Does Productive Capital A¤ect the Order of Resource Exploitation?

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Abstract

The purpose of this paper is to show that in a general equilibrium framework it is never optimal to use high cost substitute after lower cost exhaustible resource even if it is possible to accumulate productive capital. Indeed if the high cost substitute is scarce it is always optimal to consume it simultaneously with a lower cost stock. Moreover it may be optimal to consume the high cost substitute before using a lower cost resource. JEL Classi…cation Number: Q3. Keywords: Natural resources, substitute, productive capital, general equilibrium.

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1 Introduction

Kemp and Long [3] are the …rst to question the "folk theorem" that low cost natural resource stocks should be exhausted before using a high cost substitute[1], when agents discount the future. They show that in a general equilibrium framework if the marginal cost of extraction and of using the substitute are both constant then it may be optimal to use the high cost substitute before exhausting a low cost natural resource. Simultaneously extracting a resource and using a high cost substitute smooths the utility path. More recently, Amigues et al. [1] show that it may be optimal to exploit a high cost substitute strictly before using a lower cost exhaustible resource, if the ‡ow capacity of the high cost substitute is limited. They suppose that the marginal utility of one unit of this limited substitute is strictly positive at the steady state, where the population has exhausted the stocks. So the ‡ow of the substitute is scarce and since this ‡ow is unstorable, delaying the start of the exhaustible resource exploitation is a method of saving.

Lewis [5] modi…es Kemp and Long's model by allowing the extracted

[1] The high cost substitute could be a natural resource or produced with a natural resource, i.e. it could be a high cost inexhaustible resource.

resource and high cost substitute to be costlessly converted into capital. He finds that a sufficient condition for restoring the folk theorem in this model is that capital has a positive rate of return. In his conclusion he conjectures that in a general equilibrium analysis, it is optimal to use the least cost energy sources first whenever the extracted resource can be converted into productive capital[2].

If Lewis' conjecture holds, then the inclusion of productive capital in the Amigues et al. model would restore the folk theorem. The latter paper claims that the inability to save implies the failure of the folk theorem. I show that the Lewis' conjecture does not hold when the flow of substitute is bounded and scarce in the sense described above. It is always optimal to consume simultaneously the high cost substitute and one of the stocks. Moreover it may be optimal to use the high cost substitute before beginning to exhaust a lower cost stock.

Since the Amigues et al. result holds even if the economy can accumulate productive capital, their explanation for the result (based on the desire to save) is incomplete. In fact, extending their model by including the possibility

[2] Lewis suggests also that in a partial equilibrium framework the folk theorem is valid. Chakravorty and Krulce (1994) show that this conjecture does not hold when there is more than one demand for different resources.

of saving decreases but does not suppress the incentive to use the high cost substitute before exhausting lower cost stocks. The critical assumption is that the substitute is scarce, i.e. at the steady state the capacity constraint on the ‡ow of the substitute is binding. The shadow value of this constraint is thus positive. The decision maker reduces the cost of this constraint (i.e. chooses its optimal shadow value) by investing and does so by using the low cost stock early in the program and delaying the date at which he uses the substitute. The ability to invest in productive capital provides an additional mean of reducing the cost of the constraint. By extracting the low cost stock, and investing some of the output, the decision maker increases future consumption possibilities: he reduces the shadow value of the constraint on the high cost ‡ow. But the ‡ow scarcity does not implie the labor scarcity at the steady state. In this paper I suppose the labor abundance at the steady state. Investment increases future opportunities for consumption but at the same time it decreases present opportunities for work. As the shadow value of labor at the steady state is equal to zero (abundance), the decision maker has an incentive to delay extraction on the low cost stock in order to keep future opportunities to work. The only possibility to manage both ‡ow and labor constraints is to save the high cost substitute. Then, by using the high cost substitute early in the program, the decision maker delays the unemployment spell and, at the same time, delays the date at which he must rely exclusively on the substitute. Contrary to both Lewis' conjecture and to the explanation in Amigues et al., the folk theorem may fail even in a general equilibrium model with opportunities to invest in productive capital.

The model I present does not explicitly include disutility of labor. Including individual valuation of leisure alters none of the results. In this more general model, the shadow value of the constraint on the high cost ‡ow equals the di¤erence between the marginal utility of consumption and of leisure. In the steady state the agent would like to transform leisure into consumption, but is unable to do so because of the constraint on the ‡ow of the substitute. The decision maker has to perform a similar trade-o¤ in the model without disutility of labor.

I present the model and the results in the next section. The last section gives the main conclusions.

