Non-intrinsic common agency*

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Abstract

In this paper we analyse a common agency model in which agents can choose with how many principals they want to work, while principals can not condition contracts on the agent's decision to accept other contracts. In this case of "*non-intrinsic*" common agency we characterize the equilibrium. Unless the substitutability between the two outputs is very strong, optimality conditions for principals' contracts are the same as with intrinsic common agency. However, principals suffer from reciprocal competition, which with "moderate" substitutability increases the informational rent agents obtain in equilibrium.

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1 Introduction

The framework of common agency with adverse selection has many important applications in economics. For example, a retailer, privately informed on final demand, may be the common agent of several wholesale producers (Martimort, 1996). A privately informed multinational enterprise may be the (common) agent of host countries' regulators (Calzolari, 1998). An international investor may be the common agent of national tax authorities (Olsen and Osmundsen, 1998). And so on.

In standard common agency models, each principal (she) independently and simultaneously offers a contract to the agent (he)[1]. If the agent accepts the contracts, he has to perform all contractual requirements.

If the agent only has the possibility to accept either both contracts, or none of them, we are in the case known as *intrinsic common agency* [Bernheim and Whinston (1985, 1986)]. It is "intrinsic" in the sense that either the agent accepts to work with both principals or, if he refuses, he can not operate at all. This is the framework employed by the theoretical literature (e.g., Martimort, 1992 and Stole, 1992) and by the already mentioned more applied papers.

The case where the agent may decide to accept some - but not necessarily all - contracts is analysed only within the framework called *delegated common agency* (Bernheim and Whinston, 1985). In this class of models each principal does not simply propose a contract, but designs a *menu* of contracts, which are made contingent on the agent's decision to participate or not with the other principal(s)[2].

Authors generally adopt the intrinsic common agency scenario in applied works to simplify the analysis. However, the assumption that the agent is not free to choose to work only with a sub-set of principals is not always easy to justify. On the other hand, delegated common agency is in a sense less restrictive, but it is more difficult to deal with and the contractual structure of conditional offers seems at odd with some observed situations. In particular, notice that offering conditional contracts can be seen as a way to

[1] For a model with non simultaneous offers see Baron (1985).

[2] An extreme case of delegated agency is the one where principals explicitly forbid the agent to participate with other principal(s), case known as exclusive dealing (Bernehim and Whinston, 1998). A somehow similar situation is the one where principals compete for the agent, but technology makes common agency impossible (Biglaiser and Mezzetti, 1993).

increase other principals' costs by affecting the agent's willingness to work for them; this type of behaviour is usually sanctioned by antitrust laws in the manufacturer-retailer relationship[3].

In this paper we analyze a situation in which (i) the agent is free to choose with how many principals he wants to work (among those offering a contract), and (ii) no principal can condition her contract on the agent's decision to accept other contracts. In these instances the agent is offered just one contract by each principal and has the freedom to accept all, some or even none of them. This case is an intermediate case between delegated common agency and intrinsic common agency, which we label *non-intrinsic common agency*[4].

The aim of this paper is to characterize the equilibrium of this agency problem. When goods are complementary, we prove that equilibrium contracts are identical to what we have with intrinsic common agency. Therefore conclusions on the efficiency of contracts in intrinsic common agency models generalize to this case, as no additional distortion is introduced.

When goods are substitutes, optimality conditions for principals' contracts remain the same unless substitutability is "too strong". However, with non-intrinsic common agency principals suffer from reciprocal competition. Each of them has to design a contract which makes the agent participate with her whatever he does with the other(s). We show that this additional competition increases the informational rents agents obtain in equilibrium, because in the game with each principal the agent's reservation utility is - endogenously - higher.

The paper is organized as follows. In section 2 we describe the common agency benchmark model with full information and asymmetric information with cooperating principals. In section 3 we solve the model with intrinsic and non-intrinsic common agency and we compare the two cases and establish our main results.

[3]The agent's decision to participate with other principals (as well as other contract characteristics) may be non observable or non-contractible. Designing differentiated contracts is sometimes forbidden by the law or may be considered unfair with respect to other principals. For instance, when the agent is a multinational enterprise and the principals are national regulators, offering different contracts depending on whether the firm produces in other countries would be considered an extension of domestic jurisdiction outside national boundaries.

[4]Notice that when the contracting activity of one principal is essential to the profitability of the relationship with the other principal(s), then this case coincides with the one of intrinsic common agency.

