Dynamic Uncertainty and Global Warming Risk[¤]

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Abstract

When economic agents decide their optimal environmental behavior, they have to take into account non continuos evolutionary trends and irreversible changes characterizing environmental phenomena. Given the still non perfect biophisical and economic knowledge, decisions have to be taken in an uncertain framework. The paper analyses in the speci…c how agents' optimal choices are a¤ected in the presence of a future possible, but uncertain catastrophic occurrence provoked by a climate collapse due to global warming.

JEL: C5, D8, D9, O2, Q2.

Key words: Climate Change, Irreversibility, Endogenous-exogenous uncertainty, Optimal policy,

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1 Introduction

> The major concern, in my view, is the potential for abrupt and unforeseen changes in climate, particularly on a regional level. A major concern, for example, is reversal of thermohaline circulation, which could lead to enormous climatic shifs in Europe. This and similar "catastrophes" are genuinely frightening prospects, but we have no reliable way of assessing their likelihood at present. (Nordhaus, 1999, p. 10)

In spite of continuous scientific improvements the bio-physical aspects of a large number of environmental phenomena are still highly uncertain. This uncertainty increases with the spatial and temporal scale of the issue under investigation. The great scientific debate on the evolution of global temperature or on ozone depletion are just two examples of the still relevant role of uncertainty. Associated with this physical, chemical, and biological uncertainty, there is also an economic uncertainty which makes it difficult to evaluate the costs and benefits associated with environmental policy interventions, and the effectiveness of instruments to control greenhouse gas emissions[1].

The role of uncertainty in designing sound environmental policies is then complicated if one considers that natural phenomena do not generally follow linear evolutionary trends. Natural developments are characterized by radical changes that may dramatically modify living and economic conditions. As well as being non-linear, natural phenomena may also be "irreversible". Once the point of no return is exceeded, changes are impossible to reverse: one dramatic example is the extinction of animal and vegetal species, but also the possibilities of catastrophic events induced e.g. by Global Warming cannot be excluded a priori (IPCC, 1996a,b,c). Consequently, models need to be able to capture "jumps" and irreversibilities in order to predict

[1] The costs and benefits of pollution control are analysed by, e.g., Frankhauser (1995), Nordhaus (1994a), Larson and Tobey (1994), Eismont and Welsch (1996) and the contribution of Working Group III to the Second Assessment Report of the Intergovernmental Pannel on Climate Change: IPCC (1996a), while the effectiveness of instruments is considered by Peck and Teisberg (1993a), and Pizer (1997). It is also worth to see the wide literature stemming from the huge debate on flexibility mechanisms proposed in the Kyoto Protocol, see e.g. Janssen (1998), Ellerman et al. (1998), Grubb (1998), Bohm (1999) and Hahn and Stavins (1999).

and avoid dangerous divergence from equilibrium paths. Given this relevance various models have been designed with the speci…c aim of de…ning the cost of this uncertainty. A "common" approach is to quantify the value of "early knowledge", that is, the economic value of resolving the uncertainties about climate change sooner rather than later (Peck and Teisberg, 1993b; Manne and Richels, 1992). In particular, Nordhaus and Popp (1997), using the PRICE model, a probabilistic version of DICE de…ne the value of information (the cost of uncertainty) as the di¤erence between the expected value of net damages with perfect information (learn-then-act case) and that with stochastic information (act-then-learn). The approach of Nordhaus and Popp allows an estimate of the value of information not only about di¤erent "states of the world", but also about individual variables and about di¤erent modelling areas (environmental, socio-economic, technological etc.). A similar method is used by Manne (1996), who analyses the value of information about two key parameters - climate sensitivity and warming damage - using seven di¤erent Integrated Assessment Models (IAMs): CETA, DIAM, DICE, HCRA, MERGE, SLICE, Yohe. The value of the information highlighted can be relevant, but only when the probability associated with a possible "bad" state of the world is high (close to 50%) - otherwise it falls rapidly to zero.

A di¤erent perspective in accounting for uncertainty is the direct possibility o¤ered by some models to evaluate the outcome of a given action under di¤erent future (more or less likely) scenarios which could be chosen by the user. Models like FUND (Tol 1997), PAGE (Plambeck and Hope 1996) , ICAM (Dowlatabady and Kandlikar 1995) and CONNECTICUT (Yohe 1995) belong to this category.

Finally, a third approach to uncertainty, the one followed in this paper is to describe how an uncertain, but possible, future and irreversible event can in‡uence present decisions. The uncertainty stems from our ignorance on a level of global temperature required to trigger a "catastrophic" event that, once occurred, either drops utility to zero or to 1990 level (see also Gjerde, Grepperud and Kverndokk, 1998, for a similar analysis). This is done by introducing an hazard rate function in some well known IAMs which deal with Global Warming: RICE, CETA and MERGE. Furthermore, in our hypothesis, agents adjust their expectations about the uncertain future event according to two di¤erent learning processes: an exogenous one, in which information stems only from the state of the world presently observed, and an endogenous one in which information is dependent also on past history.

The numerical simulations show that, although the long-run equilibrium points, either with exogenous or endogenous uncertainty, are characterized by lower emissions than the no-uncertainty case, three of the most popular numerical IAMs respond di¤erently in designing the approach to the …nal equilibrium. In the speci…c while CETA and MERGE depict the expected sudden emission decrease below the no-uncertainty case, RICE highlights that a less prudent behavior in the short run can be possible.

The paper is organized as follows. Section 2 presents a short description of the models considered, section 3 summarizes in a simple theoretical framework the main common features of these models. Our di¤erent de…nitions of uncertainty are introduced in section 4, whereas section 5 is devoted to the presentation of the simulation results. Section 6 draws our main conclusions. A large theoretical appendix provides analytical support.

2 The three participating models

In this section we present a short description of the main characteristics of the three models we used for the practical implementation of uncertainty. They are well known Integrated Assessment Models whose structure, functioning and results are well documented and familiar to most of environmental economists.

2.1 RICE

The RICE[2] model (Nordhaus and Yang, 1996)[3] is an IAM which depicts in a general equilibrium framework the optimizing behavior of economic agents represented by six di¤erent macroregions. Each agent maximizes his intertemporal utility function given by the discounted utility of per capita consumption, where the discount rate is …xed at a level of 3% per year, over

[2] All our investigations are focused on the RICE version presented by Nordhaus and Yang in their 1996 article (see references), but to perform some sensitivity analyses and some speci…c tests on RICE, the RICE-98 model (Nordhaus and Boyer 1999a, 1999b) has been used as comparison. Among other speci…cities RICE-98 introduces a backstop technology, a new energy input in the Cobb-Douglas production function and a new form of energy-saving technological progress. Other novelties with respect to RICE are the higher regional disaggregation and the di¤erent discount rates applied to di¤erent regions. The environmental part has been improved as well consisting in a three-reservoir model calibrated to existing carbon-cycle models.

[3] See also Sanstad and Greening (1998) and Bosello, Carraro and Kemfert (1998).

a …nite time horizon[4]. Each agent/region is endowed with an initial capital stock, population and technology. Population grows exogenously and technology is assumed to be Hicks neutral for capital and labor. Moreover an exogenous rate of Autonomous Energy E¢ciency Improvement (AEEI) is applied to energy consumption. Capital accumulation, on the contrary, is endogenously determined by optimizing the ‡ow of consumption over time. The production process is represented by a Cobb-Douglas production function which depends on capital, labour and technology.

The environmental part of the model is represented by the Schneider and Thompson climate model (Nordhaus, 1991). Endogenous CO_2 emissions - by product of the production activities - are translated into temperature increase by a coupled atmosphere-ocean model with two mixed strata - an atmosphere-upper ocean and a deep ocean stratum, with parameters derived through calibration with general circulation models.

The link between the economic and the environmental part is given by the output function. In fact output is negatively a¤ected by both the environmental damage caused by global warming and the environmental protection which diverts resources away from production. A damage function and a control cost function in‡uence production and thus consumption. Agents, during the optimization procedure …nd the utility maximizing balance among production, consumption, environmental protection and pollution. A particular feature of RICE is the possibility of analyzing di¤erent strategies undertaken by nations: 1) A market approach in which there are no emission controls; 2) A cooperative approach in which all nations cooperate in a globally e¢cient way to curb CO_2 emissions and 3) A noncooperative approach in which individual nations undertake policies according to their national self-interest and ignore the spillovers of their actions on other countries.

2.2 CETA

CETA model (Peck and Teisberg, 1992)[5] depicts the relations among worldwide economic system, energy consumption, energy technology choices, global warming and global warming cost linking together four mathematical modules: the energetic submodel, the production submodel, the output allocation

[4] For obvious computational limits, all the models optimise over a …nite time horizon. Nevertheless simulations are carried well over the century, for that reason they are able to reproduce long run e¤ects and to capture long term dynamics.

[5] See also Bosello, Carraro and Kemfert (1998).

submodel and the environmental submodel. Production is a CES function which depends on labour, capital, electric and non electric energy inputs weighted by exogenous AEEI. Output, via the allocation submodel is used to satisfy the demand for investment, energy, clean environment and, finally, consumption.

The energy submodel determines energy costs which on their turn depends on energy consumption choices and on available technologies. CETA adopts the so-called backstop approach: different technologies are available, but some of them remain economically unfeasible because they include the costs of engaging in R&D. They enter the market only once the price of "old" technologies increases in response to the increasing scarcity in their base resources. CETA, in the specific, considers five electric and seven non electric technologies entering and exiting the market according to quantity and price constraints. Finally the environmental submodel determines the environmental damage. Four GHGs are modeled, of which only CO_2 is endogenous. Emissions of other gases are linked to CO_2 by a stable function.

As usual, GHGs emissions increase the GHGs' stock (i.e. GHGs' concentration) and so global temperature. A damage function then transforms the temperature increase in lower world GDP. In this framework the representative agent maximizes her/his intertemporal discounted utility derived from consumption with a 3% per annum utility discount rate.

2.3 MERGE

The MERGE model (version 2) (Manne, Mendelsohn and Richels, 1995)[6] provides an integrated framework to assess costs and benefits of environmental protection. The model is composed by three submodules: 1) Global 2200 which defines regional and global costs of emission reduction; 2) a climatic submodel which calculate the temperature increase from emissions and 3) a damage submodel which quantifies the damage for the whole system derived from global warming.

Global 2200 splits the world in five macroregions linked by international trade flows. Five commodities are traded, among them emission trading permits. Production is represented by a Cobb-Douglas structure nested in a CES framework. Arguments of the production function are labour, capital and two energy inputs. The production side is disaggregated in three sec-

[6] See also Sanstad and Greening (1998) and Bosello, Carraro, and Kemfert (1998).

tors: two energetic sectors, electric and non-electric, and one sector for all other productions. Fourteen different technologies are available to produce the three outputs according to the backstop approach. The output is then allocated among consumption, investment and energy payments.

In the climatic submodel GHGs' emission stems endogenously from energy consumption and exogenously from non-energy consumption; three GHGs are considered: CO_2, CH_4 and N_2O. According to the Maier-Reimer and Hasselmann (1987) model, emissions are translated in concentration and finally following the IPCC model in temperature increase.

The damage submodel calculate the environmental damage from global warming. MERGE considers market and non-market damages, that is the first is linked to the temperature increase by a quadratic function while the second is defined according to agents' willingness to pay. As usual, MERGE uses an aggregate optimization framework, where the solution depends on maximizing the weighted sum of regional intertemporal discounted utilities. The discount rate is 3%.

3 A simple IAM

Apart some specificities, the productive and environmental structure shared by the above IAMs can be summarized by a simple optimal growth model with exogenous given increase of population and energy use and endogenous capital accumulation. N denotes the set of regions indexed by $i = 1, 2, \ldots, N$. For each region social benefit is represented by a utility function strictly increasing and strictly concave on consumption:

$$U_i(t) = U_i(C_i(t)) \qquad i = 1, 2, \ldots, N \tag{1}$$

Consumption is shared with investment following:

$$C_i(t) = Y_i(t) - I_i(t) \qquad i = 1, 2, \ldots, N \tag{2}$$

where Y_i indicates production net of costs and benefits of environmental control, and I_i is the gross investment. Net production is given by[7]:

$$Y_i(t) = \Omega(e_i(t); T(t)) Q_i(t) \qquad i = 1, 2, \ldots, N \tag{3}$$

[7] Taking Ω separable, i.e. $\Omega(e_i; T) = \Omega_1(e_i)\Omega_2(T)$; the first $\Omega_1(e_i)$ indicates the energy function and $\Omega_2(T)$ the Greenhouse damage function, with $\Omega_1' > 0$ and $\Omega_2' < 0$ (Cesar and Zeeuw, 1995).

where Q_i is the gross production and $-(e_i; T)$ is a function accounting for benefits and costs of environmental control, expressed as a function of region i-th's CO_2 emissions, e_i; and the average global temperature increase, above the preindustrial level, T:[8] An increase in emissions increases production while an increase in temperature reduces the production that can be devoted to consumption or investment, i.e. $-_e > 0$ and $-_T < 0$[9]. Gross output is obtained using capital as the only production factor:

$$Q_i(t) = A_i(t)Q_i(K_i(t)) \quad i = 1; 2 :::::::::: N \tag{4}$$

with $Q_i' > 0$ and $Q_i'' < 0$: A_i measures overall productivity which is assumed, in all models, to develop according to an exogenous given trend. Finally, the dynamic of the stock of capital is represented by the standard linear differential equation:

$$dK_i(t) = (I_i(t) - aK_i(t))dt; \quad K_i(0) = K_{0;i}; \quad i = 1; 2 :::::::::: N \tag{5}$$

where a stands for the capital decay rate, constant over time[10]. The environmental system is completed by introducing a diffusion equation for emissions and a relationship between emissions and temperature. The global amount

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[8] Although CO_2 is the most important-popular GHG; there are other GHGs which are important for the increase of climate: i.e. CH_4; N_2O and $CFC -$ gases:

[9] Most models of the greenhouse effect specify the feedback of pollution on future welfare through both amenity and production effects, where the former typifies the fact that environmental quality is seen as a luxury good that does not influence consumption as such but can effect the well-being of individuals in society (Kverndokk, 1994; Frankhauser,1995; Gjerde et al., 1998). However, unlike with most polluting gases, GHGs such as carbon dioxide, methane and nitrous oxides are typically benign for individuals, whilst their negative impact on climate change in the long-run possibly lead to economy-wide disruptions. Therefore a deliberate choice is made to assume that the only effect of the stock of GHGs is on productive capacity (Cesar and Zeeuw, 1995; Beltratti, 1993, 1995; Hoel and Isaksen, 1995; Moretto and Tamborini, 1997).

[10] The models considered account explicitly for a capital accumulation function as (5). Furthermore carbon emissions, for each region, are set proptional to gross production:

$$e_i(t) = \sigma_i(t)[1 - \mu_i(t)]Q_i(t);$$

The emissions to output ratio $\sigma_i(t)$ declines exogenously over time due to an assumed autonomous energy efficiency increase (AEEI), and the emission can be reduced at rate $\mu_i(t) \in [0; 1]$ in every period though this is costly.

of CO_2 accumulates in the atmosphere according to the following differential equation:

$$dM(t) = (\beta e(t) - bM(t))dt; \quad M(0) = M_0 \tag{6}$$

where $e(t) = \sum_{i=1}^{N} e_i(t)$ indicates the aggregate emissions, β is the share of total emissions polluting the atmosphere (i.e. entering the atmosphere) and b is the natural assimilation rate of CO_2 mass[11]. The temperature equation describes the reaction of temperature to changes in the atmospheric concentration of CO_2: That is[12]:

$$dT(t) = \sigma[\theta h(M(t)) - T(t)]dt; \quad T(0) = T_0 \tag{7}$$

where σ is a delay parameter of temperature in response to radiative increase (per year), θ is a factor of proportionality between radiative forcing and the long-run temperature response[13] and $h(M)$ is the increase in the radiative forcing from CO_2 equivalent emissions relative to the preindustrial level[14].

Finally, as the social welfare for each region is defined as $W_i = \int_0^{\infty} e^{-rt}U_i(C_i)dt$; the cooperative solution is obtained choosing the optimal paths of greenhouse gas emissions, e_i and investment, I_i, for each of the N regions over an infinite horizon, by maximizing the following utility additive intertemporal welfare function:

$$W(M_0; T_0; K_{0;1}; K_{0;2}; \ldots; K_{0;N}) = \sum_{i=1} W_i = \int_0^{\infty} e^{-\rho t}(\sum_{i=1} U_i(C_i))dt \tag{8}$$

subject to (1), (2), (3), (4), (5), (6) and (7), for $e_i \geq 0$; $K_i(0) = K_i$; $i = 1; \ldots; N$; $M(0) = M_0$ and $T(0) = T_0$:

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[11] Many authors facing the problem of not having a nice quantitative (deterministic) assimilation function take the linear approximation as a convenient proxy (Foster, 1975, Dasgupta, 1982, Barbier and Markandya, 1990, Pethig, 1990). Others consider different, non linear specifications of the assimilation function to give an idea of the consequences of slight variations in the steady state levels of pollution and consumption (Barbier and Markandya, 1990, Cesar and Zeeuw, 1995).

[12] See Nordhaus (1991) and Hoel and Isaksen (1995) for the suggestion of this simplified equation.

[13] This factor of proportionality is uncertain, Hoel and Isaksen (1995) set $\theta = 0.75$; that is an increase of radiative forcing of $1W/m^2$ gives a long-run temperature increase of $0.75\,^{\circ}C$.

[14] In general the function $h(.)$ vary between GHGs; for CO_2 it is of type $A \ln M + B$ (where A and B are constants), see IPCC (1992).

4 Uncertainty

As the base models are all under certainty, the purpose of this section is to see how the above set-up can be modified to represent optimal management of atmospheric pollution under a form of uncertainty regarding the occurrence of undesirable events associated with the greenhouse effect. Short-run as well as long-run analytical properties of optimal pollution management are given in the Appendix which is used as reference throughout the paper.

Dealing with uncertainty two main features should be defined: the first is the nature of the uncertain (future) event (i.e. what could happen); the second is the time path for the probability that this event would happen. In our setting, the uncertain future event is defined as a "catastrophe" that, once occurred either drops utility to zero or to 1990 level. Regarding the second issue, the uncertainty stems from our ignorance on a level of global temperature required to trigger such a catastrophic event. In other words, as a result of excessive accumulation of GHGs, the undesirable event occurs as soon as the temperature level $T(t)$ crosses an uncertain critical level $X \leq \bar{T}$.

Introducing this feature in our deterministic model (8) and letting τ represents the event occurrence time, the objective function now becomes:

$$W(M_0, T_0, K_{0,1}, K_{0,2}, \ldots K_{0,N}) = E\left[\int_0^{\tau} e^{-\rho t}\left(\sum_{i=1}^{N} U_i(C_i(t))dt + e^{-\rho\tau}V(\tau)\right)\right] \tag{9}$$

where $V(\tau) = 0$ or $V(\tau) = V_{1990} \equiv V(M_0, T_0, K_{0,1}, K_{0,2}, \ldots K_{0,N}) > 0$, is the post-event benefit. The expectation operator is taken with respect to the random variable τ induced by the critical level X. In this respect two methods of implementing uncertainty in the above integrated model are considered.

