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ENVIRONMENTAL LIABILITY AND TECHNOLOGY CHOICE : A DUOPOLISTIC ANALYSIS

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Abstract

This paper focuses both on the competition process and the firms liability in environmental protection and the demonstration is made by comparing two models of safety investment. The first one shows sensitive players to their environmental liability: they seek to minimize the technologies accident risk while the second one corresponds to a much more standard choice. The players main preoccupation is about their market share even if they care about liability. Then, from a very simple duopolistic competition model with strict liability, we show, first, that the way the firms assess the environmental question is not neutral on their expected performances. Second, that the associated level of technology to the liability concern - i.e. a high level of care or a low one- have different impact on profitability. Consequently, the competitors general attitude, their beliefs and the institutional rules have strong effects on the environmental investment assessments. More precisely, the enforcing rule the players will adopt will play directly on the performance, not only of one firm, but on the whole set of industrial firms.

Key words : duopoly, environmental investment, liability theory.
JEL : K32,C72,D43,D81

1 Introduction

How environmental strict liability rules may impact on the competition process? That is the question this paper aims at answering mainly. We recall that strict liability means liability imposed without evidence of negligence. That is, the defendant may be found guilty upon a showing that his action resulted in harm, without consideration of whether or not he acted reasonably. Strict liability is usually imposed upon those who engage in abnormally dangerous or "ultra-hazardous" activities, like handling explosives, or other activities defined by statute [1]. The most significant advantage to a nuisance action is that all these damages, as well as the relaxed statute of limitations, are all (probably) recoverable by plaintiff without proof of fault or negligence on the part of the defendant. In the model presented here, strict liability is proportionally depending upon both the production scale and the technology used.

Models of competition with liability as central concern are quite few [2]. However, this question is important because big companies cannot play freely with safety: their own liability and reputation are committed towards public opinion [3]. The protective and preventive actions the managers

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[1] For example, the Comprehensive Environmental Response, Compensation and Liability Act. In four states, Of United States, Louisiana, Oklahoma, Oregon, and Washington, courts have labelled aerial application of pesticides an "ultra hazardous" or "abnormally dangerous" activity, and have imposed strict liability for damage done without requiring proof of fault.

[2] In the part 5 of this paper the links with the existing literature are analyzed.

[3] See for example Kunreuther (1991).

may undertake may have several features: subscribing policy insurance and/or choosing prevention measures bearing on the production process. In this paper, the analysis will be restricted to this latter point [4] .

The analysis of the impact of liability rules on competition stems from a very simple duopolistic competition model ruled by a strict liability regime. Two alternative ways of assessing investment prevention are studied. The first one studies competitors looking minimizing the accident risk while, within the same framework, the second one appears much more market share oriented. This comparison shows that the way risks are assessed is not neutral on the expected performances of each model.

In the first model, actions are "sequential ". That means that prevention prevails over competition because reducing liability appears important to the managers. Seeking at minimising their liability, technology has to be adapted once quantities have been chosen first. In the second one, both technology and quantities are simultaneously chosen, consequently, the model is said "simultaneous ".

From a strict mathematical point of view, may be showed sufficient and necessary conditions such that either the sequential model performs better than the simultaneous one, or the opposite. As a consequence, it is shown that the competitors' general beliefs and behaviors about liability have strong effects on the investment assessments and the working of the competition process. More precisely, the enforcing rule the players will adopt will play directly on the performance, not only of one firm, but on the whole set of industrial firms.
In §2, is explained the economic context in which the different models apply. In §3, the specific features of each model are presented. Then, in §4 are exposed the main theorems for the general case -i.e. the conditions such as either model one or two bring better results and § 5 is devoted to analyze the specific case of symmetric firms in which, model's two payoffs are always greater than model one. These results are checked by numerical simulation in the §6. §7 aims at comparing this research area to the on going stream of literature about liability.

2 Competition and risk : Preliminary considerations

In the model, two firms are competing duopolistically. They choose technologies among an infinite set of possible ones. They are playing a one shot, non co-operative Cournot-Nash game with no ex-ante communication. However, the technology set is common knowledge - that means that two independent firms may adopt the same technology. Technology means both the choice of the production process and the level of safety. Changing either the level of care or the productive technology is a change of technology in the technology set. The technology choice aims equally at controlling the pollutant emission of the firms. This choice is made ex-ante and the firms are supposed to take due care of the environment protection according to two rules. First, the firms have to respect a given pollution standard that has been fixed previously by the regulator. Second, they have to take into account the minimization of liability by an accurate technology choice. More precisely two points have to be distinguished :
- First, once chosen the technology, the firms check the effectiveness of the environment protection , and the good working of the production process. This is done under negligible monitoring costs. In the favourable case -i.e. when no defect checked - the firm may go on producing. Hence, the

[4] To put it shortly, these choice is induced by the insurance companies reluctance to take environmental risk in their portfolio.

firm's technology internalizes fully the pollutant emission. However, when unfavourable signal are perceived, i.e. the preventive process may defect. Consequently, the manager has to reduce or to cut off the production level of the firm. Tort liability is avoided then [5].
- Second, a liability rule is considered [6].The strict liability level depends on the extend of the injury, and on the production and the due care levels[7].
The timing of the game is either two steps for the sequential model -i.e. first, are chosen the quantities to bring on the market, then, the managers are optimizing the technology - or one unique step for the simultaneous model- i.e. quantities and technology are simultaneously set[8]. On this basis are compared the performance of both models. The so-called "sequential" game means that firms care more about liability, while the "simultaneous" game shows that market values (i.e. the expected gains) worth more compared to environmental liability. However, in this model too, liability concerns are not null.

The extend of damages may exceed the financial capacity of the firm. Profits may be negative when the damages caused exceed the firm cash-flows. Normally, the firm will go bankrupt without having fulfilled its commiments. In this latter case, we may imagine that the firm (or its managers) will be indebted for their entire life. The firm is fully liable when the production is undertaken under an unfavourable monitoring report. Under point one above, the firm must cut its activity back, partially or totally. If not, its manager can be held as fully responsible and his personal ownership may be engaged or penal punishment may be incurred. Consequently, our assumption will be that every time a defect is checked in the safety system, the firm manager will take suitable regulation measures. As a simplification, this corresponds to a zero production level.

To simplify the analytical framework assume, then :
Assumption1,(Full liability): In the case of an accident occurrence the firm's and the manager's responsibility are strictly linked.
This means that a responsible agent may be pointed out. However, this does not solve the potential firm's insolvency problem when to reimburse injured parties, but it prevents any speculation about the liability determination, hence, as an assumption,. the "judgement proof problem" is avoided. We remind that the "judgement proof problem" is the possibility for a defecting firm to become bankrupt and escape to its financial liability.
Assumption 2,(Manager fair behaviour):When he is faced with a faulty technology, the manager does non infringe the law or the rules and conforms its production to the government standard.
Assumption 2 means that the manager is facing two well-specified risks. The first one is a production risk linked to the working of the firm technology. The second risk is intrinsically associated to his decision choice about preventive technology and the level of production and is independent from a faulty decision.

3 The Models: The general framework

We suppose that two firms $i, i = 1, 2$ are competing as duopolists. Each firm makes decisions which controls its market production ($y = (y_1, y_2)$) and its technology choice ($\alpha = (\alpha_1, \alpha_2)$). The

[5]This assumption means that the firm will not infringe the regulation deliberately and will be liable because of some unexpected defect. However, because of a positive accident risk in his profit computation the manager has to take into account the possibility of a reduced production.

[6]For a better explanation the lector is induced to refer to our part 4 on the Related literature.

[7]We take for granted, here, the Shavell's (1984) demonstration that neither liability nor regulation is necessarily better than the other and have to be used jointly for a better control of potential harms.