2 The model

To facilitate comparison with the existing literature, we base the analysis on the already "classical" producers/retailer framework by Martimort (1996). We consider only two producers, the principals P_1, P_2 and one agent, the retailer A. Producers have the technology to obtain an intermediate input which is essential for the final output. However, they are not able to perform the final stage in production. On the contrary, the agent needs one unit of the intermediate input to produce one unit of the final good.

The agent has a piece of private information θ; it is common knowledge that $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$ and principals expect it to be distributed according to a cumulative distribution $F(\theta)$ and density $f(\theta)$.

Manufacturer i offers a non linear wholesale price $X_i(q_i)$ which the agent has to pay to receive q_i units of intermediate inputs, such that for $h = 1, 2$

$$X_h(q_h) = \begin{cases} 0 \text{ for } q_h = 0, \\ \bar{x}_h + x_h(q_h) \text{ for } q_h > 0. \end{cases} \tag{1}$$

If the agent buys nothing then he pays nothing, but if he buys a strictly positive amount of q_h then he pays a fixed fee $\bar{x}_h$ (possibly equal to zero) and a variable fee $x_h(q_h)$. The two contracts are offered simultaneously.

Following the existing literature, we do not allow for contracts of the type $\widetilde{X}(q_i, q_j)$ in which the contract proposed by principal i also depends on the quantity chosen by the agent from the other principal. This incomplete contract relationship can be justified on several grounds. For example, quantity q_j may not be contractible or even observable by principal i. Moreover, in the case at hand a contract like $\widetilde{X}_i(q_i, q_j)$ may be prohibited for antitrust reasons.

The agent's total utility is

$$U \equiv v(q_1, q_2, \theta) - X_1(q_1) - X_2(q_2) \tag{2}$$

where $v(\cdot)$ indicates the profit the retailer obtains from the final output market[5]. When the agent takes, say, only principal 1's contract, his payoff is

$$U(q_1, 0, \theta) = v(q_1, 0, \theta) - X_1(q_1) \tag{3}$$

similarly with principal 2. When the agent does not participate with any principal he gets his reservation utility which we normalize to zero. The

[5]A subscript to $v(\cdot)$ denotes a derivative with respect to the relevant variable.

agent observes the contracts and then chooses output levels q_1 and q_2. We employ the following regularity assumptions.

Assumption 1. $v(\cdot)$ is strictly concave in both intermediate inputs.

Assumption 2. The cross derivative $v_{ij}(\cdot)$ has constant sign.

Assumption 3. $v_\theta < 0$.

Assumption 4. $v_{i\theta} < 0$ for $i = 1, 2$.

Assumption 5. $v(0, 0, \theta) = 0$.

Principals' utility functions are quasi-linear and, for simplicity, the (variable) production cost of the intermediate input $c(q_i)$ is the same for both principals. Thus, the total payoff of principal i is

$$W_i = X_i(q_i) - c(q_i) \tag{4}$$

2.1 The benchmarks

It is straightforward to show that when both principals are informed and they cooperate, the input levels $\left(q_1^F(\theta), q_2^F(\theta)\right)$ are defined by the two necessary conditions for total surplus maximization[6]

$$v_i\left(q_1^F(\theta), q_2^F(\theta), \theta\right) - c'\left(q_i^F(\theta)\right) = 0$$

for $i = 1, 2$. Moreover, principals use a fixed fee to extract all the agent's rent, which is then allocated between the two principals according to an exogenous redistribution rule (or bargaining process).

When the two principals do not cooperate, the necessary conditions are still the same. However, the lack of coordination may lead to equilibria in which the agent prefers not to produce. These equilibria are eliminated with cooperation as both principals prefer to have the agent producing.

Using standard techniques (proofs are omitted), the two cooperating principals under incomplete information design wholesale prices such that quantity levels $\left(q_1^C(\theta), q_2^C(\theta)\right)$ are determined according to conditions

$$v_i\left(q_1^C(\theta), q_2^C(\theta), \theta\right) - c'\left(q_i^C(\theta)\right) + \frac{F(\theta)}{f(\theta)} v_{i\theta}\left(q_1^C(\theta), q_2^C(\theta), \theta\right) = 0$$

[6]See Martimort (1996).

for $i = 1, 2$. We thus see that in a symmetric equilibrium quantities are distorted downwards. In addition, to satisfy incentive compatibility constraints, all agents but the least efficient are left with a positive rent. Principals then trade off allocative efficiency (which requires no distortions) with distributive efficiency (minimizing the rents left to the agent).

3 Common agency

We now consider a common agency with non cooperating uninformed principals. We first address the intrinsic common agency setting, typically used in the literature. Then we characterize equilibria with non-intrinsic common agency. Finally we compare the two.