4.1 Endogenous uncertainty

If we consider that the uncertainty about X stems from the policy maker's ignorance concerning the conditions leading to a catastrophic event, we may specify his beliefs at any moment in time by a state-dependent distribution of the critical level X; i.e. $F(T) = \Pr(X < T)$;(Tsur and Zemel, 1994, 1996). In this case, as the distribution of X induces a distribution on τ, we may write (see Appendix):

$$1 - F_\tau(t) \equiv \Pr(\tau > t \mid \tau > 0) = \Pr(X > T(t) \mid X > T_0) \equiv \frac{1 - F(T(t))}{1 - F(T_0)};$$

with the hazard rate associated with τ given by:

$$\psi_\tau(t) \equiv \frac{f_\tau(t)}{1 - F_\tau(t)} = \lambda(T(t))\dot{T}(t); \quad \text{where} \quad \lambda(T(t)) = \frac{f(T(t))}{1 - F(T(t))} \qquad (10)$$

where $\lambda(T(t))$ represents the hazard function in terms of probability distribution of the critical temperature level. Defining $\Lambda(T(t)) = -\log\left[\frac{1-F(T(t))}{1-F(T_0)}\right]$; we may refer to:

$$1 - F_\tau(t) = e^{-\Lambda(T(t))} \qquad (11)$$

as the survivor function giving the upper tail area of the distribution. That is, it defines the probability of experiencing no catastrophe from the initial date up to time t (Kiefer, 1988).

4.2 Exogenous uncertainty

If the catastrophic occurrence is not entirely due to the policy maker's emission plan, with the temperature level exceeding the critical level X, it may be influenced by random (exogenous) environmental conditions (Copper, 1976; Heal, 1984; Clarke and Reed, 1994; Gjerde, et al., 1998). We may specify the policy maker's beliefs by a hazard rate function $\lambda(T(t))$ which depends only on the current average temperature level and not on overall temperature history. Under this assumption, and defining $\Lambda(T(t)) = \int_0^t \lambda(T(s))ds$; the distribution of the occurrence time τ (i.e. the surviving function) is (see Appendix):

$$1 - F_\tau(t) \equiv \Pr(\tau > t \mid \tau > 0) = e^{-\Lambda(T(t))}; \qquad (12)$$

with the hazard rate associated with τ given by:

$$\psi_\tau(t) \equiv \frac{f_\tau(t)}{1 - F_\tau(t)} = \lambda(T(t)) \qquad (13)$$

The difference between endogenous uncertainty and exogenous uncertainty is clear comparing (10) and (13). The former hazard rate does not depend on the current level of temperature $T(t)$ alone, but also on its rate of change $\dot{T}(t)$. When the temperature level does not increase, i.e. $\lambda h(M(t)) \leq T(t)$, the endogenous hazard rate vanishes and the probability of a catastrophe drops to zero. On the contrary, the exogenous hazard rate, which depends on the current temperature alone, does not vanish, so there is a positive probability of having a catastrophic event in the future.

4.3 How to implement uncertainty

According to (11) and (12) the probability of having a catastrophe depends on the hazard function linking this probability to the temperature trend. To distinguish between endogenous and exogenous uncertainty, we introduce the following hazard rate function of occurrence time:

$$\psi\tau(t) = \begin{cases} \varphi(\dot{T}(t))^{\eta}\,[\max(0, T(t) - T_0)]^{\eta - 1} & \text{for } \dot{T}(t) > 0 \\ \quad \text{where } \varphi(\dot{T}(t)) = \varphi_0 + \varphi_1 \dot{T}(t)\,; \quad \varphi_0, \varphi_1 \geq 0 & \\ 0 & \text{for } \dot{T}(t) \leq 0 \end{cases} \tag{14}$$

Equation (14) is clearly monotonically increasing or decreasing in T depending on the sign of $\eta - 1$.[15] Apart the aspects related with the distinction between exogenous and endogenous uncertainty, the above formulation, in our opinion, give a more satisfactory description of the way a climate change affects the policy-maker beliefs about a future catastrophic event. It is not only the current level of temperature at any particular moment, $T(t) - T_0$, which is important in forming the beliefs, but rather the speed at which the climate has been changing over several decades summarized by the current rate of change. In our hypotheses the probability of a catastrophe occurring is zero for the temperature level in 1990, T_0. As relation (14) shows, an increase of temperature above T_0, implies an increase of the hazard rate and consequently, following (11) and (12), the probability of avoiding a catastrophe decreases. Nevertheless, in cases of both endogenous and exogenous uncertainty, economic agents can influence the hazard rate and thus the surviving probability through emission control. By curbing emissions they are able to

[15] It is easy to check that, for $\dot{T} > 0$, the limit of the survivor function tends to zero, i.e. $\lim_{t \to \infty} e^{-\int_0^t \psi\tau(s)ds} = 0$

reduce temperature increases and so delay the eventuality of a catastrophic event.

The survivor probability $1 - F_{\tau}(t)$, is then used to weight the utility objective functions via a "correction" of the discount rate. Utility thus becomes:

$$U(t) = U_{bc}(t)(1 - F_{\tau}(t)) + U_{pc}(t)F_{\tau}(t) \tag{15}$$

The total utility at each time period, $U(t)$, is a weighted sum of the pre(before)-catastrophe utility and the post-catastrophe utility, the weights are given by the probabilities of not having and having a catastrophe respectively.

- Exogenous uncertainty

If $\varphi_1 = 0$; the probability distribution of avoiding a catastrophe does not depend on the rate of change of $T(t)$. At each time t agents make an effort to reduce emissions, according to the temperature at time t alone, without considering past evolution. Furthermore, if $\varphi_1 = 0$, the probability assumes the "typical" Weibull form (Kiefer, 1988) and only the two parameters φ_0 and η shape the hazard rate. φ_0 has been calibrated in order to have, according to the BAU in each model used, a catastrophe probability equal to 4,8% in year 2090[16].

Regarding exogenous uncertainty, two scenarios are built: the (a) scenario in which utility drops to zero after the catastrophe and the (b) scenario in which utility drops to its 1990 level. In case (a) only the first addendum of (15) holds. On the contrary, considering a post-catastrophic utility different from zero, case (b) implies that the whole sum must be considered.

Recalling that in the different models the value of φ_0 is calibrated to give a 4.8% probability of catastrophe in 2090[17], we also deal with two different values of η; namely 2.5 and 1 denoted in the following as case I and case II respectively. By equation (14), in case II we break the feedback between the

[16] This value derives from Nordhaus' study (1994b) who asked an expert panel to define subjectively the probability of a catastrophe in year 2090 in case an increase of $3^{\circ}C$ were experienced. Nordhaus' catastrophe is a loss of world GDP of more than 25%. The same estimate has been used among others by Manne (1996) and Gjerde et al. (1998) in a similar study.

[17] In the specific, the values of φ_0 for RICE, CETA and MERGE are respectively 0.00159, 0.00127 and 0.00121.

temperature increase and the probability of catastrophe. That is, when η is set equal to 1 agents cannot influence the catastrophe probability. Table 1 reassumes our simulation experiments.

Table 1

$a.I$	$U_{pc}(t) = 0; \eta = 2.5$
$a.II$	$U_{pc}(t) = 0; \eta = 1$
$b.I$	$U_{pc}(t) = 1990$ level; $\eta = 2.5$
$b.II$	$U_{pc}(t) = 1990$ level; $\eta = 1$

• Endogenous uncertainty

If, according to (14), the rate of change of temperature influences the hazard function, the parameter φ_1 has to be estimated together with φ_0. At each time t agents adjust their emission control effort according to the temperature level at time t and to its past trend, $\dot{T}(t)$. Computationally, as two unknown parameters have to be estimated, we couple the first assumption of a 4.8% probability of a catastrophe in 2090 with the further assumption that the catastrophe probability in years 2010-2020 is lower than 0,1%, a value that can be considered plausible for the occurrence of a catastrophic event in the next decade[18].

In this case, in an initial experiment we calibrate this probability to find positive values for both φ_0 and φ_1 and thus have a catastrophic probability which is always increasing as the temperature increases[19]. We are aware that these values have weak scientific foundations and above all, do not allow for complete homogeneous comparison among models. Nevertheless, as we await a sufficient set of data in order to make estimates that are statistically consistent with the learning curve $\varphi(.)$, an initial sensitivity analysis may be carried out between exogenous and endogenous uncertainty both in the short and in the long-run[20].

[18] In his study of a rapid sea-level rise from a collapse of the West Antartic ice sheet, Bentley (1997) concludes that the probability of such an event over the next century or two is in the order of 0.1%. Although this probability seems plausible for the next decade or two we believe that Nordhaus' conjecture is more reasonable if the entire century is conisdered.

[19] For RICE, CETA and MERGE, we set this probability at 0,044%, 0,045% and 0,083% respectively.

[20] A general formulation for the learning curve is $\varphi = \varphi(T(t), \dot{T}(t), Z(t), \dot{Z}(t))$, with $Z(t)$ as a vector of exogenous and/or endogenous explicative variables (Kiefer 1988).

In case of endogenous uncertainty, we restrict our analysis to the case of zero post catastrophe utility and ´ = 2:5 corresponding to the a.I exogenous uncertainty case. To denote that now agents are learning endogenously we label this case (En).

5 Simulation experiments.

It is important to clarify at the beginning what is our definition of "Base Case". To calibrate the parameters we refer to the BAU which is the situation in which agents are doing nothing to control emissions. On the opposite for our subsequent analysis of the uncertainty effects our reference "Base Case" becomes the situation of optimal emission control performed by agents, without considering the probability of a catastrophe. As CETA and MERGE give similar results, the description and comment of the outcome of the two models are presented together. RICE offers interesting peculiarities which are commented separately.

5.1 Exogenous uncertainty in CETA and MERGE

5.1.1 Emissions paths and global mean temperature

As could be expected, a possible future catastrophic event, even if uncertain, makes agents more cautious (cases a.I, b.I in figg. 1 and 2). In the two models the agents' optimal abatement effort under uncertainty is higher than the optimal abatement effort obtained without considering the possibility of catastrophic event.

When a catastrophic event is considered the optimal paths of carbon emissions are (dramatically) reduced compared to the base case. As an example, in CETA a.I case the reduction experienced is around -22% in 2010 (5,9 vs. 7,82 billion tons of emitted CO_2) and around -92% in 2100 (1,72 vs. 22,8 billion tons of emitted CO_2), in MERGE the reduction amounts to -13% in 2010 (6,67 vs. 7,71 billion tons of emitted CO_2) and to -96% in 2100 (0,46 vs. 13,73 billion tons of emitted CO_2)[21].

[21] These results are comparable with the research carried out by Gjerde et al (1998). In 2010 in their Cat1 case, corresponding to our a.I case, emissions are around 15% lower than in their NoCat case, corresponding to our base case. In 2100 similarly to our outcome emissions are nearly zero.

Yet, the analysis of cases a.I and b.I shows somewhat surprisingly no relation between the penalization induced by the catastrophic event and the emission reduction e¤orts. Emission reductions are in fact very similar independently utility drops to zero or to its 1990 level. In reality, looking at the ...gures, emissions are lower in a.I than in b.I, but the di¤erence is so small that it results hardly perceived.. However an increase of CO_2 emissions via the CO_2 accumulation, together with the temperature increase process, augment the probability of an adverse event (see tables 2 and 3)[22].

This result di¤ers from the one obtained by Gjerde et al (1998), where the magnitude of the post-catastrophe penalty is important in determining the optimal emission policy and especially for the emission level at the end of next century. This di¤erence, relies on the fact that in CETA and MERGE a very high abatement is undertaken through a process of substitution among alternative energy resources, a process which is independent from the post-catastrophe penalty.

Another result in accordance with common wisdom is that if the feedback between emissions and the catastrophe is broken (cases a.II, b.II in ...gg. 3 and 4) agents behave closely to the reference case. This is not surprising in fact agents know that a catastrophe could happen, but at the same time they know that nothing can be done to avoid it. Therefore, they perform their emission reduction e¤ort just to balance bene...ts and costs under their "real" control, i.e. according to cost and damage functions "not corrected" by the catastrophic occurrence. In fact, with ´ = 1 cases a.II and b.II coincide with a base case given nearly a 3.12% utility discount factor for CETA and MERGE.

However, an important qualitative di¤erence between CETA and MERGE can be observed. While in the former agents behave exactly as in the reference case, in the latter ...rst they reduce then they increase their emissions w.r.t. the base case. By the theoretical properties (propositions 4 and 5 in the Appendix), under a completely exogenous path for the probability of the catastrophe and depending on the penalization induced by the event, the agents may ...nd it optimal to increase or to decrease their emissions w.r.t. the non-event case. These are long-run e¤ects concerning the approach to ...nal equilibrium; if the simulation period is not long enough they may not

[22] To avoid an excessive proliferation of graphs, in the following sections we omit to present temperature trends. Nevertheless we present in tables the probability patterns which, due to the direct relationship with temperature, o¤er exactly the same qualitative information.

be captured by less sensible models. However, the prospect of a¤ecting the probability of an adverse outcome acts as an incentive to reduce emissions.

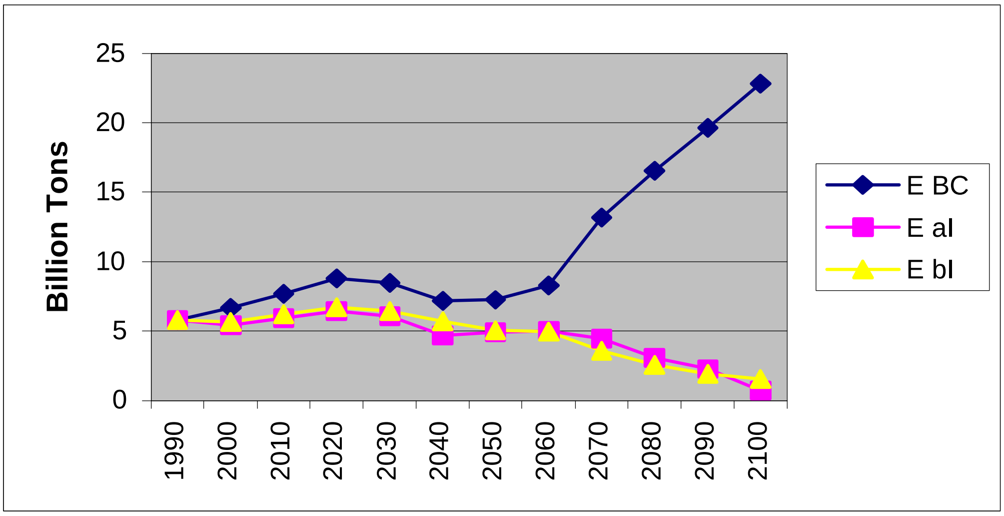

Fig. 1: CETA CO_2 emissions, Base Case and cases a.I, b.I .

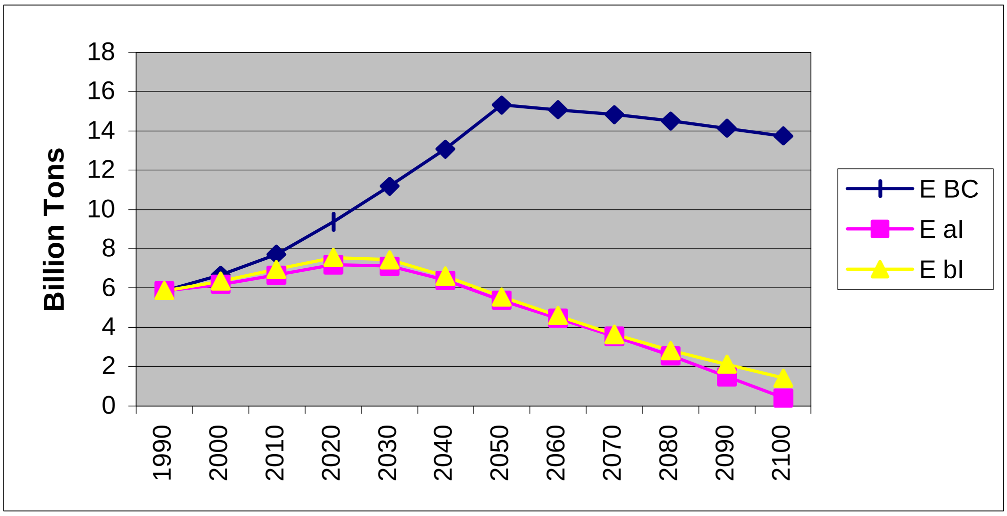

Fig. 2: MERGE CO_2 emissions Base Case, and cases a.I, b.I .

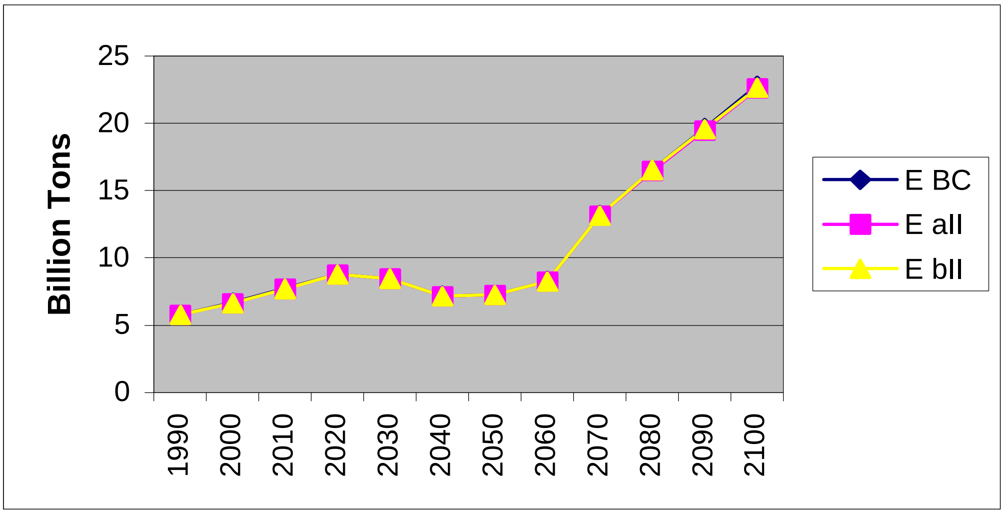

Fig. 3: CETA CO_2 emissions, Base Case and cases a.II, b.II .

