Selective Schools*

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Abstract

This paper studies how schooling admission tests affect economic performance in an economy where individuals are endowed with both academic and non academic abilities and both abilities matter for labor productivity. We develop a simple model with selective government held schools, where individuals signal their abilities by taking an admission test that sorts them into different schools. When abilities are poorly correlated in the population, as documented in the literature, a standard test based only on academic abilities is expected to be less efficient than a more balanced test, that considers both ability types. Contrary to this expectation, we show that this is not generally true, but depends both on the distribution of abilities in the population and on the marginal contribution of each ability type to individual productivity. It is also not generally true that the outcome of a more balanced test can be replicated by a sequential testing strategy, with government held schools testing academic abilities and firms testing non academic abilities on the sub-sample of graduates of elite schools.

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1 Introduction

A distinguishing feature of modern schools, especially at the upper secondary and tertiary level, is that the admission of students is based on selection criteria such as exams and admission tests, that sort applicants by quality. These

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practices are frequent in Europe and Japan, where tests prevail on prices as selection criteria (Fernandez (1998), Green, Wolf and Leney (1999)). In the US, both prices and tests have an important role in the allocation of students to colleges (Winston (1999))[1].

It is often remarked that these admission tests are designed to measure academic ability and potential success in college, not in the labor market[2]. As argued by Evans and Schwab (1997), "..standardized tests can only measure a student's ability to deal with a particular type of question and cannot measure a student's creativity or deeper problem solving skills.." (p.942), that are valued in the labor market.

The current emphasis in favor of cognitive abilities in the selection of pupils overlooks the importance both of social and of non academic abilities. According to Heckman (1999) "..current policies regarding education...focus on cognitive skills as measured by achievement or IQ tests to the exclusion of social skills, self discipline and a variety of non-cognitive skills that are known to determine success in life. The preoccupation with cognition and academic "smarts" to the exclusion of social adaptability causes a serious bias in the evaluation of many human capital interventions" (1999, p.1).

As shown by David Wise (1975), both academic and non academic abilities affect individual productivity. While academic abilities are associated to cognitive traits, non academic abilities are associated to affective traits and include initiative, decision making ability and imaginative thinking. A similar point is made by Bishop (1992) and Blackburn and Neumark (1993), who distinguish between academic and technical abilities and show that the latter are at least as important as the former for earnings growth[3].

Wise concludes his empirical investigation of the relationship between abilities and productivity by advocating more balanced admission criteria in the allocation of students to schools, that should consider not only cognitive but also affective traits. More balanced admission rules seem to be particularly relevant when individuals have different endowments of academic and non academic abilities and these abilities are poorly correlated in the population. Empirical evidence in support of this possibility is presented by Wise (1975), who finds a low correlation between the two ability types in his sample of employees of a large US manufacturing firm[4].

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[1] Venti and Wise (1982) find that college admission decisions depend both on SAT scores and on high school class rank. According to Cook and Frank (1993), the top 33 colleges in the US take up 2.5% of all SATV (SAT verbal) recipients and 42.8% of those with a SATV included from 700 to 800.

[2] "Standardized tests employed by schools lack external validation in terms of labor market skills or other subsequent outcomes. This is not particularly surprising, however, given the primary motivation for their construction. Most tests are designed to examine students on specific knowledge and to predict performance. The performance of interest, however, is often future success in schooling. This, for example, is the motivation behind the SAT scores". (Hanushek (1986), p.1154).

[3] See also Cawley, Conneely, Heckman and Vytlacil (1996). Herrnstein and Murray (1994) and Currie and Thomas (1995) recognize that individual outcomes are affected by a wide range of human talents, beside the intelligence factor "g".

[4] In his paper, Wise also quotes substantial applied psychology literature that shows similar

This paper studies how school admission tests affect economic outcomes in an economy where individuals are endowed with both academic and non academic abilities and either ability matters for labor productivity. Our main purpose is to evaluate whether admission criteria that value both academic and non academic abilities can improve economic efficiency, measured by total output produced in the economy, with respect to more standard criteria based only upon academic abilities.

The relationship between individual ability, schooling and economic performance has been studied extensively in the literature. Important contributions include the signalling models by Spence (1973) and Stiglitz (1975) and the sorting-cum-learning model by Weiss (1983). These contributions, however, focus on a single dimension of ability. As argued by Willis and Rosen (1979), the emphasis on a single dimension is probably erroneous and is equivalent to imposing large positive covariances on unobserved individual ability components.

More recent contributions that are closely related to our paper include Costrell (1994), Betts (1998) and Fernandez (1998). Costrell studies admission standards when the policy maker is egalitarian and individuals differ in their effort but not in their ability. Betts (1998) introduces heterogeneous ability in Costrell's setup and allows individuals to signal their ability by taking admission tests. Ability, however, retains a single dimension. Fernandez (1998) compares allocations by prices with allocations by tests with and without liquidity constraints. In her model, ability has a single dimension and the share of high quality schools in the economy is exogenously given.

In all these models, the main emphasis is on the supply side and on the individual decision to take and pass admission tests. Compared to these contributions, our paper innovates in a number of directions. First, we allow ability to take two different dimensions, that can be poorly correlated in the population. Second, we explicitly model the determination of the share of high quality schools in the economy. Last but not least, we model the demand side of the market and study the interactions among the decisions taken by individuals, firms and the government.

We consider a stylized economy where schools are run at no cost by the central government, that designs the schooling system to maximize total output[5]. Schools can be either comprehensive or selective. Comprehensive schools, by definition, have no admission tests. A selective schooling system consists of two types of school, the high quality and the low quality type, with admission to the high quality school based upon a precise test[6]. In this paper, schools do not produce human capital and act exclusively as signalling devices that can be used by firms to infer the expected human capital of potential employees. Hence, school quality refers to the average ability of admitted students rather than to the quality of human capital produced at school.

Firms choose between a complex and highly productive technology and a

results. See Wise (1975), footnote 14 page 354.

[5] The maximization of total output (or income) is the typical objective function used in this literature. See for instance Fernandez (1998).

[6] An alternative characterization of the two types of school is upper secondary and tertiary.

simple but less productive technology. The complex technology requires that firms sink a fixed training cost for each employee, and yields output per head equal to (expected) individual human capital. The simple technology requires no training cost and yields a fixed output per head, independent of human capital. Hence, human capital and technical complexity are complements in production[7]. Total output in this economy depends both on the share of firms adopting the complex but highly productive technology and on the expected human capital of recruited employees. When designing the schooling system and the admission test, the government takes into account both individual enrolment decisions and the selection of technology by firms.

The material in the paper is organized as follows. Section 2 provides the basic setup. Section 3 deals with the standard case of a single ability. Section 4 considers the case where individuals are endowed with both academic and non academic abilities and these abilities are poorly correlated in the population. Section 5 looks at sequential testing strategies involving employers. Conclusions and an Appendix follow.

2 Setup

Consider a small open economy populated by a constant number of risk neutral firms and individuals. Output prices are given in the international market and normalized to 1. Let the measure of both firms and individuals be 1. Each firm has a single employee, who lives and work for a single period after finishing education. In the steady state, the outflow of employees must be compensated by an equal inflow of new entrants, who have completed their time in school. By "schools" in this paper we mean upper secondary and tertiary education. Schools are run by the government at no cost[8].

The government designs the schooling system, that can be either comprehensive or selective. In a comprehensive system, there is no selection and students are allocated randomly to schools. In a selective system, schools are divided into two types, H and L, and admission to the H type is regulated by a precise admission test, that depends on the share of total slots available to students in H schools, $\Theta \in [0,1]$[9]. Two possible characterizations of H and L schools are respectively tertiary and upper secondary education and high quality and low quality colleges.