We solve these games making explicit use of indirect mechanisms described by contracts $X_i(q_i)$. We will not rely on direct mechanisms in which the agent announces his type $\hat{\theta}_i$ to principal i. The reason for this choice is that the Revelation Principle does not generally apply in common agency games with adverse selection.[7] Solving for indirect mechanisms we will follow the Martimort (1996) model making use of the technique developed in Calzolari (1998).

3.1 Intrinsic Common Agency

At the first stage, each principal makes offers one contract to the agent. In the second stage the agent either accepts both offers or neither. If he refuses both contracts, he is left with the (zero) reservation utility, and so are the principals.

The agent chooses outputs to maximize his utility, defined in (2)

$$(q_1(\theta), q_2(\theta)) \in \underset{q_1, q_2}{ArgMax} \{v(q_1, q_2, \theta) - X_1(q_1) - X_2(q_2)\} \tag{5}$$

and accepts the contracts if his (indirect) utility is not less than zero,

$$U(\theta) \geq 0 \tag{6}$$

where $U(\theta) \equiv \{v(q_1(\theta), q_2(\theta), \theta) - X_1(q_1(\theta)) - X_2(q_2(\theta))\}$. (5) is the incentive compatibility constraint while (6) is the participation constraint. Using (4), the principal problem is thus, in its original form:

[7] Martimort and Stole (1997) and Calzolari (1998).

$$(P_i)^{IC} \begin{cases} \underset{X_i(\cdot)}{Max} \int_{\underline{\theta}}^{\overline{\theta}} W(\theta)\, g(\theta)\, d\theta \\ s.t.\ (5),\ \ (6)\ \ \forall\theta \in \left[\underline{\theta}, \overline{\theta}\right]. \end{cases} \tag{7}$$

Let us consider principal i's maximization program (similarly can be done for principal j). Principal i takes as given the agent's decision concerning q_j, and anticipates that - given X_i - the agent will choose q_i to maximize his utility. Program (7) can be simplified in two steps.

First of all, let us label by $q_j(q_i, \theta)$the output level q_j which maximizes the agent's utility for any q_i. We can now define

$$\widehat{v}(q_i, \theta) \equiv v(q_i, q_j(q_i, \theta)\, \theta) - X_j(q_j(q_i, \theta)) \tag{8}$$

This is the gross utility that the agent can obtain - dealing with principal i - given the utility maximizing level of q_j. Substituting back into the agent's utility, the constraint (5) can then be rewritten as

$$q_i(\theta) \in \underset{q_i}{ArgMax} \{\widehat{v}(q_i, \theta) - X_i(q_i)\} \tag{9}$$

From this maximization one obtains the equilibrium value $q_i(\theta)$, so that $U(\theta) = \widehat{v}(q_i(\theta), \theta) - X_i(q_i(\theta))$ and therefore we obtain the first useful result:

$$X_i(q_i(\theta)) = \widehat{v}(q_i(\theta), \theta) - U(\theta) \tag{10}$$

The second step is the following[8]. The envelope theorem implies that if incentive compatibility conditions (9) are met, then

$$U_\theta = \widehat{v}_\theta(q_i(\theta), \theta) \tag{11}$$

Integrating (11) by parts, we have that

$$\int_{\underline{\theta}}^{\overline{\theta}} -U(\theta)\, g(\theta)\, d\theta = -U\left(\overline{\theta}\right) + \int_{\underline{\theta}}^{\overline{\theta}} \widehat{v}_\theta(q_i(\theta), \theta)\, G(\theta)\, d\theta \tag{12}$$

Moreover, with an envelope argument we also have $\widehat{v}_\theta(q_i(\theta), \theta) = v_\theta(\cdot) < 0$. Thus, setting to zero the utility of the agent with the highest θ, $U\left(\overline{\theta}\right) = 0$,

[8] To simplify the presentation we only use first order necessary conditions for the agent's decision and omit agent's local second order and global sufficiency conditions for incentive compatibility. We thus employ the so called second order approach which consists in using only first order (necessary) condition and checking *ex-post* that the other conditions are met.

all participation constraints (6) are satisfied. Finally, designing the contract so that $U\left(\overline{\theta}\right) = 0$ is optimal because leaving rents to the agent is costly. Employing the results (10) and (12), program $(P_i)^{IC}$ (7) becomes

$$\underset{q_i(\cdot)}{Max} \int_{\underline{\theta}}^{\overline{\theta}} \left\{ \widehat{v}\left(q_i, \theta\right) - c\left(q_i\right) + \widehat{v}_\theta\left(q_i, \theta\right) \frac{G\left(\theta\right)}{g\left(\theta\right)} \right\} g\left(\theta\right) d\theta \tag{13}$$

Notice that - as we have substituted for $X_i\left(q_i\left(\theta\right)\right)$, which incorporates $q_i(\theta)$, the agent's optimal choice - the incentive compatibility constraint (9) is automatically satisfied, and it is as if the principal could directly choose q_i. With this transformation the program looks like a standard principal-agent problem which then can be solved using standard techniques.

