Recursive intergenerational utility in global climate risk modeling[*]

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Abstract

This paper distinguishes relative risk aversion and resistance to intertemporal substitution in climate risk modeling. Stochastic recursive preferences are introduced in a stylized numeric climate-economy model using preliminary IPCC 1998 scenarios. It shows that higher risk aversion increases the optimal carbon tax. Higher resistance to intertemporal substitution alone has the same effect as increasing the discount rate, provided that the risk is not too large. We discuss implications of these findings for the debate upon discounting and sustainability under uncertainty.

Key Words: recursive utility, risk, discounting, sustainability, climate

Résumé

Ce texte étudie la différence entre l'aversion relative au risque et la résistance à la substitution intertemporelle dans la modélisation du risque climatique. Les préférences récursives stochastiques sont utilisées dans un modèle numérique stylisé utilisant les scénarios préliminaires GIEC 1998 sur l'économie et le climat. On montre qu'une aversion au risque plus forte conduit à augmenter le niveau optimal de taxation de l'énergie. Augmenter la résistance à la substitution intertemporelle a le même effet qu'augmenter le taux d'actualisation, tant que le risque n'est pas trop grand. Nous discutons les implications de ces résultats pour le débat sur l'actualisation et la durabilité sous incertitude.

Mots clés: utilité récursive, risque, actualisation, durabilité, climat

JEL: Q20, D81, D90, H43

Non-technical abstract

In this paper, we propose a modeling framework to analyse risky and intergenerational issues. We then apply it to the issue of climate change. This framework is stochastic recursive utility. It allows to represent a broader range of preferences than what is usually found in the litterature.

The structure of the paper is the following. The one-page introduction frame the issue. It requires a graduate level in economic theory to be read. Section two presents an ultra-long period model of the climate risk named ULTRAL. Section three presents the calibration of the model, where we discuss numeric values. Section four presents and discuss the results. Section five concludes. The appendix defines the model and data used. Lastly, there is the list of the 35 references.

The tool used in this paper, stochastic recursive utility, appeared in economic theory about twenty years ago. To our knowledge, it has not been previously used in integrated assessment modeling. On the theoretical side, it allow to put in perspective the debate on discounting under uncertainty.

Policy interpretations of the results should be conducted with extreme care since this more a methodological paper. The simplest insight is that the assumption of a positive expected climate damage is not needed to induce some action now. In this model, climate change is beneficial nine times out of ten, nevertheless there is some precautionary abatement.

In a sequential decision making process, the initial level of abatement was found to be sensitive the date of policy revision. The earlier is policy revision scheduled, the lower is initial carbon tax level.

The model also show that larger risk aversion strengthens optimal pollution control and that previous works possibly underestimated the sensitivity of their results to the shape of the utility function.

1. INTRODUCTION

This paper is motivated by two important economic aspects of climate change economics: intergenerational equity and risk. A classic way to introduce risk is to consider an utility function with constant relative risk aversion γ, such as:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma} \tag{1}$$

A classic way to introduce the issue of intergenerational equity is to consider an intertemporal objective function such as:

$$f(c_1, c_2) = (l_1 c_1^{1-\rho} + l_2(1-\eta) c_2^{1-\rho})^{\frac{1}{1-\rho}} \tag{2}$$

where l represents population and c consumption levels, see [2, 19, 30].

This paper focuses on γ and on the two parameters of the intertemporal objective function function: η, the discount rate for the second time period, and ρ, the resistance to intertemporal substitution.

The simplest possible model known to integrate these three parameters may be stochastic recursive utility, initially introduced by Kreps and Porteus [18]. To our knowledge, all previously published models of the climate risk literature use simpler utility functions restricted to $\rho = \gamma$ [24, 20, 16]. This situation has left open a set of question we propose to examine.

From a theoretical point of view, let us first remind that resistance to substitution ρ controls the attitude toward variations in consumption across time, under certainty. On the other hand, risk aversion γ controls the attitude toward variations in consumption across states of the world, at a given date. So the restriction $\rho = \gamma$ is suspicious since it links two conceptually different aspects of preferences. We will examine whether there is empirical evidence and economic rationale to support this constraint, or whether it is an unfortunate sequel of expected utility models.

From a technical point of view, this restriction is particularly inadequate when analyzing the question of equity under uncertainty. It prevents from considering plausible situations where society has high risk aversion together with low resistance to substitution, or vice versa. Another direct consequence is that the effect of resistance to substitution and risk aversion cannot be conveniently investigated: A change in the curvature in the utility function does not permit an unambiguous interpretation since the resistance to substitution and risk aversion move together. We will examine the effect of risk aversion on optimal climate policy and the effect of resistance to substitution on sustainability.

Section two presents an ultra-long period model named ULTRAL. Section three presents the calibration of the model, where we discuss numeric values for ρ and γ. Section four presents the results, where we study the sensitivity of optimal control strategies to specifications of the utility function, to uncertainty, and to a sustainability constraint. Section five concludes.

2. MODEL

The ULTRAL model is a stylized ultra-long period classical macroeconomic model, stochastic with a recursive utility function, where climate change due to CO_2 accumulation affects productivity and the efficiency of investment. The time step is 30 years. There are four dates t starting from 1990, therefore three periods representing current, next and future generations. There are two states of the world, H and L, referred to by the index i.

The model is coded in GAMS and solved using nonlinear optimization with MINOS or CONOPT. In this section, we will discuss the equations of ULTRAL. For convenience, they are presented all together in the annex. Equations can be divided into three blocks. The first block A.1–A.5 describes classical economic growth with a natural resource [33]. The second block A.6–A.9 computes CO_2 accumulation, concentration, temperature increase and climatic damages. The third block A.10–A.11 describes preferences.

2.1. Climate-economy

The macroeconomic block is built around a capital-labor-energy production function. Population l_t is exogenous. All variables such as capital $K_{i,t}$, energy input $E_{i,t}$ and production $Y_{i,t}$ are per-capita. Technical progress a_t is exogenous. Productivity decreases with global warming through a multiplier $F_{i,t}$. Capital depreciates and is renewed according to a fixed saving rate s_t as described in equation A.4.

The control variable $X_{i,t}$ is a multiplier to the cost of the energy input. *Ex-ante*, the reference cost of energy is p_t. The amount of energy input $E_{i,t}$ entering the production function depends upon $p_t X_{i,t}$ its *ex-post* cost. Broadly speaking, $X_{i,t} - 1$ is the percentage of energy tax over the reference price. It is constrained to be positive.

The model runs a cost-benefit analysis. The cost of control is computed as follows. At period t, capital $K_{i,t}$ is fixed from the previous period's investment. Productivity a_t and population l_t are exogenous. Equations A.3 (discussed below) and A.1 together determine production $Y_{i,t}$ and energy $E_{i,t}$. Consumption $C_{i,t}$ and investment follow since the saving rate is fixed. Economic activity decreases with $X_{i,t}$.

