

Imperfect Competition, Labour Market Distortions, and the Double Dividend Hypothesis.

Theory and Evidence from Italian Data

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Abstract

The paper explores the hypothesis of a double dividend from environmental taxation i.e. whether shifting the burden of taxation away from labour toward the environment can boost employment and increase welfare. We present a general-equilibrium model where the economy is distorted by labour taxes, monopolistic product-market competition, and union-wage bargaining. We find that employment and welfare always increase when the revenue from an the introduction of a Pigouvian tax (imposed on firms and households) is fully recycled to cut the rate of the pre-existing labour tax. Moreover, it turns out that the degree of the imperfections influences the magnitude of the effects of the reform. We also offer numerical results for the case in which the pollution tax is positive at the outset.

Keywords: double dividend, environmental tax reform, imperfect competition, welfare.

JEL classification: E62, H20, H23, H30

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1. Introduction

Various authors (see Pearce, 1991 and Oates, 1991) have argued that using the revenues from environmental taxation to reduce the rate of pre-existing distortionary taxes (such as labour taxes) can lead to a *double dividend* i.e. an improvement in the environmental quality coupled with a more efficient tax system.¹ Others (see, for example, Bovenberg and de Mooij, 1994*a,b* and Parry, 1995), however, have questioned the double dividend hypothesis, showing that an environmental tax reform typically exacerbates rather than alleviates pre-existing distortions. In particular, Bovenberg and de Mooij's argument is developed in a static general-equilibrium model which contains two distortions: pollution externalities and a distortion in the labour market due to a distortionary tax on labour income. Their focus is on the modelling of a shift of the burden of taxation away from labour towards the environment.² They assume perfect competition in the goods market and in the labour market. The consequences of this reform on employment and welfare are analysed and a double dividend effect is typically not found. More recently some studies which have addressed the issue of wage rigidities and involuntary unemployment (see, for example, Nielsen, Pedersen and Sørensen, 1995; Strand, 1995; Bovenberg and van der Ploeg, 1996; Holmlund and Kolm, 1997; Schneider, 1997; Koskela, Schöb and Sinn, 1998; and Koskela and Schöb, 1999) are typically able to generate an employment double dividend. For example, Koskela and Schöb (1999) employ a right to manage model of the labour market and find an increase in employment if unemployment benefits are taxed at a lower rate than wage-income. The reason for this result is that the burden of taxation can be shifted to the earners of non-labour income such as unemployment benefits and this works in favour of a double dividend.

¹ We refer to environmental taxes as corrective of environmental externalities (see Pigou, 1920)

² In the double-dividend literature the policy of shifting the burden of taxation away from labour to the environment has been modelled more frequently than others options (e.g. reducing the capital-income tax or indirect taxes). Unemployment and environmental degradation are serious problems for actual economies, especially in Europe, and the idea of an employment second dividend appeals to many. Furthermore, the European Commission carbon/energy tax proposal states that the revenue from the carbon-energy tax might be devoted to reduce the existing labour taxes (see, for example, Pearson and Smith, 1991).

In a similar model, Koskela, Schöb and Sinn (1998) focus on the role of factor substitution on the outcome of the reform. If the elasticity of substitution is equal to unity (Cobb-Douglas technology), they find that an environmental tax reform always boosts employment and output and that the greater the union power, the larger the positive impact of the reform on employment. If the elasticity of substitution is less than one, employment may decrease.

Recent papers (see Holmlund and Kolm, 1997, and Koskela, Schöb and Sinn, 1998) model monopolistic firms, however they do not explore the sensitivity of imperfect competition in the goods market on the double dividend outcome.

In our paper, we wish to address the issue of whether the presence of imperfect competition may be an incentive for implementing environmental tax reforms.

We address the double dividend hypothesis both in terms of increased employment and in terms of utilitarian welfare. We both offer analytical and quantitative results. In our economy the source of externalities is the use of a polluting good, and we model the tax reform as an introduction of a proportional environmental tax, both on households' consumption and on firms' use of the polluting good. The revenue from the environmental tax is then "recycled" by cutting the labour-income tax rate (or equivalently a payroll tax) in a revenue neutral fashion. In order to address the question we develop a general equilibrium model which accounts for a number of distortions (in addition to what is caused by the second-best tax system):

a) imperfect competition in the goods market, modelled as a large number of (symmetric) monopolistic competitors (we may obtain perfect competition as a special case when the elasticity of substitution between different goods goes to infinity);³

³ Models allowing for imperfectly competitive markets are suitable to describe the recent macroeconomic performances of many European countries. Furthermore they allow us to analyse the welfare effects of fiscal policy generated by mechanisms which are not captured by a perfect-competition model, such as the response to the policy of involuntary unemployment and profits (see Dixon and Rankin, 1994). There are studies which have explored how imperfect competition impact fiscal policy (see, for example, Dixon, 1987 and Dixon and Rankin, 1994, in a closed economy framework, and Dixon, 1992 in an open economy framework), however, to our knowledge, the impact of goods market imperfection on environmental tax reforms has not attracted enough attention yet. An

b) imperfect competition in the labour market, modelled as decentralised wage bargaining between unions and firms (we may obtain monopoly union and competitive wage setting as special cases when varying the power of the union);⁴

c) externality caused by the use of a polluting good as a factor of production and as a commodity consumed by households.

We model a closed economy to abstract from terms of trade effects and issues of factors mobility and to focus on the effect of imperfect competition on the outcome of the reform. Nevertheless, this framework can easily be extended to open-economy issues. Furthermore, we want to be in line with earlier double-dividend models in closed economies such as Bovenberg and de Mooij (1994a), Bovenberg and van der Ploeg (1994), Nielsen Pedersen and Sorensen (1995), Schneider (1997), Koskela and Schöb (1999), and assess whether the impact of imperfect competition in the goods market, which has not been explored in the above studies, may yield different results.

The novelty of our paper, with respect to the previous literature on environmental tax reforms in presence of imperfect competition, is fourfold.

First, we carefully solve the general equilibrium in a consistent way. When using a monopolistic competition model there is no macro-good. One cannot therefore write down the government's budget constraint in terms of a single type of public expenditure. One has to specify on which goods the government spends its tax revenue, and how those goods are chosen (see Dixon, 1987). The same is also true regarding the production or purchase of the polluting good. Since there is no macro good, one has to specify which goods can be exchanged for the polluting good (production or trade in an open economy framework). These considerations have been ignored in the previous literature on the double dividend.

exception is Barker (1998) who explores empirically the sensitivity of the double dividend to imperfect competition in the goods market.

⁴ We follow the approach of the right-to-manage model described in Layard, Nickell and Jackman (1991). The advantage of this framework is that it is able to describe real world phenomena such as the influence of unions and the price-setting behaviour of firms along with involuntary unemployment and rents formation.

Second, we allow for externalities from production and consumption simultaneously, while previous studies have treated these externalities in isolation. Indeed, if we think of the polluting good as energy, there is evidence that the part consumed by households represents a large share of the total use of energy; for example, in the UK, energy use by the households sector for domestic purposes amounted to some 28% of the total energy use in 1988 (see, Pearson and Smith, 1991). When quantifying the results it is therefore important to take into account the relative shares of households' and firms' energy use.

Third, we focus not only on the effect of changing the union power on employment (as in Koskela, Schöb and Sinn, 1998) but also on the impact of different degrees of imperfection in the goods market. Indeed it is important to assess at what extent profits rents may impact on the magnitude of the double dividend.

Fourth, we offer results not only in terms of employment as it is typically done in the previous literature, but also in terms of utilitarian welfare.

We should notice that one may take on a number of different perspectives on the double-dividend issue. We may think of the double dividend in terms of either a welfare gain (according to a social objective) or a gain for the decisive individual (median voter) or effects on the macroeconomy (such as effects on employment). Another question is what initial equilibrium we start with (when we introduce the reform). We may think of this equilibrium as either being a sub-optimal one or optimal *given the current instruments* available.⁵ One may also think of the initial system being optimal *given the preferences*. Then shocking the preferences would call for a policy reform. Our analysis is broad enough to take on either perspective.

The paper is organised as follows: section 2 introduces the model; section 3 presents the model solution; in section 4 we conduct the tax reform: we offer analytical solution to the case in which we implement a marginal increase from a zero initial environmental tax and subsequently we present

⁵ Most of the existing literature on the double dividend assumes that in the initial equilibrium either there is no environmental policy or there is a suboptimal level of environmental taxation.

quantitative results for the case of an initial positive environmental tax and non-marginal changes; section 5 concludes.

2. The Economy

Our economy consists of a large number, n , of price-setting local monopoly firms, j , each producing a different consumption good. Firms are price-takers on the factor markets, but price setters (local monopoly) on the goods market. The size of each firm, relative to the economy, is "small" so that the firm does not perceive that changes in its product price, p_j , do affect the general price level or aggregate production. With a given (predetermined) capital stock, K_j , the firms produce output, y_j , by using labour, N_j , and the polluting input, E_j^p . Each firm sets employment, taking the net wage, w , as given. The net wage is set before the employment decision is taken and is determined by bargaining between firm and the associated local union. We assume symmetry among unions and firms, so that in equilibrium each firm and each union face the same wage, same employment and same production. All unions have the same fixed stock of members who can be either full-time employed, with income equal to net wage, or unemployed, with income equal to unemployment benefits. All union members face the same probability of being laid off. Individual unions and individual firms do not take into consideration the wage bargained elsewhere and do not recognise their influence on the general level of wages and employment. Each union wishes to maximise the expected utility of its representative member. Firms are subject to an environmental tax, J^e , and without loss of generality we assume no payroll taxes (we show in section 3 that a payroll tax is equivalent to a wage tax). The polluting good may be thought of as being produced with a linear technology with the composite consumption good as input. Then the price of the polluting good, e , will be the inverse of the marginal cost of production in consumption units.

Households are of three types: employed, unemployed and shareholders. Household h derives utility from a basket of n consumption goods, $\{x_i^h\}$, and from the polluting good, E^h , which is the

polluting commodity. Households are subject to a tax on the polluting good consumed, J^e , and if employed also to a tax on labour income, J^w . The unemployed households receive unemployment benefits, b , from the government, and shareholders' income consists of profits, A , only. The government finances public spending, X^g , and unemployment benefits, with labour and environmental taxes. In what follows we list our assumptions.

A1 Households preferences For analytical tractability we assume that all households have utility of the Cobb-Douglas form in private consumption, additively separable from the externality⁶

$$U^h = \frac{1}{1-F} (X^h)^{1-F} \frac{1}{F} (E^h)^F - \cdot (\bar{E}) \quad (1)$$

where \bar{E} is the aggregate use of the polluting good and X^h is CES: $X^h = n^{\frac{1}{1-\theta}} \left(\sum_{i=1}^n (x_i^h)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}$.

A2 Households budget constraint Household h 's budget constraint is

$$\sum_{i=1}^n P_i x_i^h + (1 + J^e) e E^h = I^h \quad (2)$$

where I^h is household h 's income, which is either *i*) after-tax wage income: $(1-\tau^w)w$; *ii*) unemployment benefits b , or *iii*) shareholder income (profits): Π .

A3 Technology For analytical tractability and to focus on the effect of imperfect competition technology is assumed to be Cobb Douglas

$$Y_j = A (K_j)^\alpha (N_j)^\beta (E_j^p)^\gamma, \quad \alpha + \beta + \gamma < 1 \quad (3)$$

One may argue that the chosen technology implies a number of limitations. First, all goods are assumed to be produced with the same technology; however, there are cases in which some goods

⁶ Externalities are assumed to enter utility additively separable and thereby not affecting the marginal rate of substitution between consumption goods and energy. Thus any environmental tax reform which succeeds in affecting the aggregate externality will not alter the individuals' consumption patterns. This makes the analysis especially tractable, and we think that the effects on marginal rates of substitutions, if present, are not the most important ones in assessing an environmental tax reform.

may be more capital intensive and less labour intensive than average (as the evidence on energy intensive products typically shows). Second, we only focus on production of goods devoted to satisfy final demand, and not on intermediate productive sectors. Finally, the Cobb-Douglas specification does not permit us to capture the substitution possibilities among factors, which can indeed influence the outcome of an environmental tax reform.⁷ Nevertheless, since the focus of this paper is on the effect of market competition on the outcome of the reform, we want to keep the analysis as simple as possible to explore whether the assumption of imperfect competition changes the results of previous studies.⁸

A4 Wage bargaining The wage in each sector is determined through bargaining between the firm and the union.