Pointwise maximization then gives necessary condition,

$$\hat{v}_{q_i}\left(q_i, \theta\right) - c'\left(q_i\right) + \hat{v}_{\theta q_i}\left(q_i, \theta\right) \frac{G\left(\theta\right)}{g\left(\theta\right)} = 0 \tag{14}$$

Notice that with our transformation this necessary condition is similar to what we obtained with uninformed cooperating principals. The difference is that here we use function $\hat{v}\left(\cdot\right)$ instead of $v\left(\cdot\right)$. To obtain comparable formulations we have to transform the derivatives $\hat{v}_{q_i}$, $\hat{v}_{\theta q_i}$ in terms of $v\left(\cdot\right)$. Using the envelope theorem we have $\hat{v}_{q_i}\left(q_i, \theta\right) = v_i\left(q_1, q_2, \theta\right)$. Moreover, differentiating $\hat{v}_\theta\left(\cdot\right) = v_\theta\left(\cdot\right)$ w.r.t. q_i, gives $\hat{v}_{\theta q_i}\left(\cdot\right) = v_{\theta i}\left(\cdot\right) + v_{\theta j}\left(\cdot\right) \frac{\partial \hat{q}_j}{\partial q_i}$. Substituting back into (14), we obtain

$$v_i\left(q_1, q_2, \theta\right) - c'\left(q_i\right) + \left[v_{\theta i}\left(q_1, q_2, \theta\right) + v_{\theta j}\left(q_1, q_2, \theta\right) \frac{\partial \hat{q}_j}{\partial q_i} \right] \frac{G\left(\theta\right)}{g\left(\theta\right)} = 0$$

Totally differentiating the agent's first order condition for q_i with respect to θ one easily obtains $\frac{\partial \hat{q}_j}{\partial q_i} = \frac{v_{12}(\cdot)\, \dot{q}_j(\theta)}{v_{12}(\cdot)\, \dot{q}_i(\theta) + v_{\theta j}(\cdot)}$ and the optimality condition becomes

$$v_i\left(q_1, q_2, \theta\right) - c'\left(q_i\right) + \left[\frac{v_{\theta j}\left(\cdot\right) v_{\theta i}\left(\cdot\right) + v_{12}\left(\cdot\right) \sum_{h=1}^{2} v_{\theta h}\left(\cdot\right) \dot{q}_h\left(\theta\right)}{v_{12}\left(\cdot\right)\, \dot{q}_i\left(\theta\right) + v_{\theta j}\left(\cdot\right)} \right] \frac{G\left(\theta\right)}{g\left(\theta\right)} = 0 \tag{15}$$

Solving similarly for principal j gives the other necessary condition. The solution of this system of two differential equations provides the (candidate) equilibrium quantities in intrinsic common agency $\left(q_1^{IC}\left(\theta\right), q_2^{IC}\left(\theta\right)\right)$.

3.2 Non-intrinsic Common Agency

We now turn to the case where principals can not condition contracts on the agent's decision to accept other contracts. In other words, principal i can only propose a contract $X_i(q_i)$ as defined in (1). She can not offer a menu of contracts $\left\{X_i^{ED}(q_i), X_i^{CA}(q_i)\right\}$, as in delegated common agency, where $X_i^{ED}(q_i)$ applies if the agent is an exclusive dealer and $X_i^{CA}(q_i)$ applies if he works for both principals. In the same way, the agent cannot be forced to choose between working for both principals and not working at all. Indeed, in the second stage of the game the agent may accept both offers, only one of the two, or may refuse both.

As we have done in the previous section, we solve for principal i. Principal i now has to make sure the agent accepts her contract, whatever he does with the other principal (the quantity q_j the agent produces is given to principal i). In any case principal i wants to design a contract which the agent accepts.