Solving $Y_{i,t}$ and $E_{i,t}$ equations yields:

$$Y_{i,t}/Y^1 = (a_t p_t^{-e})^{\frac{1}{1-e}} F_{i,t}^{\frac{1}{1-e}} (K_{i,t}/K_1)^{\frac{k}{1-e}} X_{i,t}^{\frac{-e}{1-e}} \tag{3}$$

$$E_{i,t}/E^1 = (a_t F_{i,t}/p_t)^{\frac{1}{1-e}} (K_{i,t}/K_1)^{\frac{k}{1-e}} X_{i,t}^{\frac{1}{e-1}} \tag{4}$$

where superscripted parameters Y^1 and E^1 denote *ex-ante* first period values, and where K_1 is fixed. Note that for values of e much smaller than unity, production $Y_{i,t}$ is much less elastic than energy $E_{i,t}$ to changes in energy cost $X_{i,t}$. Assuming for example $t = 1$, $F_{1,i} = 1$ and $1 - e \approx 1$ in the above equations, then energy

TABLE 1.

Reduction cost curve in first period.

$X_{1,i} - 1$	0	10%	50%	100%	200%
$\Delta E_{1,i}$	0	-9%	-24%	-51%	-68%
$\Delta Y_{1,i}$	0	-0.4%	-1.7%	-2.8%	-4.5%

consumption $E_{1,i}$ is inversely proportional to the tax $X_{1,i}$, while output $Y_{1,i}$ only decreases as $X_{1,i}^{-e}$. The corresponding reduction cost curve is numerically shown in Table 1.

Equation A.3 stems implicitly from a behavioral assumption: the energy sector maximizes its profit blindly, in a sequence of static equilibria. Due to this assumption, the model can be seen as a two-stage optimization structure: economic agents optimize within a 30-years horizon, and only the central planner has an intergenerational point of view. Given that the price of output is 1 and the price of energy is $p_t X_{i,t}$, this assumption of intragenerational agent optimization implies a well-known Cobb-Douglas function property, that:

$$\frac{Y_{i,t}}{p_t X_{i,t} E_{i,t}} = e$$

This corresponds to equation A.3. The share of energy e disappear because the equation at date t is scaled with the equation at date 1. Note that equation A.1 is also scaled this way. Adjustment costs kick-in to decrease consumption when $X_{i,t}$ increases faster than 2% per year, that is 1.81 per period. This is equation A.5.

Finally, the model represents the sequentiality of the decision making process for climate policies. It is known from the start that the policy computed initially will be revised at date $t = T_R$. This is why all variables are indexed by the state of the world i and there is a constraint $X_{H,t} = X_{L,t}$ for $t \leq T_R$. This is a stochastic dynamic programming technique which would not be needed if the concept of “time unfolding on a stochastic tree” was built in the modeling languages. This sophistication will only be important when we analyze uncertainty in the last subsection 4.3, since before we will assume $T_R = 4$.

The benefits of reducing emissions are computed as follows. The carbon cycle central variable is a simple non-controversial index used by scientists and policymakers: global cumulative fossil and industrial carbon emissions since 1990. Let $M_{i,t}$ be this index computed by equation A.6 in GtC.

ULTRAL uses a black-box linear representation based upon seven major models described in [8]. At each period, these models associate CO_2 concentration levels $N_{i,t}$ with $M_{i,t}$. A linear relationship was estimated using least-square interpolation to determine α_t^0 and α_t^1 in equation A.7. Equation A.6 computes $M_{i,t}$ from $N_{i,t}$ using these linear coefficients.

Temperature increase $D_{i,t}$ is estimated in equation A.8 using a 2.5-degrees increase for a doubling of pre-industrial concentration. The 275 ppm number represents pre-industrial levels of CO_2 in the atmosphere.

Equation A.9 represents the economic effects of climate change $F_{i,t}$ as an overall mean-preserving productivity jump. It can be interpreted as an extreme irreversible event, occurring at date T_E, which changes the global climatic conditions in such a way that a major part of the capital and technological stock has to be replaced. To account for the possibility of adaptation and published impact estimates [22], the expected value of the productivity jump is zero: before T_E, the model sets $F_t = 1$. At a given date T_E, productivity drops permanently a lot with a small probability π (state H), or increases a little with a large probability $1 - \pi$ (state L). The magnitude of the fall depends quadratically upon temperature change.

2.2. Recursive utility

We now describe the representation of preferences with recursive utility. The motivation is to separate attitude towards intertemporal substitution from attitude towards risk.

Koopmans presented an early axiomatic deduction of recursive preferences [17]. He wrote $U_t = f(c_t, U_{t+1})$ and called f the aggregator function. Kreps and Porteus [18] generalized this structure to stochastic models:

$$U_t = f(c_t, \omega_t(\widetilde{U}_{t+1}))$$

where ω_t is the certainty equivalent of future utility evaluated with information available at date t. Let it be computed using $v(.)$, a von Neuman-Morgenstern utility function:

$$\omega_t(\widetilde{x}) = v^{-1}(\mathsf{E}_t v(\widetilde{x}))$$

This class of preferences separates risk aversion — tied to the certainty equivalence function ω or v — from intertemporal substitution — tied to the aggregator function f. Interestingly, they satisfy the properties of intertemporal consistency and stationarity of preferences.

The specification of f and v is not unique. Define $\widehat{f}(c, y) = v(f(c, v^{-1}(y)))$ and $\widehat{U}_t(.) = v(U_t(.))$. Then $\widehat{U}_t = \widehat{f}(c_t, \mathsf{E}_t \widetilde{\widehat{U}}_{t+1})$ define the same preferences as before.

Here we follow Epstein and Zin [10] and take f and ω_t isoelastic:

$$f(c, y) = (\alpha c^{1-\rho} + \beta y^{1-\rho})^{\frac{1}{1-\rho}} \tag{5}$$

$$\omega_t(\widetilde{x}) = (\mathsf{E}_t \widetilde{x}^{1-\gamma})^{\frac{1}{1-\gamma}} \tag{6}$$

Three positive parameters β, γ, and ρ characterize the preferences. The ρ in the aggregator function is the constant resistance to intertemporal substitution. The parameter γ is the Arrow-Pratt constant relative risk aversion. The β is next period's felicity discount factor. Since it is known that isoelastic functions tend continuously toward the logarithm when ρ or γ tend towards unity, we will simply ignore the cases $\rho = 1$ or $\gamma = 1$.