In real life, schooling systems select individuals by ability and produce human capital. Since the emphasis of the paper is on selection and signalling, we ignore

[7] Redding (1996) and Eicher (1996) have models where human capital and investment in R&D are strategic complements.

[8] Our assumption that the design and implementation of admission tests require no cost implies that there is no need to raise taxes to finance the selection procedure.

[9] We rule out noisy tests because we want to focus on the problems associated with multiple ability types. Noisy tests are discussed, among others, by Weiss (1983) and Brunello and Ishikawa (1999).

the second aspect. Therefore, H and L schools in this simple economy differ only in the average quality of their admitted students. We also ignore the interactions between individual and group human capital (peer effects). Finally, we assume that selection requires little time, and that individuals admitted to H schools become graduates of these schools within the same period. Hence, in each period of time there are Θ graduates of H schools and $1-\Theta$ graduates of L schools.

Individuals differ in their endowment of academic ability α and non academic ability β. Endowed ability is partly innate and partly the result of family background and of individual development during the early stages of education. Following Wise (1975) and Bishop (1992), academic abilities refer to cognitive traits and non academic abilities to affective traits and to technical abilities[10].

Admission tests require only nonpecuniary costs. Hence, individual wealth does not matter and there are no liquidity constraints conditioning individual choice. Ability of either type is known to each individual but is not observed either by the government or by firms. In a selective schooling system, individuals signal their ability by deciding whether to take or not to take the test. This decision takes into account both the expected returns and the net nonpecuniary costs of taking the test, that vary among individuals and are inversely related to ability[11].

Individual human h depends both on academic and on non academic abilities as follows

$$h = \alpha + \sigma\beta \tag{1}$$

where σ is strictly positive. The additive specification implies that the marginal contribution of each ability type to human capital is constant. Notice that Eq. (1) can be interpreted as a logarithmic transformation of a Cobb Douglas technology, with relative weight equal to σ. When $\sigma > 1$ the marginal contribution of non academic abilities to human capital is higher than the marginal contribution of academic abilities.

During each period, firms in this economy select a technology, recruit employees, produce output and pay wages. Since individuals live and work for only one period and there is an employee per firm, each firm needs to recruit a new employee at the beginning of each period. The recruit is a school graduate, who does not quit during his working life. There are two available technologies, C (complex) and S (simple), and firms can costlessly shift between them at the beginning of each period.

The simple technology does not require that new recruits be trained and produces a constant output flow, equal to 1 and independent of individual ability. The complex technology requires that recruits be trained at the cost λ, born by firms independently of whether training is general or firm specific[12],

[10] See OECD (1997) for a good description of literacy and cognitive abilities.

[11] We consider costs net of benefits. Heckman, Lochner and Taber (1998) find that nonpecuniary components are an important factor in educational choices.

[12] Acemoglu and Pischke (1999) show that firms can bear training costs even when human

and produces an output flow equal to the employee's human capital h. Firms are endowed with an idiosyncratic managerial ability, that affects training costs. Effective training costs are lower the higher is idiosyncratic managerial ability. Hence, firms endowed with higher managerial ability have a relative advantage in the adoption of the complex technology. Since abilities are known only to individuals, the choice of technology is based upon the comparison of expected profits. Let C and $(1-C)$ be the share of firms selecting respectively the C and the S technology. Then C is also the share of employees in the sector of C firms[13].

The decisions taken in this simple economy are in the following sequence. First, the government sets Θ, the size of the H schools sector, and the cutoff value of the admission test v^* to maximize total output in the economy. In so doing, it takes into account the effects of Θ on the decisions taken by individuals and firms. Second, individuals choose whether to take or not the admission test v. This decision depends on the comparison of the expected returns with the (net) nonpecuniary costs of investing. Third, firms choose their production technology and hire new graduates to fill their vacant jobs. Both the government and firms know the distribution of academic and non academic abilities in the population but do not observe the actual combination of abilities, that are known only to single individuals.

In practice, the outcomes of these decisions take time to materialize. In this paper, time is collapsed into a single period and equilibrium is the result of a one shot game involving individuals, firms and the government.

3 A Single Ability

Let academic abilities $\alpha \in [1,2]$ by uniformly distributed in the population of individuals and define $\beta(\alpha)$ as the function describing the relationship between non academic and academic abilities in the population. The specific case

$$\beta = \alpha \tag{2}$$

implies that individuals with high (low) α are also endowed with a high (low) β, and that the two abilities are perfectly (positively) correlated across individuals. This is equivalent to having a single ability type. In this case, individual human capital is given by $h = (1+\sigma)\alpha$.

The government chooses Θ, the size of the H schools sector and designs the admission test. Consider first a test based exclusively on academic abilities. Our assumptions that the admission test is precise and that the nonpecuniary net costs of taking the test are inversely related to academic abilities guarantee that individuals self select into the right school. Hence, the choice of Θ is equivalent to selecting a critical value of academic ability, α^*, such that all individuals

capital is fully general. The importance of skill acquisition in more productive technologies is emphasized by Caselli (1999).

[13]See Brunello and Ishikawa (1999) for a similar setup.

with $\alpha \geq \alpha^*$ are allocated to H schools and all individuals with $\alpha < \alpha^*$ are allocated to L schools. Notice that the schooling sector is selective if $\alpha^* \in (1, 2)$ and comprehensive if $\alpha^* = 1, 2$. We consider here a selective system and show at the end of this section that it always dominates a comprehensive system.

Next, consider a more balanced admission test, that is based on individual human capital h and that takes into account both academic and non academic abilities. Since individual human capital h is a linear function of academic ability α, it is clear that choosing a critical threshold α^* is equivalent to choosing a critical value of individual human capital, $h(\alpha^*)$, such that individuals with $h \geq h(\alpha^*)$ are allocated to H schools and individuals with $h < h(\alpha^*)$ end up in L schools. Therefore, the admission test based upon cognitive abilities is equivalent to a test based upon individual human capital.

With a uniform distribution of academic skills, $\Theta = 2-\ \alpha^*$ and the expected human capital of an individual who graduates from school is

$$E\left[h \mid H\right] = \frac{(1+\sigma)\int_{a^*}^{2} \alpha d\alpha}{2-\alpha^*} = (1+\sigma)\,\frac{(2+\alpha^*)}{2} \tag{3}$$

in the case of an H school and

$$E\left[h \mid L\right] = \frac{(1+\sigma)\int_{1}^{a^*} \alpha d\alpha}{\alpha^*-1} = (1+\sigma)\,\frac{(1+\alpha^*)}{2} \tag{4}$$

in the case of an L school.

With $E\left[h \mid H\right] > E\left[h \mid L\right]$ and with a sufficient supply of graduates of H schools, firms adopting the complex technology prefer to hire graduates of H schools, who have higher expected human capital than graduates of L schools. Firms adopting the simple technology, however, do not care about the type of school. Hence, the expected profits of a C firm are

$$\pi_{\mathrm{Ci}} = (1+\sigma)\,\frac{(2+\alpha^*)}{2} - w - \lambda_i \tag{5}$$

where i is the subscript for firms, w is the wage, λ is the effective training cost, that is higher the lower the firm's idiosyncratic managerial ability and $\alpha^* \in (1, 2)$. Expected profits are higher the higher the threshold α^* and the lower the idiosyncratic training cost λ. Let the latter be uniformly distributed among firms, with $\lambda \in [0, \lambda_M]$. Expected profits of a S firm are

$$\pi_S = 1 - b \tag{6}$$

where b is the exogenous reservation wage, equal to the value of leisure.