A5 Polluting good supply There is a linear transformation from the composite consumption good to the polluting good, i.e. $E^s = \phi X^e$ where $\{x_j^e\}$ are chosen so as to $\min_{x_j^e} \sum_j p_j x_j^e$ subject to

$$n^{\frac{1}{1-\theta}} \left(\sum_{i=1}^n (x_i^e)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \geq X^e.$$

A6 Government's budget constraint The government faces the budget constraint

$$G = \mathbf{J}^w w N + \mathbf{J}^e e (E^p + E^h) - b(L - N) - X^g = 0 \quad (4)$$

⁷ These limitations are even more evident if one thinks of the polluting factor as energy, which can be obviously used either as intermediate inputs and as final production (for example in the electricity industry). Also, from empirical studies of environmental tax reforms, it has turned out that complementarity/substitutability between energy, labour and capital can play an important role, together with interfuels substitution (such as coal, oil, gas), as in the case of a carbon-energy tax. For a review of the empirical studies of the double dividend see Marsiliani (1999).

⁸ For example, Bovenberg and de Mooij's (1994b) paper also presents a technology with labour, capital and polluting factor. The main difference is that we allow for price-setting firms and union bargaining.

and chooses $\{x_j^g\}$ so as to $\min_{x_j^g} \sum_j P_j x_j^g$ s.t. $n^{\frac{1}{1-\theta}} \left(\sum_{i=1}^n (x_i^g)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \geq X^g$.

3. ECONOMIC EQUILIBRIUM

In this section we solve for the economic equilibrium.⁹ Before solving for the firms' decisions we need to know the demand functions for individual commodities of the households and of the government and the polluting good sector. These are found in sections 3.1 and 3.2, respectively. The firms' decisions are solved for in section 3.3 and the bargaining equilibrium in section 3.4. We focus on how the degree of imperfect competition affects the firms' factor demand and the bargaining-equilibrium wage. Throughout we assume that the equilibrium exists and is unique. In section 4 we prove existence and uniqueness of the general equilibrium.

3.1 The households' decision problem

Maximising (1) s.t. (2) gives the households' consumption of individual commodities

$$x_k^h = (X^h / n) (P / P_k)^\theta \quad (5)$$

and of the composite commodity and polluting good

$$X^h = (1 - \mathbf{F}) I^h / P, \quad E^h = \mathbf{F} I / [(1 + \mathbf{J}^e) e] \quad (6)$$

respectively, where $P = \left(\frac{1}{n} \sum_{i=1}^n P_i^{1-\theta} \right)^{\frac{1}{1-\theta}}$ is the price index. The indirect utility function is then

$$V = P^{-(1-\mathbf{F})} [(1 + \mathbf{J}^e) e]^{-\mathbf{F}} I^h \quad (7)$$

This indirect utility function will be used to solve the wage bargaining. Notice that V is linear in income which implies risk neutrality (a consequence of Cobb-Douglas utility).

⁹ The calculations are available from the authors on request.

3.2 The government and the polluting good sector

An implication of assumption A6 is that the government's demand for any singular commodity will take the form $x_j^g = X^g(P/p_j)^{\eta/n}$, where X^g is the government's aggregate consumption bundle. Similarly, by assumption A5, the demand by the polluting good sector for commodity j takes the form $x_j^e = X^e(P/p_j)^{\eta/n}$, X^e being the aggregate consumption bundle demanded by the polluting good sector. By linearity of the transformation of the composite commodity and polluting good (assumption A5), the real price of the polluting good, e/P , is a constant ϕ .

3.3 The firms' decision problem

Since each household has the demand function for commodity j of the form (5), and since the demands by the government and the polluting good sector take the same form, each firm faces a demand function of the form $y_j = Y(P/p_j)^{\eta/n}$, where $Y = X^h + X^g + X^e$ is aggregate production, which in equilibrium is consumed by the households, the government and the polluting good sector. Thus $\eta > 1$ is the constant elasticity of demand faced by each firm. Profit maximisation by the firm will result in a price markup over marginal cost equal to $\eta/(\eta-1)$. We may interpret η as a measure of goods-market competitiveness: the greater the elasticity η the more the markup will approach unity (the perfect-competition case). Solving the firm's profit-maximisation problem, and normalising the price level to unity (i.e. $P=1$) gives the indirect profit function and the firm's demand for labour and the polluting good, as functions of the real wage, the real price of the polluting good, the environmental tax, and aggregate production

$$= nA_j = \left[\frac{1}{\theta} + (1 - \beta - \gamma) \left(1 - \frac{1}{\theta} \right) \right] \tilde{A}(Y)^{\frac{1+\eta}{\theta}} \left[(1 + \mathbf{J}^e) e \right]^{-\eta} (w)^{-1} \quad (8)$$

$$N = nN_j = \beta \left(1 - \frac{1}{\theta} \right) \tilde{A}(Y)^{\frac{1+\eta}{\theta}} \left[(1 + \mathbf{J}^e) e \right]^{-\eta} (w)^{-1} \quad (9)$$

$$E^p = nE_j^p = \left(1 - \frac{1}{\theta} \right) \tilde{A}(Y)^{\frac{1+\eta}{\theta}} \left[(1 + \mathbf{J}^e) e \right]^{-\eta-1} (w)^{-1+1} \quad (10)$$

where

$$\mathbf{J}^e = \frac{(1 - \gamma) \mu}{1 + (\theta - 1)(1 - \mu)}, \quad \mu \equiv \frac{(\theta - 1)}{1 + (\theta - 1)} \quad (11)$$

are constants, and $\tilde{A} \equiv (1 - 1/\eta)^{\mu'} / ([AK^\alpha S^\gamma] / \eta^{(1-\beta-\gamma)'} / \eta)$, and $0 < \mu \equiv \beta + \gamma < 1$. Equation (8) is the profit function, to be used in the wage bargaining problem, and equations (9) and (10) are the firm's labour- and polluting good demand respectively. Notice that an increase in the environmental tax decreases the demand of both labour and the polluting input. By premultiplying each factor by its price we obtain the factor shares. Because of our Cobb-Douglas specification and the firm being a price taker on the factor markets, these shares are *constants*. Labour's share is $\beta(1 - 1/\eta)$ and the polluting good's share is $\gamma(1 - 1/\eta)$. We see also that profit's share is $[1/\eta + (1 - \beta - \gamma)(1 - 1/\eta)]$.¹⁰ As η goes to infinity we approach competitive pricing and the factor shares are the standard Cobb-Douglas ones $(1 - \beta - \gamma, \beta, \gamma)$ so profit's share is simply $1 - \beta - \gamma$, i.e. capital's share (the inelastic factor). When η is finite profit's share is greater since now rent is incorporated. It should be noticed that these factor shares are independent of the labour-market institutions (as long as the firm is a price-taker on the factor markets), i.e. regardless we have a monopoly union or competitive wage setting. Θ is the own-price elasticity of demand for labour (i.e. the elasticity of demand with respect to the wage rate), and Γ is the cross-price elasticity of labour demand (i.e. with respect to the price of the polluting good). Each elasticity is increasing in product-market competition, that is increasing in η . Thus imperfect competition makes the demand for *each* production factor *less elastic*.¹¹ The important property of imperfect competition is that the ratio own-price elasticity to cross-price elasticity is decreasing in competition, so imperfect competition makes the own-price elasticity stronger relative to the cross-price elasticity. Finally we are able to obtain the solution for a competitive product market by letting η approach infinity: $\lim_{\eta \rightarrow \infty} \Theta = (1 - \gamma)/(1 - \mu)$ and $\lim_{\eta \rightarrow \infty} \Gamma = \gamma/(1 - \mu)$.

3.4 Wage bargaining

¹⁰Under constant returns to scale $1 - \beta - \gamma = \alpha$.

¹¹ Intuitively at the optimum an imperfectly competitive firm satisfies the condition that marginal cost is equal to marginal revenue. Then a change in marginal cost (induced by a change in factor prices) has to be accompanied by a proportionate change in marginal revenue. The less competition (i.e. the more responsive the price is with respect to output) the smaller change in output, and therefore in factor demands, is required for a proportionate change in marginal revenue.

Each union wishes to maximise the expected utility of its representative member, taking into account the unemployment probability, which is equilibrium employment divided by membership, N/L . Employed and unemployed receive the after tax wage, $(1-\tau^w)w$, and the unemployment benefit, b , respectively. Expected utility is $E[V]=\frac{N}{L}V((1-\tau^w)w)+[1-\frac{N}{L}]V(b)$. If no deal is struck, all members of the union get b , so the argument entering the Nash maximand is $E[V]-V(b)$. The firm's objective is profits $\Pi(w)$ and its fall-back position is zero. Denote the union power as ρ , then the Nash maximand is

$$\mathbf{Q} = \left[V((1-\mathbf{J}^w)w) - V(b) \right]^{\mathbf{D}\left(\frac{N(w)}{L}\right)^{\mathbf{D}}} \mathbf{A}(w)^{1-\mathbf{D}} \quad (12)$$

V is given by equation (7) and the profit function $\Pi(w)$ is given by equation (8). Then Ψ attains its global maximum at¹²

$$w = \frac{\mathbf{D} + \mathbf{1} - 1}{\mathbf{1} - 1} \frac{b}{1 - \mathbf{J}^w} \quad (13)$$

which is the bargaining-equilibrium wage. Given the symmetry of unions and firms, this is also the economy-wide wage. Thus, the bargaining-equilibrium wage is such that the net after-tax wage is a constant markup over unemployment benefits. The markup is increasing in union power, ρ , so the maximum markup is $\Theta/(\Theta-1)$ (the monopoly-union case) and the minimum is 1 (i.e. competitive wage setting). Furthermore by inspection of (11) we notice that also a decrease in goods market competition, η , increases the wage markup. Thus we are able to generate solutions for different labour-market institutions by varying ρ and different product-market structures by varying η . The environmental tax does not play any role in the bargaining.¹³ Comparative statics results based on a similar expression for the wage-equation are reported in Koskela and Schöb (1999). They show that if unemployment benefits are taxed at a lower rate than labour income, a decrease in the rate of

¹² The proof is available from the authors on request.

¹³ This is due to the Cobb-Douglas specification of the utility and production functions which gives the energy price entering multiplicatively into the indirect utility and profit functions.

labour-income tax decreases the wage level and unambiguously increases employment. It is straightforward to see that the above results are true regardless the degree of competition in the goods market and the bargaining power of the unions.

Below we show that these results also hold for a decrease in the payroll tax:

Proposition 1 *A payroll tax is equivalent to a tax on labour-income if unemployment benefits are untaxed.*

Proof: The bargaining-equilibrium after-tax wage rate is a constant markup over the unemployment benefit. The wage faced by firms is either $(\rho+\Theta-1)b/[(\Theta-1)(1-\tau^w)]$ in a wage tax system and $(\rho+\Theta-1)b(1+\tau^p)/(\Theta-1)$ in a payroll tax system. If $(1-\tau^w)^{-1}=(1+\tau^p)$ both the tax systems give the same economic behaviour and also give the same tax receipts. QED

4. FISCAL REFORM IN GENERAL EQUILIBRIUM

4.1 Existence of general equilibrium

In this section we shall solve for the general equilibrium, and present sufficient conditions for its existence. Premultiply each factor-demand equation by its price [i.e. equation (9) by w and equation (10) by $(1+\tau^e)e$] and add these to the profit equation (8) and we have

$$wN + \mathbf{A} + (1 + \mathbf{J}^e) e E^p = \tilde{A} Y \frac{1+\tau^e}{w} w^{-1+1} [(1 + \mathbf{J}^e) e]^{-1} \quad (14)$$

The left-hand side equals Y (this can be seen by adding the government's budget constraint to the aggregate household budget and using $Y=X^h+X^g+X^e$). Then (14) rearranged becomes

$$Y = \hat{A} w^{-\frac{s}{1-\mu}} [(1 + \mathbf{J}^e) e]^{\frac{-c}{1-\mu}} \quad (15)$$

where

$$\hat{A} \equiv \tilde{A}^{\frac{1+(\Theta-1)(1-\mu)}{(\Theta-1)(1-\mu)}} = \left(1 - \frac{1}{\Theta}\right)^{\frac{\mu}{1-\mu}} \left[\mathbf{A} K^\alpha \mathbf{S}^s \left(\frac{1}{1-\mu} n \frac{1-\mu-s-c}{1-\mu} \right) \right]^{\frac{1}{1-\mu}} \quad (16)$$

Substituting (15) into (8)-(10), and using (16) gives the general equilibrium quantities of profits, employment, and polluting input use by firms

$$N = \mathbf{S} (1 - 1/\Theta) \hat{A} w^{-\frac{1-c}{1-\mu}} [(1 + \mathbf{J}^e) e]^{\frac{-c}{1-\mu}} \quad (17)$$

$$E^p = ((1 - 1/\mathbf{O}) \hat{A} w^{-\frac{\mathbf{S}}{1-\mu}} [(1 + \mathbf{J}^e) e]^{-\frac{1-\mathbf{S}}{1-\mu}}) \quad (18)$$

$$\mathbf{A} = [1/\mathbf{O} + (1-\mu)(1-1/\mathbf{O})] \hat{A} w^{-\frac{\mathbf{S}}{1-\mu}} [(1 + \mathbf{J}^e) e]^{-\frac{1-\mathbf{S}}{1-\mu}} \quad (19)$$