It is possible to choose α according to the units of consumption and utility. In one equation, the recursive preferences in the model are:

$$U_t^{1-\rho} = (1-\beta)c_t^{1-\rho} + \beta(\mathsf{E}_t \widetilde{U}_{t+1}^{1-\gamma})^{\frac{1-\rho}{1-\gamma}}. \tag{7}$$

As usual in the literature, α is set to $1-\beta$. In order to explain the assumptions underlying this equality, let $\tilde{g}$ be the expected growth of utility beyond the last period, that is $\widetilde{U}_5 = (1+\tilde{g})U_4$. Then equation 7 is equivalent to:

$$\alpha(c_4/U_4)^{1-\rho} = 1 - \beta[\mathsf{E}(1+\tilde{g})^{1-\gamma}]^{\frac{1-\rho}{1-\gamma}} \tag{8}$$

The equality $\alpha = 1-\beta$ arises from the two following conditions: $U_4 = c_4$ together with $\tilde{g} = 0$. The first condition is numerically important. Our experience shows that most other formulations for α lead to non-convergence when solving for various β, γ and ρ. The second condition is a first approximation potentially leading to mathematical problems with a zero utility discount rate, as $\beta = 1$ implies $\alpha = 0$. Divergence under assumptions of zero growth and zero discounting is normal since recursive preferences generalizes classical intertemporal utility. So, the problem disappears if either some growth is assumed or, as here, utility discount rate never goes to zero.

As shown equation A.10, the certainty equivalent of next generation utility $\omega_t(\widetilde{U}_{t+1})$ is explicitly represented by the state variable V_t defined by equation A.11. In that equation, the $t = 4$ case is the final condition introduced above, where consumption level c_5 is not computed explicitly, but approximated by $c_4\frac{K_5}{K_4}$, where K_5 is itself taken from equation A.4.

Finally, the guideline to introduce population l_t was to retrieve, in the special case $\rho = \gamma$, the standard intertemporal expected utility model:

$$l_t U_t = [(1-\beta)(l_t c_t^{1-\rho} + \mathsf{E}_t \sum_j l_{t+j}\beta^j c_{t+j}^{1-\rho})]^{\frac{1}{1-\rho}}$$

3. CALIBRATION

3.1. Preferences

We now turn to the calibration of stochastic recursive utility functions. The utility discount rate is set at 1% per year in the base case. Values 0, 0.5 and 2% per year will also be discussed. This range of values is typical in the literature, see for example Portney and Weyant [30].

The other two parameters ρ and γ need more discussion.

First of all, the golden rule of optimal economic growth gives a theoretical way for estimating ρ, given η, n, g and real risk-free marginal productivity of capital r. Let it be quickly reminded. Assuming in equation 2 that $c_2 = (1+g)c_1$ and $l_2 = (1+n)l_1$, and that g, n and η are small, the marginal rate of substitution between future and present consumption is:

$$\mathrm{MRS}_{2/1} = \frac{f_{,1}}{f_{,2}} \approx 1 + \eta - n + \rho g, \tag{9}$$

It follows from the first optimality condition $1 + r = \mathrm{MRS}_{2/1}$ that $r = \eta - n + \rho g$.

Decreasing the annual utility discount rate η can be compensated with a higher ρ. With $\eta = 0$, given g in the 1 to 2 percent per year range, annual global population growth η between 1 and 1.5, and r in between 3 and 6 percent per year, this gives ρ in the 2 to 7.5 range. But these calculations are subject to controversies. For example, Cline [7] considers a wedge of market distortion reducing r by 50%, whereas Nordhaus [26] states that he does not see how anyone could convince himself that r is below 6%.

Following the seminal Kreps and Porteus' paper [18], another interesting thought experiment is to compare utility if (i) uncertainty is resolved at period 3 to utility if (ii) uncertainty is resolved at period 2:

$$(i) \quad U_1^{1-\rho} = \alpha c_1^{1-\rho} + \alpha\beta c_2^{1-\rho} + \beta^2 [\mathsf{E}\tilde{c}_3^{1-\gamma}]^{\frac{1-\rho}{1-\gamma}}$$

$$(ii) \quad U_1^{1-\rho} = \alpha c_1^{1-\rho} + \beta \left[\mathsf{E}(\alpha c_2^{1-\rho} + \beta \tilde{c}_3^{1-\rho})^{\frac{1-\gamma}{1-\rho}}\right]^{\frac{1-\rho}{1-\gamma}}$$

It can be shown that early information (ii) is preferred if and only if $\gamma > \rho$. In this sense, assuming that society exhibit a taste for early information (or a distaste for late information) would limit the range of plausible values for ρ and γ. The usual intertemporally additive expected utility model, corresponding to $\rho = \gamma$, is equivalent to assuming indifference to the timing of uncertainty resolution.

Research on saving and portfolio behavior analysis derived empirical estimates of γ and ρ directly. In a seminal paper, Hall [14] estimated the response of saving to the real interest rate, that is the elasticity of intertemporal substitution $(1/\rho)$. All estimates presented by Hall report small elasticity of substitution that may well be zero. Ogaki and Reinhart [28] argued that Hall's model was misspecified and found estimates of resistance to substitution ρ that vary between 2.2 and 3.1.

Another important issue with current ρ and γ parametrisation is the equity premium puzzle as exposed by Mehra and Prescott [21]. Mehra-Prescott and Hall models have been criticized on the ground that they fail to separate resistance to substitution from risk aversion. In doing so, Epstein and Zin [11] found estimates for ρ that vary between 1.2 and 5 and estimates for γ that vary between 0.8 and 1.3. Using a similar approach, Normandin and Saint-Amour [27] estimates for ρ and γ are in the range of 1.5. It is also noticeable that they accepted the hypothesis $\rho = \gamma$ while Epstein-Zin rejected the expected utility model.

In an experimental framework, Barsky, Juster, Kimball and Shapiro [4] reported estimates of each parameter based on individual responses to hypothetical situations. They found high heterogeneity among individuals. A striking results of their study is that risk aversion and resistance to substitution are not correlated at all across individuals.

We found no decisive empirical evidence either to reject or to accept intertemporal expected models ($\rho = \gamma$) nor to choose some specific values for ρ and γ. Arrow [1] reports that 1.5 is the currently available best-guess for both parameters. ULTRAL uses values in the 0.6 to 4 interval, with emphasis upon 1.3 and 2.0.

3.2. Socio-economic scenarios

The socio-economic parameters are calibrated using preliminary Intergovernmental Panel on Climate Change 1998 Scenarios, see the annex. There are four scenarios A1, A2, B1 and B2. Each scenario proposes a particular future. It is important to stress that these scenarios are non-mitigation scenarios with respect to climate change.