Wages w in firms C are set by Nash bargaining. Since the bargain occurs after that training costs have been sunk, bygones are bygones and the outcome is a linear combination of the reservation wage b and of the (expected) rent from the match (See Malcomson (1997)). Letting δ be the relative bargaining power of an employee and assuming that the default payoffs to the parties are equal to b for the employee and to zero for the firm, we obtain

$$w = (1-\delta)\, b + \delta\,(1+\sigma)\,\frac{(2+\alpha^*)}{2} \tag{7}$$

Eq. (7) implies that the higher the threshold α^* the higher the bargained wage in C firms. It also implies that $w > b$ if $\delta > 0$. Thus C firms pay higher wages than S firms.

Firms choose the complex technology if $\pi_C \geq \pi_S$. Using Eqs. (5) and (6), we obtain the following critical value of the training cost λ

$$\lambda^* = \frac{(1-\delta)\,(1+\sigma)\,(2+\alpha^*) - 2\,(1-\delta b)}{2} \tag{8}$$

Firms with training costs lower than λ^* choose the complex technology and firms with higher training costs prefer the simple technology. Using our assumptions on the distribution of λ, the share of firms choosing the complex technology is

$$C = \frac{\lambda^*}{\lambda_M} \tag{9}$$

The upshot of this discussion is that the share of C firms in a selective system is strictly increasing both in the threshold α^* and in the parameter σ[14].

The selection of α^* by the government and the implied allocation of individuals to H and L schools is implemented via an admission test. Let v be the score of the test and v^* the cutoff value set by the government. The stricter the test, the more selective the schooling system and the higher v^*. Admission to H schools is limited to individuals who score in the test $v \geq v^*$. The test requires that individuals spend net nonpecuniary costs, that vary with the difficulty of the test and with individual academic ability[15]. Similarly to Fernandez (1998), we model the individual score in the test as $v = V(\alpha, \Omega)$, where Ω are net nonpecuniary costs and $\frac{\partial V}{\partial \alpha} > 0$, $\frac{\partial V}{\partial \Omega} > 0$. Then $\Omega\,(\alpha, v)$ is the dual of V, with $\frac{\partial \Omega}{\partial \alpha} < 0$ and $\frac{\partial \Omega}{\partial v} > 0$, and individuals with higher academic ability α face lower net nonpecuniary costs of passing the test with cutoff value v^*.

Suppose that there are enough new graduates of H schools to fill all the available vacancies supplied by C firms. Then

$$C \leq \Theta \tag{10}$$

holds and, with random selection, the probability that a graduate of an H school finds a job in a high wage C firm is equal to $\frac{C}{\Theta}$. It follows that the net expected return from access to an H school is given by

$$\frac{C\,(\alpha^*)}{2-\alpha^*}\left\{\delta\left[(1+\sigma)\,\frac{(2+\alpha^*)}{2} - b\right]\right\} \tag{11}$$

[14] We choose λ_M to guarantee that $C \in (0,1)$

[15] We follow Spence (1973) in considering individual investment costs as nonpecuniary. See the discussion in Weiss (1983).

Notice that this return depends on the threshold α^* but does not vary with individual ability α. This is the natural consequence of the assumption that firms cannot observe individual human capital. On the other hand, the net nonpecuniary cost of taking the test monotonously decreases as individual academic ability increases. Hence, if an internal solution exists, there is a single intersection between costs and expected returns, given by

$$\frac{C(\alpha^*)}{2-\alpha^*}\left\{\delta\left[(1+\sigma)\frac{(2+\alpha^*)}{2}-b\right]\right\}=\Omega(\alpha^*,v^*) \tag{12}$$

and only individuals with academic ability $\alpha \geq \alpha^*$ take the test and pass it. As remarked by Fernandez (1998), no individual who passes the test scores more than v^*. Individuals with $\alpha < \alpha^*$ realize that their expected returns are lower than their net nonpecuniary costs of taking the test and self-select into L schools[16].

Since the left hand side of Eq. (12) is increasing and the right hand side is decreasing in α^*, the optimal cutoff value of the test v^* is strictly increasing in α^*. Using the fact that $\Theta = 2-\alpha^*$, we obtain that $\frac{\partial v^*(\Theta)}{\partial \Theta} < 0$. The smaller the size of the H schools sector, the higher the admission standard. The logic behind Eq. (12) is straightforward. The government chooses the percentage of H schools. This is equivalent to choosing α^*. Given α^*, condition (12) determines the cutoff value of the test v^*. Given v^*, individuals self select into their preferred school. The higher α^*, the fewer the slots in H schools and the stricter the test. Furthermore, the more selective the schooling system (the higher v^*), the higher the expected wage differential between graduates of H and L schools[17].

Next, we turn to the choice of optimal Θ by the government and restrict our attention to selective systems ($\alpha^* \in (1,2)$). In this choice, the government takes explicitly into account that, by choosing Θ, it affects both the cutoff value of the admission test v^* and the number of C firms. Recalling that choosing Θ is equivalent to selecting α^*, we define total output Y as

$$Y = C(\alpha^*)(1+\sigma)\frac{(2+\alpha^*)}{2}+[1-C(\alpha^*)] \tag{13}$$

Using Eqs. (8) and (9) into Eq. (13) and differentiating the outcome with respect to α^* we obtain that $\frac{\partial Y}{\partial \alpha^*} > 0$. An increase in α^* increases both the expected output of C firms and their relative share C. By increasing α^*, however, the government contemporaneously reduces Θ, the number of slots in H schools, and the constraint (10) can be violated within the domain of α.

Figure 1 illustrates these effects. In the figure, we draw the share of C firms as an increasing function of the threshold α^* ($\alpha^* \in (1,2)$), as implied by Eqs.

[16] Self-selection is confirmed by the empirical research of Venti and Wise (1982), who find that a large proportion of college applicants find admission in their "first choice" school.

[17] See Brewer, Eide and Ehrenberg (1999) for evidence on the wage premia paid to graduates of elite schools.

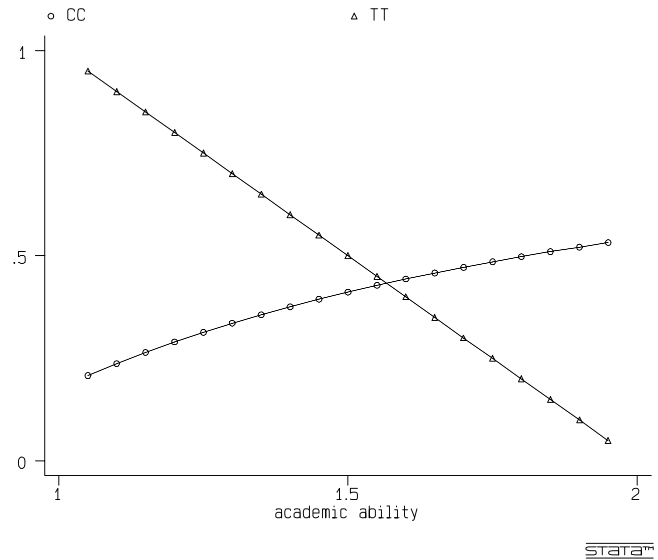

Figure 1: The CC and the TT curve.

(8) and (9). We call it the CC curve. We also draw $\Theta = 2 - \alpha^*$ and call it the TT curve. The two curves in the figure intersect only once. To the left of the intersection, $C \leq \Theta$ and total output can be increased by rising the selectivity of the schooling system. To the right of the intersection, $C > \Theta$, constraint (10) is violated and there are some C firms which cannot hire graduates from H schools, who are in short supply.

