T'(V^*) = \inf(t \geq T^* \mid V_t - V^* = 0^-)$ when the trigger V^* is hit again (for the first time) after T^*. The game ends and can then be restarted afresh.

B Appendix: Self-interested regulator and social optimum

Although the precise objective function of the regulator does not matter for the qualitative results, contrary to current wisdom, we have assumed the

[26] This probability is $P(V'; V_t) = 1$ for $\mu \leq 0$, see footnote n. 23.

public authority to be self-interested. However, it is not difficult to adapt the methodology of Dixit and Pindyck (1994, chapter 8) to the case of a regulator that maximizes the consumers' surplus.

Let p_t and Q_t be the actual price and quantity in period t. Stationary demand is $D(p)$, from which we obtain the inverse demand function $p_t = p(Q_t)$. Total quantity is produced by N productive units[27] which may differ in their efficiency. Unit n produces $q(n)$ in such a way that $Q(N) = \int_0^N q(n)dn$. Before beginning production, each unit can be activated by the public authority incurring a sunk cost I that is independent of the scale of operation. The area under the demand curve at any instant t can be seen as the flow of total social utility generated by the output flow Q_t produced by N_t productive units. This is given by:

$$\theta_t U(Q(N_t)) = \theta_t \int_0^{Q_t} p(x)dx = \theta_t \int_0^{\int_0^{N_t} q(n)dn} p(x)dx \tag{45}$$

where the multiplicative variable θ takes up the meaning of change in consumers' tastes. Recalling (6), the geometric Brownian motion follows:

$$d\theta_t = \alpha\theta_t dt + \sigma\theta_t dz_t, \qquad \text{with } \theta_0 = \theta \tag{46}$$

Therefore, taking the derivative of (45) with respect to N and assuming, for the sake of simplicity, no variable costs, the marginal social utility expresses the last unit's operating profits:

$$U'(Q(N_t))Q'(N_t)\theta_t = p(Q(N_t))q(N_t)\theta_t \equiv \pi(N_t)\theta_t \tag{47}$$

The regulator aims to maximize the expected present value of social utility net of the cost of capacity expansion. If an amount of dN_t is added to capacity at this instant a cost equal to IdN_t is incurred. The regulator's objective function can then be expressed as:

$$E_0\left\{\int_0^\infty e^{-rt}\theta_t U((Q(N_t))dt - \sum_t e^{-rt} IdN_t \mid \theta_0 = \theta\right\} \tag{48}$$

where the sum is taken over the instants when new units are introduced. Indicating by $W(N,\theta)$ the above maximized value with initial productive units N, for any given value of the shift θ, it should satisfy the following Bellman equation (Dixit and Pindyck, 1994, p.284-287):

[27] To simplify, we treat N as a continuous variable.

$$\frac{1}{2}\sigma^2\theta^2 W_{\theta\theta}(N,\theta) + \alpha\theta W_\theta(N,\theta) - \rho W(N,\theta) = -\theta U((Q(N))$$

For fixed N this can be seen as an ordinary second order differential equation in θ, the general solution of which can be expressed as:

$$W(N,\theta) = B(N)\theta^{\beta_1} + \frac{\theta U((Q(N))}{\rho - \alpha} \tag{49}$$

Whilst the second term in (49), $\frac{\theta U((Q(N))}{\rho-\alpha}$, is the expected present value of the social utility if N is held fixed at its level, the first term, $B(N)\theta^{\beta_1}$, represents the value of the comunity's ability to increase N optimally in response to the evolution of θ. In other words, it is the value placed by the society on its option to expand its productive capacity. To complete the solution we need to find the optimal capacity expansion rule. At the boundary where the marginal dN-th unit is added, the regulator decides to increase capacity only if $W_N(N,\theta)dN = IdN$: the increase in the value function must be equal to its cost (matching value condition):

$$W_N(N,\theta) \equiv B'(N)\theta^{\beta_1} + \frac{\theta U'((Q(N))Q'(N)}{\rho - \alpha} = I \tag{50}$$

Furthermore, the marginal gain and marginal cost should smooth out at the boundary (smooth pasting condition), i.e.:

$$W_{N\theta}(N,\theta) \equiv \beta_1 B'(N)\theta^{\beta_1 - 1} + \frac{U'((Q(N))Q'(N)}{\rho - \alpha} = 0 \tag{51}$$