According to the IPCC data distribution center, scenario A1 represents a world of very rapid economic growth, low population growth and rapid introduction of new and more efficient technology. In this world, people pursue personal wealth rather than environmental quality. Scenario A2 describes a world with high population growth with less concern for rapid economic development. Scenario B1 accounts for dematerialization and for the introduction of clean technologies. Scenario B2 describes a world in which the emphasis is on local solutions to economic, social, and environmental sustainability.

Parameters displayed in the annex correspond to the A1 scenario. Space constraints prevent us from reporting the four parameters set, but all numeric simulations were conducted on all four scenarios. As can be seen on Figure 1 panel **a**, future carbon emissions profiles span a large interval.

Population l_t and carbon content of energy χ_t are drawn from IPCC scenarios. Production and capital parameters are based upon order of magnitudes usually accepted in theoretical and empirical literature. The saving ratio is $s_t = 0.25$ for all t. In the production function, the share of labor is 0.6, and the share of energy is about ten times smaller than for capital, that is $k = 0.36$ and $e = 0.04$. The reduction cost curve in period 1 implicit to this is shown in Table 1.

The *ex-ante* levels of production Y^1 and energy E^1 correspond to $X_{i,1} = 1$ and are given parameters. The initial level of capital K_1 is set at three times Y^1. Finally, capital depreciation factor μ and investment accumulation ν are based upon a 5% annual depreciation rate.

Calibration of a_t and p_t was done as follows. Control $X_{i,t}$ and damage $F_{i,t}$ were set to unity, i.e. ignored. This allowed to compute values for baseline energy costs p_t by using scenario values for $Y_{i,t}$ and $E_{i,t}$ in equation A.3. Then coefficients for technical progress a_t were computed by solving equations A.3 and A.1 recursively. The effect of this calibration is that when there is no climate damage, growth tracks the mark-up scenario from IPCC.

There is a 5% probability that one degree of warming causes a 10% drop in productivity. That can be related to the EMF 14 uncertainty subgroup model comparison framed in [25]. That note suggested, with a 5% probability, to multiply the damage function by 7.8. In this framework the 10% drop is equivalent to a 1.28% unitary damage coefficient, which is in line with values used in the literature [5]. In the model, adding 3.5 degrees to the climate temperature sensitivity would have the same effect as multiplying economic damage sensitivity by a factor of about 5.8.

3.3. Baseline optimal control

As a reference optimal control scenario, we consider the $\rho = 1.3$, $\gamma = 1.3$ utility function. Policy revision and climate event occurs at the same period $T_R = T_E = 4$, for zero expected climate damage.

Panel **b** of Figure 1 displays the optimal CO_2 emissions levels with ULTRAL in each of the four IPCC markup scenarios. For each scenario, emissions rise very slowly during first period, until 2020. Then the emissions of each scenario strongly diverge. In scenario B1 for instance, there is a stabilization of emissions around 8 GtC, while in scenario A2 emissions increase sharply from 5 GtC in 2020 to 16 GtC in 2050. These results bear no implications for the timing debate, given the coarse 30-year time step.

The panel **c** of the same Figure 1 presents another view of the same results, where the vertical axis shows optimal control level $X_{i,t}$, i.e. the energy tax, instead of carbon emissions. The horizontal time scale goes only to 2050, since $X_{4,i}$ equals $X_{3,i}$ by assumption.

Both the carbon tax and the overall level of emissions are higher in scenario A2 than in any other scenarios in 1990 and 2020. Note however that this is no longer true in 2050. There is higher control in 2050 for scenario A1 than for scenario A2. This can be related to the growth effect: per capita production is very high in scenario A1, the economy expands at an average rate of about three percent to 2080. As a consequence, investment in control will be relatively less costly in the long run in scenario A1.

FIG. 1. ULTRAL reference profiles. Panel **a** Baseline carbon emissions according to IPCC preliminary 1998 scenarios. Panel **b** Optimal carbon emissions with $\rho = 1.3$ and $\gamma = 1.3$ utility function. Panel **c** Corresponding optimal control $X^*_{i,t}$

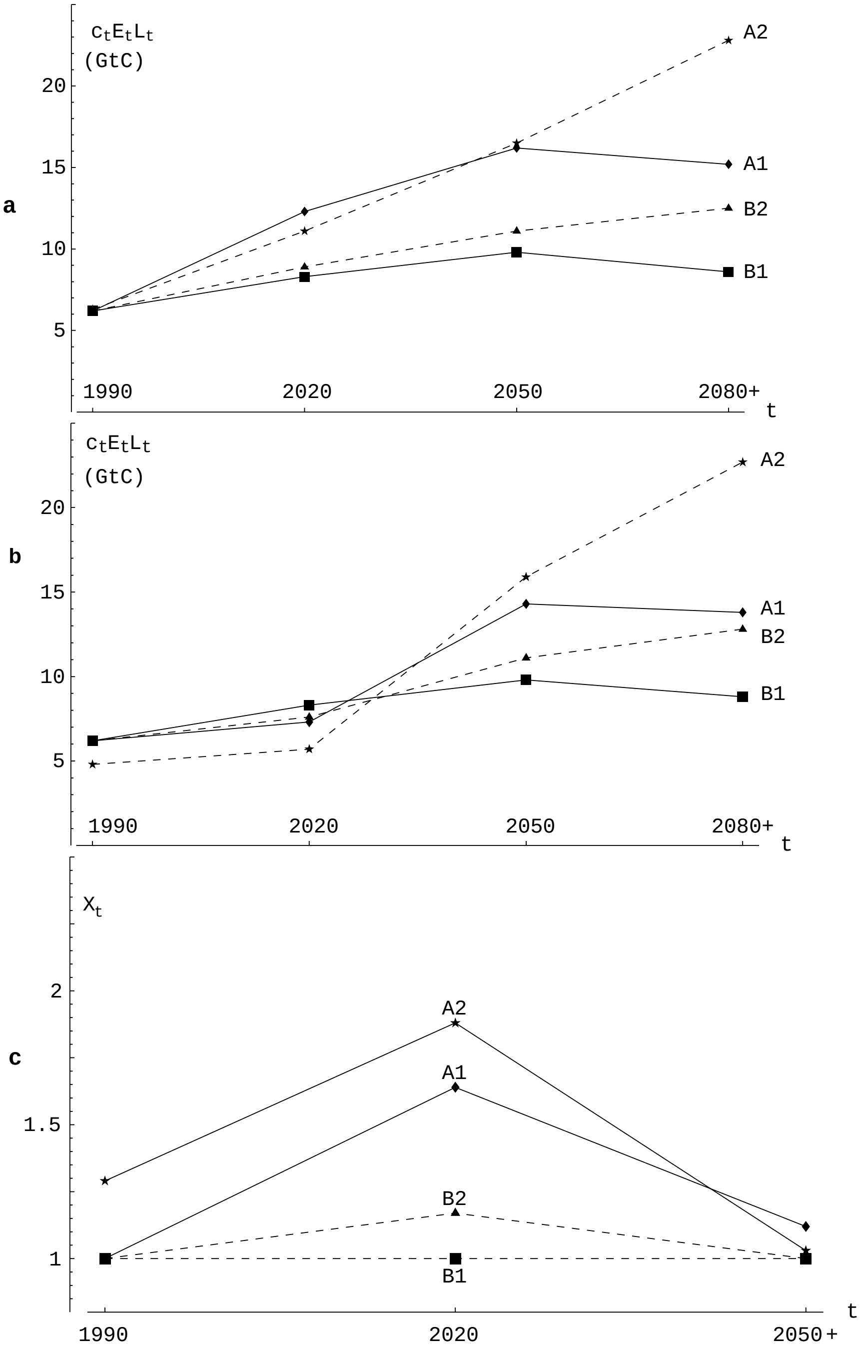
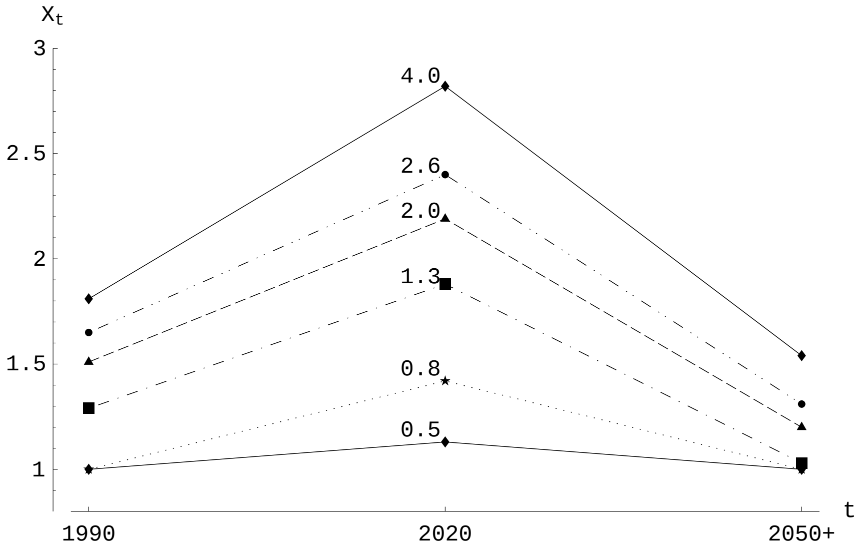