[8]See decision trees in fig. 1 and 2 for model one and fig.3 for model two.

pollution emission results naturally from production but, under normal regime, it is supposed that α internalizes the pollutant effluent ($\alpha \neq 0$). Hence, the pollution function ($e(y, \alpha)$) is supposed to respect the constraints ($\bar{e}$) imposed by a regulator, hence the market production and the technology have to respect: $e(y, \alpha) \leq \bar{e}$.
Let $Y_\alpha = \{y \in (\mathcal{R}_0^+)^2 : / : e(y, \alpha) \leq \bar{e}\}$ be the production set, while if $y = y(\alpha)$ is fixed $A = \{\alpha \in [0,1]^2 : / : e(y(\alpha), \alpha) \leq \bar{e}\}$ is the technology set. Note that in some cases the technology substitutability is not complete, so we shall consider the restriction $\bar{A} \subset A$, where $\bar{A}$ is the admissible set of technologies.
α is both a productive and a preventive technology, the higher (i.e. the nearer from 1) the more efficient it is supposed to be. This have strong consequences on the following two probability distributions:
- The first set of probabilities is formed on the working of the production process. Hence, $\rho(\alpha_i)$ is the probability that α works (with $\rho(1) < 1$ (the null risk production is impossible) and $\rho'(\alpha) \geq 0$).
- Probability of an accident occurrence : $\eta(\alpha)$ is the probability that an accident occurs (with $\eta'(\alpha) \leq 0$).
Reaching this point we can describe now the performance structure of the firm. Let us write, $\pi_i(y, \alpha_i, a)$ the profit function which may be expressed as,

$$\pi_i(y, \alpha_i, a) = y_i Pr(y) - C_i(y_i, \alpha_i, a) \qquad a = 0, 1.$$

a refers to the state nature : 0 means that technology does not work, while 1 means that it does. $Pr(y)$ is the price function, (inverse demand), $C_i(y_i, \alpha_i, a)$ is the cost function. We shall formulate specific assumptions for the cost function C_i when necessary (see § 5). If it is checked that the system may be defecting, then the production minimal level is required.

3.1 The Model basic description

The cost structure is an important factor. The choice of a given technology entails quite huge costs due first to the conception of R &D effort and second to the technological changes induced by the effective working of the innovations. Consequently, the more outstanding a technology is, the more expensive when to be implemented. Hence, the firm bears sunk costs (see §5) every time the technology is not zero.

In our analysis, the first model will be considered as liability minimizer, the second one associates in the same calculus quantity, technology and liability. This may be summarized as:

First model, ("sequential"): A duopolist seeks to maximize first a profit function (which is not the final payoff of the game), defining a set of couples composed with equilibria quantities and equilibria technology levels. Then, considering the liability functions, he will set the final equilibrium couple that will maximize his final payoff by minimising the liability level.

Second Model, ("simultaneous") : The duopolists chooses simultaneously the quantities to sell, the technology level. Hence, the payoff is maximized simultanously.
Sequential choice means that players set first quantities then technology in order to lessen their liability level about accident occurrence [9]. In the first model, quantities are chosen according a

[9] Such a behaviour may be exemplified in the computer industry for example. Environment-friendly computers are still in short supply. This year's ecological computer test awarded three producers of computers with the rating "good": Acer, Hewlett Packard and Siemens Nixdorf. "Acceptable" was awarded six times, "insufficient" only twice. The assessment included the use of halogen-free flame protection in the casing, energy-saving monitors with little radiation as well as renewable constructions. Producers of cheap home computers often ignore ecological standards. BUND had asked 20 companies to participate in the survey, but only 11 replied. Customers should pay attention

Cournot Nash game. The technology is settled then on similar basis, its intensity is ranked on a (0,1) interval, (0 for the lower technology, 1 for the higher one)[10].

3.2 Model 1

Here environmental concerns appear prior compared to production objectives. Model 1 corresponds to the so-called "sequential" model. Cournot-Nash players compete first on quantities, (step 1), then are concerned by lowering their liability impact by adapting (choosing) then their production technology.
In a first step, this Model considers that agents want to obtain a non co-operative equilibrium that maximize their expected profit function.
Player i gains $\pi_i(y(\alpha), \alpha_i, 1)$ if the choice of technology α_i is good (if it works) and the associated probability is $\rho(\alpha_i)$. In the opposite case whose probability is $1 - \rho(\alpha_i)$, player i obtains $\pi(y(\alpha), \alpha_i, 0)$. See figure 1. The players want to find $y^*(\alpha) = (y_1^*(\alpha), y_2^*(\alpha))$ the Nash equilibrium

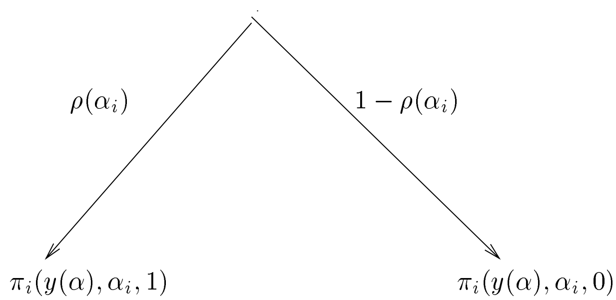

Figure 1: Step 1 for Model 1 and player i

of the game where the payoff function for player i is: :

$$\bar{\pi}_i(y, \alpha_i) = E_a \pi_i(y, \alpha_i, a) = \rho(\alpha_i)\pi_i(y, \alpha_i, 1) + (1 - \rho(\alpha_i))\pi_i(y, \alpha_i, 0). \quad (1)$$

(where $E_a(.)$ is the average expectation operator). i.e., they want to obtain for each $i = 1, 2$ the solution of:

$$\max_{y_i} \bar{\pi}_i(y, \alpha_i) = \bar{\pi}_i(y^*(\alpha), \alpha_i) = \bar{\pi}_i^*(\alpha). \quad (2)$$

with the restriction

$$y \in Y_\alpha$$

In a second step, the liability tends to be minimized. Llet be the damage function D_i.which depends both on the technology level and the production scale. In the first step $y^*(\alpha)$ and $\bar{\pi}_i^*(\alpha)$, $i = 1, 2$ are just obtained without refering to D_i Now, in this step, player i obtains a gain $\bar{\pi}_i^*(\alpha) - D_i(y_i^*(\alpha, \alpha_i)$ without accident, where

$$D_i(y_i, \alpha_i) = \rho(\alpha_i)D_i(y_i, \alpha_i, 1) + (1 - \rho(\alpha_i))D_i(y_i, \alpha_i, 0).$$

This event have a probability of $\eta(\alpha_i)$. With probability $1 - \eta(\alpha_i)$ player i obtains $\bar{\pi}_i^*(\alpha)$. See figure 2. That is they expect $\alpha^* = (\alpha_1^*, \alpha_2^*)$, the Nash equilibrium of the game where the payoff function

to long warranty periods of at least 3 years. This limited example show that some industrial sectors are much more concerned by competition than by environment protection and they may change their mind if others are doing so.

[10]These results may enlighten the debate around the "Best available technologies" (BAT) questions. For example, model one may be considered as an example of (BAT) choice while model two shows a less important effort from the manager part to reduce the firm's potential liability.