When a marginal expansion is actually carried out, the comunity loses the value of the marginal option thus exercised which is given by $-B'(N)\theta^{\beta_1}$. Via the smooth pasting condition (50) and making use of (47) we are able to write:

$$\begin{aligned} -B'(N)\theta^{\beta_1} &= \frac{(\beta_1-1)^{\beta_1-1} I^{1-\beta_1}}{\beta_1^{\beta_1}} \left(\frac{U'(Q(N))Q'(N)}{\rho-\alpha}\right)^{\beta_1} \theta^{\beta_1} \\ &= \frac{(\beta_1-1)^{\beta_1-1} I^{1-\beta_1}}{\beta_1^{\beta_1}} \left(\frac{\pi(N)\theta}{\rho-\alpha}\right)^{\beta_1} \theta^{\beta_1} \end{aligned} \tag{52}$$

Finally, defining $V = \frac{\pi(N)\theta}{\rho-\alpha}$ and substituting it in (52), we get the simplified expression:

$$-B'(N)\theta^{\beta_1} = \frac{(\beta_1 - 1)^{\beta_1 - 1} I^{1-\beta_1}}{\beta_1^{\beta_1}} V^{\beta_1} = A(V^*)V^{\beta_1} \tag{53}$$

where:

$$A(V^*) = \frac{1}{\beta_1}(V^*)^{1-\beta_1} > 0, \quad \text{and} \quad V^* = \frac{\beta_1}{\beta_1 - 1} I$$

As a representative of consumers' interests, the regulator's maximization of the discounted consumer surplus implies the maximization of the option value to revoke the contract as expressed in (13) in the text.

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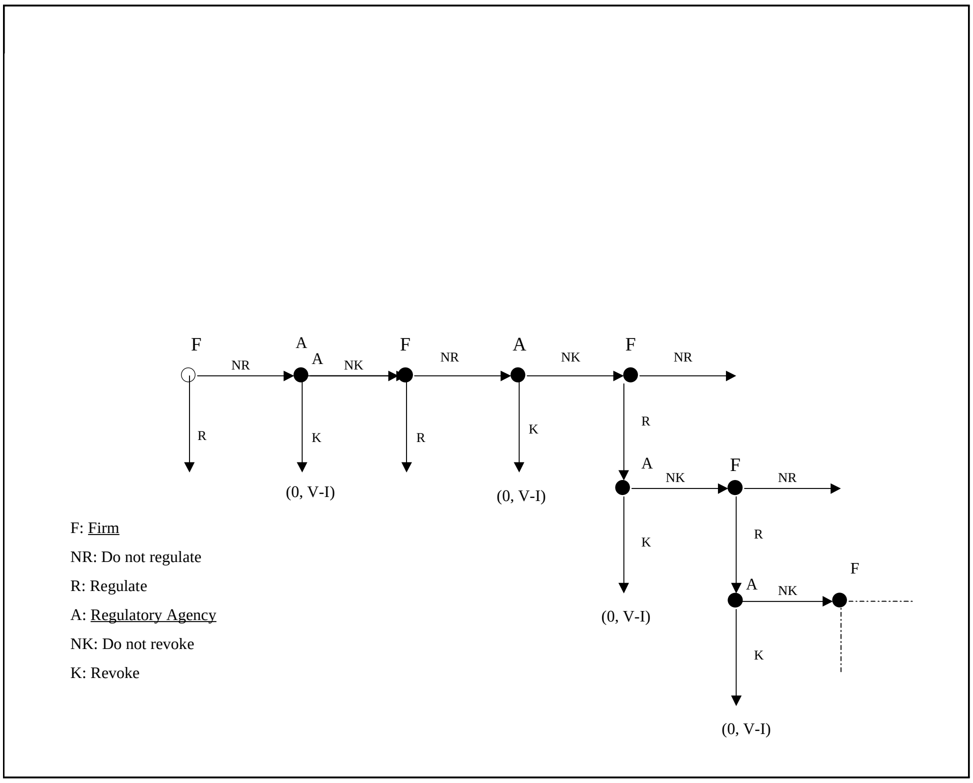

Figure 1: Discrete time representation of the game (dominant strategies)