FIG. 2. Optimal control (energy cost multiplier X_t^{*A2}) for varying relative risk aversion γ, given constant $\rho = 1.3$.



4. RESULTS

4.1. Sensitivity to utility function coefficients

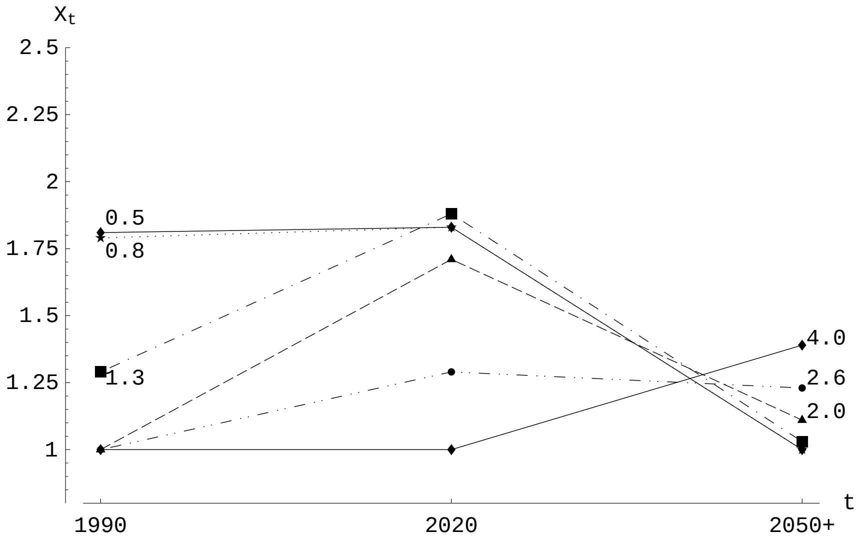
In this section, we examine the sensitivity of the optimal control profile to the three parameters of the social planner's preferences γ, ρ and β.

What is the effect of risk aversion on the optimal climate policy? Some studies [6] found that risk aversion warrants aggressive abatement action while another [24] did not find a large effect of risk aversion in his sensitivity analysis. A problem is that these results are ambiguous since they are based upon a change in the curvature of the utility function representing both resistance to substitution and risk aversion.

The isolated effect of risk aversion was first investigated in the literature on savings under uncertainty. Selden [31] and Weil [34] used recursive preferences to show that optimal precautionary savings tends to be determined solely by the elasticity of intertemporal substitution and not by the risk aversion parameter. Similar results were also found in the literature of dynamic resource management [15, 9].

Figure 2 displays the effect of relative risk aversion on the optimal climate policy in scenario A2. We consider γ varying from 0.5 to 4. Relative risk aversion leads unambiguously to an increase in the carbon tax in order to face less risk in the future. For example, increasing γ from 1.3 to 2.6 leads to increase X_1^* from 1.29 to 1.65. Doubling risk-aversion does more than doubling the optimal energy tax level.

FIG. 3. Optimal control (energy cost multiplier X_t^{*A2}) for varying resistance to intertemporal substitution ρ, given constant $\gamma = 1.3$.



With ULTRAL, the result that risk aversion increases optimal control is robust with respect to the baseline scenario and the value of ρ.

Turning to the effect of resistance to intertemporal substitution, Figure 3 displays the optimal carbon tax for different values of ρ. Numerical simulations show that increasing resistance to substitution tends to rotate the optimal control path counterclockwise, towards less control in the current period and more control in the future periods. This goes in the direction of reducing the differences of consumption levels between generations. The intuition explaining this results is therefore that more emphasis on equity — larger ρ — imply higher current emissions levels to smooth the differences with following, richer generations.

This is a discounting effect. Since control costs are borne in the short term and benefits will be received in the long term, increasing ρ implies less control in the current decade.

Indeed, as equation 9 shows, assuming a positive rate of growth, an increase in ρ increases the cost discount rate. To some extend, η and ρ have the same effect. For example, for $\rho = \gamma = 2$ and $\beta = 1$, the optimal tax rate in first period is 15%. Model runs show that lowering the discount rate to 0.5 has basically the same effect as lowering ρ from 2 to 1.3.