Equations (17)-(19) together with the households' decision rules (6), and the government's budget constraint (4) constitute the general equilibrium.¹⁴ Substitute for w in (18) by using (13), and we have an implicit function in N , τ^w , and τ^e

$$\tau(N, \mathbf{J}^w, \mathbf{J}^e) = N - \mathbf{S}(1-1/\mathbf{O}) \hat{A} (b(\mathbf{D} + \mathbf{1} - 1) / (\mathbf{1} - 1))^{-\frac{1-\zeta}{1-\mu}} [e(1 + \mathbf{J}^e)]^{-\frac{1-\zeta}{1-\mu}} (1 - \mathbf{J}^w)^{\frac{1-\zeta}{1-\mu}} = (20)$$

Next, we shall rewrite the government's budget constraint in terms of the unemployment rate. By (6) we have $(1 + \tau^e)eE^h = \sigma[(1 - \tau^w)wN + \Pi + (L - N)b] = \sigma[wN\rho/(\Theta - 1) + \Pi + Lb]$, where the last equality follows from the bargaining-equilibrium wage (13). Since by (17) and (19) $\beta(\eta - 1)\Pi = [1 + (1 - \mu)(\eta - 1)]wN$ we may write the households' consumption of the polluting good as

$$\mathbf{D}bN / (\mathbf{1} - 1) + Lb + wN(1 + (1 - \mu)(\mathbf{O} - 1)) / (\mathbf{S}(\mathbf{O} - 1)) \quad (21)$$

Next, by (18) and (19) we have $\beta(1 + \tau^e)eE^p = \gamma wN$. Using this and (21) to substitute for the polluting good in the government's budget gives another implicit function

$$\mathbf{J}^w, \mathbf{J}^e) = \mathbf{J}^w wN + \frac{\mathbf{J}^e}{1 + \mathbf{J}^e} \left(\frac{\mathbf{S}}{\mathbf{S}} wN + \frac{\mathbf{J}^e \mathbf{F}}{1 + \mathbf{J}^e} \left\{ \frac{(\mathbf{D}b + w)N}{\mathbf{1} - 1} + Lb \right\} \right) \quad (22)$$

where R is government revenue (to be held fixed during the tax reform). Equations (20) and (22) form the general equilibrium. The next proposition tells when the equilibrium exists.

Proposition 2 *Sufficient for existence of an equilibrium is that*

$$\frac{L + X^g/b}{(+ (1 - \mu) \frac{\mathbf{D}}{\mathbf{1} - 1})} \leq \left[\left(\frac{\mathbf{S}}{1 - \zeta} \right) / \left(1 + \frac{\mathbf{D}}{\mathbf{1} - 1} \right) \right]^{\frac{1-\zeta}{1-\mu}} e^{-\frac{1-\zeta}{1-\mu}} \left(1 - \frac{\zeta}{\mathbf{1} - 1} \right) \quad (23)$$

¹⁴Note that A is increasing (decreasing) in the number of firms, n , if $\alpha + \beta + \gamma < 1$ ($\alpha + \beta + \gamma > 1$), and independent of n if $\alpha + \beta + \gamma = 1$. That is economic activity is increasing (decreasing, constant) in the number of firms if there is decreasing (increasing, constant) returns to scale.

$$b \frac{D+1}{1-1} [(1+J^e) e]^{\frac{1}{1-\zeta}} \left(\mathfrak{s} \left(1 - \frac{1}{\theta} \right) \frac{\hat{A}}{L} \right)^{-\frac{1-\mu}{1-\zeta}} \geq 1 \quad (24)$$

both hold, and if the environmental tax is zero, condition (23) is also necessary. There are at most two equilibria, and if two equilibria exist one is Laffer inefficient.

Proof: See Appendix A.

Condition (23) tells that public expenditure cannot be too large in relation to the productive capacity of the economy A , obviously not a too high level of public expenditure can be funded. We may always fulfil this condition by choosing A appropriately. Condition (23) is also a necessary condition if the environmental tax is zero, i.e. when the economy relies only on wage taxation. Condition (24) states that b and L have to be large enough and is the requirement to obtain equilibrium unemployment. If (24) is fulfilled then in absence of wage taxation the economy produces unemployment, implying that a union member at the bargaining stage faces a probability of being laid off. The proposition states also that there are at most two equilibria, and if there are two, one of them is Laffer inefficient.

4.2 Double dividend in general equilibrium

We shall analyse a revenue neutral tax reform, increasing the environmental tax marginally from zero and reducing the labour tax. We shall concentrate on the situation where the labour tax is Laffer efficient, i.e. an increase in the labour tax rate would increase tax revenues (not decrease them). The following lemma states the condition for Laffer efficiency.

Lemma 1 *The labour income tax is Laffer efficient if*

$$J^w < (1 - \mathfrak{s} - \zeta) / (1 - \zeta) \quad (25)$$

Proof: Differentiating (22) with respect to τ^w , using (13) and (17) and evaluating at $\tau^e=0$ gives

$$\frac{\partial (wN)}{\partial \mathbf{J}^w} = wN - \mathbf{J}^w \frac{w}{1 - \mathbf{J}^w} N - \mathbf{J}^w w \frac{1 - (\quad)}{1 - \mu} \frac{N}{1 - \mathbf{J}^w} = \frac{wN}{1 - \mathbf{J}^w} \left(1 - \mathbf{J}^w \right) \quad (26)$$

which is positive if and only if (25) holds.

QED

The next result is the double dividend.

Proposition 3 *A newly introduced environmental tax, combined with a decrease in the labour tax so as to keep revenue neutrality, increases employment if the labour tax is Laffer efficient.*

Proof: Differentiating through the system (20) and (22) gives derivative of aggregate employment and of the labour tax when the environmental tax is increased

$$\frac{\partial N}{\partial \mathbf{J}^e} = \begin{vmatrix} -F_{\mathbf{J}^e} & F_{\mathbf{J}^w} \\ -G_{\mathbf{J}^e} & G_{\mathbf{J}^w} \end{vmatrix} \det^{-1}, \quad \frac{\partial \mathbf{J}^w}{\partial \mathbf{J}^e} = \begin{vmatrix} F_N & -F_{\mathbf{J}^e} \\ G_N & -G_{\mathbf{J}^e} \end{vmatrix} \det^{-1} \quad (27)$$

where $\det = F_N G_{\tau^w} - F_{\tau^w} G_N$, and the partial derivatives of F and G are denoted by subscripts. We have (for $\tau^e=0$): $F_{\tau^e} = \gamma N / (1 - \mu)$, $G_{\tau^e} = e(E^h + E^p)$, $F_{\tau^w} = (1 - \gamma)N / [(1 - \mu)(1 - \tau^w)]$, $G_{\tau^w} = wN / (1 - \tau^w)$, $F_N = 1$, and $G_N = \tau^w w$, then the determinant becomes

$$\det = wN \left[1 - \mathbf{J}^w \left(1 - (\quad) / (1 - \mu) \right) \right] (1 - \mathbf{J}^w)^{-1} > 0 \quad (28)$$

(which is positive by Lemma 1) and consequently (27) becomes

$$\frac{\partial N}{\partial \mathbf{J}^e} = \frac{(1 - (\quad)) e (E^h + E^p) - \mathbf{J}^w e E^p}{\det} \frac{N}{1 - \mathbf{J}^w} \frac{1}{1 - \mu} > 0 \quad (29)$$

$$\frac{\partial \mathbf{J}^w}{\partial \mathbf{J}^e} = \frac{-e E^h - e E^p \left[1 - \mathbf{J}^w \mathbf{J} / (1 - \mu) \right]}{\det} < 0 \quad (30)$$

QED

Thus, regardless of the competitive structure we have the double dividend. The way in which the competitive structure affects the size of the double dividend is given in the next proposition.

Proposition 4 *A newly introduced environmental tax, combined with a decrease in the labour tax (provided Laffer efficiency) so as to keep revenue neutrality, increases employment more (both in absolute value and in percentage units) if competition is low in the product market and/or if the unions' bargaining power is large.*

Proof: Use the expression for households' use of the polluting factor, (21), and substitute for N by the relation $\beta(1+\tau^e)eE^p=\gamma wN$ (from (18) and (19)), to obtain an expression in terms of firms' use of the polluting input

$$E^h = \mathbf{F} s E^p / (\mathbf{F} L b / [(1+\mathbf{J}^e) e]) \quad (31)$$

where

$$s \equiv \mathbf{D} / (\mathbf{D} + \mathbf{1} - 1) + 1 - \mu + 1 / (\mathbf{0} - 1) \quad (32)$$

Use (31), (28) and $\beta e E^p = \gamma w N$ in (29) (noticing that $\tau^e = 0$) and we have

$$N^{-1} \partial N / \partial \mathbf{J}^e = \left[(1-\mu) (\mathbf{F} s / \mathbf{F} + (1-\mu) L b / (wN)) \right] (1-\mu - (1-\mu) \mathbf{J}^w) \quad (33)$$

Competition structure affects s (equation (32)) as well as wN . From (32) it can be verified that $\partial s / \partial \eta < 0$ and $\partial s / \partial \rho > 0$. Next, denote $z = \{\eta, \rho\}$, then we have $d(wN)/dz = N \partial w / \partial z + w (\partial N / \partial w) (\partial w / \partial z) + w (\partial N / \partial z) = N (\partial w / \partial z) [1 + (w/N) (\partial N / \partial w)] + w (\partial N / \partial z) = -\beta N (\partial w / \partial z) / (1-\mu) + w (\partial N / \partial z)$. Since $\partial w / \partial z < 0$ and $\partial N / \partial z > 0$ for $z = \eta$, and $\partial w / \partial z > 0$ and $\partial N / \partial z = 0$ for $z = \rho$, we have $d(wN)/dz > 0$ for $z = \eta$ and $d(wN)/dz < 0$ for $z = \rho$. QED

4.3 Welfare

In this section we analyse the welfare impact of an environmental tax reform. We assume a utilitarian welfare function, a sum of the individuals' expected utilities. Using the indirect utilities of the employed, unemployed and firm owners (using equation (7) and replacing I^h by $(1-\tau^w)w$, b , and Π respectively), we obtain the social welfare function

$$(1+\mathbf{J}^e) e \left[N (1-\mathbf{J}^w) w + (L-N) b + \mathbf{A} \right] - (L+1) \cdot \left(\sum_{h=1}^{L+1} E^h \right) \quad (34)$$

We wish to distinguish between the welfare effect from efficiency gain in the tax system and the welfare effect associated with environmental quality changes. Denote welfare without the externality as $W = [(1+\tau^e)e]^{-\sigma} [(1-\tau^w)wN + (L-N)b + \Pi]$. Denote aggregate disposable income, I , as the sum of after-tax wage income, unemployment benefits, and profits: $I = I^w + I^b + I^A$, i.e. $I^w \equiv (1-\tau^w)wN$, $I^b \equiv (L-N)b$, $I^A \equiv \Pi$. Relative change in welfare due to the tax reform is

$$\left. \frac{\partial W}{\partial \mathbf{J}^e} \right|_{\mathbf{J}^e=0} = -\mathbf{F}I + \frac{I^w}{I} \left(\frac{1}{I^w} \frac{\partial I^w}{\partial \mathbf{J}^e} \right) + \frac{I^b}{I} \left(\frac{1}{I^b} \frac{\partial I^b}{\partial \mathbf{J}^e} \right) + \frac{\mathbf{A}}{I} \left(\right) \quad (35)$$

The first term on the right hand side of (35) is the loss in welfare due to the distortion induced by the environmental tax (the environmental tax distorts the individuals' consumption of the composite commodity and the polluting good). If households consume no polluting good ($\sigma=0$) this term is zero. The second term is the fraction of wage income to total income times the percentage change in wage income due to the reform. This term is positive, not because real wages increase but because employment increases. The third term is the income of the unemployed as a fraction of total income multiplied by the percentage change in unemployment income. This term is negative since the number of unemployed are reduced. The sum of the second and third terms is the net gain from moving people from unemployment into employment. Since $(1-\tau^w)w$ is unaffected by the taxes, this may be written as

$$\frac{I^b}{I} \left(\frac{1}{I^b} \frac{\partial I^b}{\partial \mathbf{J}^e} \right) = \frac{I^w}{I} \left(\frac{1}{N} \frac{\partial N}{\partial \mathbf{J}^e} \right) - \frac{I^b}{I} \frac{1}{L-N} \frac{\partial N}{\partial \mathbf{J}^e} = \frac{[(1-\mathbf{J}]}{I} \quad (36)$$

Thus the gain from moving people into employment is proportional to the wage markup, i.e. proportional to $(1-\tau^w)w \cdot b$. The fourth term of (35) is the gain in welfare due to the gain in income by the firm owners. This term is positive because the gain in profits from a decrease in the wage tax turns out to be larger than the loss in profits due to the increase in the environmental tax. It turns out that the positive terms in (35) are larger than the negative term, i.e.