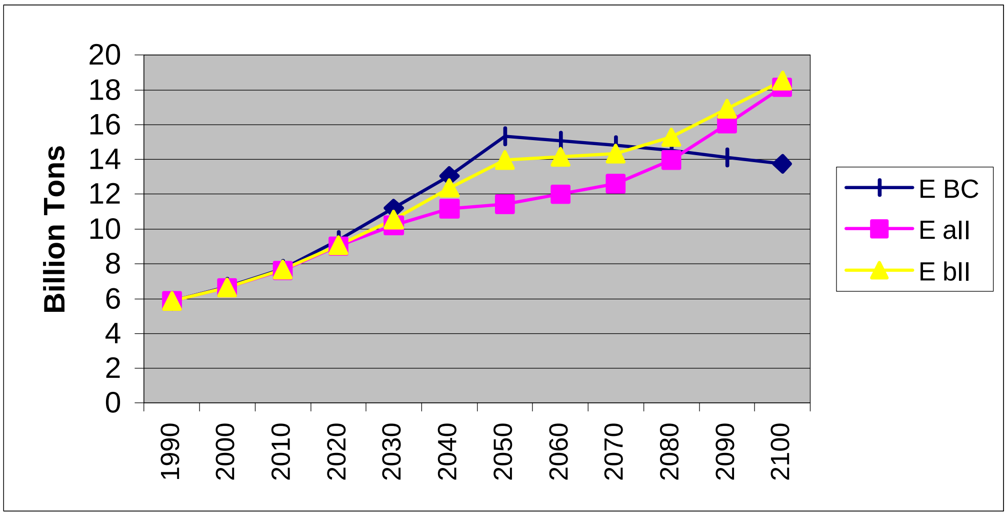

Fig. 4: MERGE CO_2 emissions, Base Case and cases a.II, b.II .

	sr a.I	*sr b.I*	*sr a.II b.II*
1990	1,0000	1,0000	1,000
2000	0,9995	0,9995	0,999
2010	0,9984	0,9984	0,997
2020	0,9965	0,9965	0,996
2030	0,9937	0,9937	0,995
2040	0,9901	0,9899	0,994
2050	0,9854	0,9852	0,992
2060	0,9800	0,9795	0,991
2070	0,9736	0,9729	0,990
2080	0,9665	0,9654	0,989
2090	0,9586	0,9572	0,987
2100	0,9500	0,9484	0,986

Tab. 2: CETA surviving probabilities (1-P_{cat}), ' = 0:00127:

	sr a.I	*sr b.I*	*sr a.II b.II*
1990	1,00000	1,00000	1,0000
2000	0,99980	0,99980	0,9988
2010	0,99918	0,99918	0,9976
2020	0,99801	0,99799	0,9964
2030	0,99616	0,99609	0,9952
2040	0,99351	0,99338	0,9940
2050	0,99004	0,98981	0,9928
2060	0,98576	0,98540	0,9916
2070	0,98073	0,98022	0,9904
2080	0,97505	0,97438	0,9892
2090	0,96884	0,96797	0,9880
2100	0,96221	0,96110	0,9868

Tab. 3: MERGE surviving probabilities (1-P_{cat}), ' = 0:00121 .

5.2 Energy-technology substitution

One speci…c feature of models like CETA and MERGE is to describe how di¤erent energy technologies enter and exit the market according to quantity and price constraints. An interesting exercise can thus be to investigate how the introduction of uncertainty in‡uences the rise and the decline of di¤erent production methods. One could expect that taking into account uncertainty would imply an even higher penalization of pollution intensive technologies. This is exactly what happens in CETA and in MERGE.

For example, comparing ...g. 5 representing the energy technology adoption path in the CETA base case with ...g. 6 corresponding to case a.I, three considerations can be made: 1) in the latter the use of fossil fuel for electricity production (EFO), characterized by a high emission coe¢cient is lower; 2) synthetic fuel (SYN) never comes into use (at least before 2100); 3) on the contrary it is substituted by the adoption of non-electric backstop technology (NEBA) which enters the market in 2060 (well 50 years sooner than in the base case). It is worth saying that CETA assumes that non-electric backstop is a carbon-free technology whereas synfuel derives from coal. Synfuel which is one of the main energy sources in the base case, totally disappears in the a.I case (as well as in the b.I case not shown), this means that the high costs of the backstop technology become sustainable in view of the high damage cost of emissions in case of a catastrophe.

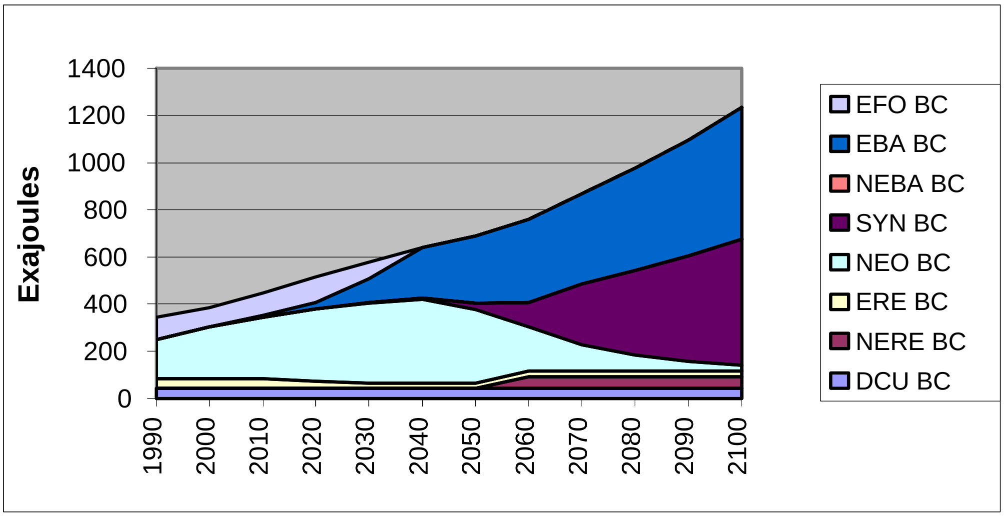

Fig. 5: CETA energy generation technologies, Base Case.

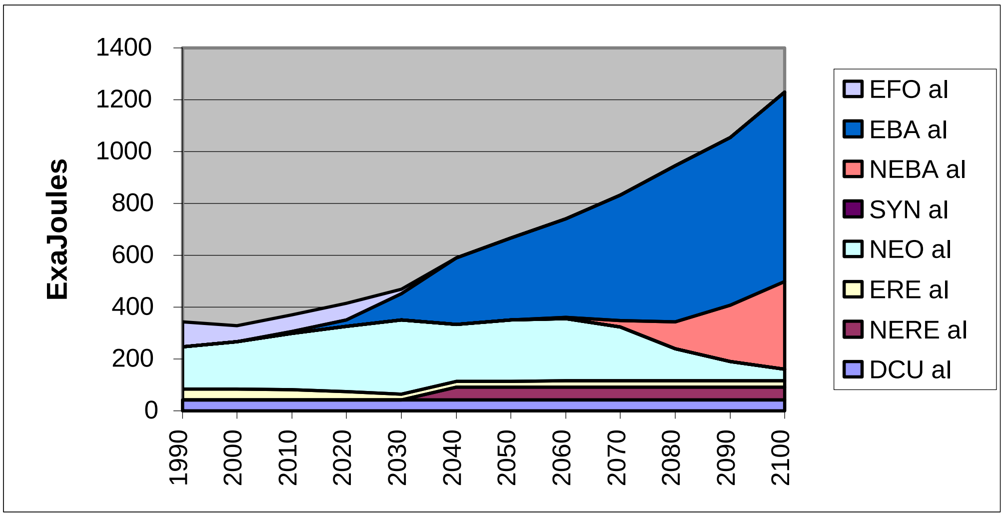

Fig. 6: CETA energy generation technologies, a.I case.

Similar considerations hold for MERGE. The model proposes nine non-electric and nine electric generation technologies. Comparing ...g. 7 corresponding to MERGE base case for non-electric technologies with ...g. 8 concerning the a.I case we can notice: 1) a general lower energy production in the second than in the ...rst scenario; 2) coal (C) that in the base case in 2100 is still the leading source of energy is replaced in the a.I case by renewable sources of energy (RN) and by the non-electric backstop technology (NEB); this latter comes into use in 2030 instead of 2070; 3) synthetic fuel that in the base case from 2030 onward increases its contribution to energy production, in the a.I scenario remains out of the market.

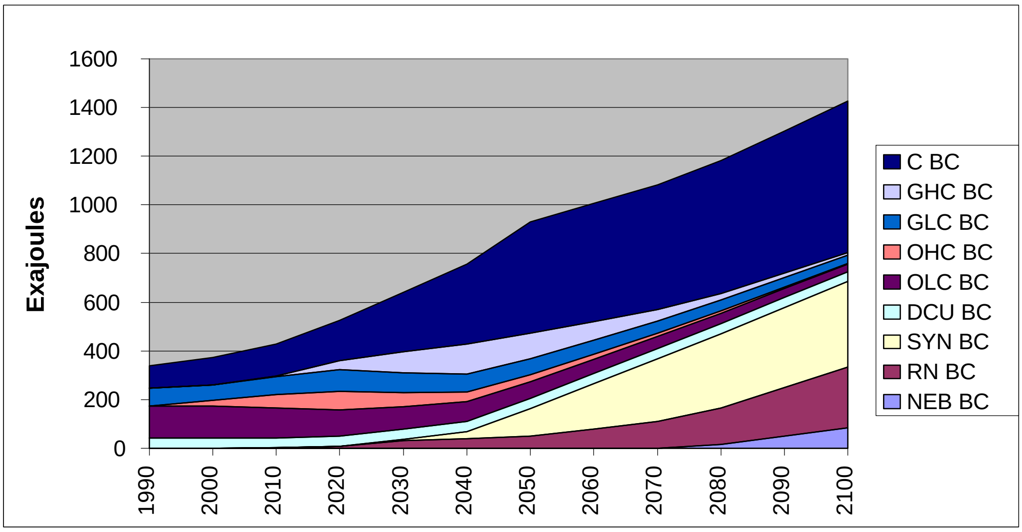

Fig. 7: MERGE non-electric energy generation technologies, Base Case.

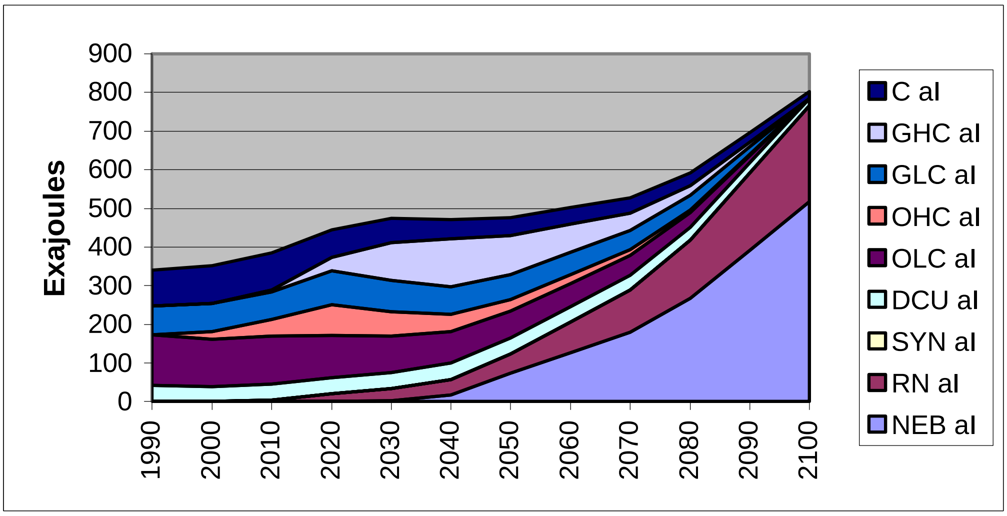

Fig. 8: MERGE non-electric generation technologies, a.I case.

Specular comments can be drawn comparing electric-energy generation technologies in the base case and in the a.I case (figg. 9 and 10 respectively). In the base case, during the 2000-2040 period, fossil fuel based technologies (new extracted gas and new extracted coal, GN and CN in figg. 9

and 10) dominate the market, then they start to decline in favor of a low-cost carbon-free technology (CFLC). On the opposite, in a.I , the same period is dominated by new extracted gas, whose use is increased w.r.t. the base case, and by a high-cost carbon-free (CFHC) technology; new extracted coal contributes only marginally to energy production. This result is in line with expectations, in fact natural gas has a lower emission coe¢cient than coal thus it is preferred because of its lower contribution to temperature increase. At the same, the high damage imposed by a catastrophic event makes it convenient the production of energy also via a carbon-free high-cost technology.

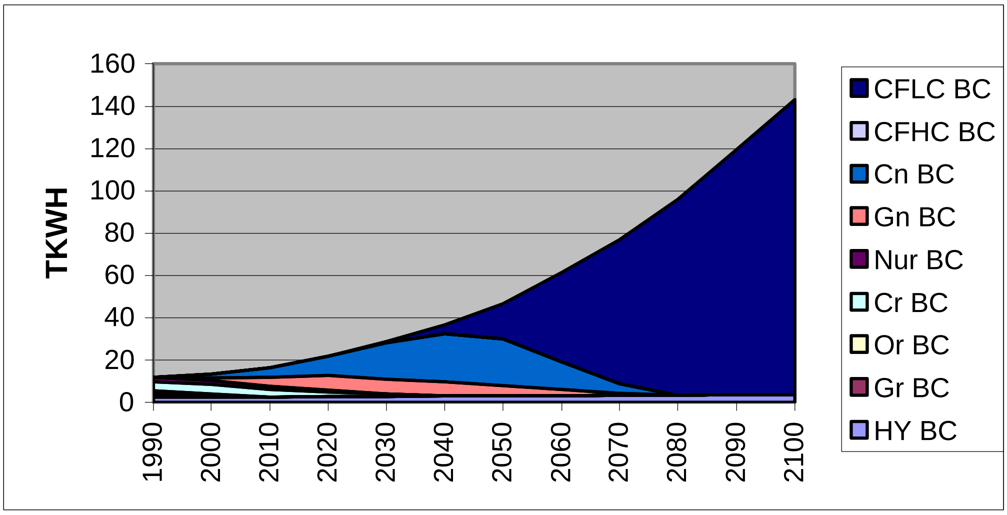

Fig. 9: MERGE electric-energy generation technologies, Base Case.

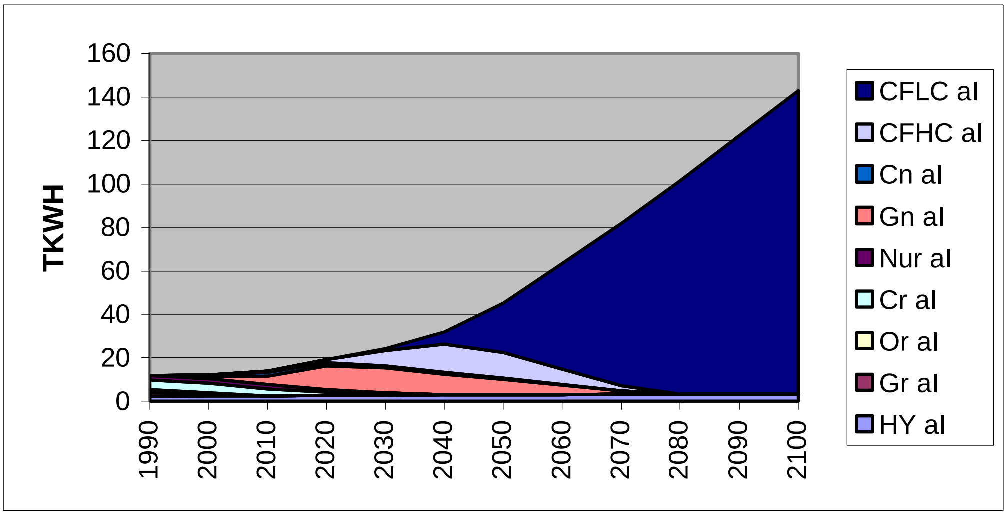

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Fig. 10: MERGE electric-energy generation technologies, a.I case.

A natural extension for the future would be to simulate di¤erent patterns of energy substitutions according to di¤erent costs reduction in the production of backstop technology and world's energy demand expansion[23].

5.3 Exogenous uncertainty in RICE

The implementation of uncertainty in RICE leads to results that could be labelled as paradoxical and totally opposite to what was obtained with CETA and MERGE. In the speci…c the "paradox" concerns three speci…c aspects:

1. The introduction of the probability of a catastrophe as shaped by cases a.I and b.I implies an evident increase of emissions above the no-uncertainty base case (see …g. 11);

2. When post catastrophe utility drops to its 1990 level (reversibility) agents are more prudent than in the case of a "zero" post catastrophic utility (irreversibility) (see …g. 11);

3. With constant hazard rate (i.e. when the probability of an adverse outcome is completely out of the agents' control - cases a.II and b.II

[23] Along these lines, Chakravorty, Roumasset and Tse (1997) develop an endogenous substitution of energy-technologies under certainty.

in …g. 12-), agents behave more conservatively than in the case of a in‡uentiable uncertainty (cases a.I and b.I in …g.11).

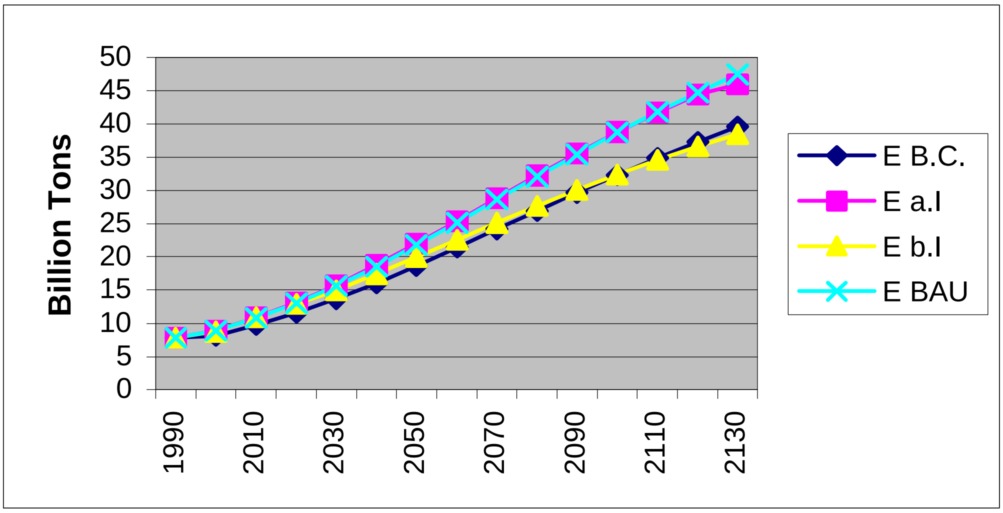

Fig. 11: RICE CO_2 emissions, Base Case, BAU and cases a.I, b.I .