Sensitivity of results to joint variations in ρ, γ and β are reported in table 2 for the B2 scenario. Comparing the $\rho = \gamma = 1.3$ cases with $\rho = \gamma = 2$, the optimal tax level does not change much: from 15% to 17% in one case, from 77%

TABLE 2.

Sensitivity of the optimal tax level at $t = 2$ to the utility function parameters ρ, γ and β.

Resistance to substitution (ρ)	2	2	1.3	1.3	2	2	1.3	1.3
Aversion to risk (γ)	2	1.3	2	1.3	2	1.3	2	1.3
Annual discount rate (gives β)	1%	1%	1%	1%	0.5%	0.5%	0.5%	0.5%
Optimal tax level ($X_2^{*B2} - 1$)	15%	0%	72%	17%	77%	18%	83%	80%

to 80% in the other. This seemingly small sensitivity disappears when only ρ or γ change. This illustrates that parameters ρ and γ tend to cancel out when they move together.

In the light of previous results, the explanation is straightforward. Increasing risk aversion leads to increase early control to prevent the climatic catastrophe. But increasing resistance to substitution in the same time leads to decrease control to favor the current generation which is the poorest in average. Thus the use of intertemporally additive expected utility model ($\rho = \gamma$) underestimates the sensitivity of the results since the two parameters move together and triggers these two opposite effects.

Three main results emerged from this section. First, increasing risk aversion γ has a positive effect on the control rate. Second, increasing resistance to intertemporal substitution ρ has the same effect as increasing the discount rate η, it decreases the initial level of control. Third and consequently, the classical expected utility model induces a bias toward a more neutral climate policy.

4.2. Discounting and sustainability under uncertainty

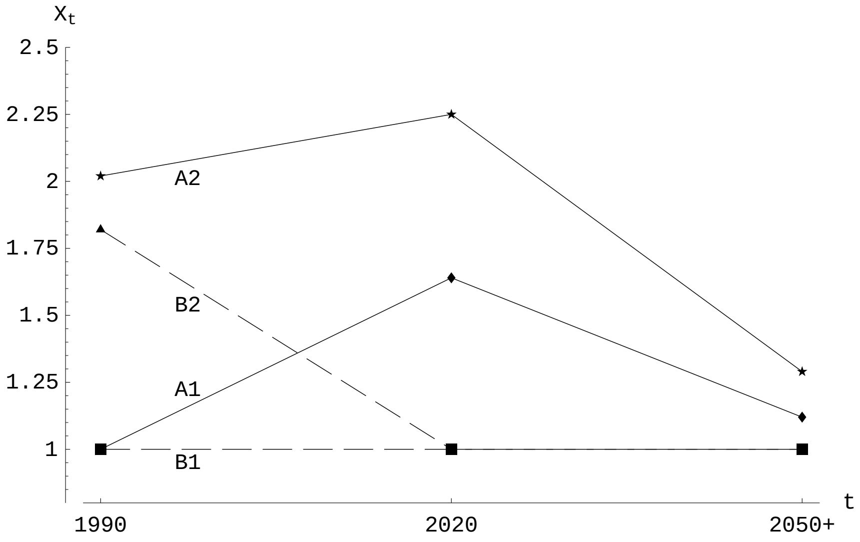
If ρ and η have the same effect, why use two parameters? The second result above suggests that there is some redundancy in the parametrisation of recursive utility function. One possibility is to eliminate time preference $\eta = 0$ and go for a resistance to substitution higher than the usual $\rho = 1$ log-of-consumption model, for example $\rho = 2$. A more widespread attitude is to restrict utility functions to the $\rho = \gamma$ subset and focus the discussion on η.

In this section, we first explore with ULTRAL the difference between ρ and η when it comes to the question of sustainability. The discussion of these results will allow to show limitations of the attitude focusing only on the discount rate.

There are many definitions of sustainability. Amongst them, let us single out Solow's [32] formalization as non-decreasing utility through time. This interpretation imposes a chain of obligation between each successive generation which can very conveniently be represented with recursive utility. Sustainability under certainty is defined $U_t \leq U_{t+1}$, for all t. A natural generalization under uncertainty discussed theoretically in [3] is:

$$U_t \leq \omega_t(\widetilde{U}_{t+1}) \tag{10}$$

FIG. 4. Optimal control (energy cost multiplier X_t^{*A2}) with the sustainability constraint, for $\rho = 1.3$ and $\gamma = 1.3$.



By using a certainty equivalent function $\omega_t(.)$, this definition allows welfare to decrease if bad luck arises, but to an extent controlled by the risk aversion parameter. Pezzey [29] suggested to consider sustainability as a prior constraint on the optimization program. It can also be used as a criteria to evaluate *ex-post* optimal paths. We conducted both analysis.

We examined, for 76 model runs using varying parameters sets, which ones were sustainable in the above sense. As could be expected, scenario A2 was the least often sustainable, followed by B2, A1 and B1 in this order. This explains why, when solving the model under the sustainability constraint 10, in the model $U_t \leq V_t$, the effect is important only for A2 and B2 scenarios (compare figure 4 with the unconstrained optima figure 1 panel **c**).

Discounting felicity at a high rate, two or three percent per year, led to unsustainable pathways with two interesting exceptions. For A1 and $\eta = 2\%$, a risk aversion of $\gamma = 2$ restored sustainability where $\gamma = 1.3$ did not. For B1, a resistance to substitution $\rho = 2$ led to sustainable optima. The latter result is interesting since $\rho = 1.3$ did not lead to sustainability. Here η and ρ have opposite consequences on the optimal path.

Our explanation is that substitution effects depend upon the absolute level of welfare, whereas discounting depends upon time. As long as there is a monotonous relationship between time and wealth, i.e. growth, substitution and discounting are broadly equivalent. But the stochastic model used here breaks this relationship for some states of the world. This is why η and ρ have opposite effects.

This result illustrates that, from the sustainability point of view, there is more in resistance to substitution than simply in discounting. Consequently, focusing the debate on intergenerational equity only on η is problematic.

This adds to other problems with the discounting approach when analyzing very large scale and long term projects. There is an endogeneity question since large scale projects affect the growth path of the economy. This path should not be considered as exogeneous.

There is also the need to introduce risk when considering long-run projects. In their recent path-breaking papers, Weitzman [35] and Gollier [12] examine directly how macroeconomic uncertainty affects the choice of the discount rate. They prove that it reduces significantly the discount rate for very long-term projects, maybe at its lowest possible value. On the other hand, usual practice is to increase the discount rate for risky project. For anthropic climate change, the correlation of risks between emissions, economic activity and impacts therefore lead to ambiguity.