Proposition 5 *A newly introduced environmental tax, combined with a decrease in the labour tax so as to keep revenue neutrality increases utilitarian welfare, if the labour tax is Laffer efficient, even if there are no externalities in the use of the polluting good (i.e. even if $\mathbb{N}(\bar{E})=0$).*

Proof: See Appendix A.

The effect of market competition on welfare could not be evaluated analytically. In section 5 we offer quantitative results for the case of an initial environmental tax greater than zero.

4.5 Interpretations of the results

The way in which imperfect competition affects the magnitude of the double dividend is through the government's budget constraint only, and depends on the relative size of the tax bases. If the tax base for the environmental tax is large relative to the tax base for the labour tax, an increase in the environmental tax can be accompanied by a larger decrease in the wage tax. The reason is that an increase in the environmental tax will give relatively large revenue, and the decrease in the wage tax will not give rise to a relatively large loss in revenue. So anything that makes the base of the environmental tax large relative to the base of the labour tax gives a greater double dividend. The same is true for the reduction in the use of the pollution good due to the tax reform. The larger the increase in the environmental tax relative to the reduction in the wage tax the greater is the reduction in the use of the polluting good. Again it is the relative size of the tax bases that determines the relative size of the changes in the tax rates.

Competition affects the relative size of the tax bases in the following way. The value of the polluting input used by firms divided by the wage bill is just a constant and is unaffected by competition (this is due to the Cobb-Douglas production function, the firm uses the value of factors in constant proportions). It is only the polluting good used by households divided by the wage bill which is affected by competition. The utility function is homothetic, and households therefore allocate constant fractions of their income to consumption of the polluting good. If all households

were wage-income earners, the ratio of household consumption good to the wage bill would be a constant, unaffected by competition. However, some households are firm owners and others are unemployed. So anything that makes profits larger and that creates larger unemployment, would give a larger use of the polluting good by households in relation to the wage bill. Imperfect competition in the goods market gives a larger profit share and lower economic activity (i.e. {lower employment) and therefore increasing the proportion of household income from profits and unemployment benefit, and the household use of the polluting good relative to the wage bill.

Similarly, the greater the bargaining power of the unions, the greater is unemployment and the greater is the non-wage income relative to the wage income, and again the greater is the polluting good used by households relative to the wage bill. Next we show this intuition formally.

Equation (20), $F(N, \tau^w, \tau^e) = 0$, gives the firms' employment decision as function of the prices (taxes), which we may write as

$$N = N^* (\mathbf{J}^w, \mathbf{J}^e) \quad (37)$$

N^* has the property that $\frac{1 - \mathbf{J}^w}{N} \frac{\partial N}{\partial \mathbf{J}^w}$ and $\frac{1 + \mathbf{J}^e}{N} \frac{\partial N}{\partial \mathbf{J}^e}$ are constants, independent of the competitive structure (i.e. independent of η and ρ). Equation (22), $G(N, \tau^w, \tau^e) = 0$, is the government budget, and substituting for $N=N^*$ gives $G(N^*(\tau^w, \tau^e), \tau^w, \tau^e) = 0$, which in turn gives the labour tax as a function of the environmental tax: $\tau^w = T(\tau^e)$. Substituting for the labour tax in (37) gives employment as a function of the environmental tax only, $N=N^*(T(\tau^e), \tau^e)$, which totally differentiated with respect to τ^e gives

$$\frac{1}{N} \frac{dN}{d\mathbf{J}^e} = \left(\frac{1 - \mathbf{J}^w}{N} \frac{\partial N^*}{\partial \mathbf{J}^w} \right) \left(\frac{d\mathbf{J}^w}{d\mathbf{J}^e} \frac{1}{1 - \mathbf{J}^w} \right) + \frac{1}{N} \frac{\partial N^*}{\partial \mathbf{J}^e} \quad (38)$$

The only way in which the competitive structure can affect $\frac{1}{N} \frac{dN}{d\mathbf{J}^e}$ is through $\frac{d\mathbf{J}^w}{d\mathbf{J}^e} \frac{1}{1 - \mathbf{J}^w}$ since the other terms are constants, independent of η and ρ . Only if the competitive structure affects the reduction in τ^w affordable by an increase in τ^e it affects the size of the double dividend.

Our Proposition 5, proving a welfare gain from the environmental tax reform, is in contrast with Bovenberg and de Mooij (1994a). Bovenberg and de Mooij results show that an environmental tax reform typically does not increase employment and welfare if there no other source of income apart from labour. In our model instead there are two sources of non-labour income, namely unemployment benefits and profits, towards which the burden of the pollution tax can be shifted. Both employed and unemployed workers together with shareholders bear the burden of the pollution tax, while only employed workers benefit from the reduction of the labour tax. Therefore, the base of the environmental tax is relatively larger than the base of the labour tax, which mitigates the tax-base erosion effect.¹⁵ Higher union power and higher price markups increase the proportion of non-labour income in relation to labour income (by increasing the number of unemployed workers and profits) and in doing so enhance the tax burden shifting effect.¹⁶ The introduction of lump-sum transfers has already been explored by Bovenberg and De mooij (1994b), while the effect of unemployment benefits has been studied by Koskela and Schöb (1999) among others. These papers find that under these circumstances, a double dividend is more likely.

Furthermore, in our economy we tax both consumers and producers on their use of the polluting good. In our framework, the wage tax is not a uniform consumption tax preferable to an environmental tax (as it would be with homothetic preferences and perfect competition, see Bovenberg and de Mooij, 1994a), but a tax enforcing a distortion on the labour market.

In our economy we also tax producers on their use of the polluting input. The welfare gain from introducing such a tax seems to contradict the Diamond and Mirrlees (1971) Production Efficiency Theorem. This is not the case, since the Production Efficiency Theorem holds only in absence of imperfections and when there is constant-returns-to-scale (or 100% profit taxation). In our economy, even if competition is perfect, we still have the welfare gain from producer taxation because of

¹⁵ This result is consistent with the one obtained by Bovenberg and de Mooij (1994b) in a perfect competition framework, when they include lump-sum transfers.

¹⁶ We assume that even in a perfectly competitive model there is some involuntary unemployment equal to the natural rate.

decreasing returns in labour and the polluting factor, $\beta+\gamma < 1$.

5. QUANTITATIVE RESULTS

5.1 The benchmark data and the calibration procedure

In this section we present quantitative results for the case in which the initial environmental tax is greater than zero. We calibrate the general equilibrium model described in section 3 on Italian macro data and compute the macroeconomic and welfare effects, which follow the imposition of a top-up environmental tax, the revenue of which is devoted to reducing the rate of the pre-existing labour-income tax in a revenue-neutral fashion. Consequently, by changing the values of the imperfect competition parameters we are able to assess the impact of different market structures. In this section, we have chosen to express the polluting good as final consumption of energy; therefore, the environmental tax of section 3 should be interpreted as an energy tax.

To make our model consistent with real data we augment the theoretical specification of section 3 in the following way. First, we replace the labour-income tax paid by the employees J^w with the effective labour-income tax J^l , which includes both the income tax paid by the employees and the social security contributions paid by the employers and employees. Indeed, the social contribution tax accounts for most of the tax burden. Second, we do not start from an equilibrium with zero energy tax, but instead split the energy tax into a pre-existing energy tax J^e and a top-up energy tax introduced by the reform J^{e*} . We choose the pre-existing energy tax to be equal to the VAT on energy expenditure, i.e. 19%. Finally, we calibrate the replacement ratio using the wage equation (13) above.

For the purpose of calibration, equation (20) can be rewritten as¹⁷

$$F'(N, \mathbf{J}^w, \mathbf{J}^e, \mathbf{J}^{e*}) \equiv \frac{N}{L} - M(1 - \mathbf{J}^l) \frac{1-\zeta}{1-\mu} (1 + \mathbf{J}^e + \mathbf{J}^{e*}) \frac{-\zeta}{1-\mu} = 0 \quad (39),$$

¹⁷ The mathematical derivation is given in Appendix C.

energy tax. The case in which the increment in the energy tax is infinitesimal is the analogue of the case for which we have provided an analytical solution in section 4. The greater the top-up energy tax, the greater the reduction in the labour-income tax and the increase in the employment rate, GDP, private consumption and welfare with respect to the pre-reform scenario. However, these effects are less than proportional.

[Table 2 approximately here]

5.3 Sensitivity analysis

In this section we assess the robustness of our results to changes in the values of selected calibration parameters (Table 3). First, we experiment with two different values of union power, one representing perfect competition in the labour market, $D=0.01$, the other close to the monopoly union model, $D=0.99$. Second, we model two different product markets, one almost perfectly competitive, characterised by a price markup of 1.01, the other highly uncompetitive, denoted by a markup of 1.25.²⁰ Finally we change the value of the production elasticities.

The sensitivity analysis shows that errors in the valuation of the union power do not seem to influence our results. However our results appear to be sensitive to measurement errors in the productivity elasticities, especially the labour elasticity and to a lesser extent to variations in the price markup.

[Table 3 approximately here]

5.4 Changing the degree of imperfection

Table 4 below reports the results from an extended analysis, which considers how values for the

²⁰ We also tried with markup values greater than 1.25 (as in Hall, 1988, for example); however, after a certain degree of imperfection, the signs of the changes in some variables were opposite to our analytical findings (due to our chosen calibration procedure) and, as such, we could not directly compare them to the other reported results.

imperfection parameters other than the benchmark case might effect the outcome of the reform. It is important to point out that in doing so we are no longer looking at the impact on the benchmark economy defined above, but on other hypothetical economies, which are characterised by different degrees of imperfection and, as such, can present different benchmark data. For example, a highly unionised economy and a highly uncompetitive product market economy will be characterised by a lower initial employment rate N/L^* (see equations (10) and (13) in section 3) and viceversa. We experiment with a range of values for the markup from 1.07, approaching perfect product market competition, to 1.25, as a measure of imperfection. Higher markup values than this make the problem no well defined, that is either the initial employment ratio or its value after the reform are greater than full employment; while values smaller than the lower bound give raise to an employment rate less than 50% of the labour force, which is obviously unacceptable on empirical grounds. We also try with a union power equal to 0.08 (approaching a competitive labour market) and a value of 0.2 (denoting imperfect labour market). We do not go above 0.2, since this leads to an initial employment rate below 50% of the labour force.

[Table 4 approximately here]

We notice that the higher the labour and goods market imperfections, the larger the double-dividend effect in terms of employment and the larger the reduction in the labour-income tax. This result is consistent with the qualitative findings from our theory and confirms the hypothesis that imperfect competition enhances the double-dividend effect of an environmental tax reform. General results are as follows: the greater the markup and the union power, the greater the after-tax employment rate, GDP and the lower the effective labour-income tax after the reform. For example, for a top-up environmental tax of 2.55% (see Table 4-b), an increase in the power of the unions from $\rho=0.08$ to $\rho=0.2$ boosts employment (in percentage changes from pre-reform equilibrium) from 4.4% to 4.8%, and GDP from 2.4% to 2.6%. An increase in the markup from 1.13 to 1.25, increases employment from 3.6% to 4.4% and GDP from 1.7% to 2.4%. Private (clean) consumption and

disposable income together with welfare always increase: the higher the markup, the greater the increase; the greater the union power, the less the increase. In particular, for the case examined above, welfare increases from 1.1% to 1.3%, if the firms' market power increases, while it assumes values from 1.4% to 1.3%, if the unions' power increases. This is a new result, since we could not analytically assess in section 4 the influence of imperfection on the magnitude of the increase in the above variables. Even if the environmental tax is positive at the outset, there is always scope for a double-dividend effect. We have also noticed that, if the market imperfections are higher, an environmental tax reform has a greater positive impact on employment and GDP. Therefore, countries characterised by a low degree of competition have greater incentives, in terms of employment and GDP, for implementing environmental tax reforms.

6. Summary and Conclusions

The paper has explored the double dividend issue within a general equilibrium model of imperfect competition with consumption and production externalities. We have focused on the double dividend in terms of employment and welfare. Imperfect competition is described by the price-setting behaviour of firms and by the presence of unions which create distortions in the labour market. We have found that several effects are at work when a revenue neutral environmental tax reform is implemented. We have specifically focused on the sensitivity of the magnitude of the double dividend to changes in the degree of goods market imperfection.

We confirm the findings of Koskela, Schöb and Sinn (1998): when one takes into account all general equilibrium effects in the economy we always find a double dividend regardless of the unions' bargaining power and the firms' product market power and its magnitude positively depends on the union power.