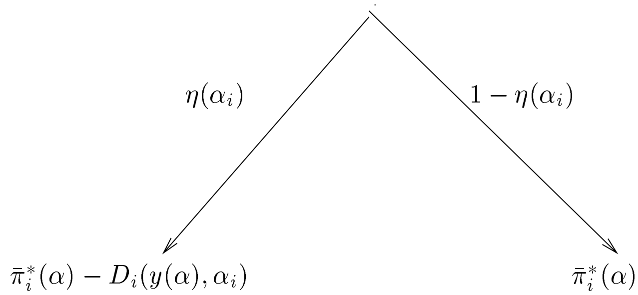

Figure 2: Step 2 for Model 1 and player i

of player i is

$$\eta(\alpha_i)(\bar{\pi}_i^*(\alpha) - D_i(y_i^*(\alpha), \alpha_i)) + (1-\eta(\alpha_i))\bar{\pi}_i^*(\alpha) = \bar{\pi}_i^*(\alpha) - \eta(\alpha_i)D_i(y_i^*(\alpha), \alpha_i))$$

i.e., they expect the solution of

$$\bar{P}_i(y^*(\alpha^*), \alpha_i^*) = \max_{\alpha_i}\{\bar{\pi}_i^*(\alpha) - \eta(\alpha_i)D_i(y_i^*(\alpha), \alpha_i))\} \tag{3}$$

with the restriction

$$\alpha \in A$$

Finally firm i proposes $(\alpha_i^*, y_i^*(\alpha_i^*))$ as a satisfactory solution of the problem (2) + (3).

3.3 Model 2

In this Model technology and quantities are chosen simultaneously. Firms seek both to lower their potential liability by choosing the more efficient technologies and to maximize their profit by fixing the quantities to bring to the market.
In this case the agent i wants to obtain $\hat{\alpha}_i$ and $\hat{y}_i$, $(\hat{y} = (\hat{y}_1, \hat{y}_2))$ a Nash equilibrium of a game where the gains of player i for each events, (technology α_i works or not, it carries an accident or not), are given in figure 3. That is, if we define

$$\begin{aligned} P_i(y, \alpha_i) &= \rho(\alpha_i)\{\eta(\alpha_i)(\pi_i(y, \alpha_i, 1) - D_i(y_i, \alpha_i, 1)) + (1-\eta(\alpha_i))\pi_i(y, \alpha_i, 1)\} \\ &+(1-\rho(\alpha_i))\{\eta(\alpha_i)(\pi_i(y, \alpha_i, 0) - D_i(y_i, \alpha_i, 0)) + (1-\eta(\alpha_i))\pi_i(y, \alpha_i, 0).\} \\ &= \bar{\pi}_i(y, \alpha_i) - \eta(\alpha_i)D_i(y_i, \alpha_i). \end{aligned} \tag{4}$$

they want to obtain $\hat{\alpha}_i$ and $\hat{y}_i$, $i = 1, 2$, solution of:

$$P_i(\hat{y}, \hat{\alpha}_i) = \max_{\alpha_i}\max_{y_i} P_i(y, \alpha_i). \tag{5}$$

with the restriction

$$e(y, \alpha) \leq \bar{e}$$

Then firm i proposes $(\hat{\alpha}_i, \hat{y}_i)$ as a satisfactory solution of the problem (5).

Remark 3.1 *It is easy to see that* $\bar{P}_i(y^*(\alpha^*), \alpha_i^*) = \max_{\alpha_i/\alpha \in A} P_i(y^*(\alpha), \alpha_i)$.

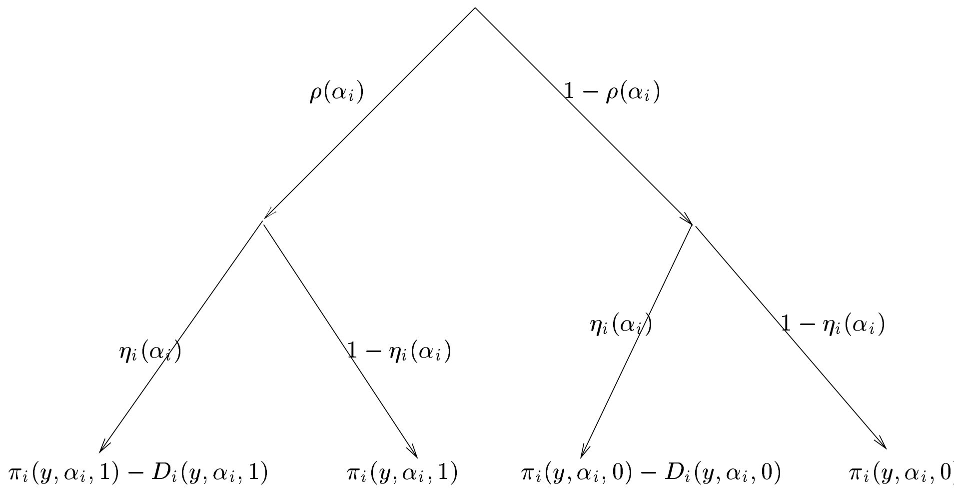

Figure 3: Model 2 for player i

Remark 3.2 Notice that finding $\hat{\alpha}_i$ and $\hat{y}_i$, $i=1,2$, solution of:

$$P_i(\hat{y},\hat{\alpha}_i)=\max_{\alpha_i}\max_{y_i} P_i(y,\alpha_i)\max_{y_i} P_i(y,\alpha_i) \qquad \text{with} \quad e(y,\alpha)\leq\bar{e}$$

is equivalent to solve

$$\max_{y_i/y\in Y_\alpha} P_i(y,\alpha_i)=P_i(\hat{y}(\alpha),\alpha_i)$$

and then

$$\max_{\alpha_i/e(\hat{y},\alpha)\leq\bar{e}} P_i(\hat{y},\alpha_i)=P_i(\hat{y}(\alpha),\hat{\alpha}_i)$$

4 General results

At stake in our research is to find sufficient conditions allowing firms to choose either computation issued from Model 1, or from Model 2. Each computation does not give similar result and may have strong environmental involvement. Hence, it is of great interest to compare their mutual efficiency.

Two situations have been examined. The first one, is a general analysis with non-symmetrical duopolistic models. It appears that efficiency of the first one compared to the second one is dependent on the relationship between damage function and probability of accident occurrence. We clearly identify the conditions to have Model 1 pre-eminent compared to Model 2 and vice-versa. Stronger results are reached with the symmetrical-case model. Indeed, it can be shown that Model 2 gives better performances than Model 1 in any circumstances. By performances are understood both the payoff functions and the higher level of safe technology. These results are then tested with numerical examples which give good illustration of the firms behavior when facing with environmental constraints.

4.1 General case

We suppose that:

- The quantities $\pi_i(y,\alpha_i,0)$ and $D_i(y_i,\alpha_i,0)$ do not depend on y. These conditions simplify computations and can be removed.

- We suppose the existence but not the unicity of Nash equilibria. In case of multiplicity of equilibria, we assume that players have a criteria to chose one of them. Sufficient conditions for existence and uniqueness of Nash equilibrium, are given in [11].
- π_i is a decreasing function of y_{-i}, $i = 1, 2$.
- D_i is an increasing function of y_i.

Let $y_1^* = y_1^*(\alpha_1, \alpha_2)$ and $y_2^* = y_2^*(\alpha_1, \alpha_2)$ be the production Nash equilibrium for Model 1. That is y_1^*, y_2^* verifies

$$\bar{\pi}_1(y_1, y_2^*, \alpha_1) \leq \pi_1(y_1^*, y_2^*, \alpha_1), \quad \bar{\pi}_2(y_1^*, y_2, \alpha_2) \leq \pi_2(y_1^*, y_2^*, \alpha_2), \quad \forall y_1, y_2, \alpha_1, \alpha_2. \tag{6}$$

Let $\hat{y}_1 = \hat{y}_1(\alpha_1, \alpha_2)$ and $\hat{y}_2 = \hat{y}_2(\alpha_1, \alpha_2)$ be the production Nash equilibrium for Model 2. That is $\hat{y}_1, \hat{y}_2$ verify

$$P_1(y_1, \hat{y}_2, \alpha_1) \leq P_1(\hat{y}_1, \hat{y}_2, \alpha_1), \quad P_2(\hat{y}_1, y_2, \alpha_2) \leq P_2(\hat{y}_1, \hat{y}_2, \alpha_2), \quad \forall y_1, y_2, \alpha_1, \alpha_2. \tag{7}$$

By (1) and (4) it is easy to see that:

$$P_i(y_1, y_2, \alpha_i) \leq \bar{\pi}_i(y_1, y_2, \alpha_i) \qquad \forall y_1, y_2, \alpha_i \quad i = 1, 2 \tag{8}$$

Let us compare now, the two models and let us study conditions insuring that the so-called BAT Model performs better than the "simultaneous" one.