Employing the previous notation [see (8), which depends on $q_j(q_i, \theta)$ as well] we define,

$$U(q_i, q_j(q_i, \theta), \theta) = \widehat{v}(q_i, \theta) - X_i(q_i) \tag{16}$$

and

$$\widehat{U}(0, q_j(0, \theta), \theta) = \max\{U(0, q_j(0, \theta), \theta), 0\} \tag{17}$$

Expression (17) denotes the outside opportunity the agent has in his relationship with principal i. Notice that in case $U(0, q_j(0, \theta), \theta) < 0$, the agent always maintains the option of not participating at all, with a reservation utility equal to 0. The participation constraint in the game with principal i becomes, in line with (3),

$$U(q_i(\theta), q_j(\theta), \theta) \geq \widehat{U}(0, q_j(0, \theta), \theta) \tag{18}$$

This is the crucial difference between this problem and P_i^{IC}. In that program the participation constraint (6) implies that each principal must set her contract making sure that the agent's *total* utility is positive. Here, what matters to the decision of the agent to work with principal i is the agent's *incremental* utility: the difference between what he would get working (also) with principal i and what he would anyway receive from principal j if he did *not* accept the contract with principal i. Notice that q_j is given to principal i (the two principals move simultaneously) and its actual value will be determined in equilibrium. Finally, with constraint (18) it is as if the agent had a type-dependent reservation utility[9] $\widehat{U}(0, q_j, \theta)$.

With the same approach employed above, we now write the program of principal i with non-intrinsic common agency,

$$(P_i)^{NIC} \left\{ \begin{array}{l} \underset{X_i(\cdot)}{Max} \int_{\underline{\theta}}^{\overline{\theta}} \{\widehat{v}(q_i, \theta) - c(q_i(\theta)) - U(\theta)\} g(\theta) d\theta \\ s.t. \ (9), \ \ (18) \ \ \forall \theta \in \left[\underline{\theta}, \overline{\theta}\right]. \end{array} \right.$$

Again, the necessary incentive compatibility condition and the envelope theorem imply (12) and then the program becomes

$$(P_i)^{NIC} \left\{ \begin{array}{l} \underset{q_i(\cdot)}{Max} \int_{\underline{\theta}}^{\overline{\theta}} \left\{\widehat{v}(q_i, \theta) - c(q_i) + \hat{v}_\theta(q_i, \theta) \frac{G(\theta)}{g(\theta)}\right\} g(\theta) d\theta - U\left(\overline{\theta}\right) \\ s.t. \ (18) \ \ \forall \theta \in \left[\underline{\theta}, \overline{\theta}\right]. \end{array} \right.$$

We now introduce the following definitions.

Complements: Goods i and j are complements when $v_{ij}(.) > 0$.

Substitutes: Goods i and j are substitutes when $v_{ij}(.) < 0$.

We can now prove our first result.

Proposition 1 *(i) Provided the two goods are either complement or their substitutability is not "too strong", setting* $U\left(\overline{\theta}\right) = \widehat{U}\left(0, q_j, \overline{\theta}\right)$ *verifies the participation constraints (18) for any* θ.

(ii) Under the above assumption, optimality conditions are the same as with intrinsic common agency (15).

Proof.

(i) Take $U(q_i, q_j, \theta) - \hat{U}(0, q_j, \theta)$ and differentiate w.r.t. θ,

$$\frac{d\left[U(\theta) - \hat{U}(0, q_j, \theta)\right]}{d\theta} = v_\theta(q_i(\theta), q_j(\theta), \theta) - v_\theta(0, q_j(0, \theta), \theta) \qquad (19)$$

The relative magnitude of the values of $q_j(\theta)$ in the two terms on the RHS depends on whether the two goods are complements or substitutes. In the

[9] A general technical analysis of this type of constraint is provided by Jullien (1997).

case of complements, given $v_{\theta i}(\cdot) < 0$, we have $\frac{d\left[U(\theta)-\hat{U}(0,q_j(\theta),\theta)\right]}{d\theta} < 0$ and the net rent $U(q_i, q_j, \theta) - \hat{U}(0, q_j(0,\theta), \theta)$ is decreasing with θ. Setting $U\left(q_i, q_j, \overline{\theta}\right) = \widehat{U}\left(0, q_j(0,\theta), \overline{\theta}\right)$ implies that the participation constraints (18) are satisfied for all the other types. It is easy to check that the same result follows whenever $q_j(0,\theta)$ is not too large relative to $q_j(\theta)$, i.e. when substitutability is not "too strong".

(ii) When the result proven in (i) holds, program $(P_i)^{NIC}$ becomes,

$$\underset{q_i(\cdot)}{Max} \int_{\underline{\theta}}^{\overline{\theta}} \left\{ \hat{v}(q_i, \theta) - c(q_i) + \hat{v}_\theta(q_i, \theta) \frac{G(\theta)}{g(\theta)} \right\} g(\theta)\, d\theta - \hat{U}\left(0, q_j(0, \overline{\theta}), \overline{\theta}\right),$$

which is identical to the program with intrinsic common agency (13) except for the constant term $-\hat{U}\left(0, q_j, \overline{\theta}\right)$. The necessary conditions (15) are then unchanged. ■

This result states that at the margin the existence of an additional possibility for the agent (i.e., to accept or not the contract with principal j) does not change his relationship with principal i relative to the case of *intrinsic* common agency. The intuition behind this result is that in equilibrium the agent will anyway accept both contracts, so that whether or not the agency is "intrinsic" does not change much in equilibrium.