Furthermore, we have achieved important new results in the analysis of environmental tax reforms with uncompetitive markets. First, we have found that the magnitude of the change in

employment is stronger, the higher the degree of imperfection in the goods market. Therefore, not only economies with a highly distorted labour market, but also economies with highly uncompetitive goods markets have greater incentives (in terms of employment and GDP) to implement environmental tax reforms. Second, we have conducted welfare analysis, and we have reached the conclusion that an environmental tax reform, of the type analysed in this paper, is always welfare improving, even if we ignore the gain from reducing the externality.

Finally, we have calibrated the theoretical model on Italian data and shown numerical evidence that our results also hold in the presence of a pre-existing positive environmental tax. In doing so, we have also provided a quantitative assessment of the macroeconomic and welfare effects of the reform and of their sensitivity to changes in imperfect competition.

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Appendix B

Computation procedure

Calculus to (39)

The households' energy expenditure and the firms' energy expenditure can be expressed as follows

$$h = \frac{1+J^e}{1+J^e+J^{e*}} \left\{ wN \left[1 - J^w + \frac{1}{S} \frac{1+(1-\mu)(O-1)}{O-1} \right] + (L-N) b \right. \quad (77)$$

$$e E^P = \frac{1}{1+J^e+J^{e*}} \frac{C}{S} wN \quad (78)$$

Substituting (77) and (78) in (20) we get

$$F = \frac{N}{L} - S \frac{A}{L} [w(1-J^l)]^{-\frac{1-C}{1-\mu}} (1-J^l)^{\frac{1-C}{1-\mu}} (1+J^e+J^{e*})^{-\frac{C}{1-\mu}} = 0 \quad (79)$$

Calculus to (41)

For $\tau^{e*}=0$ (22) becomes

$$\begin{aligned} &= \frac{R}{w(1-J^l)L} \\ &= \frac{1}{1-J^l} \left\{ J^l + \frac{J^e}{1+J^e} \left[\frac{C}{S} + \frac{1+J^e}{S} \left(\frac{1}{O-1} + 1-\mu \right) F \right] - J^e F \frac{b}{w} \right\} \\ &\quad + J^e \left(\frac{N}{L} + \frac{b}{w(1-J^l)} \right) F \end{aligned} \quad (80)$$

Computing the changes in the macroeconomic variables and in welfare

Consumption and disposable income

We use

$$(1+J^e) PX - (1+J^e+J^{e*}) e E^h = I \quad (81)$$

$$(1 + \mathbf{J}^e) PX = (1 - \mathbf{F}) I \quad (82)$$

$$\frac{(1 + \mathbf{J}^e) PX}{(1 + \mathbf{J}^e + \mathbf{J}^{e*}) e E^h} = \frac{1 - \mathbf{F}}{\mathbf{F}} \quad (83)$$

$$(1 + \mathbf{J}^e) PX = \frac{1 - \mathbf{F}}{\mathbf{F}} (1 + \mathbf{J}^e + \mathbf{J}^{e*}) e E^h \quad (84)$$

therefore

$$\frac{)X}{X} = \frac{1 + \mathbf{J}^e + \mathbf{J}^{e*}}{1 + \mathbf{J}^e} \frac{e E_1^h - e E_0^h}{e E_0^h} + \frac{\mathbf{J}^{e*}}{1 + \mathbf{J}^e} \quad (85)$$

GDP

Use

$$\frac{Y}{L} = H \left(\frac{N}{L} \right)^s \left(\frac{E}{L} \right)^c L^{\mu-1} \quad (86)$$

then the changes in GDP are

$$\frac{)Y}{Y} = \left(\frac{N_1/L}{N_0/L} \right)^s \left(\frac{E_1^P/L}{E_0^P/L} \right)^c - 1 \quad (87)$$

Welfare

Welfare is easily recovered as

$$= \left(\frac{1 + \mathbf{J}^e}{1 + \mathbf{J}^e + \mathbf{J}^{e*}} \right)^F \frac{\frac{\mathbf{D} + \mathbf{2} - 1}{\mathbf{2} - 1} \left(\frac{\mathbf{C}}{\mathbf{S}} + \frac{\mathbf{F}}{\mathbf{2} - 1} \right) \frac{1}{1 - \mathbf{J}_1^l} \frac{N_1}{L} + \mathbf{F} + \mathbf{F} \frac{\mathbf{D}}{\mathbf{2} - 1}}{\frac{\mathbf{D} + \mathbf{2} - 1}{\mathbf{2} - 1} \left(\frac{\mathbf{C}}{\mathbf{S}} + \frac{\mathbf{F}}{\mathbf{2} - 1} \right) \frac{1}{1 - \mathbf{J}_0^l} \frac{N_0}{L} + \mathbf{F} + \mathbf{F} \frac{\mathbf{D}}{\mathbf{2} - 1}} \quad (88)$$

Appendix C

Data sources and definitions

N/L: employment rate as percentage of labour force. All workers, males and females. Source: National Accounts

E^h : households' total energy expenditure. It includes expenditure for heating and other domestic uses, and private transport. Calculated from aggregated expenditure on all fuels. Current prices, aggregate households. Source: BEN and National Accounts

I: households disposable income. Current prices, aggregate households Source: National Accounts

τ^w : labour-income tax. Calculated as INCTAX over I. Where INCTAX is: income-tax revenue. Current prices. Source: National Accounts. See Modigliani et al. (1986) and Mendoza, Razin and Tesar (1994)

τ^s : social security contribution tax. Calculated as SS over W. where, SS: total social security contributions. Current prices. Source: National Accounts; and W: pre-tax nominal wage bill. Current prices. Dependent workers, males and females. Source: National Accounts

τ^l : effective labour-income tax. Calculated as in Mendoza, Razin and Tesar (1994), using the formula:
$$\tau^l = (\tau^w + \tau^s) / (1 + \tau^s)$$

α : capital production elasticity. Estimated as in Marsiliani (1997)

β : labour production elasticity. Estimated as in Marsiliani (1997)

γ : energy production elasticity. Estimated as in Marsiliani (1997)

Θ : wage elasticity of labour demand. Recovered as in Marsiliani (1997)

$\eta/(\eta-1)$: price markup. Recovered as in Marsiliani (1997)

ρ : union power. Recovered as in Marsiliani (1997)

Annex 1: Italian data, annual average 1986-1995

Year	U	E ^h	I	INCTAX	τ^w	SS	W	τ^p	τ^l
1986	11.125	32876959	671631	93515	0.139236	112162	291903	0.384244	0.37817
1987	11.975	34358883	735247	101336	0.137826	119887	318951	0.375879	0.373365
1988	12.05	36315778	803716	120608	0.150063	132510	350043	0.378553	0.383457
1989	12	39515774	884856	132727	0.149998	151147	377193	0.400715	0.393166
1990	11.4	44279033	982605	147622	0.150235	170344	422049	0.403612	0.394587
1991	10.925	53320295	1078315	161668	0.149927	185521	461255	0.402209	0.393761
1992	10.65	54757648	1145750	177636	0.155039	198078	482295	0.410699	0.401034
1993	10.25	58922005	1137344	196964	0.173179	203049	484802	0.418829	0.417251
1994	11.275	59241605	1186468	187569	0.15809	204883	490271	0.417897	0.406227
1995	11.975	64453862	1265867	193817	0.15311	218188	501066	0.435448	0.410017

Legend:

U: unemployment rate as percentage of labour force. All workers, males and females.

Source: National Accounts

E^h: households total energy expenditure. It includes expenditure for heating and other domestic uses, and private transport. Calculated from aggregated expenditure on all fuels. Current prices, aggregate households. Source: BEN and National Accounts

I: households disposable income. Current prices, aggregate households

Source: National Accounts

INCTAX: income-tax revenue. Current prices.

Source: National Accounts

τ^w : labour-income tax. Calculated as INCTAX over I. See Modigliani et al. (1986) and Mendoza, Razin and Tesar (1994)

SS: total social security contributions . Current prices
.Source: National Accounts

W: pre-tax nominal wage bill. Current prices. Dependent workers, males and females.
Source: National Accounts

τ^p : payroll tax. Calculated as SS over W

τ^l : effective labour-income tax. Calculated as in Mendoza, Razin and Tesar (1994), using the formula: $\tau^l = (\tau^w + \tau^p) / (1 + \tau^p)$

Table 2 *The macroeconomic and welfare effects of environmental tax reforms (Δ =percentage changes from base)*

	$\tau^{e*}=\varepsilon$	$\tau^{e*}=0.8\%$	$\tau^{e*}=2.5\%$	$\tau^{e*}=5\%$	$\tau^{e*}=10\%$
a) Effective labour tax	38.99	38.7	37.99	37.2	35.8
τ^l (%)					
$\Delta\tau^l$ (%)	-0.01	-0.3	-1.01	-1.8	-3.2
b) Employment rate	88.0142	88.9631	91.2648	93.8616	98.1984
N/L (%)	0.01613	1.094	3.71	6.661	11.589
Δ N/L (%)					
c) ΔGDP (%)	0.00889	0.599	2.022	3.604	6.192
d) Δ Private Consumption and Disposable Income (%)	0.00535	0.359	1.212	2.159	3.71
e) Δ Welfare (%)	0.00494	0.327	1.108	1.956	3.889

|

Table 3 Sensitivity analysis of the calibration procedure (Δ =percentage changes from base)

	1) Central case	2) Union power $\rho=0.01$ $\rho=0.99$		3) price markup $m=1.01$ $m=1.25$		4) labour elast. $\beta=0.57$ $\beta=0.50$		5) energy elast. $\gamma=0.021$ $\gamma=0.001$	
a) Effective labour tax									
τ^l (%)	37.99	37.998	37.962	38.121	37.859	37.583	38.345	37.8	38.147
$\Delta\tau^l$ (%)	-1.01	-1.002	-1.038	-0.879	-1.141	-1.417	-0.655	-1.2	-0.853
b) Employment rate									
N/L (%)	91.2648	91.2416	91.361	90.8343	91.703	92.849	89.879	91.905	90.757
Δ N/L (%)	3.71	3.684	3.819	3.221	4.208	5.51	2.135	4.438	3.133
c) ΔGDP (%)	2.022	2.008	2.082	1.755	2.295	3.116	1.05	2.423	1.711
d) Δ Private Consumption and Disposable Income (%)	1.212	1.203	1.247	0.949	1.477	1.813	0.673	1.441	1.032
e) Δ Welfare (%)	1.108	1.099	1.144	0.846	1.373	1.709	0.572	1.337	0.929

Table 4 *Changing the degree of competition*

a) Effects of a top-up environmental tax of 0.1‰

	$\eta=5$ markup=1.25	$\eta=8.6542$ markup=1.13	$\eta=9.9$ markup=1.112	$\eta=15$ markup=1.07
$\rho=0.08$	N/L*=73.5137 N/L=73.5281 Δ N/L=0.01958 Δ GDP=0.0108 Δ I=0.00653 Δ W=0.00612 $\tau^l = 38.9947$	N/L*=95.7671 N/L=95.782 Δ N/L=0.01555 Δ GDP=0.0085 Δ I=0.00528 Δ W=0.00488 $\tau^l = 38.9957$	N/L*=99.9415 N/L=99.9565 Δ N/L=0.015 Δ GDP=0.00826 Δ I=0.00516 Δ W=0.00475 $\tau^l = 38.9959$	N/L*>full employment
$\rho=0.12$	N/L*=66.4955 N/L=66.509 Δ N/L=0.0203 Δ GDP=0.0112 Δ I=0.00648 Δ W=0.00607 $\tau^l = 38.9945$	N/L*=88.0 N/L=88.0142 Δ N/L=0.01613 Δ GDP=0.0088 Δ I=0.00535 Δ W=0.00494 $\tau^l = 38.9956$	N/L*=92.0627 N/L=92.077 Δ N/L=0.01553 Δ GDP=0.00854 Δ I=0.0051 Δ W=0.00469 $\tau^l = 38.9957$	N/L*>full employment
$\rho=0.2$	N/L*=55.0888 N/L=55.1007 Δ N/L=0.0216 Δ GDP=0.0119 Δ I=0.0063 Δ W=0.00589 $\tau^l = 38.9941$	N/L*=74.974 N/L=74.9868 Δ N/L=0.01707 Δ GDP=0.0094 Δ I=0.00524 Δ W=0.00484 $\tau^l = 38.9953$	N/L*=78.7839 N/L=78.7968 Δ N/L=0.01637 Δ GDP=0.00904 Δ I=0.0051 Δ W=0.0047 $\tau^l = 38.9955$	N/L*=88.2536 N/L=88.2668 Δ N/L=0.01495 Δ GDP=0.00824 Δ I=0.00474 Δ W=0.00433 $\tau^l = 38.9959$

Legend:

- ρ denotes union power
- η is the price elasticity of output demand
- N/L* is the pre-reform equilibrium employment in %;
- N/L is the post-reform equilibrium employment in %;
-) N/L denotes percentage changes in equilibrium employment
-) GDP denotes percentage changes in GDP
-) I denotes percentage changes in disposable income (and consumption)
-) W denotes percentage changes in private welfare
- τ^l is the effective labour tax

b) Effects of a top-up environmental tax of 2.55%

	$\eta=5$ markup=1.25	$\eta=8.6542$ markup=1.13	$\eta=9.9$ markup=1.112	$\eta=15$ markup=1.07
$\rho=0.08$	N/L*=73.5137 N/L=76.7656 $\Delta N/L=4.424$ $\Delta GDP=2.412$ $\Delta I=1.451$ $\Delta W=1.347$ $\tau^l = 37.802$	N/L*=95.7671 N/L=99.219 $\Delta N/L=3.604$ $\Delta GDP=1.9647$ $\Delta I=1.217$ $\Delta W=1.113$ $\tau^l = 38.019$	N/L*=99.9415 N/L>full employment $\tau^l = 38.005$	N/L*>full employment
$\rho=0.12$	N/L*=66.4955 N/L=69.5326 $\Delta N/L=4.567$ $\Delta GDP=2.4906$ $\Delta I=1.438$ $\Delta W=1.333$ $\tau^l = 37.764$	N/L*=88 N/L=91.2648 $\Delta N/L=3.71$ $\Delta GDP=2.0224$ $\Delta I=1.2116$ $\Delta W=1.1079$ $\tau^l = 37.991$	N/L*=92.0627 N/L=95.3603 $\Delta N/L=3.5819$ $\Delta GDP=1.9524$ $\Delta I=1.1754$ $\Delta W=1.0718$ $\tau^l = 38.025$	N/L*>full employment
$\rho=0.2$	N/L*=55.0888 N/L=57.7572 $\Delta N/L=4.844$ $\Delta GDP=2.641$ $\Delta I=1.402$ $\Delta W=1.2981$ $\tau^l = 37.691$	N/L*=74.974 N/L=77.9053 $\Delta N/L=3.909$ $\Delta GDP=2.1317$ $\Delta I=1.1938$ $\Delta W=1.0902$ $\tau^l = 37.938$	N/L*=78.7839 N/L=81.7572 $\Delta N/L=3.774$ $\Delta GDP=2.0575$ $\Delta I=1.1609$ $\Delta W=1.0573$ $\tau^l = 37.974$	N/L*=88.2536 N/L=91.3217 $\Delta N/L=3.476$ $\Delta GDP=1.8946$ $\Delta I=1.0859$ $\Delta W=0.9824$ $\tau^l = 38.053$

Legend: see Legend of Table 4a

c) Effects of a top-up environmental tax of 10%

	$\eta=5$ markup=1.25	$\eta=8.6542$ markup=1.13	$\eta=9.9$ markup=1.112	$\eta=15$ markup=1.07
$\rho=0.08$	N/L*=73.514 N/L=83.4819 $\Delta N/L=13.5918$ $\Delta GDP=7.2312$ $\Delta I=4.3499$ $\Delta W=3.9438$ $\tau^l = 35.4$	N/L*=95.7671 N/L>full employment $\tau^l = 35.9$	N/L*=99.9415 N/L>full employment $\tau^l = 36$	N/L*>full employment
$\rho=0.12$	N/L*=66.4955 N/L=75.7757 $\Delta N/L=13.95613$ $\Delta GDP=7.4392$ $\Delta I=4.2945$ $\Delta W=3.8886$ $\tau^l = 35.3$	N/L*=88 N/L=98.1984 $\Delta N/L=11.58909$ $\Delta GDP=6.1924$ $\Delta I=3.7096$ $\Delta W=3.306$ $\tau^l = 35.8$	N/L*=92.0627 N/L>full employment $\tau^l = 35.9$	N/L*>full employment
$\rho=0.2$	N/L*=55.0888 N/L=63.2151 $\Delta N/L=14.75127$ $\Delta GDP=7.8555$ $\Delta I=4.1698$ $\Delta W=3.7644$ $\tau^l = 35.1$	N/L*=74.974 N/L=84.252 $\Delta N/L=12.37495$ $\Delta GDP=6.6077$ $\Delta I=3.7006$ $\Delta W=3.297$ $\tau^l = 35.7$	N/L*=78.7839 N/L=88.2699 $\Delta N/L=12.04053$ $\Delta GDP=6.4312$ $\Delta I=3.6286$ $\Delta W=3.2253$ $\tau^l = 35.8$	N/L*=88.2536 N/L=97.7904 $\Delta N/L=10.80613$ $\Delta GDP=5.7774$ $\Delta I=3.3113$ $\Delta W=2.9092$ $\tau^l = 36.1$

Legend: see Legend of Table 4a

APPENDIX A: Proof of Propositions 2 and 5

Proof of Proposition 2

Define the employment rate, v , as

$$\frac{N}{L} = \mathfrak{s} \left(1 - \frac{1}{\mathbf{0}} \right) \frac{\hat{A}}{L} \left(\frac{\mathbf{D} + \mathbf{1} - \mathbf{1}}{\mathbf{1} - \mathbf{1}} b \right)^{-\frac{1-\zeta}{1-\mu}} [e(1+\mathbf{J}^e)]^{-\frac{\zeta}{1-\mu}} (1-\mathbf{J}^w) \quad (42)$$

where the right-hand side is equation (20), where we have substituted for w by using (13).

Alternatively we may write (42) as

$$F \equiv < - (1-\mathbf{J}^w) \frac{1-\zeta}{1-\mu} (Sb)^{-\frac{1-\zeta}{1-\mu}} = 0 \quad (43)$$

$$\text{where } S \equiv \left(\frac{\mathbf{D} + \mathbf{1} - \mathbf{1}}{\mathbf{1} - \mathbf{1}} \right) [e(1+\mathbf{J}^e)]^{\frac{\zeta}{1-\zeta}} \left(\mathfrak{s} \left(1 - \frac{1}{\mathbf{0}} \right) \frac{\hat{A}}{L} \right)^{-\frac{1-\mu}{1-\zeta}} \quad (44)$$

Next, we shall rewrite the government's budget constraint, (22), by using the fact that government revenue, R , equals unemployment benefit plus public goods provision, i.e. $R = (L-N)b + X^g$. Then we have

$$\frac{\mathbf{J}^e}{1+\mathbf{J}^e} \frac{\zeta}{\mathfrak{S}} wN + \frac{\mathbf{J}^e \mathbf{F}}{1+\mathbf{J}^e} \left\{ \frac{\mathbf{D}}{\mathbf{1} - \mathbf{1}} bN + Lb + wN \frac{1 + (\mathbf{0} - \mathbf{1})}{\mathfrak{S}(\mathbf{0} - \mathbf{1})} \right\} = (L - \quad (45)$$

Dividing by bL and premultiplying by $(1-\tau^w)$ and rearranging gives

$$\mathbf{J}^w z \frac{N}{L} + \frac{N}{L} a = (1-\mathbf{J}^w) \left[\frac{1 + (1-\mathbf{F})\mathbf{J}^e}{1+\mathbf{J}^e} + \frac{X^g}{bL} \right] \quad (46)$$

where

$$z \equiv \frac{\mathbf{D}}{\mathbf{1} - \mathbf{1}} \frac{1 + (1-\mathbf{F})\mathbf{J}^e}{1+\mathbf{J}^e} > 0 \quad (47)$$

and

$$i \equiv 1 + \frac{\mathbf{J}^e \mathbf{F}}{1 + \mathbf{J}^e} \frac{\mathbf{D}}{\mathbf{1} - \mathbf{1}} + \frac{\mathbf{J}^e}{1 + \mathbf{J}^e} \frac{\mathbf{D} + \mathbf{1} - \mathbf{1}}{\mathbf{1} - \mathbf{1}} \left\{ \frac{\mathbf{C}}{\mathbf{S}} + \mathbf{F} \frac{1 + \mathbf{1} (\mathbf{0} - \mathbf{1})}{\mathbf{S} (\mathbf{0} - \mathbf{1})} \right\} > \quad (48)$$

Rewrite (46) in terms of the employment rate v , then

$$G \equiv < - \frac{1 - \mathbf{J}^w}{a + z \mathbf{J}^w} H = 0 \quad (49)$$

where

$$H \equiv \frac{1 + (1 - \mathbf{F}) \mathbf{J}^e}{1 + \mathbf{J}^e} + \frac{X^g}{bL} \quad (50)$$

Equation (43) gives the employment rate v as a function of the wage tax τ^w . It has the following properties as $\tau^w = \{0, 1\}$

$$\begin{aligned} <(0) &= (Sb)^{-\frac{1-\zeta}{1-\mu}} \\ <'(0) &= -\frac{1-\zeta}{1-\mu} (Sb)^{-\frac{1-\zeta}{1-\mu}} \end{aligned} \quad (51)$$

$$\begin{aligned} <(1) &= 0 \\ <'(1) &= 0 \end{aligned} \quad (52)$$

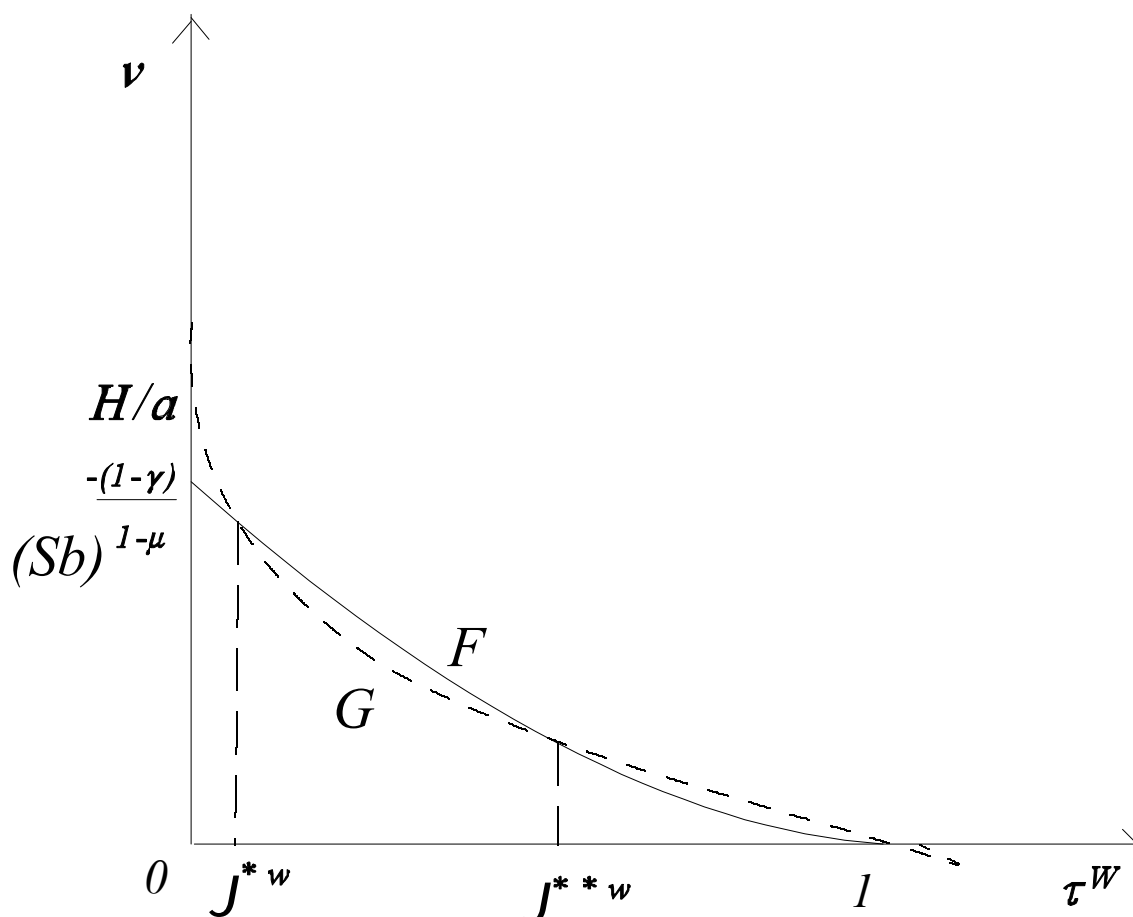
Similarly, equation (49) gives the employment rate as a function of the wage tax, with the following properties

$$\begin{aligned} <(0) &= H/a \\ <'(0) &= -H(a+z)/a^2 \end{aligned} \quad (53)$$

$$\begin{aligned} <(1) &= 0 \\ <'(1) &= -H/(a+z) \end{aligned} \quad (54)$$

Both function F and G decreasing and convex in τ^w , and take on value zero at $\tau^w=1$. Function F has zero slope at $\tau^w=1$, G not. If $H/a > (Sb)^{-(1-\gamma)/(1-\mu)}$ and F and G cross, they must cross exactly twice, like in Figure 1.¹

Figure 1: The General Equilibrium



In this situation we have two equilibria, one Laffer efficient (τ^{*w}) and one Laffer inefficient (τ^{**w}). For the Laffer efficient equilibrium to be well defined we require that employment is less than 100% at this tax rate, i.e. $v(\tau^{*w}) < 1$. Sufficient for this to be true is that F intersects the vertical axis at $v \leq 1$,