We do not see how analysis of sustainability could avoid to deal with non-marginal differences in the welfare levels. More general models of intergenerational utility, such as the stochastic recursive framework, seems to us more appropriate. It appears that increasing the discount rate is not the same as increasing the resistance to intertemporal substitution.

4.3. Sensitivity to uncertainty

In this section we present the sensitivity of optimal control to uncertainty in ULTRAL.

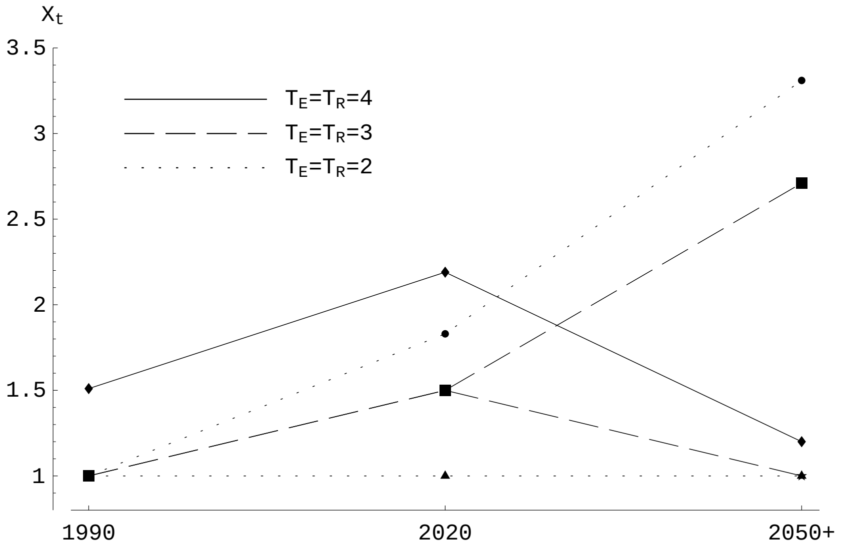
Results up to this point assumed $T_R = T_E = 4$, in other words that bifurcations occurs after 2050. We examine now how an earlier climatic event affects the optimal control. Still assume $T_R = T_E$, that policy is revised in the same period as climate change happens. Figure 5 illustrates optimal strategies in a sequential framework.

In a sequential framework, the central result is near-term policy: How does the possibility of sooner climate event affect the optimal first-period control? Results show that an later settlement of climate change uncertainties implies a higher control in the short run. This result, which Ha-Duong, Grubb and Hourcade already found in [23], calls some discussion.

At first, the potential for earlier climatic event implies larger total lifetime impacts since they come sooner and thus last longer. This could lead to increase control in the current period. But early damage also allows for a better adaptation strategy since efforts may be developed sooner. There is thus also a substitution effect between *ex post* and *ex ante* control. Figure 5 shows that the substitution effect seems to dominate.

To confirm this hypothesis, we examined the difference between X_1^* sensitivity to T_E and its sensitivity to T_R. This needs some theoretical comments first.

FIG. 5. Optimal control strategies $X^*_{A2,t}$ for different period of uncertainty resolution, assuming that policy revision and climatic event coincide, for $\rho = 1.3$ and $\gamma = 2$.



In many situations, assuming $T_E = T_R$ is realistic. In some financial markets for example, the movements of exchange rates (T_E) are observed in real time by operators which adjust their portfolio within minutes (T_R). For climate change, this is different.

There is ongoing climatic research and it is hopeful that scientists and policy-makers will know well in advance the severity of climate change. This suggests that precautionary action is possible, and gives background to the assumption $T_R \leq T_E$ of early perfect foresight. On the other hand, ULTRAL does not exclude either the opposite situation $T_R > T_E$. This is a way to introduce inertia into the model. International decision procedures usually involves a large time lag. Another source of time lag is the natural variability of climate which complicate detection and attribution of anthropic climate change. This is why in ULTRAL there is no a priori relation between T_E, the date of climatic event, and T_R, the date of policy revision.

Table 3 summarizes sensitivity of first-period optimal control X_1^{*A2} to the uncertainty parameters T_E, T_R and risk aversion γ, with $\rho = 1.3$ in the A2 scenario.

Results are very sensitive to T_R indeed. Compare the line $T_R = 4$ with $T_R = 3$. There is much lower initial control with earlier policy revision. For the lower values of risk aversion $\gamma \leq 2$, earlier learning may even lead to no effort at all. This remains when comparing $T_R = 4$ with $T_R = 2$. This a standard wait and see effect which can be linked to the theory of irreversibility. The prospect to be able to adapt policies sooner gives an incentive to differ control.

TABLE 3.

Sensitivity of optimal initial tax ($X_1^{*A2} - 1$) to uncertainty parameters T_R, T_E and γ.

	$\gamma = 3.9$			$T_E = 4$		
	$T_E = 2$	$T_E = 3$	$T_E = 4$	$\gamma = 1.3$	$\gamma = 3.9$	$\gamma = 6.0$
$T_R = 2$	24%	27%	28%	0	28%	48%
$T_R = 3$	01%	11%	10%	0	10%	65%
$T_R = 4$	55%	74%	81%	29%	81%	81%

Results confirm the hypothesis that the substitution effect dominates the damage effect. Table 3 shows that optimal control is much less sensitive to T_E than to T_R. Chancey Starr has a saying he calls the Noah principle of decision under uncertainty: “Predicting rains does not count, building ark does”.

Studying the sensitivity of results to uncertainty parameters with recursive utility is all the more interesting that it most published work to date has been led with a logarithmic intertemporal expected utility function [20, 24, 16]. Yet Gollier, Julien and Treich [13] have shown that the logarithmic case is precisely the limit case where better information has no effect on the level of current consumption in a stock pollutant model (take $\gamma = 1$ in their proposition 1). With respect to this initial motivation of our work, numerical simulations led with parameters close to 1 were non conclusive.

5. CONCLUSION

In this paper, we considered the generalized Kreps-Porteus stochastic recursive preferences that permit to cut the link between risk aversion γ and resistance to intertemporal substitution ρ. They were used to build a stylized intergenerational integrated assessment model, ULTRAL.

The model has zero climate damages on average. This highlight a simple insight: the assumption of a positive expected climate damage is absolutely not needed to induce some precautionary abatement in the optimal path.

Another characteristic of the model is the distinction between the date of policy revision T_R and the date of effective climate change T_E. It was found that the initial level of abatement is much more sensitive to the former than to the latter: The earlier is policy revision scheduled, the lower is initial carbon tax level.

On the theoretical side, stochastic recursive utility contributes directly to the discussion on discounting in cost-benefit analysis [2, 19, 30] by providing an unifying framework for the main different points of view. For example, many climate change models use the discounted sum of logarithmic expected utility, corresponding to the $\rho = \gamma = 1$ case. In this paper, we analyzed the sensitivity of the optimal energy consumption path to the specifications of the utility function.