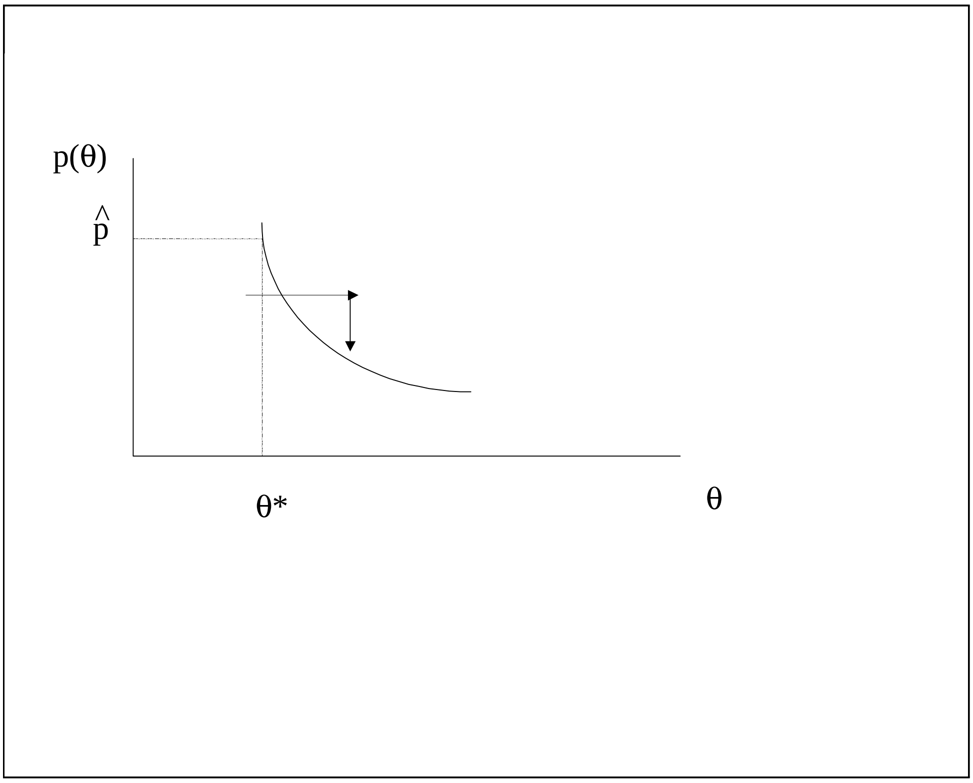

Figure 2: Price regulation under threat of revocation

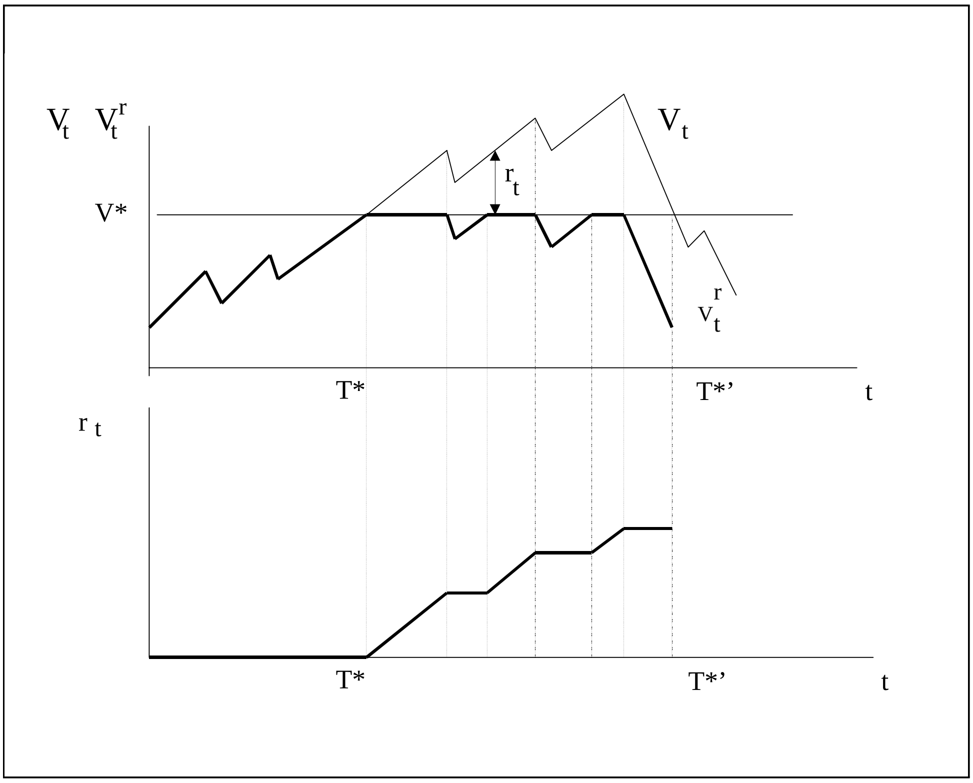

Figure 3: Threat game timing