¹If $H/a < (Sb)^{-(1-\gamma)/(1-\mu)}$ public expenditure is too small in relation to the government revenue from the energy tax, and there is no Laffer efficient wage tax. The wage tax has to be Laffer inefficient in this situation. If $H/a = (Sb)^{-(1-\gamma)/(1-\mu)}$ we have two equilibria. One in which the wage tax is zero, and one in which the wage tax is positive but Laffer inefficient. Both equilibria raise the same revenue. We shall not explore these cases further.

i.e. that $Sb \geq 1$. We require that G lies below F for values of the wage tax between the Laffer efficient level and the Laffer inefficient level. Sufficient for this being the case is that G lies below F at the Laffer maximum wage tax. The Laffer optimal wage tax is (see Lemma 1)

$$\mathbf{J}^w = \frac{1-\mu}{1-\zeta} \quad (55)$$

Function F at the Laffer optimal tax is

$$F = \left(\frac{\mathbf{S}}{1-\zeta} \right)^{\frac{1-\zeta}{1-\mu}} (Sb)^{-\frac{1-\zeta}{1-\mu}} \quad (56)$$

Function G at the Laffer optimal tax is

$$G = \frac{\mathbf{S}}{1-\zeta} H / \left(a + z \frac{1-\mu}{1-\zeta} \right) \quad (57)$$

Thus we need to show that

$$\frac{\frac{\mathbf{S}}{1-\zeta} H}{a + z \frac{1-\mu}{1-\zeta}} \leq \left(\frac{\mathbf{S}}{1-\zeta} \right)^{\frac{1-\zeta}{1-\mu}} (Sb)^{-\frac{1-\zeta}{1-\mu}} \quad (58)$$

or

$$\frac{H}{a + z \frac{1-\mu}{1-\zeta}} \leq \left(\frac{\mathbf{S}}{1-\zeta} \right)^{\frac{1-\zeta}{1-\mu}} (Sb)^{-\frac{1-\zeta}{1-\mu}} \quad (59)$$

First, $a+z(1-\mu)/(1-\zeta)$ is increasing in τ^e . Second H is decreasing in τ^e , and S is increasing in τ^e . The larger τ^e is the more likely the condition is fulfilled.

For sufficiency it is thus enough to prove the condition at $\tau^e=0$. First we have

$$H|_{\mathbf{J}^e=0} = 1 + \frac{X^g}{Lb} \quad (60) \quad a + z \frac{1-\mu}{1-\zeta} \Big|_{\mathbf{J}^e=0} = 1 + \frac{1-\mu}{1-\zeta} \frac{\mathbf{D}}{\mathbf{1}-1} \quad (61)$$

and

$$Sb|_{J^e=0} = \left(1 + \frac{\mathbf{D}}{\mathbf{2}-1}\right) e^{\frac{-\zeta}{1-\zeta}} \left(\mathbf{s} \left(1 - \frac{1}{\mathbf{0}}\right) \frac{\hat{\mathbf{A}}}{L}\right)^{-\frac{1-\mu}{1-\zeta}} \quad (62)$$

Then (59) becomes

$$\frac{\frac{X^g / (Lb)}{1-\mu} \frac{\mathbf{D}}{1-\zeta}}{\frac{\mathbf{D}}{1-\zeta} \frac{1}{\mathbf{1}-1}} \leq \left(\frac{\mathbf{s}}{1-\zeta}\right)^{\frac{1}{1-\mu}} \left(1 + \frac{\mathbf{D}}{\mathbf{1}-1}\right)^{-\frac{1-\zeta}{1-\mu}} e^{\frac{-\zeta}{1-\mu}} \mathbf{s} \left(1 - \frac{1}{\mathbf{0}}\right) \quad (63)$$

Premultiply both sides by L and divide by $1-\gamma$ to obtain (23).

QED

Proof of Proposition 5

Differentiating (34) with respect to τ^e , and premultiplying by $(1+\tau^e)^\sigma$ we have

$$(1+\mathbf{J}^e)^{\mathbf{F}} \frac{\partial \tilde{W}}{\partial \mathbf{J}^e} = -\frac{\mathbf{F}}{1+\mathbf{J}^e} I + \frac{\partial I}{\partial \mathbf{J}^w} \frac{\partial \mathbf{J}^w}{\partial \mathbf{J}^e} + \frac{\partial I}{\partial N} \frac{\partial N}{\partial \mathbf{J}^e} \quad (64)$$

where $I \equiv (1-\tau^w)wN + (L-N)b + \Pi$. First we need to find the derivative of I with respect to τ^w , holding N constant. Since $\Pi = [1+(1-\mu)(\eta-1)]\beta^{-1}(\eta-1)^{-1}wN$ (which follows from (17) and (19)), we have

$$\left. \frac{\partial \mathbf{A}}{\partial \mathbf{J}^w} \right|_N = \frac{\frac{1}{\mathbf{0}} + (1-\mu) \left(1 - \frac{1}{\mathbf{0}}\right)}{\mathbf{s} \left(1 - \frac{1}{\mathbf{0}}\right)} \frac{w}{1-\mathbf{J}^w} N = \frac{\mathbf{A}}{1-\mathbf{J}^w} \quad (65)$$

Then, since $(1-\tau^w)w$ is independent of τ^w (which follows from (13)), we have

$$\frac{\partial I}{\partial \mathbf{J}^w} = \frac{\mathbf{A}}{1-\mathbf{J}^w} \quad (66)$$

Next, since $(1-\tau^w)w$ is also independent of N we have

$$\frac{\partial I}{\partial N} = (1-\mathbf{J}^w)w - b + \frac{\mathbf{A}}{N} \quad (67)$$

Substituting (66) and (67) into (64), and premultiplying by $(1-\tau^w)$ and by the determinant (28), gives

$$\begin{aligned}
 (1 - \mathbf{J}^w) \det \frac{\partial \tilde{W}}{\partial \mathbf{J}^e} \Big|_{\mathbf{J}^e=0} &= -\mathbf{F} (1 - \mathbf{J}^w) I \det + \mathbf{A} \frac{\partial \mathbf{J}^w}{\partial \mathbf{J}^e} \det \\
 &+ \left[(1 - \mathbf{J}^w) w - b + \frac{\mathbf{A}}{N} \right] (1 - \mathbf{J}^w) \frac{\partial N}{\partial \mathbf{J}^e} \det
 \end{aligned} \tag{68}$$

Substituting for the derivatives by using (29) and (30), and for the determinant using (28) gives

$$\begin{aligned}
 (1 - \mathbf{J}^w) \det \frac{\partial \tilde{W}}{\partial \mathbf{J}^e} \Big|_{\mathbf{J}^e=0} &= -\mathbf{F} I w N \left(1 - \frac{1 - (\mathbf{J}^w)}{1 - \mu} \right) + \mathbf{A} \left[-e \bar{E} + e E^p \frac{\mathbf{J}^w \mathbf{S}}{1 - \mu} \right] \\
 &+ \left[(1 - \mathbf{J}^w) w N - b N + \mathbf{A} \right] \frac{(1 - (\mathbf{J}^w)) e \bar{E} - \mathbf{S} e E^p}{1 - \mu}
 \end{aligned} \tag{69}$$

First we have

$$(1 - \mathbf{J}^w) w N - b N = \left(\frac{\mathbf{D} + \mathbf{1} - 1}{\mathbf{1} - 1} - 1 \right) b N = \frac{\mathbf{D}}{\mathbf{1} - 1} b N = \frac{\mathbf{D} (1 - \mathbf{J}^w)}{\mathbf{D} + \mathbf{1} - 1} w N \tag{70}$$

where the first and last equality follows from (13). Next, substitute (70) into (69) and collect the terms involving $e \bullet$ and $e E^p$, then we have

$$\begin{aligned}
 (1 - \mathbf{J}^w) \det \frac{\partial \tilde{W}}{\partial \mathbf{J}^e} \Big|_{\mathbf{J}^e=0} &= -\mathbf{F} I w N \left(1 - \frac{1 - (\mathbf{J}^w)}{1 - \mu} \right) + \frac{e \bar{E}}{1 - \mu} \left(\mathbf{S} \mathbf{A} + (1 - (\mathbf{J}^w)) \frac{\mathbf{D} (1 - \mathbf{J}^w)}{\mathbf{D} + \mathbf{1} - 1} w N \right) \\
 &- e E^p \frac{\mathbf{S} (1 - \mathbf{J}^w)}{1 - \mu} \left[\mathbf{A} + \frac{\mathbf{D}}{\mathbf{D} + \mathbf{1} - 1} w N \right]
 \end{aligned} \tag{71}$$

Since $\Pi = w N / (\Theta - 1)$ (follows from (17), (19) and (11)) we have

$$\begin{aligned}
 (1 - \mathbf{J}^w) \det \frac{\partial \tilde{W}}{\partial \mathbf{J}^e} \Big|_{\mathbf{J}^e=0} &= -\mathbf{F} I w N \left(1 - \frac{1 - (\mathbf{J}^w)}{1 - \mu} \right) + \frac{e \bar{E} w N}{1 - \mu} \left(\frac{\mathbf{S}}{\mathbf{1} - 1} + (1 - (\mathbf{J}^w)) \frac{\mathbf{D} (1 - \mathbf{J}^w)}{\mathbf{D} + \mathbf{1} - 1} \right) \\
 &- \frac{e E^p w N}{1 - \mu} \mathbf{S} (1 - \mathbf{J}^w) \left[\frac{1}{\mathbf{1} - 1} + \frac{\mathbf{D}}{\mathbf{D} + \mathbf{1} - 1} \right]
 \end{aligned} \tag{72}$$

Substituting for $e \bullet$ by using (34) and rearranging we have

$$\begin{aligned}
 (1-\mathbf{J}^w) \det \frac{\partial \tilde{W}}{\partial \mathbf{J}^e} \Big|_{\mathbf{J}^e=0} &= -\mathbf{F}IwN \left(1 - \frac{1-(\)\mathbf{J}^w}{1-\mu} \right) + \frac{\mathbf{F}LbwN}{1-\mu} \left(\frac{\mathbf{S}}{\mathbf{1}-1} + (1-(\)) \frac{\mathbf{D}(1-\mathbf{J}^w)}{\mathbf{D}+\mathbf{1}-1} \right) \\
 &+ \frac{eE^PwN}{1-\mu} \left[\frac{\mathbf{S}}{\mathbf{1}-1} \left(\mathbf{F} \frac{\mathbf{S}}{(\)} + \mathbf{J}^w \right) + \frac{\mathbf{D}(1-\mathbf{J}^w)}{\mathbf{D}+\mathbf{1}-1} \left(1 - (-\mathbf{S} + \mathbf{F} \frac{1-(\)}{(\)} \mathbf{S}) \right) \right]
 \end{aligned} \tag{73}$$

Next, since by (6) $\sigma I = eE^h$, we have (by (31))

$$\mathbf{F}I = \mathbf{F} \frac{\mathbf{S}}{(\)} eE^P + \mathbf{F}Lb \tag{74}$$

substituting (74) into (73) gives

$$\begin{aligned}
 (1-\mathbf{J}^w) \det \frac{\partial \tilde{W}}{\partial \mathbf{J}^e} \Big|_{\mathbf{J}^e=0} &= \frac{\mathbf{F}LbwN}{1-\mu} \left(\frac{\mathbf{S}}{\mathbf{1}-1} + (1-(\)) \frac{\mathbf{D}(1-\mathbf{J}^w)}{\mathbf{D}+\mathbf{1}-1} - (1-\mu - (1-(\))\mathbf{J}^w) \right) \\
 &+ \frac{eE^PwN}{1-\mu} \left[\left(\frac{\mathbf{S}}{\mathbf{1}-1} - (1-\mu - (1-(\))\mathbf{J}^w) \right) \mathbf{F} \frac{\mathbf{S}}{(\)} + \frac{\mathbf{S}\mathbf{J}^w}{\mathbf{1}-1} \right. \\
 &\quad \left. + \frac{\mathbf{D}(1-\mathbf{J}^w)}{\mathbf{D}+\mathbf{1}-1} \left(1 - (-\mathbf{S} + \mathbf{F} \frac{1-(\)}{(\)} \mathbf{S}) \right) \right]
 \end{aligned} \tag{75}$$

or by rearranging

$$\begin{aligned}
 (1-\mathbf{J}^w) \det \frac{\partial \tilde{W}}{\partial \mathbf{J}^e} \Big|_{\mathbf{J}^e=0} &= \frac{\mathbf{F}LbwN}{1-\mu} \left(\frac{1}{\mathbf{0}-1} + (1-(\)) \frac{\mathbf{D}(1-\mathbf{J}^w)}{\mathbf{D}+\mathbf{1}-1} + (1-(\))\mathbf{J}^w \right) \\
 &+ \frac{eE^PwN}{1-\mu} \left[\left(\frac{1}{\mathbf{0}-1} + (1-(\))\mathbf{J}^w \right) \mathbf{F} \frac{\mathbf{S}}{(\)} + \frac{\mathbf{S}}{\mathbf{1}-1} \mathbf{J}^w \right. \\
 &\quad \left. + \frac{\mathbf{D}(1-\mathbf{J}^w)}{\mathbf{D}+\mathbf{1}-1} \left(1 - (-\mathbf{S} + \mathbf{F} \frac{1-(\)}{(\)} \mathbf{S}) \right) \right]
 \end{aligned} \tag{76}$$

Equation (76) is positive.