This result depends on the relationship between the two goods. Let us see why it must be so. When dealing with principal i, what matters to the agent is the difference between the utility he gets in equilibrium and $\widehat{U}(0, q_j(0,\theta), \theta)$, the utility he gets when participating with her only [see (18)]. This difference is monotonic when the two goods are complements, as $v_\theta(0, q_j(0,\theta), \theta)$ is certainly smaller in absolute value than $v_\theta(q_i(\theta), q_j(\theta), \theta)$. With substitutes, this certainty vanishes, and we can have a non-monotonic, type-dependent participation constraint[10]. Whenever this happens, we may have a distortion in the incentive provided to the agent, given that now the incremental utility an agent gets accepting a contract no longer increases with the agent's efficiency; less efficient agents may find themselves in a stronger position.

The intuition might be the following. Principal i would like to have an agent who has a large value of v_i. Given that $v_{i\theta} < 0$, she tends to prefer

[10]When susbstitutability is extremely strong, we can have a monotonically increasing equilibrium payoff ($\frac{d\left[U(\theta)-\hat{U}(0,q_j,\theta)\right]}{d\theta} > 0$). The intermediate case might be analysed on the basis of Jullien (1997), although we leave this study to potential future research.

efficient, low-θ agents. However, if v_{ij} is negative and "large", principal i tends to prefer an agent who produces small quantities of the other output, and thus maybe - somehow paradoxically - less efficient agents. Efficient agents are *per se* desirable, but (with $v_{ij} < 0$) they become less desirable as they produce a large quantity of the other output. If the latter effect prevails, their equilibrium rent might be lower than the one of less efficient types.

To improve the intuition, let us consider a couple of more specific cases where θ is a cost parameter. Assume that q_i and q_j are homogeneous goods and the cost function is $C = \theta(q_i + q_j)^\beta$. Here $v_\theta\left(q_i\left(\theta\right), q_j(\theta), \theta\right) = -C_\theta\left(q_i(\theta), q_j(\theta), \theta\right) = -\left[q_i(\theta) + q_j(\theta)\right]^\beta$ while $v_\theta\left(0, q_j(0,\theta), \theta\right) = -\left[q_j(0,\theta)\right]^\beta$. Now (19) becomes $-\left[q_i(\theta) + q_j(\theta)\right]^\beta + \left[q_j(0,\theta)\right]^\beta$, which is always negative with non-increasing marginal costs ($\beta \leq 1$) because $q_j(\theta) \geq q_j(0,\theta)$ but may become negative in the opposite case[11]. Notice that expression (19) is negative whenever in equilibrium $q_i(\theta) + q_j(\theta) > q_j(0,\theta)$ which seems to represent the most plausible case even with substitutability.

An analogous story can be told with heterogeneous goods. In this case (19) is negative when there are economies of scope, but may be positive if diseconomies of scope are large.

Let us now turn to the problem of whether the agent gains relative to the case of intrinsic common agency. To this end we first need to establish a preliminary result[12].

Lemma 2 *If the goods are complements $v(q_1, q_2, \theta) > v(q_1, 0, \theta) + v(0, q_2, \theta)$ and if they are substitutes $v(q_1, q_2, \theta) < v(q_1, 0, \theta) + v(0, q_2, \theta)$.*

Proof. The statement with complements can be written as

$$\left[v(q_1, q_2, \theta) - v(q_1, 0, \theta)\right] - \left[v(0, q_2, \theta) - v(0, 0, \theta)\right] > 0$$

This in turn may be written as

$$\int_{u=0}^{q_2} v_2(q_1, u)du - \int_{u=0}^{q_2} v_2(0, u)du > 0$$

which becomes

$$\int_{u=0}^{q_2} \left[v_2(q_1, u) - v_2(0, u)\right] du > 0$$

[11]Notice that here we are taking equilibrium output values, which in turn depend on β.

[12]This result is a restatement of Proposition 4B1 by Baumol *et al.* (1982).

and then - following the same procedure,

$$\int_{s=0}^{q_1}\int_{u=0}^{q_2} v_{21}(s,u)dsdu > 0$$

which holds when $v_{21} > 0$, proving the result. The case of substitutes is completely analogous. ■

The following result shows that the increase in the reservation utility due to the option to accept one contract only allows at least some agents to extract a larger rent.