The behavior of the economy when Eq. (10) does not hold is described by the following Proposition

Proposition 1 *When there is a single ability type and the schooling system is selective, the optimal share of H schools, determined by the threshold α^*, is given by the condition $C = \Theta$.*

PROOF: We have shown above that, when $C < \Theta$, total output Y and the share of C firms, C, are increasing in the threshold α^*. By increasing α^*, the government contemporaneously reduces Θ and increases C up to $C = \Theta$. Next, suppose that $C > \Theta$. In this case at least some C firms will not be able to fill with certainty their vacancies with graduates of H schools, because of the limited supply of these graduates. With random selection, a firm C can hire a graduate of H schools with probability $\frac{\Theta}{C}$ and a graduate of L schools with probability $1-\frac{\Theta}{C}$, and the expected profits from recruitment are

$$\pi_{\mathrm{Ci}} = (1+\sigma)\,\frac{(1+a^*)}{2} + \frac{\Theta}{C}\left(\frac{1+\sigma}{2}\right) - w - \lambda_i$$

It follows that the critical value of λ, λ^*, is equal to

$$\lambda^* = \frac{(1-\delta)(1+\sigma)\left[(1+\alpha^*) + \frac{2-\alpha^*}{\frac{\lambda^*}{\lambda_{MAX}}}\right] - 2(1-\delta b)}{2}$$

Total differentiation of above equation yields $\frac{\partial\lambda^*}{\partial\alpha^*} < 0$. Hence, an increase in the critical value of α^* reduces the share of C firms when $C > \Theta$.

Total output when $C > \Theta$ is given by

$$Y = C(1+\sigma)\,\frac{(1+\alpha^*)}{2} + \left(\frac{1+\sigma}{2}\right)(2-\alpha^*) + 1 - C$$

Because $\frac{\partial C}{\partial\alpha^*} < 0$ when $C > \Theta$, straightforward differentiation shows that a reduction in α^* increases total output Y. Therefore, the government increases α^* if the economy starts from $\Theta > C$ and reduces it if it starts from $\Theta < C$. It follows that the optimal value of α^* is at $\Theta = C$. □

Proposition 1 plays a key role in the paper. Figure 2 illustrates the optimal allocation for $\alpha^* \in (1, 2)$.

An implication of Proposition 1 is that in the optimal allocation all the graduates of H schools are certain to find a job in a high wage C firm. Therefore there is complete segregation of graduates of L schools in S firms[18] and both "over-education" ($C < \Theta$) and "under-education" ($C > \Theta$) are inefficient. We can also establish the following:

Corollary 2 *Both the optimal share of H schools, Θ, and the optimal share of C firms, C, are decreasing functions of the bargaining power of employees δ and increasing functions of the marginal productivity of non academic abilities σ.*

PROOF: The optimal size of the H schools sector, $\Theta = 2 - \alpha^*$, and the optimal share of high tech firms, C, are given by the condition $C = \Theta$ and by Eqs. (8) and (9). The comparative statics properties follow from straightforward differentiation.□

It is useful to illustrate some of these properties using Figure 3. Consider for instance an increase in σ, that can occur because of the undergoing changes in the organization of production and job design. According to Lindbeck and

[18] Notice that complete segregation depends on the demand side of the market rather than on the presence of peer effects, as for instance in Epple and Romano (1998). Segregation by skill is discussed also by Kremer and Maskin (1997).

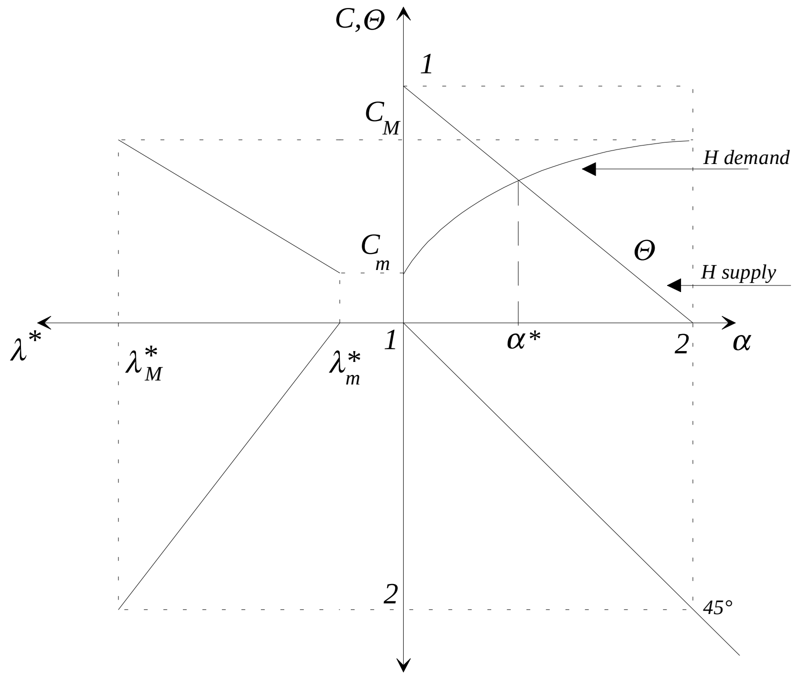

Figure 2: The optimal selection of Θ

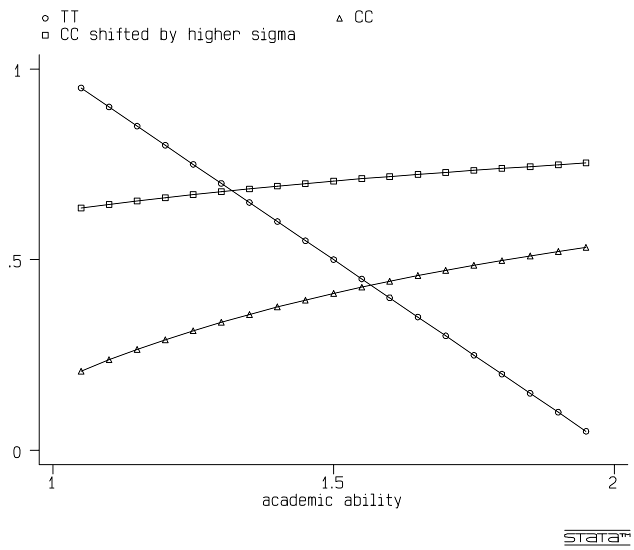

Figure 3: Comparative statics.

Snower (1999), the transition from a Tayloristic to an holistic way of organizing production, that includes job rotation and multi-skilling, implies that non academic abilities such as social skills, communication, initiative, creativity and collaborative skills rapidly increase in importance. An increase in σ shifts the CC line upwards, leading initially to an excess demand for graduates of H schools ($C > \Theta$). It follows from Proposition 1 that the government will react to this excess demand by reducing the threshold α^* and by increasing the share of individuals admitted to high quality schools or to higher education. As a consequence, C falls along the new CC curve and Θ increases up to the point where $C = \Theta$.

To summarize, when there is a single ability type or, equivalently, academic and non academic abilities are perfectly correlated across individuals, the optimal schooling policy leads to the equality of the supply of and the demand for graduates of H schools. In equilibrium, individuals endowed with higher academic abilities are employed by firms paying higher wages and producing higher (expected) output. On the other hand, less fortunate individuals are trapped in the low wage sector.