Theorem 4.1 *if*

$$\bar{\pi}_1(\hat{y}_1, \hat{y}_2, \alpha_1) \leq \bar{\pi}_1(\hat{y}_1, y_2^*, \alpha_1) - \eta(\alpha_1) D_1(y_1^*, \alpha_1)$$

then

$$P_1(\hat{y}_1, \hat{y}_2, \alpha_1) \leq P_1(y_1^*, y_2^*, \alpha_1).$$

That means that the Model 1 performs better than the Model 2, for player 1, at least in choosing the production level, and for all α.

Proof: By (8),

$$P_1(\hat{y}_1, \hat{y}_2, \alpha_1) \leq \bar{\pi}_1(\hat{y}_1, \hat{y}_2, \alpha_1). \tag{9}$$

By hypothesis,

$$\bar{\pi}_1(\hat{y}_1, \hat{y}_2, \alpha_1) \leq \bar{\pi}_1(\hat{y}_1, y_2^*, \alpha_1) - \eta(\alpha_1) D_1(y_1^*, \alpha_1) \tag{10}$$

by (6),

$$\bar{\pi}_1(\hat{y}_1, y_2^*, \alpha_1) \leq \bar{\pi}_1(y_1^*, y_2^*, \alpha_1) \tag{11}$$

then by (9), (10) and (11), we have:

$$\begin{aligned} & P_1(\hat{y}_1, \hat{y}_2, \alpha_1) \leq \bar{\pi}_1(\hat{y}_1, \hat{y}_2, \alpha_1) \leq \\ & \bar{\pi}_1(\hat{y}_1, y_2^*, \alpha_1) - \eta(\alpha_1) D_1(y_1^*, \alpha_1) \leq \\ & \bar{\pi}_1(y_1^*, y_2^*, \alpha_1) - \eta(\alpha_1) D_1(y_1^*, \alpha_1) = P_i(y_1^*, y_2^*, \alpha_1) \end{aligned}$$

As we want to show. ■

Theorem 4.2 *if*

$$\bar{\pi}_2(\hat{y}_1, \hat{y}_2, \alpha_2) \leq \bar{\pi}_2(y_1^*, \hat{y}_2, \alpha_2) - \eta(\alpha_2) D_2(y_2^*, \alpha_2)$$

then

$$P_2(\hat{y}_1, \hat{y}_2, \alpha_2) \leq P_2(y_1^*, y_2^*, \alpha_2).$$

that means that the Model 1 performs better than the Model 2, for player 2, at least in choosing the production level, and for all α*.*

The proof is analogous to that one of theorem 4.1. Let us analyze now the "simultaneous" Model. As previously, we draw the following two theorems:

Theorem 4.3 *if*

$$\bar{\pi}_1(y_1^*, y_2^*, \alpha_1) \leq \bar{\pi}_1(y_1^*, \hat{y}_2, \alpha_1)$$

then

$$P_1(y_1^*, y_2^*, \alpha_1) \leq P_1(\hat{y}_1, \hat{y}_2, \alpha_1).$$

That means that Model 2 performs better than the Model 1 for player 1 at least in choosing the production level, and for all α*.*

Proof: As $\hat{y}$ is a Nash equilibrium for P_i, we have that

$$P_1(\hat{y}_1, \hat{y}_2, \alpha_1) \geq P_1(y_1^*, \hat{y}_2, \alpha_1) \tag{12}$$

then, by assumption and (12), we have,

$$\begin{aligned} &P_1(\hat{y}_1, \hat{y}_2, \alpha_1) \geq P_1(y_1^*, \hat{y}_2, \alpha_1) \\ &= \bar{\pi}_1(y_1^*, \hat{y}_2, \alpha_1) - \eta(\alpha_1) D_1(y_1^*, \alpha_1) \\ &\geq \bar{\pi}_1(y_1^*, y_2^*, \alpha_1) - \eta(\alpha_1) D_1(y_1^*, \alpha_1) = P_1(y_1^*, y_2^*, \alpha_1) \end{aligned}$$

As we want to prove. ∎

Theorem 4.4 *if*

$$\bar{\pi}_2(y_1^*, y_2^*, \alpha_2) \leq \bar{\pi}_2(\hat{y}_1, y_2^*, \alpha_2)$$

then

$$P_2(y_1^*, y_2^*, \alpha_2) \leq P_2(\hat{y}_1, \hat{y}_2, \alpha_2).$$

That means that the Model 2 performs better than the Model 1 for player 2 at least in choosing the production level, and for all α*.*

The proof is analogous that one of theorem 4.3.

Remark 4.1 *The hypothesis of theorem 4.1 is that*

$$\bar{\pi}_1(\hat{y}_1, y_2^*, \alpha_1) - \bar{\pi}_1(\hat{y}_1, \hat{y}_2, \alpha_1) \geq \eta(\alpha_1) D_1(y_1^*, \alpha_1) \tag{13}$$

This inequality means:

- $y_2^* < \hat{y}_2$ *(because it must be* $\bar{\pi}_1(\hat{y}_1, y_2^*, \alpha_1) - \bar{\pi}_1(\hat{y}_1, \hat{y}_2, \alpha_1) \geq 0$*).*

- *If function* $\bar{\pi}_1$ *is continuous in* y_2*, then either,* $A > 0$ *exists such that* $\hat{y}_2 - y_2^* > A$ *(in order to make the positive quantity* $\bar{\pi}_1(\hat{y}_1, y_2^*, \alpha_1) - \bar{\pi}_1(\hat{y}_1, \hat{y}_2, \alpha_1)$ *big enough; or* $D_1(y_1^*, \alpha_1)$ *or* $\eta(\alpha_1)$ *must be small enough.*

Taking into account that $\pi_i(y, \alpha_i, 0)$ *does not depend on* y*, condition (13) means that,*

$$\eta(\alpha_1) \le \frac{\rho(\alpha_1)(\pi_1(\hat{y}_1, y_2^*, \alpha_1, 1) - \pi_1(\hat{y}_1, \hat{y}_2, \alpha_1, 1))}{D_1(y_1^*, \alpha_1)}$$

We can obtain a similar interpretations for theorem 4.2.
Remind that theorem 4.3 assumption is that,

$$\bar{\pi}_1(y_1^*, y_2^*, \alpha_1) \le \bar{\pi}_1(y_1^*, \hat{y}_2, \alpha_1)$$

this assumption is equivalent to,

$$y_2^* > \hat{y}_2.$$

So the sufficient conditions to have Model 2 performing better than Model 1 are weaker than those one to have Model 1 better than Model 2.