2 The model

The economy contains N identical agents each with one unit of labor. N is normalized to 1. Agents can produce capital by extracting a homogenous natural resource or by using a high cost substitute. Agents obtain the natural resource from exhaustible deposits. The level of the stock i at time t is $Y_i(t)$ and the quantity extracted from it is $y_i(t)$. There are two natural resource stocks[3], $i = 1; 2$. In order to produce one unit of consumption with one unit of resource from stock i the population has to furnish ¹$_i$ units of labor. I assume that the instantaneous level of the high cost substitute is bounded from above by $\overline{x}$. The high cost substitute produces one unit of consumption using ´ units of labor, with ´ > ¹$_2$ > ¹$_1$. As Lewis (1982), I introduce a storage technology where one unit of capital, which may be either stored or consumed. Capital growths at the exogeneous rate ±. Note thate the extraction costs are not a¤ected by the accumulation of capital. At time t the stock of capital is $K(t)$.

The individual utility function $U(c(t))$ is twice continuously di¤erentiable, strictly increasing in consumption c and concave[4]. Consumption is an essen-

[3] The case with more stocks does not provide additional insights.

[4] It is straightforward to show that my results hold with a more general utility function which depends on leisure as Amigues et al. [1] provided that the cross derivatives are non

tial good, i.e. $\lim_{c \searrow 0} u' = +\infty$, so it is not optimal that $c(t)$ fall to 0. The instantaneous marginal utility is always positive and when the representative agent extracts only $\bar{x}$ (the maximal level) we have $1 - \eta\bar{x} > 0$, so the high cost substitute is scarce. The rate of discount r is assumed constant and strictly positive and with $r > \delta$[5].

The program of the planner who seeks to maximize intertemporal welfare is:

$$\max_{\{x(t);y_1(t);y_2(t);c(t);t \geq 0\}} \int_0^{+\infty} u(c(t))e^{-rt}dt$$

subject to

(1) $$\dot{K}(t) = \delta K(t) + x(t) + \sum_{i=1}^{2} y_i(t) - c(t)$$

(2) $$K(t) \geq 0 \text{ and } K(0) = 0$$

(3) $$\dot{Y}(t) = -y_i(t) \qquad i = 1;2$$

(4) $$y_i(t) \geq 0; \qquad i = 1;2$$

(5) $$Y_i(t) \geq 0 \text{ and } Y_i(0) = Y_i^0; \qquad i = 1;2$$

(6) $$x(t) \geq 0 \text{ and } \bar{x} - x(t) \geq 0$$

negative.

[5] This assumption prevents unrealistic optimal consumption paths.

(7) $$1 - \eta x(t) - \sum_{i=1} \mu_i y_i(t) \geq 0.$$

Let $\rho(t)$ denote the multiplier associated with the law of motion of capital (1) and $\lambda_i(t)$ the scarcity rent associated with deposit i. The Lagrangian of the planner's problem is L

$$L = u(c(t))e^{-rt} + \rho(t)(\delta K(t) + x(t) + \sum_{i=1} y_i(t) - c(t)) + \beta(t)K(t) - \sum_{i=1} \lambda_i(t)y_i(t)$$

$$+ \sum_{i=1} \nu_i(t)y_i(t) + \gamma_i(t)Y_i(t) + \underline{\alpha}(t)x(t) + \bar{\alpha}(t)(\bar{x} - x(t)) + \sigma(t)(1 - \eta x(t) - \sum_{i=1} \mu_i y_i(t));$$

where $\beta(t)$, $\nu_i(t)$, $\gamma_i(t)$, $\underline{\alpha}(t)$, $\bar{\alpha}(t)$ and $\sigma(t)$ are associated with the nonnegativity constraints on respectively (2), (3), (4), (5), (6) and (7).

The optimal program $\{x^*(t); y_1^*(t); y_2^*(t); c^*(t); t \geq 0\}$ satisfies the first order conditions:

(8) $$e^{-rt}u'(c(t)) - \rho(t) = 0;$$

(9) $$\rho(t) + \underline{\alpha}(t) - \bar{\alpha}(t) - \eta\sigma(t) = 0;$$

(10) $$\rho(t) - \lambda_i(t) + \nu_i(t) - \mu_i\sigma(t) = 0 \qquad i = 1; 2;$$

and the complementary slackness conditions. The evolution of the shadow value of capital $\rho(t)$ and of the scarcity rent $\lambda_i(t)$ are:

$$(11) \qquad \dot{\rho}(t) = -\delta\rho(t) - \beta(t)$$

$$(12) \qquad \dot{\lambda}_i(t) = -\gamma_i(t) \qquad i = 1; 2$$

and the transversality condition is $\lim_{t \to +\infty} \rho(t)K(t) = 0$.

There is a unique solution to this program in which it is optimal to accumulate capital at the beginning and then to exhaust it in infinite time (see Lewis [4]). This implies that $\beta(t) = 0 \ \forall t > 0$ and with (11) that

$$(13) \qquad \rho(t) = \rho_0 e^{-\delta t}.$$

Proposition 1 For all optimal plans there exists an interval over which it is optimal to extract simultaneously the low cost resource and the high cost substitute.