So far, we have assumed that the optimal design of the public schooling system when schools do not produce human capital consists of separating schools into two types and of choosing both the relative size of H schools and the admission test, that allocates individuals to schools by ability. An alternative option is to have a comprehensive schooling system, that pools all individuals in a single type of school, without any admission standard. We next show that, if the government maximizes total output, a separating system is always more productive than a comprehensive system. Notice that a pooling system takes place when the threshold α^* is set equal to its extreme values, 1 or 2. In either case, all individuals go to the same comprehensive school, that pools all the available academic and non academic abilities. With pooling, expected human capital is

$$E[h] = 1.5 + 1.5\sigma \tag{14}$$

and the share of C firms is determined by

$$\lambda^* = (1-\delta)[1.5 + 1.5\sigma] - (1 - \delta b) \tag{15}$$

Finally, total output with pooling Y_P is given by

$$Y_P = C(1.5 + 1.5\sigma) + 1 - C \tag{16}$$

Is it straightforward to show that, by choosing an internal value of α and by creating as a consequence a separation between H and L schools, the government can increase total output in the economy. The reason why this happens is that, by making the schooling system selective, the government allows individuals with high academic and non academic abilities to signal their quality to firms. Since firms hiring graduates of H schools expect higher human capital when schools are selective, there are in equilibrium more firms adopting the complex but highly productive technology. Thus, total output is higher when $\alpha^* \in (1, 2)$.

4 Two abilities

In the previous section we have studied schooling admission tests when the correlation between ability types (academic and non academic) in the population is equal to 1. This is equivalent to having a single ability type. In the current section, we study the economic implications of alternative admission tests when the two ability types are poorly correlated in the population, as found by Wise in his empirical research. To sharpen the contrast with the previous section, we assume that this correlation is equal to zero.

Clearly, there are many functions $\beta(\alpha)$ that describe the relationship between non academic and academic abilities and yield zero correlation between α and β in the population. Figure 4 considers three examples of distributions of individual human capital in the population, based on Eq. (1) and on the additional assumption that the marginal contribution of each ability type to individual human capital is equal ($\sigma = 1$).

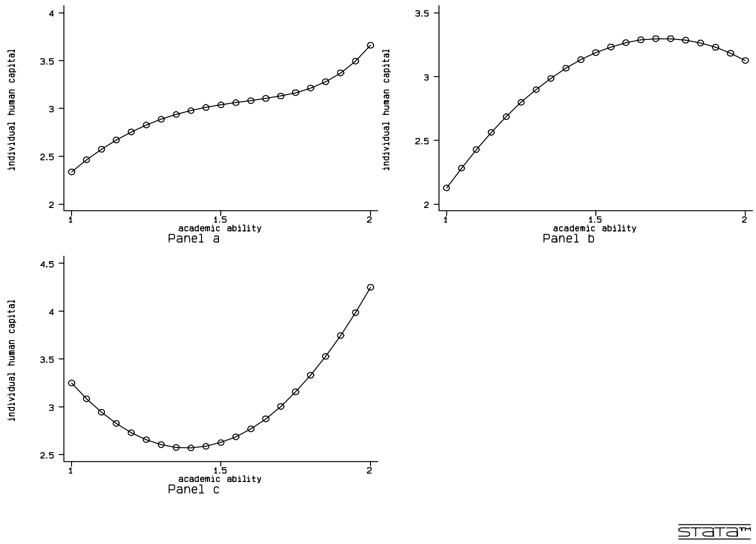

Figure 4: Individual human capital. Three examples.

In Panel (a), non academic ability β is a highly nonlinear function of academic ability and is described by a polynomial of the fifth order in α[19]. This functional form implies the coexistence in the same population of two groups of individuals: in the former group, individuals with low non academic ability can have both low and high academic ability. In the latter group, individuals with high non academic ability can be endowed either with medium or with very high academic ability.

In Panel (b), β is a hump shaped function of α. In words, individuals with both lower and higher than average academic ability have a poorer endowment of non academic ability, either because they specialize in the former (academic types) or because they are poorly endowed of both abilities. Specialization before the admission test can occur when individuals who are well endowed of academic abilities under-invest in the development of non academic skills. The underlying idea is that initiative, creativity, leadership and technical abilities require academic abilities such as intelligence and knowledge. Individuals with

[19] $\beta = 1.74\alpha^5 - 10.3\alpha^4 + 25.56\alpha^3 - 35.46\alpha^2 + 28.42\alpha - 8.62$

a high endowment of academic ability, however, are not necessarily as well endowed with non academic skills. Finally, Panel (c) is based on the assumption that β is a U-shaped function of α[20]. Contrary to the previous case, in this case individuals with both lower and higher than average academic ability have a richer endowment of non academic ability.

Notice that only in Panel (a) is individual human capital a monotonic function of academic ability α. Recall that an admission test based on academic abilities consists of choosing a threshold value of α, α^*, such that all individuals with $\alpha \geq \alpha^*$ are allocated to H schools. Since individual human capital h is a monotonic function of α, the selected threshold also allocates to H schools all individuals with human capital h higher than $h(\alpha^*)$. Therefore, an admission standard based upon h yields the same outcome of an admission standard based exclusively on α.

The situation in Panels (b) and (c) is different. In Panel (b) individuals with very high academic ability have a lower human capital than individuals with intermediate to high academic ability. In Panel (c), individuals with very low academic ability have higher human capital than individuals with intermediate academic ability. In either case, an encompassing test based upon h, that considers both ability types, and a test based on academic ability α yield a different allocation of individuals to schools, and only the former test is capable of allocating *all* individuals with high human capital to H schools. A standard test based only on academic ability is less efficient because, by ignoring non academic ability, it allocates some individuals with relatively high human capital to L schools (Panel (c)), and some individuals with relatively low human capital to H schools (Panel (b)). In these cases, an encompassing test does better (yields higher total output) than a standard test.

Whether individual human capital is a non monotonic function of α depends both on the function $\beta(\alpha)$ and on the value of σ, the marginal contribution of non academic abilities to h. For example, an increase of σ from 1 to 2 transforms the relationship in Panel (a) of Figure 4 from monotonic to non monotonic. Therefore, an increase in the relative importance of non academic abilities for individual productivity can affect the relative efficiency of standard admission tests.

While the examples in Figure 4 are specific to the selected functional forms, the result is general: when the relationship between individual human capital and academic ability is non monotonic, an encompassing test that considers both ability types yields higher output than a standard test. The intuition is simple. By allocating to H schools the individuals with higher potential productivity, the encompassing test increases both the expected human capital of graduates of these schools and the share of firms adopting complex technologies. Therefore, output increases.

The advantages of an encompassing test can be better appreciated by analyzing in more detail the case corresponding to Panel (b) in Figure 4. A hump-shaped relationship between individual human capital (and wages) and

[20] $\beta = 1.125(4\alpha^2 - 12\alpha + 10)$

academic ability is consistent with recent evidence produced by Ashenfelter and Rouse (1999), who have examined the returns to education by test score quartile. These authors use a standardized measure of the Armed Force Qualification Test ($AFQT$), a test of cognitive skills that constitutes a subset of the $ASVAB$ tests administered in 1980 to the sample of individuals included in the National Longitudinal Survey of Youth ($NLSY$), and find that returns to education are higher for intermediate values than for either low or high values of the test score.

We provide additional evidence by extracting from the $ASVAB$ scores in the $NLSY$ data used by Ashenfelter and Rouse two common factors, that explain 78% of the variance of these scores. The former factor, that includes the scores of the subtests on coding speed, numerical operations, paragraph comprehension, math knowledge, word knowledge and arithmetic reasoning, focuses mainly on testing cognitive abilities α. The latter factor, that includes the scores of the subtests on auto and shop information, electronics, mechanics and general science, focuses mainly on testing technical abilities β. We plot in Figure 5 the values taken by these two factors by putting the first factor (α) on the abscissa and the second factor (β) on the ordinate. The scatter diagram clearly suggests the presence of a hump shaped relationship between the two components. The continuous lines in the diagram are the best quadratic fit and the associated confidence intervals.