Remark 4.2 *It is easy to see that if, by instance,*

$$P_1(y_1^*(\alpha), y_2^*(\alpha), \alpha_1) \le P_1(\hat{y}_1(\alpha), \hat{y}_2(\alpha), \alpha_1),$$

then,

$$\eta(\alpha_1) \ge \frac{\bar{\pi}_1(y_1^*(\alpha), y_2^*(\alpha), \alpha_1) - \bar{\pi}_1(\hat{y}_1(\alpha), \hat{y}_2(\alpha), \alpha_1)}{D_1(y_1^*(\alpha), \alpha_1) - D_1(\hat{y}_1(\alpha), \alpha_1)}$$

As we suppose that $\pi_i(y, \alpha_i, 0)$ *and* $D_i(y_i, \alpha_i, 0)$ *do not depend on* y*, this last expression becomes:*

$$\eta(\alpha_1) \ge \frac{\pi_1(y_1^*(\alpha), y_2^*(\alpha), \alpha_1, 1) - \pi_1(\hat{y}_1(\alpha), \hat{y}_2(\alpha), \alpha_1, 1)}{D_1(y_1^*(\alpha), \alpha_1, 1) - D_1(\hat{y}_1(\alpha), \alpha_1, 1)}$$

that means, as $\eta(\alpha_i) < 1$ *that*

$$\pi_1(y_1^*(\alpha), y_2^*(\alpha), \alpha_1, 1) - D_1(y_1^*(\alpha), \alpha_1, 1) \le \pi_1(\hat{y}_1(\alpha), \hat{y}_2(\alpha), \alpha_1), 1) - D_1(\hat{y}_1(\alpha), \alpha_1, 1)$$

Remark 4.3 *By Remarks 4.1 and 4.2 we obtain sufficient conditions to have,*

- $P_i(y_1^*(\alpha), y_2^*(\alpha), \alpha_i) \le P_i(\hat{y}_1(\alpha), \hat{y}_2(\alpha), \alpha_i) \qquad i = 1, 2 \qquad \forall \alpha$*, which are:*

$$\begin{aligned} & y_i^* > \hat{y}_i \qquad i = 1, 2 \\ & \frac{\pi_1(y_1^*(\alpha), y_2^*(\alpha), \alpha_1, 1) - \pi_1(\hat{y}_1(\alpha), \hat{y}_2(\alpha), \alpha_1, 1)}{D_1(y_1^*(\alpha), \alpha_1, 1) - D_1(\hat{y}_1(\alpha), \alpha_1, 1)} \le \eta(\alpha_1) \\ & \frac{\pi_2(y_1^*(\alpha), y_2^*(\alpha), \alpha_2, 1) - \pi_2(\hat{y}_1(\alpha), \hat{y}_2(\alpha), \alpha_2, 1)}{D_2(y_2^*(\alpha), \alpha_2, 1) - D_2(\hat{y}_2(\alpha), \alpha_2, 1)} \le \eta(\alpha_2) \end{aligned} \tag{14}$$

When $\alpha_i = \alpha_i^*$ *for* $i = 1, 2$ *we have that* $P_i(y_1^*(\alpha^*), y_2^*(\alpha^*), \alpha_i^*)$ *is the optimal payoff for the players in Model 1. If conditions (14) are checked for* α^**, Model 2 performs better than Model 1 when players make the best in this Model.*

We can also obtain necessary and sufficient conditions to have,

- $P_i(y_1^*(\alpha), y_2^*(\alpha), \alpha_i) \geq P_i(\hat{y}_1(\alpha), \hat{y}_2(\alpha), \alpha_i) \qquad i = 1, 2$, *which are:*

$$\begin{array}{l} \hat{y}_i > y_i^* \qquad i = 1,2 \\ \dfrac{\pi_1(\hat{y}_1(\alpha), \hat{y}_2(\alpha), \alpha_1, 1) - \pi_1(y_1^*(\alpha), y_2^*(\alpha), \alpha_1, 1)}{D_1(\hat{y}_1(\alpha), \alpha_1, 1) - D_1(y_1^*(\alpha), \alpha_1, 1)} \leq \eta(\alpha_1) \leq \\ \qquad \dfrac{\rho(\alpha_1)(\pi_1(\hat{y}_1, y_2^*, \alpha_1, 1)) - \pi_1(\hat{y}_1, \hat{y}_2, \alpha_1, 1)}{D_1(y_1^*, \alpha_1)} \\ \dfrac{\pi_2(\hat{y}_1(\alpha), \hat{y}_2(\alpha), \alpha_2, 1) - \pi_2(y_1^*(\alpha), y_2^*(\alpha), \alpha_2, 1)}{D_2(\hat{y}_2(\alpha), \alpha_2, 1) - D_2(y_2^*(\alpha), \alpha_2, 1)} \leq \eta(\alpha_2) \leq \\ \qquad \dfrac{\rho(\alpha_2)(\pi_2(y_1^*, \hat{y}_2, \alpha_2, 1)) - \pi_2(\hat{y}_1, \hat{y}_2, \alpha_2, 1)}{D_2(y_2^*, \alpha_2)} \end{array} \tag{15}$$

When $\alpha_i = \hat{\alpha}_i$ for $i = 1, 2$ we have that $P_i(\hat{y}_1(\hat{\alpha}), \hat{y}_2(\hat{\alpha}), \hat{\alpha}_i)$ is the optimal payoff for the players in Model 2. If conditions (15) are checked for $\hat{\alpha}$, when the players in this Model choose the technology $\hat{\alpha}$, the Model 1 performance is better than the one of Model 2 when in this Model players make the best.

4.2 Results in the symmetrical case

From now, we suppose without lost of generality, that:

$$\pi_i(y_1, y_2, \alpha_i) = y_i Pr(y) - C(y_i, \alpha_i)$$

where $Pr(y)$ is the price and C is the cost payed for player i for production and the technology choice.

Remark 4.4 It is known that in the symmetrical case where the payoff function are concave, there exists a Nash equilibrium with the property, $y_1^* = y_2^*$, $\hat{y}_1 = \hat{y}_2$, $\alpha_1^* = \alpha_2^*$ and $\hat{\alpha}_1 = \hat{\alpha}_2$.

We suppose that players are going to choose, Nash equilibria with the property of remark 4.4.

Theorem 4.5 *If $e(y_1^*, y_2^*, \alpha_1) < \bar{e}$, and $e(\hat{y}_1, \hat{y}_2, \alpha_1) < \bar{e}$, then:*

$$y^* \geq \hat{y}$$

Proof: By definition of $\bar{\pi}_i$ and P_i, (1) and (4) respectively, y^* and $\hat{y}$ verify:

$$\frac{\partial \bar{\pi}}{\partial y_i}(y^*, y^*, \alpha_i) = 0, \qquad \frac{\partial \bar{\pi}}{\partial y_i}(\hat{y}, \hat{y}, \alpha_i) = \rho(\alpha_i)\eta(\alpha_i)\frac{\partial D_i}{\partial y_i}(\hat{y}_i, \alpha_i) \tag{16}$$

As $\bar{\pi}_i$ is a concave function of y_i, we have that $\frac{\partial \bar{\pi}}{\partial y_i}$ is a decreasing function of y_i. So

$$\begin{array}{l} \dfrac{\partial \bar{\pi}}{\partial y_i}(x, y_{-i}^*, \alpha) \geq 0 \quad \text{iff} \quad x \leq y_i^* \\ \dfrac{\partial \bar{\pi}}{\partial y_i}(x, y_{-i}^*, \alpha) \leq 0 \quad \text{iff} \quad x \geq y_i^* \end{array} \tag{17}$$

As $\rho(\alpha_i)\eta(\alpha_i)\frac{\partial D_i}{\partial y_i}(\hat{y}_i, \alpha_i) \geq 0$ we obtain by (17) that $\hat{y} \leq y^*$. ∎

Remark 4.5 *Note that in the case above the sufficient conditions of theorem 4.1 and 4.2 are not satisfied anymore by the players, because $y_* \geq \hat{y}$ and $\bar{\pi}_i$ is a decreasing function on y_{-i}.*
If conditions of theorems 4.3 and 4.4 are satisfied, by theorem 4.4, then,

$$P_i(y_1^*(\alpha_1^*, \alpha_2^*), y_2^*(\alpha_1^*, \alpha_2^*), \alpha_i^*) = P_i(y_1^*, \alpha_1^*) \qquad i = 1, 2,$$

and then,

$$P_i(y_1^*, y_2^*, \alpha_i^*) \leq P_i(\hat{y}_1, \hat{y}_2, \alpha_i^*) \leq P_i(\hat{y}_1, \hat{y}_2, \hat{\alpha}_i)$$

under this hypothesis Model 2 performs better than Model 1.
It would be interesting if α^* (optimal technology for Model 1) and $\hat{\alpha}$ (optimal technology for Model 2) could be compared. We are going to show and discuss this fact in same examples.