QED

APPENDIX D: Solving for the economic equilibrium

Households

Maximising (1) s.t. (2) w.r.t. x_i gives the FOC

$$U_{x_i^h} = n^{\frac{1}{1-\theta}} \left(\sum_i (x_i^h)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}-1} (x_i^h)^{\frac{\theta-1}{\theta}-1} = \lambda p_i \quad (89)$$

where λ is the Lagrange multiplier. Dividing the FOC for x_i^h by the FOC for x_k^h gives

$$x_i^h = \left(\frac{p_k}{p_i} \right)^{\theta} x_k^h \quad (90)$$

Taking both sides to the power of $(\theta-1)/\theta$ and summing over i gives

$$\sum_i (x_i^h)^{\frac{\theta-1}{\theta}} = \sum_i p_i^{1-\theta} p_k^{\theta-1} (x_k^h)^{\frac{\theta-1}{\theta}} \quad (91)$$

Taking both sides to the power of $(\theta-1)/\theta$ and premultiplying by $n^{1/(1-\theta)}$ gives

$$n^{\frac{1}{1-\theta}} \left(\sum_i (x_i^h)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} = n^{\frac{1}{1-\theta}} \left(\sum_i p_i^{1-\theta} \right)^{\frac{\theta}{\theta-1}} p_k^{\theta} x_k^h \quad (92)$$

The LHS of (92) is the composite commodity X^h , and part of the RHS is the price index P , thus (92)

is

$$X^h = n P^{-\theta} p_k^{\theta} x_k^h \quad (93)$$

Rearrange (93) to obtain the demand function

$$x_k^h = \frac{X^h}{n} \left(\frac{P}{p_k} \right)^{\theta} \quad (94)$$

Polluting good sector

Solving the minimisation problem in assumption A5 gives first-order conditions of the same form as

(89), i.e.

$$n^{\frac{1}{1-\theta}} \left(\sum_i (x_i^e)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} (x_i^e)^{\frac{\theta-1}{\theta}-1} = \mathbf{8}P_i \quad (95)$$

Following the same steps (90)-(94) gives the demand function for the polluting good sector, which is of the same form as the one of the households, i.e.

$$x_k^e = \frac{X^e}{n} \left(\frac{P}{P_k} \right)^{\theta} \quad (96)$$

Government

Solving the government's minimisation problem (assumption A6) gives first-order conditions of the same form as (89) and (94)

$$n^{\frac{1}{1-\theta}} \left(\sum_i (x_i^g)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} (x_i^g)^{\frac{\theta-1}{\theta}-1} = \mathbf{8}P_i \quad (97)$$

Following the same steps as of the household and polluting good sectors, gives the demand function for the government

$$x_k^g = \frac{X^g}{n} \left(\frac{P}{P_k} \right)^{\theta} \quad (98)$$

Firms

Firms minimise costs, subject to the level of y_j

$$\begin{aligned} C_j(Y_j) &= \min_{N_j, E_j^p} wN_j + \hat{e}E_j^p \\ \text{s.t. } &\hat{A}N_j^{\alpha} (E_j^p)^{\beta} \geq Y_j \end{aligned} \quad (99)$$

where $\hat{e} \equiv (1+\tau^e)e$ and $\hat{A} \equiv AK^{\alpha}$. Equivalently we may substitute for N_j by using the constraint in (99), then we have

$$C_j(Y_j) = \min_{E_j^p} w \hat{A}^{-\frac{1}{\mathfrak{s}}} Y_j^{\frac{1}{\mathfrak{s}}} (E_j^p)^{-\frac{\mathfrak{c}}{\mathfrak{s}}} + \hat{\epsilon} E_j^p \quad (100)$$

The FOC w.r.t. E_j^p is

$$-\frac{\mathfrak{c}}{\mathfrak{s}} w \hat{A}^{-\frac{1}{\mathfrak{s}}} Y_j^{\frac{1}{\mathfrak{s}}} (E_j^p)^{-\frac{\mathfrak{c}}{\mathfrak{s}}-1} + \hat{\epsilon} = 0 \quad (101)$$

or equivalently

$$E_j^p = \left(M \left(\frac{w}{\hat{\epsilon}} \right)^{\frac{\mathfrak{s}}{\mu}} Y_j^{\frac{1}{\mu}} \right) \quad (102)$$

where $\mu = \beta + \gamma$ and

$$M \equiv \hat{A}^{-\frac{1}{\mu}} \left(\frac{\mathfrak{c}}{\mathfrak{s}} \right)^{\frac{\mathfrak{s}}{\mu}} \frac{1}{\mathfrak{c}} \quad (103)$$

Since the ratio of the marginal products is equal to the relative factor price

$$\frac{\mathfrak{s}}{\mathfrak{c}} \frac{E_j^p}{N_j} = \frac{w}{\hat{\epsilon}} \quad (104)$$

we have

$$N_j = \mathfrak{s} M \left(\frac{w}{\hat{\epsilon}} \right)^{-\frac{\mathfrak{c}}{\mu}} Y_j^{\frac{1}{\mu}} \quad (105)$$

Substituting (102) and (105) into $C(\cdot)$, gives the cost function

$$C(Y_j) = M (\mathfrak{s} + \mathfrak{c}) Y_j^{\frac{1}{\mu}} w^{\frac{\mathfrak{s}}{\mu}} \hat{\epsilon}^{\frac{\mathfrak{c}}{\mu}} \quad (106)$$

The firm's profit function is

$$\mathbf{A}_j = \max_{p_j} p_j(Y_j) Y_j - C(Y_j) \quad (107)$$

Since households, the polluting good sector and the government have the same structure of their demand functions, each firm faces demand of the form

$$p_j(Y_j) = \left(\frac{Y}{nY_j} \right)^{\frac{1}{\mathbf{0}}} P \quad (108)$$

where in equilibrium $Y = X^h + X^e + X^g$. Each firm takes the aggregate production Y , and the price index P , as beyond its own control. Then the FOC w.r.t. y_i is

$$\left(1 - \frac{1}{\mathbf{0}} \right) p_j(Y_j) = C'(Y_j) \quad (109)$$

At this stage we may look at the symmetric equilibrium and make the normalisation $P=1$. Substituting for the derivative of (106) gives

$$\left(1 - \frac{1}{\mathbf{0}} \right) Y_j^{-\frac{1}{\mathbf{0}}} \left(\frac{Y}{n} \right)^{\frac{1}{\mathbf{0}}} = M \hat{e}^{\frac{\mathbf{c}}{\mu}} w^{\frac{\mathbf{s}}{\mu}} Y_j^{\frac{1-\mu}{\mu}} \quad (110)$$

or rearranged

$$Y_j^{\frac{1}{\mu}} = \left[\left(1 - \frac{1}{\mathbf{0}} \right) / M \right]^{\frac{\mathbf{0}}{\mathbf{0}(1-\mu) + \mu}} \left(\frac{Y}{n} \right)^{\frac{1}{\mathbf{0}(1-\mu) + \mu}} \hat{e}^{-\frac{\mathbf{c}}{\mu} \frac{\mathbf{0}}{\mathbf{0}(1-\mu) + \mu}} w^{-\frac{\mathbf{s}}{\mu} \frac{\mathbf{0}}{\mathbf{0}(1-\mu) + \mu}} \quad (111)$$

Defining $\tilde{M} \equiv \left[\left(1 - \frac{1}{\mathbf{0}} \right) / M \right]^{\frac{\mu(\mathbf{0}-1)}{\mathbf{0}(1-\mu) + \mu}}$ we get the firm's profit and factor demand equations

$$\mathbf{A}_j = \left[\frac{1}{\mathbf{0}} + (1 - \mathbf{s} - \mathbf{c}) \left(1 - \frac{1}{\mathbf{0}} \right) \right] \tilde{M} \left(\frac{Y}{n} \right)^{\frac{1+\mathbf{c}}{\mathbf{0}}} \hat{e}^{-\mathbf{c}} w^{-1+\mathbf{s}} \quad (112)$$

$$N_j = \mathbf{s} \left(1 - \frac{1}{\mathbf{0}} \right) \tilde{M} \left(\frac{Y}{n} \right)^{\frac{1+\mathbf{c}}{\mathbf{0}}} \hat{e}^{-\mathbf{c}} w^{-1} \quad (113)$$

$$E_j^P = \left(1 - \frac{1}{\mathbf{0}} \right) \tilde{M} \left(\frac{Y}{n} \right)^{\frac{1+\mathbf{c}}{\mathbf{0}}} \hat{e}^{-\mathbf{c}-1} w^{-1+\mathbf{s}} \quad (114)$$

Next, defining $\tilde{\mathbf{A}} \equiv \tilde{M} n^{\frac{(\mathbf{0}-1)(1-\mu)}{1+(\mathbf{0}-1)(1-\mu)}}$ substituting for M in (112)-(114) and aggregating gives (8)-

(10).

Nash bargaining

$\Psi'(w)=0$ gives the global maximum, because Ψ is concave up to a point w^* , and $\Psi'(w)<0$ for $w>w^*$.

We prove concavity by verifying that $\Psi''(w)<0$ for $w>w^*$.

First, for any function the following is true

$$\frac{d}{dw} \left(\frac{Q(w)}{Q(w)} \right) = \frac{Q''(w)}{Q(w)} - \left(\frac{Q'(w)}{Q(w)} \right)^2 \quad (115)$$

or rearranged

$$\frac{Q''(w)}{Q(w)} = \left(\frac{Q'(w)}{Q(w)} \right)^2 + \frac{d}{dw} \left(\frac{Q'(w)}{Q(w)} \right) \quad (116)$$

Next, taking logarithms of (12) gives

$$\ln Q(w) = (1-D) \ln w + D \ln [(1-J^w)w-b] \quad (117)$$

Differentiating (117) w.r.t w gives

$$\frac{Q'(w)}{Q(w)} = \frac{1-D}{w} + D \frac{1-J^w}{(1-J^w)w-b} \quad (118)$$

Differentiating (118) w.r.t w gives

$$\frac{d}{dw} \left(\frac{Q'(w)}{Q(w)} \right) = -\frac{1-D}{w^2} - D \left[\frac{1-J^w}{(1-J^w)w-b} \right]^2 \quad (119)$$

(118) may be written as

$$\frac{Q'(w)}{Q(w)} w = 1-D + D \frac{(1-J^w)w}{(1-J^w)w-b} \quad (120)$$

and (119) may be written as

$$w^2 \frac{d}{dw} \left(\frac{Q'(w)}{Q(w)} \right) = -(1-D-1) - D \left[\frac{(1-J^w)w}{(1-J^w)w-b} \right]^2 \quad (121)$$

Substitute (120) and (121) into (116) to get

$$\frac{Q''(w)}{Q(w)} w^2 = \left[1-D-1 + D \frac{(1-J^w)w}{(1-J^w)w-b} \right]^2 - (1-D-1) - D \left[\frac{(1-J^w)w}{(1-J^w)w-b} \right]^2 \quad (122)$$

Define $A \equiv 1-1-D$ and $B \equiv \frac{(1-J^w)w}{(1-J^w)w-b}$, then (122) becomes

$$\frac{Q''(w)}{Q(w)} w^2 = (DB-A)^2 + A - DB^2 \quad (123)$$

and (120) becomes

$$\frac{Q'(w)}{Q(w)} w = DB - A \quad (124)$$

Thus Ψ is decreasing for $B < A/\rho$. Denote $w^*: B(w^*) = A/\rho$, and write $B = A/\rho - \epsilon$, so that when $\epsilon = 0$, we have $w = w^*$ then (123) becomes

$$\begin{aligned} \frac{Q''(w)}{Q(w)} w^2 &= D^2, ^2 + A - D(A/D - ,)^2 \\ &= -D(1-D), ^2 - \frac{A}{D}(A-D) + 2, A \end{aligned} \quad (125)$$

Since $A-\rho = \Theta-1 > 0$, $\Psi'' < 0$ for $\epsilon \leq 0$, i.e. for $B \geq A/\rho$. Thus for $w \leq w^*$, Ψ is concave, and for $w > w^*$, Ψ is decreasing (i.e. $\Psi' < 0$). Thus Ψ attains a global maximum at $\Psi' = 0$.