We characterize the hump shaped relationship between β and α with the following functional relationship[21]

$$\beta = 1.125\left(6a - 2a^2 - 3\right) \tag{17}$$

It can be easily verified that Eq. (17) implies that the population covariance between the two abilities is equal to zero.

Consider first a standard test based on academic ability. As discussed in the previous section, in a selective schooling system ($\alpha^* \in (1,2)$) the admission test consists of choosing a single threshold value of academic abilities α^* such that only the individuals with $\alpha \geq \alpha^*$ pass the test and are allocated to H schools. As a result of the allocation, expected human capital is given by

$$E\left[h \mid H\right] = \frac{\int_{\alpha^*}^{2}\left[1.125\sigma\left(6\alpha - 2\alpha^2 - 3\right) + \alpha\right] d\alpha}{2 - \alpha^*}$$

$$= 1 + 0.375\sigma + \alpha^*\left(0.5 + 1.875\sigma\right) - 0.75\sigma\alpha^{*2} \tag{18}$$

for graduates of H schools and by

[21] The constant term before parentheses guarantees that total non academic ability in the population, measured by $\int_1^2 \beta\left(\alpha\right) d\alpha = 1.5$, is equal to the case of perfectly correlated abilities discussed in the previous section of the paper.

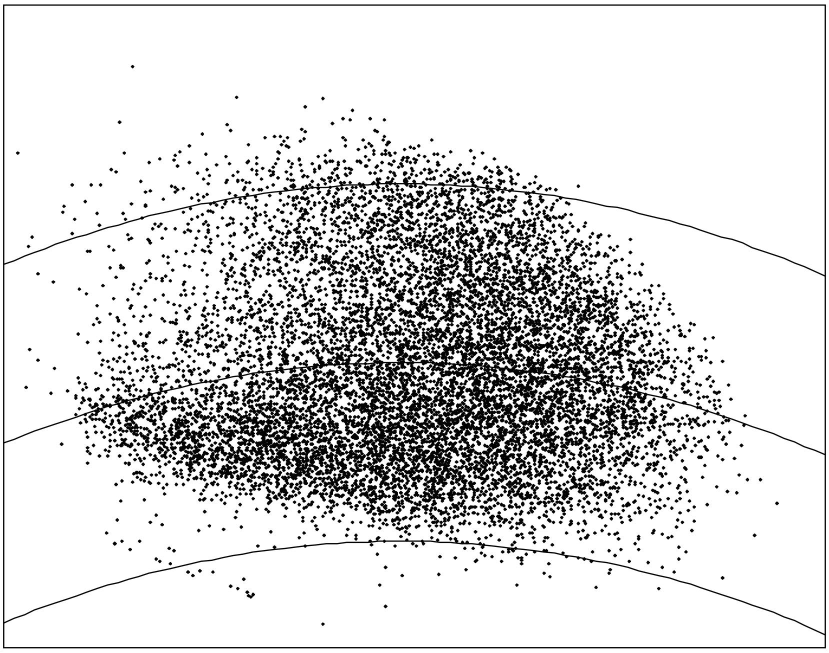

Figure 5: Plot of the two common factors

$$E[h \mid L] = \frac{\int_1^{\alpha^*} \left[1.125\sigma \left(6\alpha - 2\alpha^2 - 3\right) + \alpha\right] d\alpha}{\alpha^* - 1}$$

$$= 0.5 - 0.75\sigma + \alpha^* (0.5 + 2.625\sigma) - 0.75\sigma\alpha^{*2} \tag{19}$$

for graduates of L schools.

The difference between $E[h \mid H]$ and $E[h \mid L]$ is positive if $\alpha^* < \alpha^{'} = 1.5 + \frac{0.67}{\sigma}$. When $\alpha^* \geq \alpha^{'}$, the selective schooling system collapses because C firms find more profitable to hire graduates from L schools. In this event, no rational individual will take and pass the admission test to enter H schools. When $\alpha^* < \alpha^{'}$, firms prefer to hire graduates from H schools and the share of firms adopting the complex technology is

$$\lambda^* = (1 - \delta) \left[1 + 0.375\sigma + \alpha^* (0.5 + 1.875\sigma) - 0.75\sigma\alpha^{*2}\right] - (1 - \delta b) \tag{20}$$

A comparison of Eq. (20) with Eq. (8) shows that the critical value λ^* is higher when there is a single ability (or when abilities are perfectly and positively correlated).

Given the optimal choice of Θ by the government, which selects the critical value of academic ability, α^*, the individual choice of taking and passing the admission test when $C \leq \Theta$ is based upon the comparison of expected costs with expected benefits. The marginal individual, who is just indifferent between investing and not investing, is identified by the following condition

$$\frac{C(\alpha^*)}{2 - \alpha^*} \left\{\delta \left[1 + 0.375\sigma + \alpha^* (0.5 + 1.875\sigma) - 0.75\sigma\alpha^{*2}\right] - \delta b\right\} = \Omega(\alpha^*, v^*) \tag{21}$$

Notice that the left hand side of Eq. (21) is increasing in α^*, while the right hand side is monotonically decreasing in α^*. As in the previous section, there is a single intersection between expected returns and individual costs.

Quite in contrast to the case of a single ability, however, total output, defined as

$$Y = C(\alpha^*) \left[1 + 0.375\sigma + \alpha^* (0.5 + 1.875\sigma) - 0.75\sigma\alpha^{*2}\right] + 1 - C(\alpha^*) \tag{22}$$

has an interior maximum given by

$$\alpha^M = \frac{0.5 + 1.875\sigma}{1.5\sigma} \tag{23}$$

if $\sigma > 0.44$.

An important feature of Eq. (23) is that $\alpha^M < \alpha^{'}$. Thus, when total output has an interior maximum, the government always chooses Θ, the size of H schools, to obtain that $E[h \mid H] > E[h \mid L]$. Notice that equation (23) corresponds to the optimal government choice only if $C \leq \Theta$. This is shown in the following Remark:

Remark 3 *When the schooling system is selective and Eq. (17) holds, the optimal size of the H schools sector Θ cannot be lower than the optimal number of C firms.*

Proof: See the Appendix.

Let α^{θ} be the value of the threshold that guarantees that $C = \Theta$. Then it follows that the optimal threshold is[22]

$$\alpha^{*} = \min\left(\alpha^{M}, \alpha^{\theta}\right) \tag{24}$$

We compare total output in the selective schooling system (Eq. (22)) with total output in a comprehensive system (Eq. (16)). It turns out that the former dominates the latter if $\alpha^{*} < \alpha^{'}$. As shown above, this condition always holds.

The interior maximum α^{M} is a decreasing function of σ. Hence, the more productive is non academic ability, the lower the optimal threshold selected by the government and the less selective the admission test. The reason is that, when non academic abilities matter for production, having a very selective schooling system based on a standard admission test could attract to H schools only the academically talented, who are relatively poorly endowed of non academic abilities. Total output can be increased in these circumstances by reducing the critical threshold α^{*} below the value that yields $C = \Theta$.

Therefore the optimal design of a selective schooling system could involve having an excessive supply of graduates of H schools ($C < \Theta$). In this case, over-education is efficient and a selective schooling system does not imply complete segregation, because a share $1 - \frac{C}{\Theta}$ of graduates of H schools ends up in a S firm. Without complete segregation, good performance at school does not necessarily imply good labor market performance[23].

An alternative to testing only academic ability is to design admission tests that focus on individual human capital, which depends on both ability types. The rationale for this alternative selection procedure is that, by focusing exclusively on one ability, H schools risk to include academic talents with limited valuable non academic skills and relatively low productive potential. Schools are aware of this risk. As argued by Cook and Frank (1993), for example, elite schools in the US try to select students by considering many personal qualities that happen also to predict success on the job.