Remark 4.6 *Theorem gives a comparison between the optimal quantities y in the case they verify $e(y, \alpha) < \bar{e}$. Because only two players have been considered we can easily see that when equilibria for both players verify $e(y, \alpha) = \bar{e}$, Models 1 and 2 are identical. When in one of the model quantities verify the equality and in the other one the strict inequality, the comparison is very difficult and it depends on the specific data problem.*

5 Model 1 vs Model 2

If by instance firm 1 decides to follow Model 1 and firm 2 decides to follow Model 2. That means, that the firms solve the following problem: Find $\bar{y}_1, \bar{y}_2$ such that:

$$\bar{\pi}_1(y_1, \bar{y}_2) \leq \bar{\pi}_1(\bar{y}_1, \bar{y}_2), \quad P_2(\bar{y}_1, y_2) \leq P_2(\bar{y}_1, \bar{y}_2) \quad \forall y_1, y_2. \tag{18}$$

Here, to obtain comparison results we formulate some assumption for C_i, D_i and Pr that we specify in each theorem. We consider in this section that $e(\hat{y}, \hat{\alpha}) < \bar{e}$, $e(\bar{y}, \bar{\alpha}) < \bar{e}$.

Theorem 5.1 *If $\dfrac{\partial Pr}{\partial y_1}(y_1, y_2) = \dfrac{\partial Pr}{\partial y_2}(y_1, y_2) < 0$, $\dfrac{\partial^2 C}{\partial y_i^2} > 0$, and $\dfrac{\partial^2 C}{\partial \alpha_i \partial y_i} > 0$, then,*

$$\bar{\alpha}_1 > \bar{\alpha}_2 \qquad \text{or} \qquad \bar{y}_1 \geq \bar{y}_2$$

Proof: The pair $\bar{y}_1, \bar{y}_2$ must verify:

$$\bar{y}_1 \frac{\partial Pr}{\partial y_1}(\bar{y}) + Pr(\bar{y}) - \frac{\partial C}{\partial \bar{y}_1}(\bar{y}_1, \alpha_1) = 0$$

$$\bar{y}_2 \frac{\partial Pr}{\partial y_2}(\bar{y}) + Pr(\bar{y}) - \frac{\partial C}{\partial \bar{y}_2}(\bar{y}_2, \alpha_2) = \eta(\alpha_2) \frac{\partial D_2}{\partial y_2}(\bar{y}_2, \alpha_2).$$

So,

$$0 < \eta(\alpha_2) \frac{\partial D_2}{\partial y_2}(\bar{y}_2, \alpha_2) = \bar{y}_2 \frac{\partial Pr}{\partial y_2}(\bar{y}) - \bar{y}_1) \frac{\partial Pr}{\partial y_1}(\bar{y}) + \frac{\partial C}{\partial y_1}(\bar{y}_1, \alpha_1) - \frac{\partial C}{\partial y_2}(\bar{y}_2, \alpha_2) \tag{19}$$

If we suppose

$$\bar{\alpha}_1 < \bar{\alpha}_2 \qquad \text{and} \qquad \bar{y}_1 \leq \bar{y}_2, \tag{20}$$

as $\dfrac{\partial C}{\partial y_i} > 0$, $\dfrac{\partial^2 C}{\partial y_i^2} > 0$, and $\dfrac{\partial^2 C}{\partial \alpha_i \partial y_i} > 0$, we have,

$$\frac{\partial C}{\partial \bar{y}_1}(\bar{y}_1, \alpha_1) - \frac{\partial C}{\partial \bar{y}_2}(\bar{y}_2, \alpha_2) < 0. \tag{21}$$

As $\frac{\partial Pr}{\partial y_1}(y_1, y_2) = \frac{\partial Pr}{\partial y_2}(y_1, y_2) < 0$ we have,

$$(\bar{y}_2 - \bar{y}_1)\frac{\partial Pr}{\partial y_i}(\bar{y}) < 0. \tag{22}$$

Then (21) + (22) contradicts (19). This contradiction comes from assumption (20). So, this is not true, then it must be $\bar{\alpha}_1 > \bar{\alpha}_2$ or $\bar{y}_2 - \bar{y}_1 \geq 0$. And the theorem is proved. ∎

The following theorem allows to compare between $\hat{y}_i$ and $\bar{y}_i$.

Theorem 5.2 *If*

$$\bar{y}_1 \geq \bar{y}_2,$$

$$\frac{\partial^2 C}{\partial y_i^2} > 0$$

$$\frac{\partial Pr}{\partial y_1}(y_1, y_2) = \frac{\partial Pr}{\partial y_2}(y_1, y_2) < 0$$

$$\frac{\partial^2 D}{\partial y_i^2} > 0$$

$$z_1 + z_2 > y_1 + y_2 \Rightarrow \frac{\partial Pr}{\partial y_i}(y_1, y_2) > \frac{\partial Pr}{\partial y_i}(z_1, z_2)$$

then

$$\bar{y}_2 < \hat{y}_2, \qquad \textit{and} \qquad \bar{y}_1 > \hat{y}_1.$$

Proof: $\bar{y}_2$ and $\hat{y}_2$ verify:

$$\hat{y}_2\frac{\partial Pr}{\partial y_2}(\hat{y}) + Pr(\hat{y}) - \frac{\partial C}{\partial y_2}(\hat{y}_2, \alpha_2) = \eta(\alpha_2)\frac{\partial D_2}{\partial y_2}(\hat{y}_2, \alpha_2$$

$$\bar{y}_2\frac{\partial Pr}{\partial y_2}(\bar{y}) + Pr(\bar{y}) - \frac{\partial C}{\partial y_2}(\bar{y}_2, \alpha_2) = \eta(\alpha_2)\frac{\partial D_2}{\partial y_2}(\bar{y}_2, \alpha_2\frac{\partial D_2}{\partial y_2}(\bar{y}_2, \alpha_2).$$

Suppose that $\bar{y}_2 > \hat{y}_2$.
So, as $\frac{\partial^2 D}{\partial y_i^2} > 0$, we have,

$$\begin{aligned} 0 < \eta(\alpha_2)(\frac{\partial D_2}{\partial y_2}(\bar{y}_2, \alpha_2) - \frac{\partial D_2}{\partial y_2}(\hat{y}_2, \alpha_2)) = \\ \bar{y}_2\frac{\partial Pr}{\partial y_2}(\bar{y}) - \hat{y}_2\frac{\partial Pr}{\partial y_2}(\hat{y}) + Pr(\bar{y}) - Pr(\hat{y}) + \frac{\partial C}{\partial y_2}(\hat{y}_2, \alpha_2) - \frac{\partial C}{\partial y_2}(\bar{y}_2, \alpha_2) \end{aligned} \tag{23}$$

As $\frac{\partial^2 C}{\partial y_i^2} > 0$, we have that

$$\frac{\partial C}{\partial y_2}(\hat{y}_2, \alpha_2) - \frac{\partial C}{\partial y_2}(\bar{y}_2, \alpha_2) < 0, \tag{24}$$