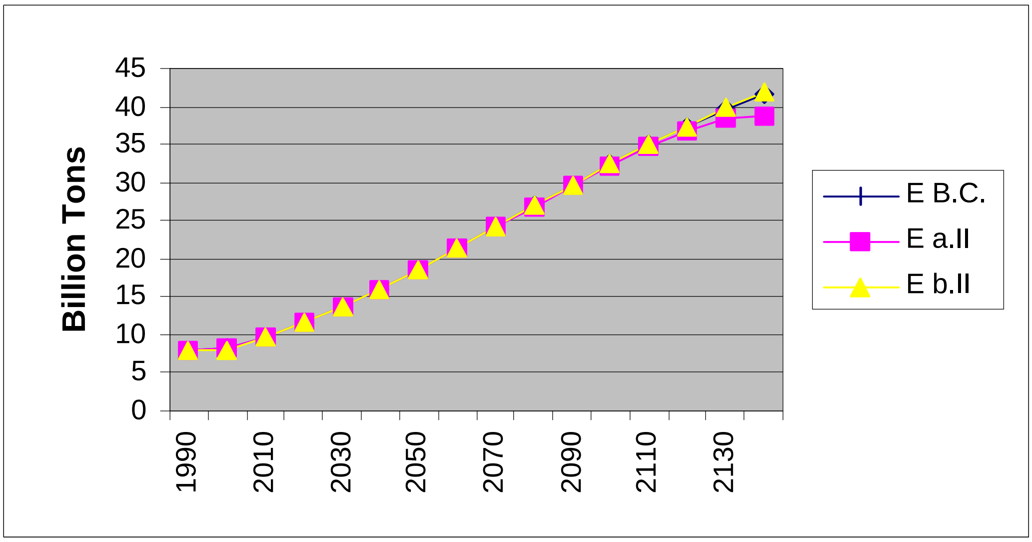

Fig. 12: RICE CO_2 emissions, base case and cases a.II, b.II .

	sr a.I	sr b.I	sr a.II b.II
1990	1,00000	1,00000	1,0000
2000	0,99989	0,99989	0,9984
2010	0,99957	0,99957	0,9968
2020	0,99879	0,99879	0,9952
2030	0,99720	0,99722	0,9937
2040	0,99449	0,99453	0,9921
2050	0,99027	0,99038	0,9905
2060	0,98419	0,98448	0,9889
2070	0,97590	0,97654	0,9874
2080	0,96510	0,96632	0,9858
2090	0,95155	0,95365	0,9842
2100	0,93510	0,93843	0,9827

Tab. 4: RICE surviving probabilities (1-P_{cat}); ' = 0:00159:

Although, according to the theoretical properties, under exogenous (and endogenous) uncertainty short-run emissions may be higher than in the case of no-uncertainty (propositions 3 and 6 in the Appendix), these results can be explained by intuition. As long as emissions are a by-product of the production activity the only way to reduce emissions and to delay a future catastrophic event is trough a decrease in output. However, while the costs of lower emissions in terms of reduced probability of a catastrophe are certain and experienced immediately the bene…ts of such a reduction, due to the inertia of the environmental system in transforming lower emissions in a lower increase in temperature, are experienced far in the future and, most importantly, still uncertain (in fact, if emissions are decreased, as far as temperature is increasing, the probability of the catastrophic occurrence is lowered but not avoided). Given sure current bene…ts from emissions and uncertain future bene…ts of pollution control, RICE just says that the …rst outweigh the second and a less conservative behavior can be observed (see tab. 4 for RICE surviving probability paths).

For how long? Although this being a typical short-run result, in RICE this behavior takes place for all the simulation period considered, that is over the entire next century.

Why we do not observe this result in CETA and MERGE? The answer can be ascribed to three di¤erent aspects: the di¤erent parametrization of the damage functions, the presence of backstop technologies and …nally to the pattern of capital accumulation. Concerning the …rst respect, di¤erent parametrization of the damage function contribute to vary the weight of future damages and bene…ts in‡uencing their optimal balance. On the other hand backstop technologies can o¤er a way to curb emissions alternative to a lower level of economic activity, via the switching from pollution intensive technologies to cleaner ones. Accordingly, even if these cleaner technologies may be costly, due to their environmental e¢ciency they perform strong results in terms of lower emissions at a cost in any case much lower than the cost imposed by a slump in economic activity. In this case a more conservative behavior can be sustained. Finally capital which enters directly the production function, responds positively to uncertainty which may increase the bene…t of present accumulation and production. If it responds too positively it may increase output and then emissions with respect to the no-uncertainty case. We address speci…cally these issues in the next subsection.

The explanation of why a lower post catastrophic penalization induces agents to a safer behavior with respect to a higher penalization, is strictly connected to the previous point. If agents behave safely, but the catastrophe happens anyway, which is always possible with exogenous uncertainty, they will be in the position of having sustained costs fruitlessly. In case of a "zero" post catastrophic utility, the cost of the useless prudent behavior (or in other words the cost of agents' valuation error) is compensated by a zero bene…t, whereas if utility drops to its 1990 level, the cost of the safe useless behavior is compensated by a higher bene…t. Therefore, when a less conservative behavior induced by uncertainty is observed - which recalling the previous arguments means that the current bene…ts from emissions are higher then the uncertain future bene…ts from pollution control - we expect that an increase of future bene…ts (which is exactly what happens if post catastrophic utility moves from zero to the 1990 level) leads to an increase of the willingness to undertake conservation (see proposition 7 in the Appendix).

To conclude, we comment on the results of RICE in a.II and b.II where the feedback between temperature and the catastrophic event is broken. From …g. 12 and according to what was observed in CETA and MERGE, emissions are almost identical to or slightly lower than the base case emissions. At …rst sight it seems strange that a catastrophic event beyond the agents' control may induce more cautious behavior than a controllable one. However, we are comparing a situation in which agents can do nothing to in‡uence the undesired event with one in which they can. In the latter case and only in the latter, agents may judge that it could be optimal for them to postpone emission controls (taking fewer safety precautions in the short run), as they know that in any case they will have the real possibility to act more safely in the future. This possibility obviously does not exist under a completely exogenous path for the catastrophic probability (see eq. (52) in the Appendix).

Again we stress that RICE's counter-intuitive results depend on the definition of uncertainty which is based on the assumption of exogenous upgrading of the probability of an adverse outcome and on the parametrization of the hazard function from Nordhaus' estimate of a 4,8% catastrophe probability in 2090[24]. Modifying this last assumption, it is possible to obtain more "standard" results. To get a conservative behavior and a sound reaction to the level of penalization (greater prudence in response to greater catastrophic damage), we should increase the value of φ_0. In the case at hand, fig. 13 depicts RICE's outcomes setting φ_0 at 0.2, well above the previous value 0.00159, which corresponds to the obviously unrealistic catastrophe probability of 99.8% in 2090 in BAU. Besides the base case, fig. 13 also shows two other cases: A.I and B.I (denoted in capital letters to remind us that we are now dealing with different values of φ_0).

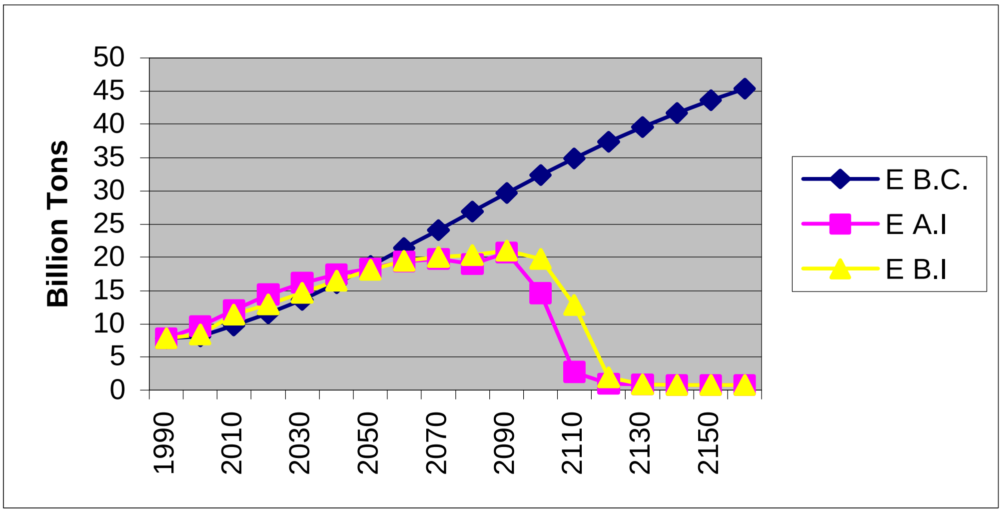

Fig. 13: RICE CO_2 emissions, Base Case and cases A.I, B.I .

[24] As already mentioned 4.8% was the average probability given by a panel of experts. The average opinion of the non-enviromental economists was 0.4%, while the opinion of natural scientists was 12%.

The high probability associated with the undesired event now induces greater caution by economic agents who are willing to reduce emissions below their no-uncertainty level. But again the high costs imposed by e¤ective pollution control (really able to reduce the temperature increase process) makes it optimal to postpone precautionary intervention close to certainty of non-return and to act less conservatively at the outset. Yet, we underline that, due to environmental inertia and a greater amount of short run emissions, the emission path under the silly assumption of $'_0 = 0{:}2$ is still unable to lower the exogenous probability of having a catastrophe before 2090; only after 2100 do we obtain signi…cative reductions. The probability patterns for A.I and B.I are reported in Table 5.

	sr A.I	***sr B.I***
1990	1,00000	1,00000
2000	0,98631	0,98631
2010	0,94775	0,94775
2020	0,85512	0,86013
2030	0,69137	0,70580
2040	0,47681	0,50179
2050	0,26844	0,29704
2060	0,11925	0,14123
2070	0,04085	0,05234
2080	0,01063	0,01478
2090	0,00209	0,00314
2100	0,00033	0,00050

Tab. 5: RICE surviving probabilities ($1\text{-}P_{cat}$), $' = 0{:}2$.

RICE results are not sensitive to the perception of the likelihood of a future adverse outcome generated by an increase in average temperatures. We follow up this issue in the section devoted to the comparison between exogenous uncertainty and endogenous uncertainty.

5.4 Further sensitivity analyses.

In this section we present some further sensitivity analyses, speci…cally focused on the RICE model, aimed at shedding more light on its counterintuitive behavior and to test also the robustness of our intuitions. In the specific we first analyze the reaction of RICE if we change assumptions regarding: the backstop technologies, the endogenous capital accumulation process, the discount rate (this last analysis in the broader context of an inter model comparison) and lately the shape of the damage function.

An indirect possibility to test the reaction of the "structure" of RICE in the presence of backstop technologies is given by its latest development: RICE-98 (Nordhaus and Boyer 1999) which models the existence of a non-carbon backstop technology available at $500 per ton of carbon-energy. RICE-98 is very different from the RICE-96 version in many respects so a deep analysis of the impact of backstop technology would require more accuracy than the one performed here, in any case interesting insights could be found. As shown in fig. 14 if we apply in RICE-98 the exogenous uncertainty defined in the case a:I we can notice how the presence of the backstop technology is not influential in lowering emissions below their optimal level in a non-uncertainty context. On the contrary, exactly as in RICE, the implementation of uncertainty increases emissions above the non-uncertainty case. Although CETA and MERGE offer several different backstop options instead of the only one offered by RICE-98, these results support our intuition that the different behavior of RICE (and of RICE-98) with respect to CETA and MERGE relies more on other aspects than on the presence of backstop options. It's anyway interesting to note the difference among the emissions path of RICE-98 and the emissions of RICE. RICE-98 performs the typical "bell-shaped" emissions pattern of a model with energy-saving technological progress that reduces the amount of carbon emissions per unit of output at given input prices as opposite to the constantly increasing emissions of RICE that does not include this kind of opportunity[25].

[25] It is worth to note that after 2130 RICE-98, in the case of uncertainty, reduces emissions below their level in the case of non-uncertainty.

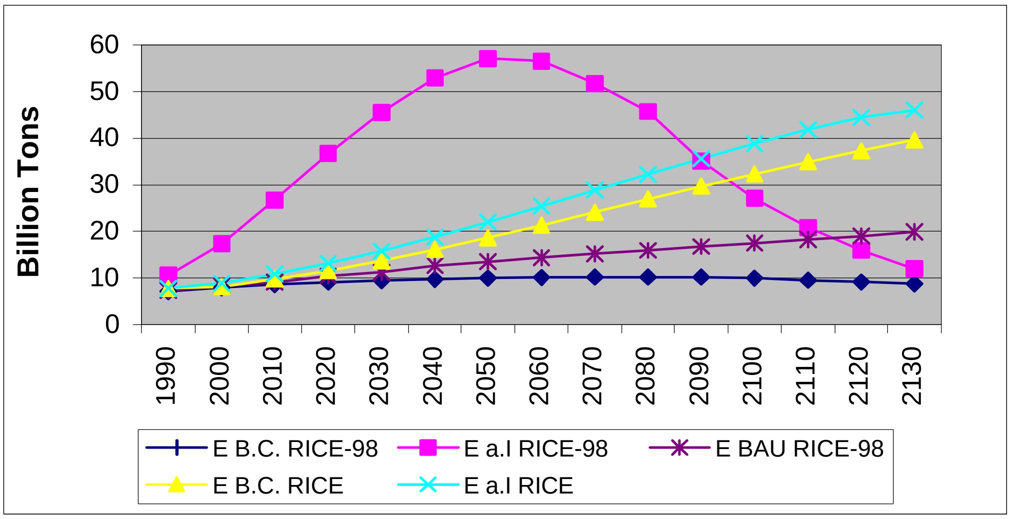

Fig. 14: RICE-98 and RICE CO_2 emissions, B.C., a.I cases (BAU for RICE-98).

A possible way to test the relevance of the capital accumulation process in determining RICE's outcomes is to remove the "capital e¤ect" substituting RICE's Cobb-Douglas production function with a completely exogenous pattern for income/output and to modify accordingly the utility objective function. As represented in …g. 15, when income and thus capital accumulation are exogenous, RICE "behaves well" in response to the implementation of the a:I uncertain framework. Emissions in the presence of uncertainty are now not only lower than in the case of non-uncertainty, but even without any "perverse" e¤ects in the short run.

The intuition behind this result is that, under uncertainty, when income is completely exogenous, costs in terms of pollution control become lower. Costs are represented by lower levels of present consumption which should be compensated in the future by a lower catastrophic probability and then expected environmental damage. If capital accumulation is endogenous, agents can decide to allocate resources among consumption, abatement and investment. Intertemporal investment decisions which increase capital stock, on the one hand decrease present consumption in favor of future consumption, on the other hand increase emissions and temperature. The increase in temperature, increases the probability of the catastrophe in all future instants of time and thus reduces future bene…ts deriving from the sacri…ces in term of postponed consumption. Accordingly agents react decreasing abatement in order to increase present level of consumption (see appendix proposition 8).

Regarding discount rate, in a non-catastrophic-certain world, all the models considered perform a higher (lower) emissions reduction effort if it is decreased (increased). As said, costs are sustained in the present whereas benefits are experienced in the future, thus lowering the discount rate, that corresponds to increase the importance of the future, makes agents more willing to undertake early sacrifices.

We observe the same behavior the uncertainty is completely exogenous, i.e. when there is no feed-back between temperature and catastrophe and the probability of having the adverse event is out of agents' control (cases a:II, b:II). Here agents face exogenous paths for present costs and future benefits, accordingly each time the weight of the future for example increases (the discount rate is lowered), present sacrifices are more sustainable.

Interestingly, models react differently to changes in the discount rate when the probability of a catastrophe is under agents' control as it happens when agents adjust their decisions according to exogenous or endogenous learning processes.

In particular, while CETA and MERGE offer the "typical" result - higher (lower) emissions reduction the lower (higher) the discount rate - RICE gives the opposite outcome. This is due (see appendix section 7.6 for analytical considerations) to the contrasting action of two counteracting effects: the first is the "typical" response to the variation of the discount rate, according to what, emissions decrease (increase) as the discount rate decreases (increases). The second effect is due to capital accumulation, i.e. capital accumulation responds inversely to changes in the discount rate. If it decreases, then investment, capital accumulation, output and finally emissions tend to increase. In CETA and MERGE the first effect outweigh the second, while in RICE the second outweigh the first. In RICE uncertainty that as said may make early sacrifices fruitless allows a strong capital accumulation as the present benefit of production overcomes the future benefit of abatement.

Once again fig. 15 highlights how, removing from RICE this "capital effect", the model reacts traditionally to a lowering of the discount rate even in an uncertain framework. If for example the discount rate is set equal to 1% emissions lie always below their trend if discount rate is set at 3%.

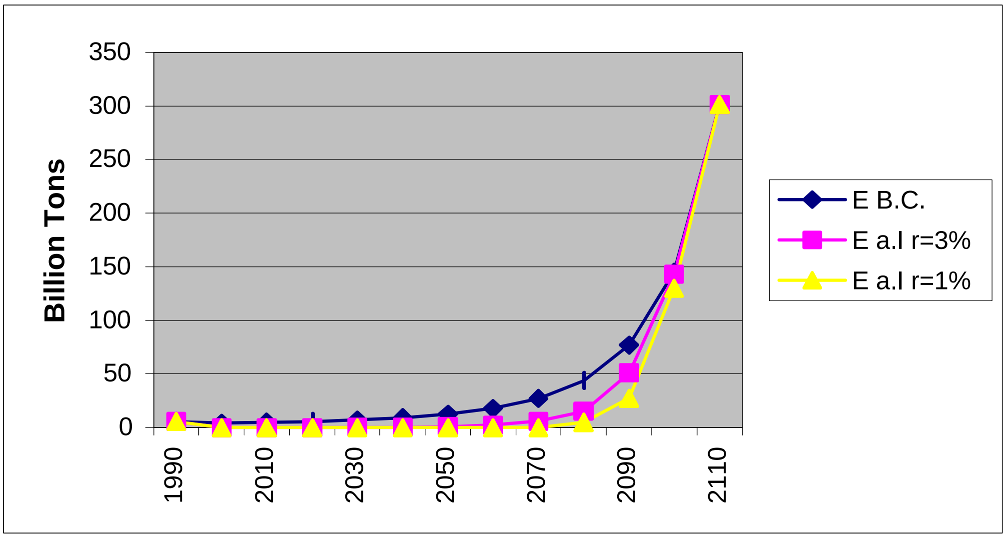

Fig. 15: RICE CO_2 emissions, B.C. and a.I cases with discount rate at 3% and 1%.

To conclude this section we describe how RICE reacts to changes in the shape of the damage function. As an experiment we set its exponent up to 3.5 instead of the original 2.5. As damages are increased, we expect a stronger emissions reduction in both cases of non-uncertainty and uncertainty. As depicted in …g. 16 this e¤ectively happens, but, while in the case of non-uncertainty emissions reductions are evident, in the uncertainty case a:I they are just slightly lower than the previous case in which the exponent was 2.5. Thus, increasing the weight of damages we obtain the result to increase the "paradoxical" outcome of RICE. As an extreme simpli…cation we can say that RICE behavior could be attributed to its great di¢culty in decreasing emissions. If we perform the same experiment with RICE-98, that shares part of the rationales of RICE, but at the same introduces a backstop technology and an additional source of technical progress, we obtain always higher emissions under uncertainty than under non-uncertainty, but in any case less divergent (see …g. 17).