An admission test based on h requires that the government maximizes total output by setting a threshold value of h, h^{*}, and that individuals with $h \geq h^{*}$ pass the test and are admitted to H schools, while individuals with $h < h^{*}$ do not take the test and are assigned to L schools. The selection of the threshold h^{*} can

[22] When $\sigma \leq 0.44$, both α^{M} and α^{θ} are larger than 2 and there is no interior solution.

[23] Card and Krueger (1992) and Hanushek (1986) present interesting evidence on this point.

be better illustrated by considering Panel (b) of Figure 4, that plots individual human capital when the correlation between abilities in the population is zero, Eq. (17) holds and σ is equal to 1.

Since the choice of h^* in the figure is equivalent to the selection of a point on the vertical axis, it can immediately be seen that, corresponding to such a point, there could be two rather than a single critical value in the domain of academic abilities α. Furthermore, because the relationship between the two abilities α and β is nonlinear, there exists a value of h, $\bar{h}$, such that $\bar{h} = h(2)$, where $h(2)$ is the value of individual human capital corresponding to the maximum value of academic ability $\alpha = 2$. The nonlinear relationship between h and α also implies that, while the distribution of α is uniform, the distribution of h is not and has a discontinuity at $h = \bar{h}$. The density of h, $f(h)$, requires that we also compute the maximum value of h, h_m, and that we order h from its minimum value $(1 + 1.125\sigma)$ to its maximum h_m. Following Ventsel (1983), we obtain

$$f(h) = \frac{4}{\sqrt{16 + 216\sigma + 243\sigma^2 - 144\sigma h}} \tag{25}$$

where $\frac{\partial f(h)}{\partial h} > 0$. With this density, the expected human capital of graduates of H schools is given by

$$E[h \mid H] = \frac{\int_{h^*}^{\bar{h}} hf(h)\,dh + \int_{\bar{h}}^{h_m} 2hf(h)\,dh}{\int_{h^*}^{h_m} f(h)\,dh} \tag{26}$$

if $h^* < \bar{h}$ and by

$$E[h \mid H] = \frac{\int_{h^*}^{h_m} 2hf(h)\,dh}{\int_{h^*}^{h_m} f(h)\,dh} \tag{27}$$

if $h^* \geq \bar{h}$.

In either case, it can be shown that $E[h \mid H]$ is an increasing function of the threshold h^*. By increasing this threshold, the government increases both $E[h \mid H]$, C and total output Y. As shown in Proposition 1, this implies that the optimal share of H schools is $C(h^*) = \int_{h^*}^{h_m} f(h)dh$, where the right hand side of this equality is the size of the H schools sector. Therefore, the government can increase the threshold h^* up to the point where the demand for graduates of H schools by C firms is equal to the supply of this type of graduates. Once the threshold value is determined by this equilibrium condition, the cutoff value of the admission test v^* is defined in the usual way and only requires that individuals can be ordered by their (net) nonpecuniary cost of taking the admission test[24].

[24] As in the previous section, we need to assume that the score of the test is increasing in individual human capital and in the (net) nonpecuniary costs of the test.

When the government selects h^* and h^* turns out to be in the open interval $\left(\bar{h}, h_m\right)$, there are two critical values of academic abilities, α_1 and α_2, corresponding to the selected threshold. While individuals endowed with values of α between these critical values take and pass the admission test, both individuals with low academic abilities ($\alpha < \alpha_1$) and individuals with high academic abilities ($\alpha > \alpha_2$) do not take or pass the test. The latter group is selected out because it is endowed with little but valuable non academic ability.

We compare the outcomes of the encompassing test and of the standard test based only on academic abilities by solving the model for a given configuration of the underlying parameters. Table 1 shows the results. In the table, we allow σ to vary and keep the remaining parameters fixed. We find that total output is always higher when the admission test is based upon h than when it is based upon α. Output is higher because testing h rather than α increases both the expected human capital of graduates from H schools and the number of firms adopting complex technologies. While some individuals admitted in H schools when the test is on h rather than on α can have lower academic abilities, they certainly have a higher combination of both abilities.

Table 1. Model solutions with alternative values of σ

σ	0.75	1	1.25	0.75	1	1.25
		Test on α			Test on h	
α^*	1.69	1.58	1.56	1.75, 1.85	1.56, 1.87	1.45, 1.90
$E[h\|H]$	2.90	3.26	3.62	2.91	3.28	3.65
$E[h\|L]$	2.60	2.89	3.18	2.58	2.87	3.15
C	0.08	0.30	0.44	0.10	0.31	0.45
Y	1.16	1.68	2.15	1.19	1.71	2.19

Note: parameters used for the solutions: $\delta = 0.6$, $b = 0.5$, $\lambda_M = 1.7$.

The percentage output gain associated to testing h rather than α clearly depends on the values assigned to the underlying parameters of the model. Based on the values used in Table 1, this gain ranges from 2.6% of total output when $\sigma = 0.75$ to 1.8% when $\sigma = 1$ to 1.9% when $\sigma = 1.25$. A decline in λ_M from 1.7 to 1.4, however, is sufficient to raise the output gain from 1.8% to 4.4% when $\sigma = 1$. To put these numbers in their right perspective, consider that public educational expenditure as a percentage of GDP in the OECD area was about 4.7% in 1994[25]. It follows that the efficiency losses associated to testing only academic skills when Eq. (17) holds can be substantial and sufficient to justify the additional costs that the design and the implementation of more balanced and complicated admission procedures require[26].

[25] OECD, Education at a Glance, 1997.

[26] The use of the multiplicative technology $h = \alpha\beta^\sigma$ amplifies the variations of individual human capital in the population and sharpens the decline of h when α is high and β is low. Our qualitative results, however, remain unchanged.

We conclude that lack of covariance in the population between academic and non academic abilities is not sufficient to claim that admission tests based on individual human capital yield superior outcomes than more traditional admission tests based only on academic abilities. Whether or not a more balanced admission procedure in the allocation of talents to schools is superior depends both on the distribution of abilities in the population and on their contribution to individual productivity. It follows that an adequate knowledge of both is of paramount importance to design efficient admission policies.

5 Testing by firms

The design of an encompassing test on h is potentially difficult. First, schools are run by academics, who are particularly good at testing academic traits but perhaps not as good as firms at testing non academic abilities. Second, a test on individual human capital requires that each ability be properly weighted. Since the weight is the marginal productivity of non academic abilities, it is natural to expect that firms have a relative advantage over schools in grasping the importance of non academic abilities for economic production.

In this situation, an alternative testing strategy when individual human capital is a non monotonic function of academic ability α is to have schools test academic abilities and firms test non academic abilities conditional on the information provided by schools. The interesting question is whether this testing strategy can replicate the aggregate outcome achieved by an encompassing test on human capital. If the answer is affirmative, the emphasis placed by schools on academic abilities makes good economic sense. Schools are simply doing what they are best at, but their testing activity should be supplemented when necessary with additional testing of non academic abilities by firms.

Given the one-shot nature of our model, we exclude screening, that requires time, and assume that firms can implement a recruitment test on non academic abilities at no cost. The cutoff value μ^* of the recruitment test μ is an increasing function of β^*, the threshold value of non academic abilities, and is designed to have only individuals with $\beta \geq \beta^*$ to take and pass the test. While the individual decision to take and pass the test can be modeled as in Eq. (12), the threshold β^* is set by profit maximizing individual firms rather than by a benevolent central planner.