Because it has been supposed that $\bar{y}_1 \geq \bar{y}_2$, by theorem 4.4, $\hat{y}_1 = \hat{y}_2$ and because we have $\bar{y}_2 > \hat{y}_2$ that implies that $\bar{y}_1 > \hat{y}_1$. Then, by assumption we have $0 > \dfrac{\partial Pr}{\partial y_2}(\hat{y}) > \dfrac{\partial Pr}{\partial y_2}(\bar{y})$, so, we get,

$$\bar{y}_2 \frac{\partial Pr}{\partial y_2}(\bar{y}) - \hat{y}_2 \frac{\partial Pr}{\partial y_2}(\hat{y}) < 0, \tag{25}$$

and

$$Pr(\bar{y}) - Pr(\hat{y}) < 0. \tag{26}$$

So (24) + (25) + (26) is a contradiction of (23). So it must be $\bar{y}_2 < \hat{y}_2$.
Now, we prove that $\bar{y}_1 > \hat{y}_1$. $\bar{y}_1$ and $\hat{y}_1$ verify:

$$\hat{y}_1 \frac{\partial Pr}{\partial y_1}(\hat{y}) + Pr(\hat{y}) - \frac{\partial C}{\partial y_1}(\hat{y}_1, \alpha_1) = 0$$

$$\bar{y}_1 \frac{\partial Pr}{\partial y_1}(\bar{y}) + Pr(\bar{y}) - \frac{\partial C}{\partial y_1}(\bar{y}_1, \alpha_1) = 0.$$

So

$$\bar{y}_1 \frac{\partial Pr}{\partial y_1}(\bar{y}) - \hat{y}_1 \frac{\partial Pr}{\partial y_1}(\hat{y}) + Pr(\bar{y}) - Pr(\hat{y}) + \frac{\partial C}{\partial y_1}(\hat{y}_1, \alpha_1) - \frac{\partial C}{\partial y_1}(\bar{y}_1, \alpha_1) = 0 \tag{27}$$

Suppose $\bar{y}_1 < \hat{y}_1$, by assumption, is,

$$\begin{aligned} &\tfrac{\partial C}{\partial y_1}(\hat{y}_1, \alpha_1) - \tfrac{\partial C}{\partial y_1}(\bar{y}_1, \alpha_1) > 0 \\ &Pr(\bar{y}) - Pr(\hat{y}) > 0 \\ &\bar{y}_1 \tfrac{\partial Pr}{\partial y_1}(\bar{y}) - \hat{y}_1 \tfrac{\partial Pr}{\partial y_1}(\hat{y}) > 0. \end{aligned} \tag{28}$$

Then (28) contradicts(27). So, the supposition is false and $\bar{y}_1 > \hat{y}_1$. ∎

Remark 5.1 Let us analyse the case where a player (player 1) chooses Model 1 ($\bar{y}_1$) and the other one (player 2) chooses Model 2 ($\bar{y}_2$). We can compare this situation with the case in which both player choose Model 2 ($\hat{y}$). By theorems 5.1 and 5.2 we have that if $\bar{y}_1 > \bar{y}_2$, then $\bar{y}_1 > \hat{y}_1$ and $\bar{y}_2 < \hat{y}_2$.
With this result we can deduce a comparison between $\bar{\pi}_1(\bar{y}, \alpha_1) - \eta(\alpha_1) D_1(\bar{y}_1, \alpha_1)$ and $\bar{\pi}_1(\hat{y}, \alpha_1) - \eta(\alpha_1) D_1(\hat{y}_1, \alpha_1)$.
In fact, as $\bar{y}_1 > \hat{y}_1$, we have,

$$C(\hat{y}_1, \alpha_1) + \eta(\alpha_1) D_1(\hat{y}_1, \alpha_1) > C(\bar{y}_1, \alpha_1) + \eta(\alpha_1) D_1(\bar{y}_1, \alpha_1)$$

We cannot compare $\hat{y}_1 Pr(\hat{y})$ with $\bar{y}_1 Pr(\bar{y})$, but if for example the price is a linear function of y_i and C and D grows "enough" as a function of y_i, we can obtain

$$P_1(\hat{y}, \alpha_1) > P_1(\bar{y}, \alpha_1)$$

So player 1, having chosen Model 1 (while his companion was chosen Model 2) obtains a smaller payoff if he would have chosen Model 2.

6 Examples

We test these models in a simple example in order to compare the optimal technology chosen. In this example,

- $\rho(\alpha_i) = 1 - (1 + \alpha_i)^{-k}$
- $\eta(\alpha_i) = e^{-M\alpha_i}$
- $\pi_i(y, \alpha_i, 1) = y_i(A - B(y_1 + y_2)) - 1/2F\alpha_i^2 - 1/2y_i^2, \qquad \pi_i(y, \alpha_i, 0) = -1/2F\alpha_i^2$
- $D_i(y_i, \alpha_i, 1) = 1/2N(1 - \alpha_i)y_i^2 \qquad D_i(y_i, \alpha_i, 0) = 1/2N(1 - \alpha_i)$
- $e(y, \alpha) = (y_1 + y_2) + (2 - \alpha_1 - \alpha_2)\gamma$

In these examples the parameters A and B are related to the price. Hence, $A - B(y_1 + y_2)$ is the price. $1/2F\alpha_i^2$ is the fixed cost and $1/2y_i^2$ the variable cost. M is related with η, the probability of accidents. When M grows up, the probability of accident goes down, i.e., η is a decreasing function of M. k is related with ρ, the probability that the technology works, when k grows up this probability grows up, i.e. ρ is a increasing function of k. F is related with C, the cost to pay by the choice of production and technology. C is an increasing function of F. N is related with D, the damage function, D is an increasing function of N. Finally, γ, β and $\bar{e}$ are related with the constraint of pollution. The different values of $\bar{e}$ give the top level of pollution.
In these examples we can easily verify that for both models the respective profit functions are concave functions of y_i. Their concavity as a function of α_i was checked graphically. In order to prove unicity, we have verified that there exists only one element verifying necessary conditions from equilibria.

6.1 Example 1

When $A = 100$, $B = 1$, $M = 3$, $k = 5$, $N = 20$, $F = 2000$, $\gamma = 0.5$ and $\bar{e} = 20$, 41 and 51 we obtain the following results:

	Model 1=2	**Model 1**	**Model 2**	**Model 1**	**Model 2**
	$\bar{e}$=20 (S)	$\bar{e}$=41 (S)	$\bar{e}$=41 (NS)	$\bar{e}$=51 (NS)	$\bar{e}$=51 (NS)
$y_1 = y_2$	9.7227	20.3248	19.8692	25	19.8642
$\alpha_1 = \alpha_2$	0.4455	0.6497	0.6437	0.7123	0.6437
$P_1 = P_2$	304.7191	306.5612	315.27	168.61	315.27

6.2 Example 2

When $A = 800$, $B = 20$, $M = 1$, $k = 20$, $N = 100$, $F = 5000$, $\gamma = 0.5$ and $\bar{e} = 26$ or 27, we obtain the following results:

	Model 1	**Model 2**	**Model 1**	**Model 2**
	$\bar{e}$=26 (S)	$\bar{e}$=26 (NS)	$\bar{e}$=27 (NS)	$\bar{e}$=27 (NS)
$y_1 = y_2$	12.9157	10.6871	13,1147	10.6871
$\alpha_1 = \alpha_2$	0.8315	0.7163	0.8482	0.7163
$P_1 = P_2$	1236.10	1849.7	1168.33	1849.7

Here, the pollution for Model 1 is 26.3807 and for Model 2 is 21,6578.
In the same example when $M = 3$, we obtain:

	Model 1	**Model 2**	**Model 1**	**Model 2**
	$\bar{e}$=26 (S)	$\bar{e}$=26 (NS)	$\bar{e}$=27 (NS)	$\bar{e}$=27 (NS)
$y_1 = y_2$	12.7972	11.6096	13.1147	11.6096
$\alpha_1 = \alpha_2$	0.5944	0.5668	0.6070	0.5668
$P_1 = P_2$	2163.29	2492.29	2057.52	2492.29

Stronger results are reached with the symmetrical-case model. Indeed, it can be shown that model 2 gives better performances than model 1 in any circumstances. By performances are understood the payoff functions; however, the level of safe technology is higher with model 1 that with model 2. These results are then tested with numerical examples which give good illustration of the firm's behaviour under environmental constraints. It appears that in equilibrium, the model 1 with higher preventive technology raises higher production level compared to model 2.