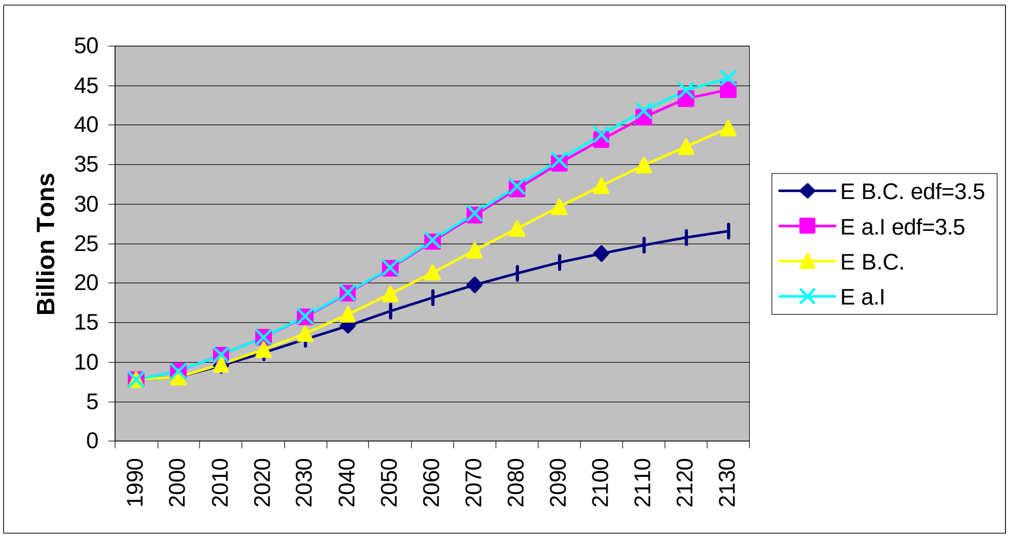

Fig. 16: RICE CO_2 emissions varying the exponent of the damage function.

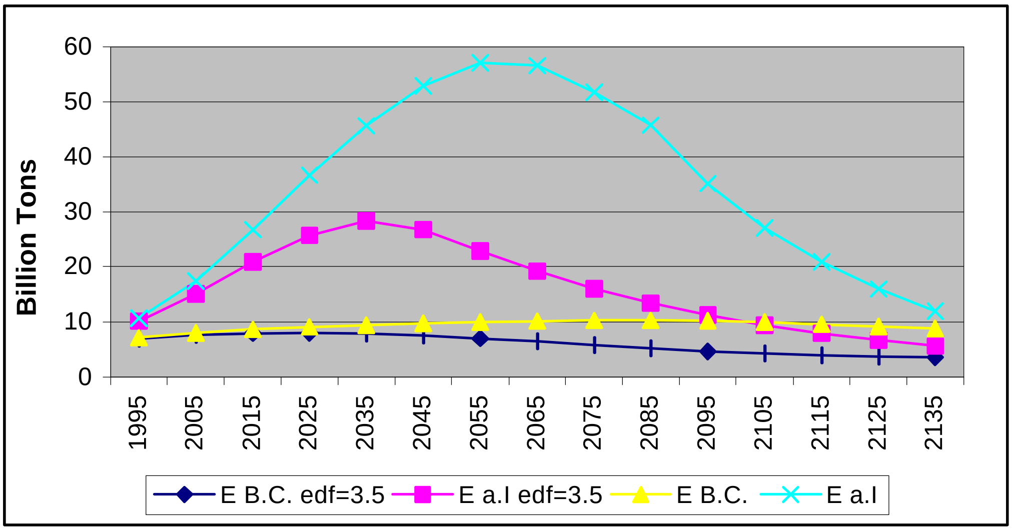

Fig. 17: RICE-98 CO_2 emissions varying the exponent of the damage function.

5.5 Endogenous uncertainty and expected value of learning information on the nature of the catastrophic event.

We apply our structure for endogenous uncertainty, as expressed in (14), to the a.I case only. First of all, we just describe (without showing graphs) the comparison between the results obtained with endogenous uncertainty and the no-uncertainty base case. We note that CETA and MERGE give lower emissions in the short-run, as well as in the long-run, while RICE, whose short-run behavior is “very extended”, shows a less prudent emission path. Long-run emissions are always lower under endogenous uncertainty than long-run emissions under no-uncertainty; nevertheless less conservation could be observed in the short-run (propositions 2 and 3 in the Appendix). Comments on the speci…c results obtained can be directly drawn from the comments previously made regarding exogenous uncertainty.

In the following, we compare the exogenous uncertainty renamed as (Ex) with the endogenous uncertainty named as (En). Not surprisingly, the three models cover all the possible range of results. In RICE the endogenous “learning e¤ect” does not exert any sensible consequence on the emission pattern (see …g. 18), in CETA we observe …rst higher emissions then lower emissions with respect to the (Ex) case (see …g. 19), and …nally in MERGE the endogenous “learning e¤ect” implies lower emissions, though only in the long-run (see …g. 20). Table 6 shows surviving probabilities.

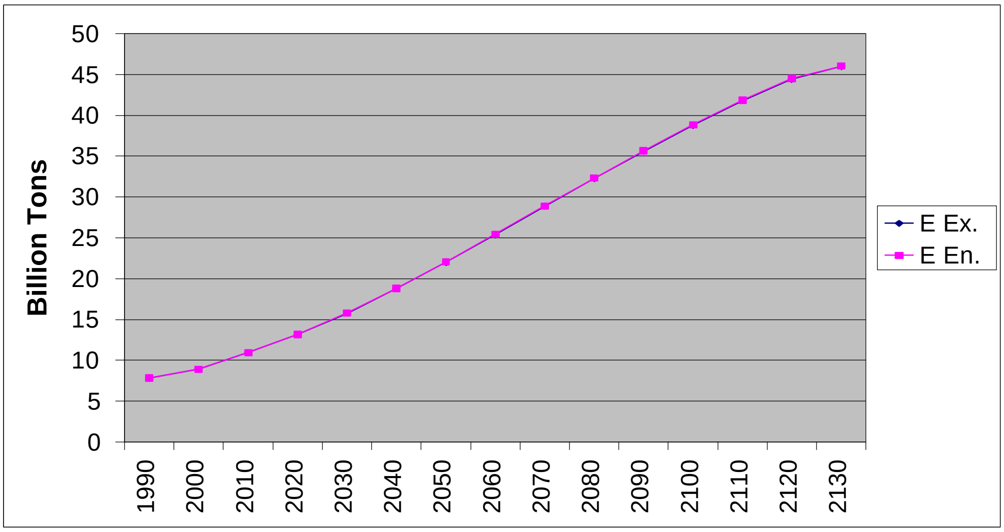

Fig. 18: RICE CO_2 emissions a.I case, with ex. and end. learning.

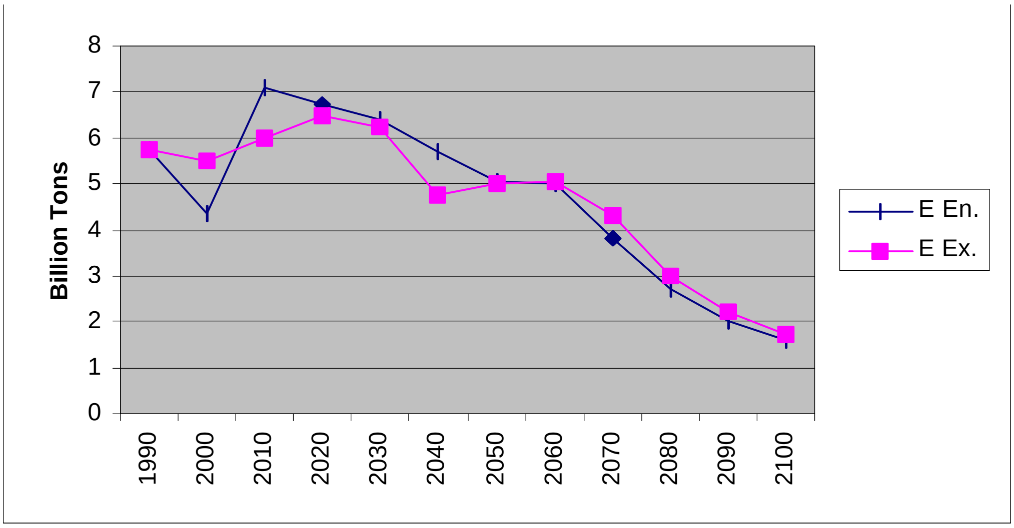

Fig. 19: CETA CO_2 emissions, a.I case with ex. and end. learning.

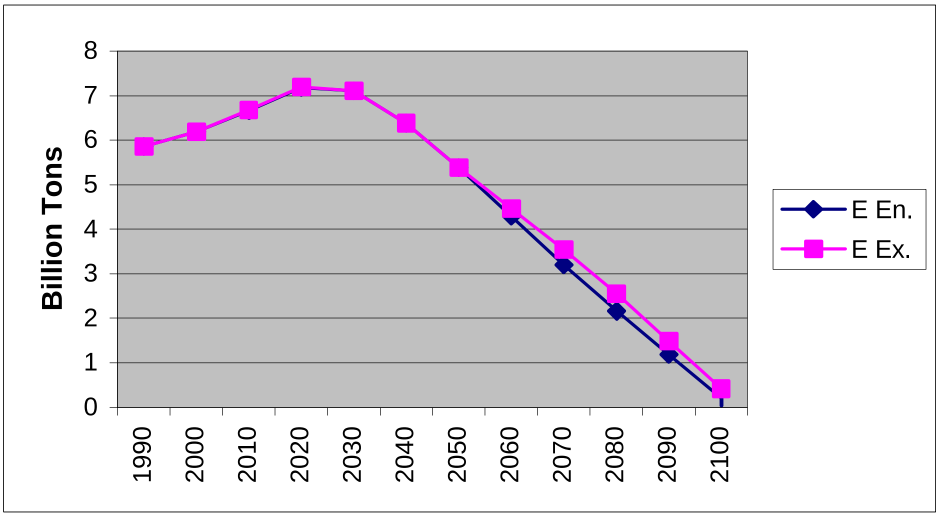

Fig. 20: MERGE CO_2 emissions, a.I case with ex. and end. learning .

	CETA		*MERGE*		*RICE*	
	sr Ex.	*sr En.*	*sr Ex.*	*sr En.*	*sr Ex.*	*sr En.*
1990	1,0000	1,0000	1,0000	1,0000	1,0000	1,0000
2000	1,0000	1,0000	0,9998	0,9998	0,99989	0,99989
2010	0,998	0,998	0,9992	0,9991	0,99957	0,99956
2020	0,996	0,995	0,9980	0,9979	0,99879	0,99868
2030	0,994	0,996	0,9962	0,9961	0,9972	0,99692
2040	0,99	0,994	0,9935	0,9934	0,99449	0,99392
2050	0,985	0,99	0,9900	0,9899	0,99027	0,98934
2060	0,98	0,986	0,9858	0,9857	0,98419	0,98283
2070	0,974	0,98	0,9807	0,9807	0,9759	0,97407
2080	0,966	0,974	0,9751	0,9751	0,9651	0,96279
2090	0,959	0,966	0,9688	0,9690	0,95155	0,9488
2100	0,95	0,959	0,9622	0,9626	0,9351	0,93199

Tab. 6: Surviving probabilities, a.I case, with ex. and end. learning.

Summarizing, the temperature learning process exerts some e¤ects that do not necessarily go in the same unambiguous direction. The outcomes seem to depend on the structure of the models used, in particular on the environmental bene…t and damage functions, as well as on the climate models through which the emissions in‡uence the rate of change of the temperature.

It should be recalled that the difference between endogenous and exogenous uncertainty depends on the policy maker's unawareness of the exact occurrence conditions, rather than on the ecosystem's intrinsic stochastic nature. In fact, although in both cases the emission policy applied by the planner influences the probability of a catastrophic outcome, under endogenous uncertainty, the process describing how information accumulates over time is revised the closer the event becomes. Therefore, given statistical confidence in the learning curve $\varphi(.)$ in (14), dynamic use of the information on the probability of a catastrophe occurring may represent a potential gain for the policy maker. In this respect, the difference between the expected value of discounted utility obtained under endogenous uncertainty and the expected value of discounted utility under exogenous uncertainty, can be taken as an attempt to determine the magnitude of such gain. We term this difference the "Expected Value of Dynamic Information" (EVDI). That is, the amount that an "informed" decision maker would be willing to pay for being certain that a catastrophic event will occur, depends (only) on his ignorance of the critical temperature (pollution) state at which the event occurs; the learning about this critical value is given by $\varphi(.)$.

There is an evident analogy of the EVDI with the EVPI (Expected Value of Perfect Information), proposed by Peck and Teisberg (1993b), which is defined as the amount that a policy maker would be willing to pay for being informed immediately about evolution in the uncertainty. However, the two measures differ as the latter compares, at the beginning of the planning period, the expected values over a distribution of states of the world (where each state is obtained as the world is known before a policy is applied) with the expected value obtained if one and only one policy is adopted across all possible states of the world, without revising the distribution of states over time[26].

[26] Many papers present estimates of the value of information about climate parameters, (see for example Peck and Teisberg, 1993b; Manne and Richels, 1992; Manne, 1996; Nordhaus and Popp, 1997), but only Yohe (1996) and Gjerde et al. (1998) present estimates of EVPI for castrophic losses. In a model of exogenous uncertainty, Gyerde et al. (1998) achieve estimates for EVPI where each state represents a particular estimate of the probability of a catastrophic event ocurring by year 2090 and the states are uniformly distributed.

Table 7

	EVLI
RICE	-184.8 b$
MERGE	16.9 b$
CETA	465.1 b$

The difference yields an estimate of EVDI equal to -184.8 billion dollars for RICE, 16.9 for MERGE and 465.1 for CETA. In spite of its methodological construction, not surprisingly EVDI turns out to be negative for RICE. This result is basically due to two causes: first, the time horizon chosen for ensuring simulation reliability is too short to show up the more conservative behavior induced by endogenous uncertainty in the long run; second, the presence of missing variables in the learning curve. On this point, as we have seen earlier, a more general formulation of $'(:)$ should involve a vector of both exogenous and endogenous explicative variables, $Z(t)$; i.e. $' = '(T(t); \bar{T}(t); Z(t); Z(t))$; statistically significative.

6 Conclusions

This study analyses the consequences for optimal CO_2 emissions when explicit attention is paid to the possibility that a catastrophic event will occur. This is done by introducing a hazard rate function in some well known IAMs dealing with Global Warming: RICE, CETA and MERGE. We distinguish between endogenous and exogenous uncertainty. In both cases we have a learning process, but under endogenous uncertainty temperature (pollution) history matters and the critical temperature level triggering the unfavorable event must lie above all the temperature levels reached in the past, under exogenous uncertainty knowledge about the future possible catastrophic event stems from present information alone.

As already suggested by many studies, our simulation results show that the probability of high-consequence irreversible outcomes can be a good argument for reducing GHG emissions below the level determined by merely considering continuous damage. The final equilibrium point accounting for exogenous or endogenous uncertainty is characterized by lower emissions than the no-uncertainty case. However, taking these catastrophic impacts into account, we show that three of the most popular integrated numerical assessment models respond differently in designing the approach to final equilibrium. Specifically, while CETA and MERGE depict a sudden emission decrease below the no-uncertainty case, RICE points out that less prudent behavior in the short run can be possible. These differences depend crucially on three factors: first, on the ex-ante probabilisation of the catastrophic event (the higher the probability, the greater the caution), second on the reaction of capital accumulation and thus of the production function to uncertainty, third on the possibility to curb emissions efficiently.. In this last respect, it is worth to notice how the introduction of an energy-saving technological progress (see simulation on RICE-98) is effective in reducing emissions even in the RICE model where the presence of a backstop option seems to exert a much lower influence on emissions than in MERGE and CETA. On the contrary, a lower influence on agent's behavior seems to be due to the amount of the catastrophic damage.

A first natural improvement on this study will be to investigate different forms of knowledge accumulation in order to implement a more realistic learning process governing agents' decisions. This will require better parametrization of the hazard rate equation and explicit consideration of conditional probabilities. Another interesting field would be to explore the relationship between uncertainty and technological progress. In our analysis we have shown that, in the presence of backstop technologies, uncertainty works in favor of low polluting production methods. It could be useful to test this outcome in a context of endogenous technical progress (all the models considered incorporate a more or less exogenous technical progress), driven by R&D able to modify the substitution relationships among different inputs.

7 Appendix: Some theoretical results

Although in principle the general model presented in the text can be solved, and the impact of an increase of CO_2 on the climate analyzed, for the purpose of this section to be a theoretical reference we proceed solving a simplified version of it.

First, we do not consider the effect of capital accumulation, then benefit can be represented in terms of consumption C and of the total atmospheric concentration of polluting gas M[27]. Second, we consider a long-run relationship between the concentration of CO_2 equivalent emissions and the emissions, i.e. $M(t) = M(\sum_{i=1}^{N} e_i(t); \beta; b)$; that allows us to simplify (7) as:

$$dT(t) = [g(e(t)) - \sigma T(t)]\,dt; \qquad T(0) = T_0; \tag{16}$$

where $g(e)$ summarizes the impact of total emissions to the long-run temperature, with $g'(e) > 0$; $g(0) = 0$ and $e(t) = \sum_{i=1}^{N} e_i(t)$.[28] In equation (16) the temperature is introduced in order to reflect the depreciation process due to the thermal inertia of oceans. Yet, for sake of simplicity, taking account of (2), (3) and (4) we specify here a separable net benefit function of the form:

$$U_i(C_i(t)) \equiv U_i[\Omega(e_i(t); T(t))Q_i(t)] = B_i(e_i(t)) - D_i(T(t)) \tag{17}$$

where $B_i(e_i)$ is increasing and strictly concave in e_i, $D_i(T)$ is nondecreasing and convex in T and the output is normalized to one, i.e. $Q_i(t) = 1$ for all $t \geq 0$[29]. $D_i(T)$ represents the direct environmental costs of climatic change. This function essentially links the effects from the changes in CO_2 to average global temperature and then evaluates in money terms the various costs imposed on society. We also postulate the existence of a limiting temperature level $\bar{T}$ above which the whole environmental system is bound to collapse.

In our simplified setting a plan consists of the greenhouse gas emissions, $e_i(t)$; for each region and the associated state process, $T(t)$; $t \geq 0$: This plan is feasible if, for all $t \geq 0$, equation (16) is satisfied, $e_i(t)$ is piecewise continuous and nonnegative, and the temperature $T(t) \leq \bar{T}$:

[27] This is equivalent to assume an exogenous stream of gross production over time.

[28] This is the simplest accumulation function for temperature used here for expositional clarity.

[29] Equation (17) can be obtained assuming a linearly separable utility function U togheter with a linearly separable Ω function.

In addition to direct costs, the planner must also consider the consequences of a possible occurrence, at any temperature level $T(t)$; of an environmentally catastrophic event. In other words, as a result of excessive accumulation of CO_2; an undesirable event occurs as soon as $T(t)$ crosses a critical level $X \leq \bar{T}$: If a catastrophic event happens, we observe a sudden change in welfare that we model as an utility loss. Thus, immediately after the event, the post-event benefit equals $V(X)$; where V is the value of optimal plan with a utility loss. Formally:

$$V(T) = \max_{e_1(t);\ldots;e_N(t)} \int_{\tau}^{\infty} e^{-\rho(t-\tau)} \left(\sum_{i=1} B_i(e_i(t)) - D_i(T(t)) - L_i(t) \right) dt \qquad (18)$$

subject to (16), $e_i(t) \geq 0$; $T(t) \leq \bar{T}$ and $T(\tau) = T$: Where ρ is the time rate of discount and $L_i(t) \geq 0$ is the amount of penalty inflicted to each region in a post-catastrophic world. It is worth noting that the nature of catastrophic costs is inherently different and independent from the environmental costs induced by a continuous climate-feedback effect represented by $D_i(T(t))$: Finally, we assume that the policy maker is fully informed of the magnitude of this sudden change in utility.