The first result is that larger risk aversion strengthens optimal pollution control. Second, larger resistance to substitution — more emphasis on intergenerational equity — rotates the optimal control path toward less control in the current decade and more control in the future. This compares with an increase in the utility discount rate, but the effects differ from a sustainability point of view. Third and finally, low resistance to substitution ρ with high risk aversion γ leads to the largest carbon tax levels. Constraining $\rho = \gamma$ leads to underestimating the sensitivity of results to the shape of the utility function.

APPENDIX: THE ULTRAL MODEL

Variables

$X_{i,t}$	Energy cost multiplier (Control variable)
$Y_{i,t}$	Production (PIB per capita per year in kUS$ 90)
$K_{i,t}$	Capital stock (kUS$ 90 per capita)
$E_{i,t}$	Energy input (toe per capita per year)
$C_{i,t}$	Consumption (kUS$ 90 per capita per year)
$R_{i,t}$	Adjustment costs factor (% of investment)
$M_{i,t}$	Begin-of-period cuMulative global emissions (GtC)
$N_{i,t}$	Begin-of-period CO_2 mixing ratio (ppmv)
$D_{i,t}$	Equilibrium temperature increase (° C)
$F_{i,t}$	Climate damage (fraction of $Y_{i,t}$)
$U_{i,t}$	Utility (k dol. 90 per capita)
$V_{i,t}$	Certainty equiValent of next generation's utility (kUS$ 90 p.c.)

Equations

ULTRAL maximizes $U_{H,1}$ under constraints A.1-A.11, the stochastic dynamic programming constraint $X_{L,t} = X_{H,t}$ for all $t \leq T_R$ and the final stationarity condition $X_{i,4} = X_{i,3}$.

$$
\begin{aligned}
Y_{i,t}/Y^1 &= a_t F_{i,t} (K_{i,t}/K_1)^k (E_{i,t}/E^1)^e && \text{(A.1)}\\
C_{i,t} &= (1 - s_t R_t) Y_{i,t} && \text{(A.2)}\\
Y_{i,t}/Y^1 &= p_t X_{i,t} E_{i,t}/E^1 && \text{(A.3)}\\
K_{i,t+1} l_{t+1} &= (\mu K_{i,t} + \nu s_t Y_{i,t}) l_t && \text{(A.4)}\\
R_{i,t+1} &= \max(1, |(X_{i,t}/X_{i,t-1})^{\frac{1}{\delta}} - 1|/\lambda) && \text{(A.5)}
\end{aligned}
$$

$$
\begin{aligned}
M_{i,t+1} &= M_{i,t} + \delta \chi_t E_{i,t} l_t && \text{(A.6)}\\
N_{i,t} &= \alpha_t^0 + \alpha_t^1 M_{i,t} && \text{(A.7)}\\
D_{i,t} &= 2.5(N_{i,t} - 275)/275 && \text{(A.8)}\\
F_{i,t} &= \begin{cases} 1 & \text{if } t < T_E \\ 1 - \Delta D_{i,t}^2 & \text{if } t \geq T_E \text{ and } i = H \\ 1 + \pi/(1-\pi)\, \Delta D_{i,t}^2 & \text{if } t \geq T_E \text{ and } i = L \end{cases} && \text{(A.9)}\\
U_{i,t} &= [(1-\beta)(l_t/l_4)^\rho C_{i,t}^{1-\rho} + \beta V_{i,t}^{1-\rho}]^{\frac{1}{1-\rho}} && \text{(A.10)}\\
V_{i,t} l_t &= \begin{cases} U_{i,t+1} l_{t+1} & \text{if } t \neq T_R \text{ and } t \neq 4 \\ \left[\pi (l_{t+1} U_{H,t+1})^{1-\gamma} + (1-\pi)(l_{t+1} U_{L,t+1})^{1-\gamma}\right]^{\frac{1}{1-\gamma}} & \text{if } t = T_R \\ C_{i,4} l_4 (\mu K_{i,4} + \nu s_4 Y_{i,4})/K_{i,4} & \text{if } t = 4 \end{cases} && \text{(A.11)}
\end{aligned}
$$

Parameters (A1 scenario)

k	Elasticity of capital	0.36
e	Elasticity of energy	0.04
l_t	World population	5.26, 7.49, 8.70, 7.72 Giga
a_t	Technical progress coefficient	1.00, 1.81, 3.69, 5.52
s_t	Saving ratio	0.25
p_t	Baseline increase in the energy input cost	1.00, 1.28, 2.05, 2.70
Y^1	Production per capita in 1990 without damages	3.97 k$90
E^1	Energy per capita in 1990 without reduction	1.07 toe
K_1	Initial capital	11.92 k$90
μ	Inter-period depreciation of capital	0.215
ν	Inter-period accumulation of investment	16.14
λ	Maximum X annual variation without adj. costs	0.02
β	Inter-period utility discount factor	0.74
ρ	Resistance to intertemporal substitution	1.3
γ	Relative risk aversion	2.0
δ	Duration of a period	30 years
χ_t	Carbon content of energy	1.10, 1.03, 0.68, 0.39 tC/toe
α_t^0	Intercept of the carbon-cycle relation	354, 371, 390, 297 ppmv
α_t^1	Slope of the carbon-cycle relation	0, 0.154, 0.129, 0.278 ppmv/toe
T_E	Date of impact: productivity jump Event	4
T_R	Date of policy Revision	4
π	Probability for a negative impact	0.05
Δ	Magnitude of damage for 1° C warming	0.1

Scenarios

Data from IPCC preliminary 1998 scenarios used for calibration. Year 2080 values are linearly interpolated except for emissions. Energy Intensity in B1 taken from A1.

			1990	2020	2050	2080	2100
A1	Population	10^9	5.3	7.5	8.7	7.7	7.1
	GNP per capita	1990k$	4.0	7.5	20.8	50.5	74.9
	Energy Intensity	MJ/$	11.3	8.8	5.5	4.2	3.3
A2	Population	10^9	5.3	8.2	11.3	12.4	13.1
	GNP per capita	1990k$	4.0	5.0	7.3	8.9	9.9
	Energy Intensity	MJ/$	12.8	12.4	10.0	7.4	5.7
B1	Population	10^9	5.3	7.8	8.9	7.9	7.2
	GNP per capita	1990k$	4.0	6.2	12.8	31.4	46.7
	Energy Intensity	MJ/$	11.3	8.8	5.5	4.2	3.3
B2	Population	10^9	5.3	7.7	9.4	10.0	10.4
	GNP per capita	1990k$	4.0	6.6	11.7	18.5	22.6
	Energy Intensity	MJ/$	12.9	8.5	6.0	4.8	4.0

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