Proof. In order to demonstrate this result it is sufficient to consider the case with only one stock i. First of all, it is never optimal to produce a positive quantity of substitute less than the capacity $\bar{x}$. Indeed, if it were optimal to set $0 < x(t) < \bar{x}$ then $\underline{\alpha}(t) = \bar{\alpha}(t) = 0$. If $\sigma(t) = 0$ (9) contradicts

(13). If $\sigma(t) > 0$, then $\nu_i(t) = 0$ and so (12) and (10) are incompatible. Hence there are only three possibilities of phases of extraction: $x(t) = 0$ and $y_i(t) > 0$ (i.e. $\underline{\alpha}(t) \geq 0$, $\bar{\alpha}(t) = \nu_i(t) = 0$), $x(t) = \bar{x}$ and $y_i(t) > 0$ (i.e. $\bar{\alpha}(t) \geq 0$, $\underline{\alpha}(t) = \nu_i(t) = 0$) or $x(t) = \bar{x}$ and $y_i(t) = 0$ (i.e. $\bar{\alpha}(t) \geq 0$, $\underline{\alpha}(t) = 0$ and $\nu_i(t) \geq 0$) respectively called phase 1_i, 2_i and 3. To prove the proposition it is sufficient to prove that phase 1_i cannot be immediately followed by phase 3. During phase 1_i (9) becomes $\rho(t) + \underline{\alpha}(t) - \eta\sigma(t) = 0$ and (10) $\rho(t) - \lambda_i(t) - \mu_i\sigma(t) = 0$. Since $\underline{\alpha}(t)$ must be non negative since $\eta > \mu_i$

$$\lambda_i(t) \leq \frac{\eta - \mu_i}{\eta}\rho_0 e^{-\delta t} < \rho_0 e^{-\delta t}. \quad (14)$$

During phase 3 (10) becomes $\rho(t) - \lambda_i(t) + \nu_i(t) = 0$. Since $\nu_i(t)$ must be non negative

$$\lambda_i(t) \geq \rho_0 e^{-\delta t}. \quad (15)$$

Since $\lambda_i(t)$ is constant if $Y_i > 0$ and decreases after the date of exhaustion time T (see (12)) and moreover at T, $\lambda_i(t)$ is either continuous or it jumps down (Seierstad and Sydsaeter [6] page**), equation (14) and (15) cannot both hold. QED

Proposition 1 implies that there is always a phase along the optimal extraction path during which it is optimal to extract simultaneously the high cost substitute and the stock[6].

Proposition 2 For a nonempty subset of initial stocks it is optimal to extract the high cost substitute strictly before using a lower cost stock.

Proof. First of all, it is never optimal to extract simultaneously both stocks. Simultaneous extraction and equation (10) for $i = 1;2$ implies that $\sigma(t)$ must be constant over this phase because $\nu_i(t) = 0$ and $\lambda_i(t) = \lambda_i$ for $i = 1;2$. If $\sigma(t)$ is constant, then (10) implies $\rho(t)$ constant, which contradicts (13). It is never optimal to produce a positive quantity of substitute less than the capacity $\bar{x}$. See the proof of the proposition 1. Let the initial stocks be such that $\lambda_2(0) \in (\frac{\eta-\mu_2}{,}\rho_0;\rho_0)$ and $\lambda_1(0) \in (\frac{\eta-\mu_1}{,}\rho_0;\rho_0)$ and moreover $\lambda_1(0) > \lambda_2(0)$ because the first is lower cost. Over this nonempty set[7] of scarcity rents it is never optimal to extract a stock without also using the substitute at capacity. Indeed the first inequality in (14) is false $\forall t > 0$, so each point of this set the optimal extraction path is composed of phase 2_1 followed by phase 2_2 and then phase 3. We know that there exists a one-to-

[6] If the initial level of stock i is sufficiently large phase 1_i belongs to the optimal path otherwise it is optimal to begin directly with phase 2_i.

[7] There are other pairs of rents which verify the proposition 2 but my purpose is to show only that it happens.

one relation[8] between the set of stocks and the set of rents, thus completing the proof.

3 Conclusion

When the high cost substitute is scare it is not always optimal to exhaust low cost resource stocks before making use of the substitute, even if the population can accumulate productive capital. Saving capital increases the future availability of consumption. When the population accumulates capital simultaneously it "saves leisure". There is a trade o¤ between accumulating capital as quickly as possible by extracting only the least cost stock and working as much and long as possible by exploiting the substitute. This trade o¤ implies my results. In my model the value of leisure is zero when the population consumes only the substitute, but of course it is easy to generalize this feature by allowing an endogenous value of leisure.

[8] Amigues et al. [1] characterize this relation without capital accumulation but it is straightforward to show that the properties of this relation do not change in my model.

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