7.1 Endogenous uncertainty

We assume that X is uncertain; this uncertainty stems from our ignorance concerning the occurrence conditions of a catastrophic event rather than from the intrinsic stochastic nature of the ecosystem. We then specify the policy maker's beliefs at any instant of time by a state-dependent distribution and density function of the critical level X:

Assumption 1. $F(T) = \Pr(X < T)$ and $f(T) \equiv dF(T)/dT$ where $\bar{T}$ is the upper support of the distribution of X while T_0 is the lower support.

Letting τ represent the event's occurrence time, the distribution of X induces a distribution on τ: Given that the catastrophe has not occurred at time $t = 0$, by assumption 1 the expected benefit generated by emissions control is:

$$W(T_0) = \max_{e_1(t);\ldots;e_N(t)} E\left[\int_0^{\tau} e^{-\rho t} \left(\sum_{i=1} B_i(e_i(t)) - D_i(T(t)) \right) dt + e^{-\rho\tau} V(T(\tau)) \right] \qquad (19)$$

subject to (16), (18), $e_i(t) \geq 0; i = 1, \ldots, N$; $T(t) \leq \bar{T}$ and $T(0) = T_0$. The expectation operator is taken with respect to the random variable τ.[30]

From (19), we allow for utility in the post-event world to be different from utility in a pre-event world. If we consider irreversible events the utility will drop to zero for all time after the catastrophe. For these events $B_i(e_i(t)) - D_i(T(t)) - L_i(t) = 0$, and the social welfare reduces to zero $V = 0$ (Clark and Reed, 1994). If, on the contrary, we assume that $L_i(t) = 0$ for all $t \geq 0$ there are no disutility costs associated with the catastrophic event, and W simply reduces to the nonevent, "n", case. That is, (19) and (18) coincide and are equal to the value of the optimal plan when no event can interrupt.

$$W^n(T_0) \equiv V(T_0) = \max_{e_1(t),\ldots,e_N(t)} \int_0^\infty e^{-\rho t}\Big(\sum_{i=1} B_i(e_i(t)) - D_i(T(t))\Big)dt \qquad (20)$$

As the process evolves over time, the distribution of the trigger value X, and therefore of the stopping time τ, is modified to account for incoming information. At each time t, the distribution of X, given that the event has not yet occurred, depends on the history of the temperature $T(t)$ up to t. In particular X must lie above $\max_{s \in [0,t]} \{T(s)\}$; otherwise the event would have occurred at some time s before t. As pointed out by Tsur and Zemel (1994) rather significantly, this complicates the optimal emission decision, since the expected benefit in (19) depends on all history up to time t. Fortunately, the program can be greatly simplified as the temperature trajectory $T(t)$ evolves monotonically in time. At least one of the optimal $T(t)$ trajectories corresponding to (19) is indeed monotonic, so that we are allowed to restrict our attention to it[31], [32]. If the temperature evolves monotonically nondecreasing we get $\max_{s \in [0,t]} \{T(s)\} = T(t)$; while if it evolves monotonically nonincreasing $\max_{s \in [0,t]} \{T(s)\} = T_0$.

[30] Tsur and Zemel, 1996 refer to (19) as the general uncertainty problem.

[31] In the absence of decay rate σ, the temperature level cannot decrease. Therefore, the information that arrives over time cannot affect decisions prior to the event's occurrence: the temeperature trajectory is always monotonic nondecreasing (Long, 1975).

[32] A detailed proof of the monotonicity property is found in Tsur and Zemel (1994, p.407). As the utility function, $\sum_{i=1}^{N} B_i(e_i(t)) - D_i(T(t)))$ and the distribution function, $F(T)$, do not depend explicitly on time, the program is autonomous. Knowing at any state level along the optimal trajectory that the catastrophic event has not yet occurred cannot motivate a change in early decisions. That is, prior the occurrence there is no reason to modify the optimal plan.

If a nonincreasing $T(t)$ trajectory is chosen, it is known at the outset that the catastrophic event will never occur and the objective function in (19) reduces to the nonevent case (20). On the other hand, for a nondecreasing temperature trajectory, the distribution of the occurrence time τ; induced by that of X; is given by:

$$1 - F_\tau(t) \equiv \Pr(\tau > t \mid \tau > 0) = \Pr(X > T(t) \mid X > T_0) \equiv \frac{1 - F(T(t))}{1 - F(T_0)};$$

with density:

$$f_\tau(t) = \frac{f(T(t))\,[g(e(t)) - \sigma T(t)]}{1 - F(T_0)}.$$

The hazard rate associated with τ is given by:

$$\psi_\tau(t) \equiv \frac{f_\tau(t)}{1 - F_\tau(t)} = \lambda(T(t))\dot{T}(t); \quad \text{where} \quad \lambda(T(t)) = \frac{f(T(t))}{1 - F(T(t))} \tag{21}$$

Making use of an indicator function that assumes the values one or zero depending on whether the argument is true or false, we are able to write the expectation in (19) as an infinite horizon integral:

$$E\left[\int_0^\infty e^{-\rho t}\left(\sum_{i=1}^{N} B_i(e_i(t)) - D_i(T(t))\right) I(\tau > t)\,dt + e^{-\rho\tau} V(T(\tau))\right]$$

As $E[I(\tau > t)] = 1 - F_\tau(t)$; the expectation can be rewritten in the simpler form:

$$\int_0^\infty e^{-\rho t}(1 - F_\tau(t))\left(\sum_{i=1}^{N} B_i(e_i(t)) - D_i(T(t))\right) dt + \int_0^\infty e^{-\rho t} f_\tau(t))V(T(t))\,dt$$

Moreover, as long as $1 - F_\tau(t) = \frac{1 - F(T(t))}{1 - F(T_0)}$; by defining $\Omega(T(t)) = -\log\left[\frac{1 - F(T(t))}{1 - F(T_0)}\right]$ and $d\Omega(T(t)) \equiv \lambda(T(t))dT(t)$; the planner's maximization problem for a nondecreasing temperature trajectory becomes:

$$W(T_0) = \max_{e_1(t),\ldots,e_N(t)} \left\{ \int_0^\infty e^{-(\rho t + \Omega(T(t)))} \left(\sum_{i=1}^{N} B_i(e_i(t)) - D_i(T(t))\right) dt \right.$$

$$+\int_0^{\infty} e^{-(\rho t+\Omega(T(t)))}\lambda(T(t))(g(e(t))-\sigma T(t))V(T(t))dt \Bigg\} \qquad (22)$$

subject to (16), (18), $e_i(t)\geq 0;\ i=1;\ldots;N;\ T(t)\leq \bar{T}$ and $T(0)=T_0$.[33] As $\lambda(T(t))$ represents the hazard function in terms of probability distribution of the temperature's critical level, we may refer to $1-F_\tau(t)=e^{-\Omega(T(t))}$ as the survivor function which defines the probability of experiencing no catastrophe from the initial date up to time t. If we also assume a nondecreasing hazard function, i.e. $\lambda'(T(t))\geq 0$; with nondecreasing trajectories, the probability that the catastrophic event will occur if the temperature is slightly increased from $T(t)$ to $T(t)+dT(t)$; given that it has not occurred at level $T(t)$; does not decrease with T: An increase in the average global temperature above the pre-industrial level means that the policy maker believes more strongly in the occurrence of a catastrophic event.

The interpretation of (22) is straightforward, it represents the weighted sum of the stream of pre-catastrophe and post-catastrophe benefits over the entire planning horizon. As already mentioned, it should be stressed that (22) differs with respect to (19), the two coincide only for nondecreasing temperature trajectories.

Finally, it is important to observe that from (21) the hazard rate of the event occurrence time, $\psi_\tau(t)$; does not depend on the current level of temperature $T(t)$ alone, but also on its rate of change $\dot{T}(t)$: When $g(e(t))\leq \sigma T(t)$ the temperature level does not increase and the hazard rate must vanish as the probability of a catastrophe drops to zero. Then the optimal policy becomes the riskless one.

7.2 Characterization of the optimal emissions paths

For the sake of simplicity and without losing in generality we add the following assumption:

Assumption 2: The overall impact of emissions on the temperature is quasi-linear, i.e. $g'(e(t))=1$.

The current-value Hamiltonian and Lagrangian functions for the problem (22) are as follows:

[33] Tsur and Zemel (1996) refer to this as the auxiliary problem to distinguish it from the general uncertainty case (19).

$$H(T(t);e_1(t);\ldots e_N(t);\mu(t)) = e^{-\Lambda(T(t))}\Big[\sum_{i=1}^{N}(B_i(e_i(t)) - D_i(T(t)))$$

$$+\lambda(T(t))(g(e(t)) - \sigma T(t))V(T(t))\Big] - \mu(t)[g(e(t)) - \sigma T(t)] \tag{23}$$

and:

$$L(T(t);e_1(t);\ldots e_N(t);\mu(t);\alpha(t);\gamma_i(t);\ldots\gamma_N(t)) = H(t)+\alpha(t)(\bar{T} - T(t))+\sum_{i=1}^{N}\gamma_i(t)e_i(t) \tag{24}$$

where $-\mu(t)$ is the shadow cost or the cooperative valuation of the increase in temperature, and $\alpha(t);\ \gamma_i(t);\ i = 1,\ldots,N$ are the current value Lagrange multipliers associated with the constraints $T(t) \leq \bar{T}$ and $e_i(t) \geq 0;\ i = 1,\ldots,N$ respectively.

Necessary conditions for the optimal emissions paths, assuming interior solutions, include the usual first order conditions $\partial L/\partial e_i = 0$; giving[34]:

$$e^{-\Lambda(T(t))}[B_i'(e_i(t)) + \lambda(T(t))V(T(t))] = \mu(t) - \gamma_i(t);\quad i = 1,\ldots,N. \tag{25}$$

and the dynamic condition of the costate variable $-(\dot{\mu} - \rho\mu) = -\partial L/\partial T$; which yields:

$$\dot{\mu}(t) = \mu(t)(\rho+\sigma) - e^{-\Lambda(T(t))}\Big\{\lambda(T(t))\sum_{i=1}^{N}(B_i(e_i(t)) - D_i(T(t))) \tag{26}$$

$$+\lambda(T(t))V(T(t))\Big[\lambda(T(t))\dot{T}(t) + \sigma\Big] + \sum_{i=1}^{N}D_i'(T(t))$$

$$-\dot{T}(t)[\lambda'(T(t))V(T(t)) + \lambda(T(t))V'(T(t))]\Big\} - \alpha(t);$$

[34] It is important to note that for nondecreasing temperature trajectories the emission policy characterized by (25) and (26) is an optimal policy conditional upon the catastrophic event not having been yet occurred.

Furthermore, the following complimentary slackness conditions should be satisfied:

$$\alpha(t) \geq 0; \ \alpha(t)(\bar{T} - T(t)) = 0; \ \gamma_i(t) \geq 0; \ \gamma_i(t)e_i(t) = 0; \ i = 1, ..., N. \quad (27)$$

and also the Arrow type transversality condition at infinity:

$$\lim_{t \to \infty} e^{-\rho t} \mu(t) T(t) = 0 \quad (28)$$

Conditions (25) describe the optimal emissions path for each region by balancing short-term social benefits against long-term social costs. Notice that these marginal benefits are equalized across regions. The expected marginal increase in benefits due to a higher emissions rate must equal the expected change in the social shadow cost of the increase in temperature induced by higher emissions. However, as the policy-maker cannot predict if and when the catastrophic event will occur, each region's marginal social benefits will account also for the future utility losses time the cumulated probability of this event happening in the interval $(t, t + dt)$, i.e. $e^{-\Lambda(T(t))}\lambda(T(t))V(T(t))$[35].

Focusing on the interior solution for the model (with $e_i(t) > 0; \ i = 1, ..., N$ and $T(t) > 0$); from the first order conditions (25) we can evaluate the derivatives of the Hamiltonian with respect to $e_i(t)$, $T(t)$ and $\mu(t)$:

$$\frac{\partial^2 H}{\partial e_i^2} = e^{-\Lambda(T(t))} B_i''(e_i(t)) < 0$$

$$\frac{\partial^2 H}{\partial e_i \partial \mu} = -1 < 0$$

$$\frac{\partial^2 H}{\partial e_i \partial T} = e^{-\Lambda(T(t))} \Big\{ -\lambda(T(t))[B_i'(e_i(t)) + \lambda(T(t))V(T(t))]$$

$$+ [\lambda'(T(t))V(T(t)) + \lambda(T(t))V'(T(t))] \Big\} ?$$

[35] The unconditional probability of the event occurring in the interval $(t, t + dt)$ is the product of the probability that no event has occured by time t, $e^{-\Lambda(T(t))}$, and the probability of an event occurring in the interval, $\lambda(T(t))$.

It is clear that the first two derivatives are always negative. However, the sign of H_{eT} is ambiguous.. A higher temperature increases the probability of a catastrophe (i.e. the survivor function, $e^{-\Lambda(T(t))}$; will decrease), which reduces current expected marginal benefits, but this increase reduces future utility losses induced by the marginal change in the policy-maker beliefs about the catastrophe. In conclusion, the first order conditions above give the short-run derived demand functions for emissions as implicit functions of T and μ; i.e. $e_i(t) = e_i(T(t); \mu(t))$, $i = 1; ::::N$; with:

$$\frac{de_i(t)}{d\mu(t)} = -\frac{H_{ee}}{H_{e\mu}} < 0; \qquad \text{and} \qquad \frac{de_i(t)}{dT(t)} = -\frac{H_{ee}}{H_{eT}} \; ?$$

Equation (26) represents the dynamic of the shadow valuation of the increase in temperature along the optimal emissions paths. Higher emission rates in period t will increase the average global temperature in all future periods and, then, also the probability distribution for a catastrophe. A higher future probability of a catastrophe makes utility losses more likely and hence bring social costs. From equations (27), (26) and the transversality condition (28), the shadow cost of an increase in the temperature at time t can be expressed as the expected weighted sum of all future discounted marginal costs experienced by all regions. That is:

$$\mu(t)e^{\Lambda(T(t)} = e^{(\rho+\sigma)t+\Lambda(T(t)} \Bigg\{ \int_t^{\infty} e^{-((\rho+\sigma)s+\Lambda(T(s))} \Bigg[\lambda(T(s)) \sum_{i=1} [B_i(e_i(s)) - D_i(T(s))] \tag{29}$$

$$+ \sum_{i=1} D_i'(T(s)) + \lambda(T(s))V(T(s))[\lambda(T(s))\dot{T}(s) + \sigma]$$

$$- \dot{T}(s)[\lambda'(T(s))V(T(s)) + \lambda(T(s))V'(T(s))] \Bigg] ds \Bigg\}$$

Several effects determine the shadow cost of the temperature. Apart for the usual marginal damages associated with an increase in temperature in all future dates independent of whether a catastrophe occurs or not, there are terms indicating the costs associated with the event occurrence. The first term, for example, gives the discounted sum of the pre-catastrophe benefits time the conditional probability that the catastrophe occurs between t and t + dt. In other words, it indicates the costs, in term of utility loss, associated with the marginal change in the policy-maker beliefs about future occurrence of a catastrophe. The third and fourth terms share the same meaning about post-catastrophe utility (Gjerde et al, 1998).

Let now us look at the situation in which $L_i(t) = 0$ for all $t \geq 0$. In this case there are no disutility costs associated with the catastrophe and W simply reduces to the nonevent, "n", case. That is, (25) and (26) reduce to:

$$B_i'(e_i(t)) = \mu^n(t) - \gamma_i(t); \qquad i = 1, 2, \ldots, N; \tag{30}$$

and:

$$\dot{\mu}^n(t) = \mu^n(t)(\rho + \sigma) - \sum_{i=1} D_i'(T(t))) - \alpha(t) \tag{31}$$

When deciding on optimal emissions paths, the policy maker no longer considers the possibility of a catastrophic event since there are no costs associated with this event. The policy maker behaves as though he were in a deterministic world paying attention only to the current climate-feedback effects.

7.3 Long-run emissions paths

In this section we concentrate the analysis on the equilibrium (or steady state) of the optimal emission policy under uncertainty. The analysis closely follows that of Tsur and Zemel (1996, 1998). Together with assumptions 1 and 2 we add:

Assumption 3: Although each region has different damage functions they share the same benefit function, i.e. $B_i(e_i(t)) = B(e_i(t))$ for $i = 1, \ldots, N$.

Assumption 3 together with condition (25) imply that $e(t) = Ne(t)$. An equilibrium state refers to the T member of the $(T; \mu)$ pair for which $\dot{\mu}(t) = \dot{T}(t) = 0$. Recalling that (22) and (19) coincide only for nondecreasing temperature trajectories, and that if a nonincreasing $T(t)$ trajectory is chosen the optimal control is equivalent to the nonevent case, Tsur and Zemel (1996, pag. 1301) prove the following proposition:

Proposition 1 : Let $T(t)$ be the optimal temperature process corresponding to the (endogenous) uncertain problem (22). Then: (i) $T(t)$ increases while passing through temperature levels below $\hat{T}^e$; (ii) $T(t)$ decreases while passing through temperature levels above $\hat{T}^n \geq \hat{T}^e$; (iii) the interval $[\hat{T}^e; \hat{T}^n]$ consists of the steady state of $T(t)$:

The temperature level $\hat{T}^n$ is the nonevent unique steady state defined by:

$$\begin{cases} \hat{T}^n = \hat{T}^\infty & \text{if} \quad L^n(\hat{T}^\infty) > 0 \\ \hat{T}^n = 0 & \text{if} \quad L^n(0) < 0 \\ L^n(\hat{T}^n) = 0 & \text{otherwise} \end{cases}$$

where $L^n(T) = (\rho + \sigma)B'(\sigma T/N) - \sum_{i=1}^{N} D'_i(T)$ is a function obtained from (16), (30) and (31) setting $\mu^n(t) = \lambda(t) = 0$; and recalling that emissions at a positive steady state are $e = \sigma T/N > 0$; hence $\gamma_i(t) = 0;\ i = 1; \ldots N$. As long as B is increasing and concave in $e = \sigma T/N$ and D_i are nondecreasing and convex in T, then $L^n(T)$ is decreasing in T with a unique possible root in the closed interval $[0; \hat{T}^\infty]$.