Proposition 3 *In the NIC program, under the same conditions as in Proposition 1, the following holds:*

(i) All agents get a utility level greater or equal to the one they get under intrinsic *common agency* $\left[(P_i)^{IC}\right]$.

(ii) With substitutability all agents get a utility level strictly greater than the one they get under intrinsic *common agency.*

(iii) With complementary outputs, agents get the same utility they get under intrinsic *common agency.*

Proof. [13]

(i) Trivially follows from (18).

(ii) The agents' rents are the sum of two components: the rent accrueing to the worst type (in the present case, $\widehat{U}(0, q_j(0,\bar{\theta}), \bar{\theta})$ for principal i and analogously for principal j) and a term depending on q_i and q_j. As equilibrium output levels are the same in NIC as with intrinsic common agency, the variable component is the same in the two cases and the rents may differ only if the utility levels of the worst agent in the two cases differ.

As with intrinsic common agency $U(\bar{\theta}) = 0$, we only have to prove that with substitutability $U(\bar{\theta}) = \widehat{U}(0, q_j(0,\bar{\theta}), \bar{\theta}) > 0$. The proof proceeds in two steps. First, we show that it is impossible to have both $U(0, q_j(0,\bar{\theta}), \bar{\theta}) < 0$ and $U(q_i(0,\bar{\theta}), 0, \bar{\theta}) < 0$. Then we prove that $U(0, q_j(0,\bar{\theta}), \bar{\theta}) \geq 0$ implies $U(q_i(0,\bar{\theta}), 0, \bar{\theta}) > 0$, so that $\widehat{U} > 0$.

First step. Assume on the contrary that $U(0, q_j(0,\bar{\theta}), \bar{\theta}) < 0$ and $U(q_i(0,\bar{\theta}), 0, \bar{\theta}) < 0$, which imply two things. First, $U(\bar{\theta}) = 0$, and then $U(0, q_j(0,\bar{\theta}), \bar{\theta}) + U(q_i(0,\bar{\theta}), 0, \bar{\theta}) < 0$.

[13] The strategy we follow is somehow related to the one employed by Ivaldi and Martimort (1994) in a different context.

By the definition of $U(\bar{\theta})$ we have

$$U(q_i(\bar{\theta}), q_j(\bar{\theta}), \bar{\theta}) = \max_{q_i, q_j} \left[v(q_i, q_j, \bar{\theta}) - \bar{x}_i - x_i(q_i) - \bar{x}_j - x_j(q_j) \right] = 0 \quad (20)$$

The definition of $U(0, q_j(\bar{\theta}), \bar{\theta})$ is

$$U(0, q_j(0, \bar{\theta}), \bar{\theta}) = \max_{q_j} \left[v(0, q_j, \bar{\theta}) - \bar{x}_j - x_j(q_j) \right] \quad (21)$$

and analogously for $U(q_i(0, \bar{\theta}), 0, \bar{\theta})$. From (20) we can obtain an expression for $\bar{x}_i + \bar{x}_j$ which, substituted in $U(0, q_j(0, \bar{\theta}), \bar{\theta}) + U(q_i(0, \bar{\theta}), 0, \bar{\theta}) < 0$, gives

$$\max_{q_j} \left[v(0, q_j, \bar{\theta}) - x_j(q_j) \right] + \max_{q_i} \left[v(q_i, 0, \bar{\theta}) - x_i(q_i) \right] +$$

$$- \max_{q_i, q_j} \left[v(q_i, q_j, \bar{\theta}) - x_i(q_i) - x_j(q_j) \right] < 0 \quad (22)$$

This is impossible. In fact, Lemma 2 shows that, due to substitutability, $v(q_i, q_j, \bar{\theta}) - x_i(q_i) - x_j(q_j) < v(0, q_j, \bar{\theta}) - x_j(q_j) + v(q_i, 0, \bar{\theta}) - x_i(q_i)$ which holds *a fortiori* when we take the maximum of both sides. Therefore, we cannot have both $U(0, q_j(0, \bar{\theta}), \bar{\theta}) < 0$ and $U(q_i(0, \bar{\theta}), 0, \bar{\theta}) < 0$; at least one of these expressions must be non-negative.