7 Related Literature

This paper comes from two different streams of the environmental literature. The first stream corresponds to the debates around the impact of environmental investment on the firm performance. In their optimistic and controversial analysis Porter and Van der Lind (1995) have shown that for big companies environmental prevention may induce quite favourable effects. This is particularly due to the endogenous and positive trend that innovation may generate. However, the Palmer, Oates, Portney (1995)'s answer shows that this question remains open because of the high level of investments associated to environment protection and the uncertain financial returns managers may expect to withdraw. Hence, the question may be conceived as: does environmental protection bind the firm's performances and efficiency or, in the opposite, does it induce firms to improve them? At first sight, regulating environment is expensive from several viewpoints. First, public expenses in order to assess and monitor pollution generated by uncertainty are incurred and may be quite high. Second, costly environmental investments and harms involve increasing social costs. However, from the Porter and Van der Lind (1995)'s standpoint, these investments may improve the raw material and energy management, the productivity of the firms, lessen their insurance premium and improve their image in their customers' opinion. Nevertheless, if in the long run such investments are revealed to be necessary, in the short run, environmentalist corporate strategies may be upset by free riders competitors using ecological dumping[11]. Therefore, the sensitivity of the firm to environmental concern is becoming a strategic variable and the effective impact on the private agent performances tends to remain an open question.

The second stream this paper is related equally with the literature on the firm risky investment. For example, we can mention Strand (1994), De Meza (1986), Baumol-Oates (1988), for the extension to environmental considerations. Focusing on the actions managers want to take in order to minimize the social cost of accidents may induce to think that comparative behaviour between competitive firms on the liability question is out of scope. Indeed, most of the literature about liability bears upon the relationships between the injured and the injuring party. Generally, the authors are studying how the managers could be induced to take efficient actions for being protected against accidents using several liability rules. In this research field, directions are quite numerous.

[11] We can refer to the Alison Butler's paper of 1992.

Our paper is inspired mainly from De Meza (1986)'s contribution about competition between firms choosing risky investments. In De Meza's model, the uncertainty comes from the variable input prices when durable plants have to be installed. In our model, competition is oligopolistic (duopolists), and the risks come from environmental accident with liability consequences. Hence, uncertainty sources are twofold. The first one comes from the supply side - i.e. under assumption 2, firms do not know if their competitors and themselves will be able to meet fully the demand. The second one comes from the accident risk itself. Generally, in the literature, these factors are not linked.

Another source may be found in the Huber and Litan (1991)'s book on the impact of liability law on the firms strategic behaviour. Broadly speaking, liability rules may act as a deterrent innovation factor because of the high potential consequences of faults (in products, processes). Different liability rules involve different attitudes towards safety and innovation but, this paper has been simplified to the unique strict liability case.

Of course, our mathematical model is much more deeply related to the liability literature. Hence, about this question, Beard (1990), Shavell (1986), Strand (1992), with somewhat different assumptions have shown that, generally, too little care is taken when markets are unregulated.

We did not mention here the case of victim precaution which is sometimes dealt in the literature, (Landes and Posner (1987), Hylton (1996)). It is shown generally that the damages or the associated fine due to the injurer are liable to be less than when victim precaution is impossible. It seems that this refinement here does not change too much our results. Indeed, under a strict liability rule, the injurer has an incentive to take care of its potential victim. As Landes and Posner (1987 chap.3) notice it, strict liability is efficient only in the case when there is no reasonable measure of care that a potential victim can take to avert the accident or to reduce the probability of its occurrence. This may be often the rule, in the chemical and in the energy industry - for example to the case of nuclear plants.

An important remaining problem to deal with is the "judgement proof problem" - i.e. the case when the injurer's assets are so low that he is unable to pay for losses due to insufficient care and prefers to go bankrupt and avoid legal liabilities for such damages. When liability for environmental spills is imposed on firms with insufficient wealth, those firms may file for bankruptcy and are becoming "judgement proof" as soon as a major accident occurs. Otherwise, firms engaged in hazardous activity may cause damages that exceed their own assets, becoming "judgement proof" it can avoid liability for such damages. This possibility may induce huge companies to choose strategically to minimize their exposure to liability in reducing, for example, the level of its production plants and multiplying the subsidiary companies. Doing so, they can escape their full liability in the harm occurrence. The "judgement proof" question has been voluntary underestimated on the grounds that, if generalized, this policy may induce reaction from the public and give the State incentives to have this practice forbidden.

It is a long time now that literature is aware that the damage value may deeply exceed the financial resources of the firm. For example, when the legal system is characterised by strict liability, an injurer may become bankrupt because the damage costs are in excess compared to its resources (capital and cash-flow). Bankruptcy limits the ability of any liability system to actually implement the optimal damage awards. This involves fundamental policy problems from the regulator viewpoint. Indeed, the questions the liability literature tackles usually with are twofold. This has been raised by Summers (1983) and Shavell (1986), (1987) and is relevant for our analysis. The second one is not relevant here, because it bears the moral hazard effect between injurers and injured people. Our problem is limited to an investment assessment question

[12]. Focusing on the first one, according to the mentioned authors, potential insolvency leads the firms to reduce care levels under strict liability. This result has been put under question in the nineties by several critical papers. Beard (1990), Kolstad, and alii (1990) have taken another look to the relationship between safety choice and bankruptcy and it appears that potential injurers may take too much or too little precaution in their activities. Our paper may help to give a simple explanation to such an apparent paradoxical behaviour. Indeed, the competition organization process may induce non co-operative players to promote an over or under-safety investment choice.

8 Conclusion

From a theoretical viewpoint, this very simple model tends to show that it is quite difficult to give definitive an answer to the debate about the efficiency of environmental investments under a strict liability regime. At stake in our research is the necessity to define sufficient conditions allowing firms to choose either computation issued from model 1, (i.e. the sequential model expressing care for liability) or from model 2, (i.e. the "simultaneous" or market oriented model). It can be shown that both computations do not give similar result and have a strong environmental involvement because the chosen technology will differ. It is then of great interest to compare their mutual efficiency. Two situations have been examined. The first one, is a general analysis with asymmetric duopolist models. It appears that the efficiency of the first one compared to the second one is dependent on the relationship between the damage function and the probability of an accident occurring. We clearly identify conditionunder which model 1 is pre-eminent compared to model 2 and vice-versa.

Except in the symmetrical case in which the so-called simultaneous choice opposed to the sequential (or "best available technology") model one gives better performance than the sequential one, the general case gives to the probability of accident bounds a great importance in the decision model. Going further, it can be said that managers could assess and compare the performances of each model and achieve their choice on this basis. Divergences in results, as seen in the general case, gives insight on the controversial question about the efficiency of environmental investment. Hence, when liability is involved, the performance of each model is depending on the objective production conditions and the nature of technology, in such a way that conclusions about the superiority of a given model compared to the other one may be done on some technology interval. This result may enlighten why it is difficult to bring definitive results in this area of environmental theory.

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