On the other hand, $\hat{T}^e$ is the unique steady state of problem (22), defined by:

$$\begin{cases} \hat{T}^e = \hat{T}^n & \text{if} \quad L^e(\hat{T}^n) > 0 \\ \hat{T}^e = 0 & \text{if} \quad L^e(0) < 0 \\ L^e(\hat{T}^e) = 0 & \text{otherwise} \end{cases}$$

where $L^e(T) = L^n(T) - \lambda(T)\rho[W(T) - V(T)]$ and $W(T) \equiv [NB(\sigma T/N) - \sum_{i=1}^{N} D_i(T)]/\rho$ represents the benefit obtained under the steady state policy $e = \sigma T > 0$ without the event occurring. Since $\lambda(T)$ is nondecreasing and the undesirability of the catastrophic event implies $W(T) - V(T) \geq 0$; we also obtain that $L^e(T)$ decreases. In particular with $\lambda(\hat{T}) > 0$ and $W(\hat{T}) - V(\hat{T}) > 0$; we find that $L^e(\hat{T}) < L^n(\hat{T}) = 0$ and then $\hat{T}^e < \hat{T}^n$[36]: Having characterized the nonevent plan and the one of problem (22), the optimal temperature trajectory under uncertainty can also be characterized, as:

- For all starting temperatures $T_0 > \hat{T}^n$, the optimal trajectory $T(t)$ is the nonevent optimal trajectory which decreases asymptotically toward $\hat{T}^n$;

[36] Indeed, even when $\hat{T}^n = \hat{T}^\infty$; the steady state of the two cases are different as long as $L(\hat{T}^\infty) - \lambda(\hat{T}^\infty)\rho[W(\hat{T}^\infty) - V(\hat{T}^\infty)] < 0$:

- For all starting temperatures $T_0 < \hat{T}^e \leq \hat{T}^n$; the optimal trajectory $T(t)$ is found by solving (22), which increases asymptotically toward $\hat{T}^e$;

- For all starting temperatures $T_0 \in [\hat{T}^e; \hat{T}^n]$; the optimal trajectory is to remain constant at T_0 forever.

In words, it is seen that the optimal process under uncertainty converges to the boundaries $[\hat{T}^e; \hat{T}^n]$ from any initial state outside this interval and remains constant when initiated at any state within the interval. As stated in proposition 1 and discussed in Tsur and Zemel (1996), the steady state interval is due to the difference between the functions $L^e(T)$ and $L^n(T)$; that is by the term $\lambda(T)\rho[W(T) - V(T)]$ which measures the expected benefit loss from an event occurring immediately following a policy that implies emissions above the steady state level.

To conclude, from (25), (26) and using the definition of $L^e(T)$; we are able to rescue the shadow cost of temperature at every steady state:

$$\Delta^e e^{-\Lambda(\hat{T}^e)} = \frac{\left[\lambda(\hat{T}^e)(NB(\sigma\hat{T}^e/N) - \sum_{i=1}^{N} D_i(\hat{T}^e)) + \sum_{i=1}^{N} D_i'(\hat{T}^e))\right]}{\rho + \sigma} \qquad (32)$$

Finally, as long as the equilibrium interval is a consequence of an occurrence hazard rate depending on the pollution history as well as on its current trend, an immediate implication of proposition 1 is that:

Proposition 2 : For a starting temperature $T_0 < \hat{T}^n$; endogenous uncertainty always implies more conservation in the long-run.

7.4 Short-run emissions paths

While the effects of the threat of occurrence of environmental catastrophes always entails more conservation in the long-run, we show in this section that, under a quite general condition, uncertainty may induce less conservation in the short-run. The general condition is that the instantaneous net benefits resulting from an increase in temperature is much greater than the social cost of such an increase. This can only occur in the early stages of the planning horizon when the probability associated with a catastrophic event is very low.

For nondecreasing temperature trajectories a full analysis of each region's optimal path of emissions can be derived substituting (26) in (25). While assumptions 1 and 2 still hold, in this section we add:

Assumption 4: The post-catastrophe benefit function drops to zero (irreversible event), i.e. $B_i(e_i(t)) - D_i(T(t)) - L_i(t) = 0.$

By assumption 4, as in Cropper (1976), Clarke and Reed (1994) and Tsur and Zemel (1998), the utility will drop to zero for all times after the catastrophic event, then $V = 0.$ With these simplifying assumptions, (25) and (26) reduce to:

$$e^{-\Lambda(T(t))}B_i'(e_i(t)) = \mu(t) - \gamma_i(t); \qquad i = 1, 2, \ldots, N; \tag{33}$$

and:

$$\dot{\mu}(t) = \mu(t)(\rho + \sigma) - e^{-\Lambda(T(t))}\left\{\lambda(T(t))\sum_{i=1}^{N}(B_i(e_i(t)) - D_i(T(t))) + \sum_{i=1}^{N} D_i'(T(t))\right\} - \alpha(t). \tag{34}$$

The first term on the r.h.s. of (34) contributes to growth of $\mu(t)$; while the second term contributes to its decline over time. An additional increase in temperature later on in the future inflicts marginal damage over a shorter horizon. Thus, it is possible for the shadow cost to decline over time. Uncertainty about future climate-feedbacks increase the marginal damage which consists of two terms. While the second term inside the bracket is the usual marginal damage following an increase in temperature the first one is the expected cost, in term of utility loss, by a marginal change in the policy maker beliefs about the event occurs in the interval $(t, t + dt)$. Together, the two terms, multiplied by the survivor function up to t, form the current value of the expected marginal loss in utility induced by higher temperature. Rearranging (34) we obtain:

$$\dot{\mu}(t) = \dot{\mu}^n(t) - \left[e^{-\Lambda(T(t))}\lambda(T(t))\sum_{i=1}^{N}(B_i(e_i(t)) - D_i(T(t))) \right. \tag{35}$$

$$-\left[1 - e^{-\Delta(T(t))}\right] \sum_{i=1} D_i'(T(t)) \Bigg] - \alpha(t).$$

where $\dot{\mu}^n(t)$ is the time derivative of the shadow cost corresponding to the nonevent case, (31).

Furthermore, as $H_{eT} = -e^{-\Delta(T(t))}\lambda(T(t))B_i'(e_i(t))$; in this case the signs of the derivatives of the demand functions for emissions in each region with respect to T and μ are as expected. Both an increase in the temperature and an increase in the social shadow price of the temperature push emissions down. That is:

$$\frac{de_i(t)}{d\mu(t)} = -\frac{H_{ee}}{H_{e\mu}} < 0; \qquad \text{and} \qquad \frac{de_i(t)}{dT(t)} = -\frac{H_{ee}}{H_{eT}} < 0$$

What can we say about the slope of the emission path in each region around zero? To answer this question we first differentiate (33) with respect to t:

$$\frac{de_i(t)}{dt} = \frac{\left[\dot{\mu}(t) + \mu(t)\lambda(T(t))\dot{T}(t)\right]}{H_{ee}} \tag{36}$$

To obtain (36), we have considered that the optimal temperature trajectory under uncertainty always increases when initiated at $T_0 < \hat{T}^e$; so that $e_i(t) > 0$ gives $\gamma_i(t) = 0$ for all $t > 0$. Valuing (36) at $t = 0$ and taking account that $\Delta(T_0) = 0$ and that, as $T_0 < \bar{T}$, $\alpha(0) = 0$, we get[37]:

$$\left.\frac{de_i(t)}{dt}\right|_{t=0} = \frac{\dot{\mu}^n(0)}{B_i''(e_i(0))} - \frac{\lambda(T_0)H(T_0)}{B_i''(e_i(0))} \tag{37}$$

where $H(T_0; e_1(0); ::::e_N(0); \mu(0)) \equiv \sum_{i=1}^{N}(B_i(e_i(0)) - D_i(T_0)) - \mu(0)\dot{T}(0)$ is the Hamiltonian evaluated at zero. The overall sign of the emission path around zero is given as a combination of the two terms on the r.h.s. of (37). The first term is the slope of the optimal emission path when no event can occur. Its sign depends on the fact that the discounted sum of marginal

[37] All the models considered share high levels of initial emissions, $e_i(0) \gg 0$; for all regions.

damage that an increase of temperature inflicts over the future is lower or greater than the discounted value of marginal damage at zero. That is:

$$\dot{\mu}^n(0) = (\rho + \sigma)\int_0^{\infty} e^{-(\rho+\sigma)t}\sum_{i=1}^{N} D_i'(T(t))dt - \sum_{i=1}^{N} D_i'(T_0) \qquad (38)$$

If, for example, we had a linear damage function such that $D_i'(T(t))$ were constant and equal to $D_i'(T_0)$ for all $t > 0$; the shadow cost of temperature would reduce to $\frac{1}{(\rho+\sigma)}\sum_{i=1}^{N} D_i'(T_0))$ and then $\dot{\mu}^n(0) = 0$; i.e. the optimal shadow cost would be flat. If $D_i'(T(t)) > D_i'(T_0)$ for all $t > 0$; the first term in (38) turns out to be larger than $\sum_{i=1}^{N} D_i'(T_0)$; and the shadow cost path would be upward sloping. Given a convex damage function, an optimized nondecreasing temperature trajectory ensures $D_i'(T(t)) > D_i'(T_0)$ for all $t > 0$; and is thus a sufficient condition for having the nonevent optimal emission path downward sloping.

The second term on the r.h.s. accounts for the influence of path interruption. Since both the Hamiltonian and the hazard function are nondecreasing in T; it may be able to modify the overall sign of the optimal emission path under uncertainty, for example making it positive $\dot{e}_i(0) > 0$: However, even if this is not the case the effect of uncertainty induces, at least for the earlier periods, a level of emissions not inferior to what would be obtained if we not consider the possibility of a future catastrophic event. To obtain $\dot{e}_i(0) > 0$ depends on $H(T_0) \equiv \sum_{i=1}^{N}(B_i(e_i(0)) - D_i(T_0)) - \mu(0)\dot{T}(0) > 0$: The first component of the Hamiltonian is simply the sum of the stream of pre-catastrophe benefits at time zero, based on the current level of temperature and the current emission policy for each region taken at that time. The second component of the Hamiltonian represents the rate of change of temperature value corresponding to the optimal emissions paths $e_i(t)$; $i = 1, 2, \ldots, N$: Therefore, unlike the first term, which relates to the current-benefit effect of policy e_i, the second term can be viewed as the future-cost effect of policy e_i: These two effects are competing in that if a particular policy decision e_i is favorable to the current benefit, then it will involve a sacrifice, due to high temperature, in future benefits. Although not sufficient, a necessary condition for endogenous uncertainty to give less conservation in the short-run is that the current-benefit effect outperforms the future-cost effect. We are now in position to state the following proposition[38].

[38] In the numerical simulations, even though the nonevent emissions are always rising,

Proposition 3 : Endogenous uncertainty and irreversible events may induce less conservation in the short-run.

Finally, in the case of reversible events equation (37) changes as:

$$\left.\frac{de_i}{dt}\right|_{t=0} = \frac{\underline{\mu}^{n}(0)}{B_i''(e_i(0))} - \frac{\lambda(T_0)H(T_0)}{B_i''(e_i(0))} - \frac{\lambda(T_0)V(T_0)[\lambda(T_0)\bar{I}(0) + \sigma]}{B_i''(e_i(0))} \quad (39)$$

where the third term on the r.h.s. tends to increase even further the emissions.

7.5 Exogenous uncertainty

As shown, the equilibrium interval emerges because the uncertainty problem reduces to two distinct problems according to whether the temperature increases or decreases with time. This is obviously due to the type of uncertainty introduced, which reflects our ignorance with regard to the exact location of the critical level of temperature X: Decreasing trajectories cannot trigger the catastrophic event and uncertainty does not affect the expected benefits. Only increasing trajectories matter.

However, in the consumption/pollution trade-off literature, the importance of catastrophic environmental outcomes has been in general incorporated assuming that such events are triggered by the ecosystem's intrinsic stochastic nature. See, among others, Copper (1976), Heal (1984), Clarke and Reed (1994), Torvanger (1997) and Gjerde, Grepperud and Kverndokk (1998). These works assume that the catastrophic occurrence is not entirely due to the policy maker's emission plan driving the pollution level above the critical level, but that it can be influenced by random (exogenous) environmental conditions. Operatively, to introduce this form of uncertainty we replace assumption 1 by the following:

Assumption 1': The probability of a catastrophic event is described by a hazard rate function $\lambda(T(t))$ which depends only on the current average temperature level and not on the complete temperature history.

optimal emission paths of RICE under uncertainty outperform the nonevent ones, at least for the beginning of the planning horizon.

Under assumption 1', the distribution of the occurrence time τ becomes:

$$1 - F_\tau(t) \equiv \Pr(\tau > t \mid \tau > 0) = e^{-\int_0^t \lambda(T(s))ds},$$

with density:

$$f_\tau(t) = \lambda(T(t))e^{-\int_0^t \lambda(T(s))ds}$$

The hazard rate associated with τ is given by:

$$\psi_\tau(t) \equiv \frac{f_\tau(t)}{1 - F_\tau(t)} = \lambda(T(t)) \tag{40}$$

We also assume that $\lambda(T(t))$ is nondecreasing in T, with $\lambda'(T(t)) < \infty$ in $[0, \bar{T}]$. Comparing (21) and (40) highlights the difference between endogenous uncertainty and exogenous uncertainty. The former hazard rate does not depend on the current level of temperature $T(t)$ alone, but also on its rate of change $\dot{T}(t)$. Then when the temperature level does not increase, i.e. $g(e(t)) \leq \sigma T(t)$, the endogenous hazard rate vanishes, as the probability of a catastrophe drops to zero. On the contrary, the exogenous hazard rate which only depends on current temperature does not vanish and there is a positive probability of having a catastrophic event in the future.

Now, defining the relationship $\Omega(t) = -\log\left[e^{-\int_0^t \lambda(T(s))ds}\right]$ with $d\Omega(t) \equiv \lambda(T(t))dt$, the planner's maximization problem (19) reduces to:

$$W(T_0) = \max_{e_1(t),\dots,e_N(t)} \left\{ \int_0^\infty e^{-(\rho t + \Omega(t))} \left(\sum_{i=1}^N B_i(e_i(t)) - D_i(T(t)) \right) dt + \int_0^\infty e^{-(\rho t + \Omega(t))} \lambda(T(t))V(T(t))dt \right\} \tag{41}$$

subject to (16), (18), $e_i(t) \geq 0$, $i = 1, \dots, N$, $T(t) \leq \bar{T}$ and $T(0) = T_0$. By virtue of (40), maximization of (41) describes the expected benefit regardless of whether the temperature trajectory increases or decreases. That is, *Exogenous uncertainty cannot give rise to equilibrium intervals.*

The current-value Hamiltonian and Lagrangian functions for the problem (41) reduce to:

$$H(T(t), e_1(t), \dots, e_N(t), \mu(t)) = e^{-\Omega(t)} \left(\sum_{i=1}^N (B_i(e_i(t)) - D_i(T(t))) \right.$$

$$+ \lambda(T(t))V(T(t))) - \mu_1(t)(g(e(t)) - \sigma T(t)) - \mu_2(t)\lambda(T(t)) \tag{42}$$

and:

$$L(t) = H(t) + \alpha(t)(\bar{T} - T(t)) + \sum_{i=1} \gamma_i(t)e_i(t) \tag{43}$$

Under assumptions 2 and 3, necessary conditions for optimal emission paths, assuming interior solutions, include the first order conditions:

$$e^{-\Lambda(t)}B'(e(t)) = \mu_1(t) - \gamma_i(t); \tag{44}$$

and the dynamic condition of the costate variables:

$$\dot{\mu}_1(t) = \mu_1(t)(\rho + \sigma) - e^{-\Lambda(t)}\Big\{\sum_{i=1} D_i'(T(t)) \tag{45}$$

$$- [\lambda'(T(t))V(T(t)) + \lambda(T(t))V'(T(t))]\Big\} - \mu_2(t)\lambda'(T(t)) - \alpha(t);$$

and:

$$\dot{\mu}_2(t) = \mu_2(t)\rho - e^{-\Lambda(t)}\Big\{(NB(e(t)) - \sum_{i=1} D_i(T(t))) + \lambda(T(t))V(T(t))\Big\} \tag{46}$$

The complimentary slackness conditions (27) as well as the transversality condition (28) still hold. Yet a transversality condition for $\Lambda(t)$ should be added:

$$\lim_{t \to \infty} e^{-\rho t}\mu_2(t)\Lambda(t) = 0 \tag{47}$$

Since the hazard rate function does not vanish when the temperature decreases, $-\mu_2(t)$ in (46) indicates the shadow value of a catastrophe, that is the price associated with changes in the probability of its happening. By integrating with (47) we obtain:

$$\mu_2(t) = \int_t^{\infty} e^{-(\rho s + \Lambda(s))}\Big\{(NB(e(s)) - \sum_{i=1} D_i(T(s))) + \lambda(T(s))V(T(s))\Big\}\, ds$$

Then, it measures the current discounted expected net gain, in utility terms, of not having the catastrophe. Making the substitutions $\xi_1(t) = \mu_1(t)e^{\Lambda(t)}$ and $\xi_2(t) = \mu_2(t)e^{\Lambda(t)}$, the above dynamic conditions simplify as:

$$\dot{\xi}_1(t) = \xi_1(t)(\rho + \sigma + \lambda(T(t))) \tag{48}$$

$$- \left\{ \sum_{i=1} D_i'(T(t)) - [\lambda'(T(t))V(T(t)) + \lambda(T(t))V'(T(t))] \right\} - \xi_2(t)\lambda'(T(t)) - \alpha(t);$$

and:

$$\dot{\xi}_2(t) = \xi_2(t)(\rho + \lambda(T(t))) - \left\{ NB(e(t)) - \sum_{i=1} D_i(T(t))) + \lambda(T(t))V(T(t)) \right\} \tag{49}$$

For this problem, an equilibrium state refers to the T member of the (T, ξ_1, ξ_2) triple for which $\dot{\xi}_1(t) = \dot{\xi}_2(t) = \dot{T}(t) = 0$. Yet, by the positivity of emissions at a positive steady state $\gamma_i(t) = 0$, $i = 1, \ldots, N$. We are now ready to state the following proposition (Tsur and Zemel, 1998):

Proposition 4 : The optimal temperature process $T(t)$ corresponding to the (exogenous) uncertain problem (41), converges monotonically to a, possibly not unique, steady state level $\hat{T}^{ex}$.

The temperature level $\hat{T}^{ex}$ is defined by:

$$\begin{cases} \hat{T}^{ex} = T^{\infty} & \text{if} \quad L^{ex}(T^{\infty}) > 0 \\ \hat{T}^{ex} = 0 & \text{if} \quad L^{ex}(0) < 0 \\ L^{ex}(\hat{T}^{ex}) = 0 & \text{otherwise} \end{cases}$$

where:

$$L^{ex}(T) = L^{n}(T) + \lambda(T(t))B'(\sigma T/N)$$

$$+ \lambda'(T)V(T) + \lambda(T)V'(T) - \lambda'(T)\left[\frac{\rho W(T) + \lambda(T)V(T)}{\rho + \lambda(T)}\right]$$

For irreversible events (assumption 4) with $V = 0$; the above function reduces to the simplified form:

$$L^{ex}(T) = L^n(T) + \lambda(T(t))B'(\sigma T/N) - \lambda'(T)\frac{\rho W(T)}{\rho + \lambda(T)} \tag{50}$$

The roots of (50) satisfy equation (46) of Clark and Reed (1994) and equation (3.1) of Tsur and Zemel (1998) which identify the long-run equilibrium level of temperature. Furthermore, if we take a constant hazard rate so that $\lambda'(T) = 0$; the arguments for the uniqueness of the nonevent steady state $\hat{T}^n$ apply for $L^{ex}(T)$ as well, and $L^{ex}(T) = L^n(T) + \lambda B'(\sigma T/N)$ can have at most one root in $[0; \hat{T}^{\infty}]$. For $T \leq \hat{T}^n$ when $L^n(T) \geq 0$; also $B'(\sigma T/N)$ is positive and then $L^{ex}(T)$ cannot vanish. Thus, to find a root of $L^{ex}(T)$ the temperature must progress monotonically in time above the nonevent steady state level $\hat{T}^n$[39]: Summarizing we can state:

Proposition 5 : If the pollution policy does not affect the occurrence probability (exogenous uncertainty) and events are irreversible, the optimal temperature management may give less conservation in the long-run.