Let us now turn to the *second step.* Given the previous result, take $U(q_i(0, \bar{\theta}), 0, \bar{\theta}) \geq 0$. To obtain an expression for $\bar{x}_j$, consider the following. In the game with principal j the worst type agent will only get his reservation utility. Condition $U(q_i(\bar{\theta}), q_j(\bar{\theta}), \bar{\theta}) = \widehat{U}(q_i(\bar{\theta}), 0, \bar{\theta})$ for principal j can be rewritten as

$$\max_{q_i, q_j} \left[v(q_i, q_j, \bar{\theta}) - \bar{x}_i - x_i(q_i) - \bar{x}_j - x_j(q_j) \right] = \max_{q_i} \left[v(q_i, 0, \bar{\theta}) - \bar{x}_i - x_i(q_i) \right]$$

or,

$$\bar{x}_j = \max_{q_i, q_j} \left[v(q_i, q_j, \bar{\theta}) - x_i(q_i) - x_j(q_j) \right] - \max_{q_i} \left[v(q_i, 0, \bar{\theta}) - x_i(q_i) \right] \quad (23)$$

Substituting $\bar{x}_j$ into (21) we have

$$U(0, q_j(0, \bar{\theta}), \bar{\theta}) = \max_{q_i} \left[v(q_i, 0, \bar{\theta}) - x_i(q_i) \right] + \max_{q_j} \left[v(0, q_j, \bar{\theta}) - x_j(q_j) \right] -$$
$$- \max_{q_i, q_j} \left[v(q_i, q_j, \bar{\theta}) - x_i(q_i) - x_j(q_j) \right]$$

Notice that $v(q_i,0,\bar\theta)-x_i(q_i)+v(0,q_j,\bar\theta)-x_j(q_j) > v(q_i,q_j,\bar\theta)-x_i(q_i)-x_j(q_j)$ for given output levels, because of the substitutability. This holds *a fortiori* when we take the maximum of both sides, and therefore $U(0,q_j(0,\bar\theta),\bar\theta) > 0$.

This second step implies that $U(0,q_j(0,\bar\theta),\bar\theta)$ and $U(q_i(0,\bar\theta),0,\bar\theta)$ are both *strictly* positive, which proves the point.

(iii) Concerning complementary outputs, Proposition 1 shows that the net rent $(U-\widehat{U})$ is decreasing in θ. It is then optimal to set conditions (18) with equality for $\bar\theta$. We now show that the value of $\widehat{U}(\bar\theta)$ (the maximum outside option for type $\bar\theta$) is zero in the game against at least one principal. As a consequence the equality of equilibrium rent with the (zero) outside opportunity implies that type $\bar\theta$ gets the same (zero) rent with intrinsic and non-intrinsic common agency (conditions (18) are satisfied with equality for $\bar\theta$, and for at least one of the two the r.h.s. is zero) thus concluding the proof.

To prove that $\widehat{U}(\bar\theta)=0$ with at least one principal, assume the contrary, i.e. that $\widehat{U}(0,q_j(0,\bar\theta),\bar\theta) = \max_{q_j}\left[v(0,q_j,\bar\theta)-\bar{x}_j-x_j(q_j)\right] > 0$ and $\widehat{U}(q_i(0,\bar\theta),0,\bar\theta) = \max_{q_i}\left[v(q_i,0,\bar\theta)-\bar{x}_i-x_i(q_i)\right] > 0$. Using (23), we can obtain $\bar{x}_j$ and $\bar{x}_i$; summing up we get

$$-(\bar{x}_j+\bar{x}_i)+\max_{q_i,q_j}\left[v(q_i,q_j,\bar\theta)-x_i(q_i)-x_j(q_j)\right]=$$

$$=\max_{q_i}\left[v(q_i,0,\bar\theta)-x_i(q_i)\right]+\max_{q_j}\left[v(0,q_j,\bar\theta)-x_j(q_j)\right]-$$

$$-\max_{q_i,q_j}\left[v(q_i,q_j,\bar\theta)-x_i(q_i)-x_j(q_j)\right] \tag{24}$$

The r.h.s. is negative, given Lemma 2, in the same way as (22) is not (notice that here we consider the opposite case of complementary outputs). As a consequence the l.h.s. in (24) is also negative. But this implies $U(q_i(\bar\theta),q_j(\bar\theta),\bar\theta) < 0$ which is impossible. This establishes a contradiction.

■

Point (ii) of this Proposition indeed shows that the competition between the principals leaves the agent with a utility level strictly greater than the one he gets under *intrinsic* common agency. When outputs are substitutes, increasing the output required by one principal makes operating with the other principal more costly. As a consequence the threat to leave one principal and contract only with the other is credible.

On the contrary with complements the threat to participate only with the other principal becomes non-credible because the agent himself prefers to produce with both. The rent is then the same the agent can get when he is obliged to deal with both of them.

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