Although this result may at first sight appear surprising it is in line with the findings of Clark and Reed (1994), Torvanger (1997), Tsur and Zemel (1998) and Gjerde, et al. (1998). If the pollution policy does not affect the probability of a catastrophe the policy maker is encouraged to emit as much as possible before the ecosystem collapses. In the case where $V = 0$ and $\lambda'(T) = 0$; the allocation problem (41) is equal to the nonevent problem (20) with the difference that $(\rho + \lambda)$ replaces ρ as the effective discount rate.

As irreversibility of future events makes it rational to redistribute utility from the far to the near future, it is also interesting to see the emissions in the short-run under exogenous uncertainty. Differentiating (44) with respect to t and evaluating it at $t = 0$ we get:

$$\frac{de(t)}{dt}\bigg|_{t=0} = \frac{\mu_1^n(0)}{B''(e(0))} - \frac{\mu_2(0)\lambda'(T_0) - \mu_1(0)\lambda(T_0)\Upsilon(0)}{B''(e(0))} \tag{51}$$

or:

$$\frac{de(t)}{dt}\bigg|_{t=0} = \frac{\mu_1^n(0)}{B''(e(0))}$$

[39] Clark and Reed (1994) and, Tsur and Zemel (1998) show that this result holds even for a more general hazard rate $\lambda'(T) > 0$:

$$-\frac{\lambda'(T_0)\int_0^\infty e^{-(\rho s+\Lambda(s))}\left\{NB(e(s))-\sum_{i=1}^N D_i(T(s))\right\}ds-\mu_1(0)\lambda(T_0)\dot{T}(0)}{B''(e(0))}$$

While the shadow cost of an increase in temperature affects negatively the emission paths, the shadow value of (not having) the catastrophe affects the emission paths positively. Again, we do not have clear cut results, in particular, we may still state that:

Proposition 6 : Exogenous uncertainty and irreversible events may induce less conservation in the short-run.

However, introducing a constant hazard rate the ambiguity of Proposition 6 disappears, the above derivative reduces to:

$$\left.\frac{de(t)}{dt}\right|_{t=0}=\frac{\dot{\mu}_1(0)}{B''(e(0))}-\frac{\mu_1(0)\lambda\dot{T}(0)}{B''(e(0))} \tag{52}$$

which is always negative. That is, less conservation in the long-run is balanced by more conservation in the short-run when the catastrophic event is irreversible and uncertainty is exogenous.

We end this section extending (51) to the case of reversible events:

$$\left.\frac{de(t)}{dt}\right|_{t=0}=\frac{\dot{\mu}_1(0)}{B''(e(0))}-\frac{\mu_2(0)\lambda'(T_0)-\mu_1(0)\lambda(T_0)\dot{T}(0)}{B''(e(0))} \tag{53}$$

$$+\frac{\lambda'(T_0)V(T_0)+\lambda(T_0)V'(T_0)}{B''(e(0))}$$

where the third term on the r.h.s. tends to reduce even further the emissions[40]. The above arguments and the comparison of (39) and (53), allow us to state the following proposition.

Proposition 7 Reversible events have countervailing effects on short-run emissions: tends to increase emissions under endogenous uncertainty and to decrease emissions under exogenous uncertainty.

[40] Indeed, this result holds even when $V'(T_0)<0$ as long as $\lambda'(T_0)V(T_0)+\lambda(T_0)V'(T_0)>0$.

7.6 Comparative analysis

As already mentioned, the optimal time paths for the temperature level and its shadow cost, as well as the long-run state equilibrium, depend on the parameters of the problem. In particular, the effects of the change in the discount rate ρ can be analyzed with the help of comparative static and dynamic analysis. However, in general a higher discount rate reduces the importance of future benefits or costs relative to current ones, with the effect of inducing less conservative behavior in the long-run. In the short-run, on the other hand, we may experience counterintuitive behavior..

The effects from changes in the discount rate ρ on the steady state are obtained by propositions 1 and 4. It is immediate to note that $dL^n(T)/d\rho$, $dL^e(T)/d\rho$ and $dL^{ex}(T)/d\rho$ are all positive, so that $d\hat{T}^n/d\rho$, $d\hat{T}^e/d\rho$ and $d\hat{T}^{ex}/d\rho$ are also positive. By giving less weight to the net benefits accruing in the future, the stock of pollutant increases as the temperature in the long-run equilibrium increases.

This result is generally accompanied by an increase in emissions over the entire optimal path. To see this, let us consider the first order conditions (33) (or (44)). With a nondecreasing temperature trajectory and a nondecreasing shadow cost $\mu(t)$ the marginal net benefits $B'(e(t))$ are bounded above, say by:

$$B'(e(t)) = \mu(t)e^{\alpha(T(t))} \leq \hat{\mu}^e e^{\alpha(\hat{T}^e)} \qquad \text{for all } t > 0 \qquad (54)$$

where $\hat{\mu}^e e^{\alpha(\hat{T}^e)}$ is given in (32). From (29) and assumption 3, the shadow value of the temperature at time t is expressed as:

$$\mu(t)e^{\alpha(T(t))} = e^{(\rho+\sigma)t+\alpha(T(t))} \int_t^{\infty} e^{-((\rho+\sigma)s+\alpha(T(s)))}\Big[\lambda(T(s))\Big(NB(e(s)) - \sum_{i=1}^{N} D_i(T(s))\Big) + \sum_{i=1}^{N} D_i'(T(s))\Big]ds \qquad (55)$$

Combining the terms, using $L^e(T) = (\rho+\sigma)B'(\sigma T/N) - \sum_{i=1}^N D_i'(T) - \lambda(T)\rho W(T)$ and $W(T) \equiv [NB(\sigma T/N) - \sum_{i=1}^N D_i(T)]/\rho$; we conclude that :

$$B'(e(t)) \leq eq.(32) = B'(\sigma \hat{T}^e/N) \qquad \text{for all } t > 0. \qquad (56)$$

Thus an increase in the discount rate will increase the emissions path.

However, examining (55), if on the one hand an increase in ρ involves a descrease in the temperature shadow cost and hence increased emissions, on the other hand a temperature increase generated in this way leads to a fall in the survivor function implying a higher shadow cost and lower emissions. Although we have shown that in the long-run the former effect predominates over the latter, in the short-run the opposite may occur (recalling propositions 3 and 6). We prove this by constructing an increasing emissions plan yielding a shadow cost lower than the optimal one and responding positively to an increase in the discount rate. For some arbitrary small constants $h > 0$ and $\delta > 0$; we define the emissions plan:

$$e^{\delta h}(t) = \begin{cases} \frac{\sigma T_0}{N} + \delta & \text{for } 0 \leq t < h \\ \frac{\sigma T(h)}{N} & \text{for } t \geq h \end{cases} \tag{57}$$

With this plan, for all $t \leq h$; $T(t) - T_0 = \int_0^t [\sigma T_0 + \delta N - \sigma T(s)]ds = \delta N t + o(\delta t)$;[41] the temperature rises in interval $0 \leq t < h$ then remains stable on reaching, T_h. The shadow cost at time zero associated with (57) is:

$$\begin{aligned} \mu^{h\delta}(0) = & \int_0^h e^{-((\rho+\sigma)s+\Lambda(T(s))}[\lambda(T(s))(NB(\sigma T_0/N + \delta)) \\ & - \sum_{i=1} D_i(T(s))) + \sum_{i=1} D'_i(T(s))]ds + e^{-((\rho+\sigma)h+\Lambda(T(h))}\mu^e(h) \end{aligned} \tag{58}$$

where $\mu^e(h)$ is the shadow cost (32), obtained under the plan $e^{h\delta}(t) = \frac{\sigma T_h}{N}$ for $t \geq h$: We are now able to reverse the disequality in (55) and state that for a nondecreasing temperature trajectory and a nondecreasing shadow cost $\mu(t)$ the marginal net benefits at zero, $B'(e(0))$, are bounded below by:

$$B'(e(0)) = \mu(0) \geq \mu^{h\delta}(0) \tag{59}$$

Expanding the first term on the r.h.s. of (58) up to the order of approxi-

[41] The $o(\delta t)$ term arises from a Taylor series expansion; it suggests that the smaller δt is, the more accurate the approximation.

mation $o(\delta h)$ we obtain:[42]

$$\int_0^h e^{-((\rho+\sigma)s+\Delta(T(s)))}\left[\lambda(T(s))(NB(\sigma T_0/N+\delta))-\sum_{i=1} D_i(T(s)))+\sum_{i=1} D_i'(T(s))\right]ds \tag{60}$$

$$= \psi^e(0)[1-e^{-(\rho+\sigma)h}]+[\lambda(T_0)(B'(\sigma T_0/N)]N\delta h+o(\delta h)$$

Considering the second term on the r.h.s. of (58) as a function of T, it can be expanded as:

$$e^{-((\rho+\sigma)h+\Delta(T(h))}\psi^e(h) = e^{-(\rho+\sigma)h}\psi^e(0)-\lambda(T_0)\psi^e(0)N\delta h+\frac{d\psi^e(0)}{dT_0}N\delta h+o(\delta h) \tag{61}$$

Substituting (60) and (61) in (58), we get the simplified expression:

$$\mu^{h\delta}(0)-\psi^e(0)=\frac{\lambda(T_0)L^e(T_0)}{\rho+\sigma}N\delta h+\frac{d\psi^e(0)}{dT_0}N\delta h+o(\delta h) \tag{62}$$

Recalling, from proposition 1, that $T_0<\hat{T}^e$ implies $L^e(T_0)>0$ and then $\frac{d\psi^e(0)}{dT_0}>0$; the r.h.s. of (62) is always positive. The effect of a change in the discount rate follows by differentiating (62) with respect to ρ: Observing that $\frac{d\psi^e(0)}{dT_0}$ responds negatively and the term $\frac{\lambda(T_0)L^e(T_0)}{\rho+\sigma}$ responds positively, there exist $h>0$ and $\delta>0$ such that $\frac{d\mu^{h\delta}(0)}{d\rho}>0$: Thus, by (59), an increase of ρ might reduce the optimal emissions plan $e(0)$ in the short-run. In the case of certainty, i.e. $\lambda(T_0)=0$; the $h\delta-$shadow cost always responds negatively to an increase in ρ reconciling (59) with (56) and increasing the emissions path.

7.7 Endogenous capital accumulation

So far we have assumed that the output accumulate according to an exogenous given trend, and in the specific we have normalized the output in each

[42] First, define $F(e;T)=e^{-\Delta(T)}[\lambda(T)(NB(\sigma T_0/N+\delta)-\sum_{i=1}^N D_i(T))+\sum_{i=1}^N D_i'(T)]$: Second, we expand in Taylor series of order δt; i.e. $F(e;T)=F(\sigma T_0/N;T_0)+F_e\delta+F_T N\delta t+o(\delta t)$: Third, integrating over h and dropping the terms of order greater than δh we obtain $\int_0^h e^{-(\rho+\sigma)t}F(e;T)dt=\frac{F(\sigma T_0/N;T_0)}{\rho+\sigma}(1-e^{-(\rho+\sigma)h})+\frac{F_e\delta}{\rho+\sigma}(1-e^{-(\rho+\sigma)h})+o(\delta h)$: Finally, expanding $(1-e^{-(\rho+\sigma)h})=(\rho+\sigma)h+o(h)$ we get (60).

region to one. We now proceed considering an endogenous accumulation of output assuming that the gross production is obtained using capital as the only production factor:

$$Q_i(t) = Q_i(K_i(t)) \tag{63}$$

with $Q_i' > 0$ and $Q_i'' < 0$: Then, production net of costs and benefits of environmental control becomes:

$$Y_i(t) = \Omega(e_i(t); T(t))Q_i(t) \; ; \tag{64}$$

The dynamic of the stock of capital is represented by the linear differential equation:

$$dK_i(t) = [I_i(t) - aK_i(t)]dt; \quad K_i(0) = K_{0;i}; \tag{65}$$

where a stands for the capital decay rate, constant over time and equal for each region, and $I_i(t)$ is the gross investment.

Finally, as production is shared between consumption and investment, i.e. $Y_i(t) = C_i(t) + I_i(t)$; taking account of (63), (64) and (17) we still specify a separable net benefit function of the form:

$$\begin{aligned} U_i(C_i(t)) &\equiv U_i[\Omega(e_i(t); T(t))Q_i(K_i(t)) - I_i(t)] \\ &= [B_i(e_i(t)) - D_i(T(t))]Q_i(K_i(t)) - C_i(I_i(t)) \end{aligned} \tag{66}$$

where $B_i(e_i)$ is increasing and strictly concave in e_i, $D_i(T)$ is nondecreasing and convex in T and the output is normalized so that $Q_i(K_i(0)) = 1$[43]. Yet, $C_i(I_i)$ represents the direct costs, in term of utility, of the investment which reduces current level of consumption, with $C_i(0) = 0$; and $C_i'(I_i) > 0$.

Keeping assumptions 1, 2 and 4, the current-value Hamiltonian and Lagrangian functions are as follows:

$$H(t) = e^{-\delta(T(t))} \sum_{i=1}^{N} [(B_i(e_i(t)) - D_i(T(t)))Q_i(K_i(t)) - C_i(I_i(t))]$$

$$- \mu^K(t)[g(e(t)) - \sigma T(t)] + \sum_{i=1}^{N} p_i(t)[I_i(t) - aK_i(t)] \tag{67}$$

[43] Again equation (66) can be obtained assuming a linearly separable utility function U togheter with a linearly separable Ω function.

and:

$$L(t) = H(t) + \alpha(t)(\bar{T} - T(t)) + \sum_{i=1} \gamma_i(t)e_i(t) \tag{68}$$

where $-\mu^K(t)$ is still the shadow cost of the increase in temperature and $p_i(t),\ i = 1,\ldots,N$ are the shadow or social cost of net capital formation in each region. Further, $\alpha(t),\ \gamma_i(t),\ i = 1,\ldots,N$ are the current value Lagrange multipliers associated with the constraints $T(t) \le \bar{T}$ and $e_i(t) \ge 0;\ i = 1,\ldots,N$ respectively.

Necessary conditions for the optimal social emissions and investment paths can be written as:

$$e^{-\Lambda(T(t))}[B_i'(e_i(t))Q_i(K_i(t))] = \mu^K(t) - \gamma_i(t); \quad i = 1,\ldots,N. \tag{69}$$

and:

$$e^{-\Lambda(T(t))}[C_i'(I_i(t))] = p_i(t); \quad i = 1,\ldots,N; \tag{70}$$

along with the dynamic condition for the costate variables:

$$\dot{\mu}^K(t) = \mu^K(t)(\rho+\sigma) - e^{-\Lambda(T(t))}\left\{\lambda(T(t))\sum_{i=1}[B_i(e_i(t)) - D_i(T(t))]Q_i(K_i(t)) + \sum_{i=1} D_i'(T(t))Q_i(K_i(t)) - \lambda(T(t))\sum_{i=1} C_i(I_i(t))\right\} - \alpha(t); \tag{71}$$

and:

$$\dot{p}_i(t) = p(t)(\rho + a) - e^{-\Lambda(T(t))}[B_i(e_i(t)) - D_i(T(t))]Q_i'(K_i(t)); \quad i = 1,\ldots,N. \tag{72}$$

Furthermore, besides the complimentary slackness conditions (27) and the transversality condition (28), we add the Arrow type transversality conditions at infinity $\lim_{t\to\infty} e^{-\rho t}p_i(t)K_i(t) = 0;\ \ i = 1,\ldots,N.$

Focusing on condition (71) there are several ways to show the difference between the shadow cost of the temperature with capital accumulation and without capital accumulation. From equations (71), (27) and the transversality condition (28), the shadow cost of an increase in the temperature at time t can be expressed as the expected weighted sum of all future discounted marginal costs experienced by all regions. That is:

$$\mu^K(t)e^{\Lambda(T(t))} = \tag{73}$$

$$e^{(\rho+\sigma)t+\Lambda(T(t))}\int_t^{\infty} e^{-((\rho+\sigma)s+\Lambda(T(s)))}\left\{\left[\lambda(T(s))\sum_{i=1}^{N}[B_i(e_i(s)) - D_i(T(s))]Q_i(K_i(s)) + \sum_{i=1}^{N} D_i'(T(s))Q_i(K_i(s) - \lambda(T(s))\sum_{i=1}^{N} C_i(I_i(s))\right] ds\right\}$$

Comparing (73) and (29) highlights the difference between endogenous accumulation and exogenous accumulation. Apart for the usual marginal damages associated with an increase in temperature in all future dates the former shadow cost includes both the costs and benefits, in term of utility, associated with the marginal change in the policy-maker beliefs about future occurrence of a catastrophe. In the specific, the future utility losses induced by a marginal increase of the probability of a catastrophe are saved or even over-saved if the society devolves part of its production for accumulation. Substituting (73) in (69) and rearranging we are able to write:

$$B_i'(e_i(t)) = \frac{\mu^K(t)e^{\Lambda(T(t))}}{Q_i(K_i(t))} = \mu(t)e^{\Lambda(T(t))} \tag{74}$$

$$+e^{(\rho+\sigma)t+\Lambda(T(t))}\int_t^{\infty} e^{-((\rho+\sigma)s+\Lambda(T(s)))}\left\{\left[\lambda(T(s))\sum_{i=1}^{N}[B_i(e_i(s)) - D_i(T(s))]\frac{\Delta Q_i(K_i(s))}{Q_i(K_i(t))} + \sum_{i=1}^{N} D_i'(T(s))\frac{\Delta Q_i(K_i(s))}{Q_i(K_i(t))} - \lambda(T(s))\sum_{i=1}^{N}\frac{C_i(I_i(s))}{Q_i(K_i(t))}\right] ds\right\}$$

where $\mu(t)e^{\Lambda(T(t))}$ is the shadow cost without endogenous capital accumulation given by (29) and $\Delta Q_i(K_i(s)) = Q_i(K_i(s)) - Q_i(K_i(t))$; for $i = 1,\ldots,N$; which are positive for nondecreasing optimal capital trajectories. From (74), simple considerations allow us to state:

Proposition 8 Endogenous capital accumulation and irreversible events may induce less conservation, at least in the short-run.

To conclude, it easy to see from (74) that in the nonevent case, i.e. ¸$(T(s)) = 0$; the exogenous capital accumulation always induces more conservation

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