When the relationship between academic and non academic abilities is as described in Panel (b) of Figure 4, individual firms have two options: first, they can ignore the signal offered by the schooling system, based on a test of academic abilities, because the test excludes individuals with relatively low α and a relatively high and useful β and includes individuals with high α and relatively low β. By so doing, firms admit to the recruitment test all new graduates, independently of the school they are coming from. The second option is to use the information provided by schools and to admit to the recruitment test only graduates of H schools, who have higher academic ability than graduates of L schools. In either case, only C firms, that use the complex technology, are interested in the test. Therefore, the recruitment test is conditional on the selection of the technology. We consider these two options in turn.

The former option has drastic implications on the schooling system. Since firms admit to the recruitment test all graduates, independently of the school they are coming from, the schooling system can only be comprehensive, because the signal provided by selective schools is useless in the labor market. When Eq. (17) holds, the expected human capital of individuals taking and passing the test on β organized by C firms is

$$E\left[h|\beta^{*}\right]=\frac{\int_{\alpha_{1}\left(\beta^{*}\right)}^{\alpha_{2}\left(\beta^{*}\right)}\left[\alpha+1.125\sigma\left(6\alpha-2\alpha^{2}-3\right)\right]}{\alpha_{2}\left(\beta^{*}\right)-\alpha_{1}\left(\beta^{*}\right)}d\alpha \tag{28}$$

As shown in Figure 6 below, the selection of the threshold β^{*} is equivalent to choosing two values of α, $\alpha_{1}\left(\beta^{*}\right)$ and $\alpha_{2}\left(\beta^{*}\right)$, that solve $\beta^{*}=1.125\left(6\alpha-2\alpha^{2}-3\right)$. It turns out that $\alpha_{1}\left(\beta^{*}\right)=\frac{3}{2}-\frac{1}{6}\sqrt{\left(27-16\beta^{*}\right)}$ and $\alpha_{2}\left(\beta^{*}\right)=\frac{3}{2}+\frac{1}{6}\sqrt{\left(27-16\beta^{*}\right)}$.

Notice that profit maximization is equivalent to the maximization of expected human capital $E\left[h|\beta^{*}\right]$. Since $E\left[h|\beta^{*}\right]$ increases in the threshold β^{*}, each firm will try to push the threshold to the maximum value of β, β_{m}. This is clearly inconsistent with the fact that the number of individuals who pass the recruitment test must be at least equal to the number of C firms. In the absence of opportunistic behavior, consistency is guaranteed by adding the following constraint to profit maximization

$$\alpha_{2}\left(\beta^{*}\right)-\alpha_{1}\left(\beta^{*}\right)=\frac{1}{3}\sqrt{\left(27-16\beta^{*}\right)}=\frac{\left[\left(1-\delta\right)E\left[h|\beta^{*}\right]-\left(1-\delta b\right)\right]}{\lambda_{M}} \tag{29}$$

where the left hand side of Eq. (29) is the supply of individuals with $\beta\geq\beta^{*}$ and the right hand side is the demand of these individuals by C firms. Since firms can increase profits by increasing β^{*}, the constraint holds as a strict equality.

In the latter option, firms use the information provided by schools in the design of the test of non academic abilities. As above, the choice of the threshold β^{*} consists of selecting the two critical values $\alpha_{1}(\beta^{*})$ and $\alpha_{2}(\beta^{*})$, where $\alpha_{1}<\alpha_{2}$. There are two possibilities: first, the threshold α^{*} on academic abilities is set by the government below $\alpha_{1}(\beta^{*})$. Hence, α^{*} is not binding the decisions of firms and Eqs. (28) and (29) hold, subject to the constraint $\frac{3}{2}-\frac{1}{6}\sqrt{\left(27-16\beta^{*}\right)}>\alpha^{*}$, that ensures that government policy does not affect the selection of the threshold β^{*}.

Second, the threshold α^{*} is binding and higher than or equal to $\alpha_{1}(\beta^{*})$. In this case, the expected human capital of individuals who pass the recruitment test is

$$E\left[h|H,\beta^{*}\right]=\frac{\int_{\alpha^{*}}^{\alpha_{2}\left(\beta^{*}\right)}\left[\alpha+1.125\sigma\left(6\alpha-2\alpha^{2}-3\right)\right]d\alpha}{\alpha_{2}(\beta^{*})-\alpha^{*}} \tag{30}$$

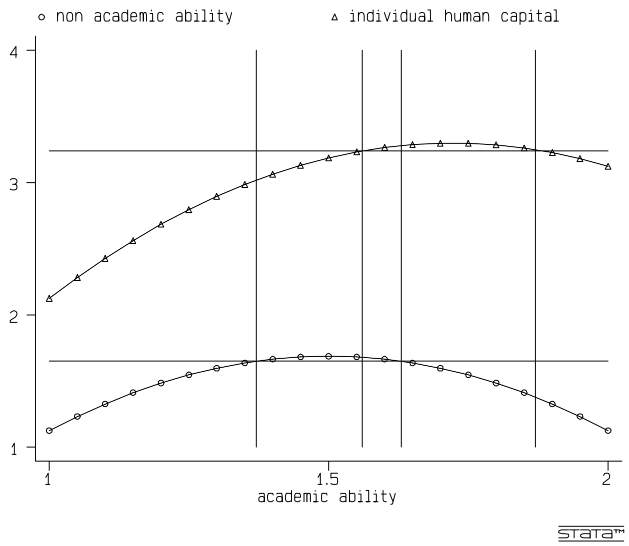

Figure 6: Testing non academic abilities

where the upper limit α_2 is determined by

$$\alpha_2(\beta^*) - \alpha^* = \frac{(1-\delta)\, E\left[h|H, \beta^*\right] - (1-\delta b)}{\lambda_M} \tag{31}$$

Eq. (31) shows that the choice of β^* must guarantee that there are enough graduates of H schools that pass the test to fill the C vacancies posted by high tech firms. This equation determines the threshold β^* and the total number of C firms as functions of α^*. The government, that moves first, takes into account the behavior of firms and sets the optimal threshold α^* to maximize total output. Individuals decide whether to take the academic admission test by considering the fact that employment in high wage C firms require that they also pass the test on non academic abilities after graduation. When C firms recruit only graduates of H schools, the following remarks apply.

Remark 4 *In equilibrium, the share of individuals taking the admission test based on academic abilities is equal to C, the share of firms adopting the complex technology.*

PROOF: The government selects α^* to maximize total output Y. The admission test based on α^* admits to H schools all individuals with $\alpha \geq \alpha^*$. Conditional on this test, firms select a threshold β^* and recruit only graduates of H schools with $\beta \geq \beta^*$. When Eq. (17) holds, this is equivalent to establishing two critical values of α, α_1 and α_2, and to exclude from recruitment all individuals with $\alpha < \alpha_1$ and $\alpha > \alpha_2$, who have low non academic ability. Since individuals are rational and know their own abilities, they take into account the test on β when deciding whether to gain admission to H schools. In particular, individuals endowed with $\alpha > \alpha_2$ (when α^* is binding) and with $\alpha < \alpha_1$ and $\alpha > \alpha_2$ (when α^* is not binding), who expect not to pass the recruitment test because of their relatively low β, do not take the admission test as well, because taking this test does not lead to a job in a high wage firm. It follows that, in equilibrium, the share of graduates of H firms is equal to the share of recruits by C firms. $\square$

Remark 5 *When α^* is binding, all graduates from H schools take the recruitment test. There is complete self selection of individuals to schools and firms.*

This is a consequence of the former Remark. When firms use the signal provided by schools, the presence of a test on β induce all individuals with $\alpha \geq \alpha^*$ but $\beta < \beta^*$ to select the L school sector.

Remark 6 *When firms test non academic abilities, the share of slots available in H schools is always equal to C, the share of firms adopting complex technologies. Therefore over-education can never be optimal.*

A rational government realizes that, when firms test graduates of H schools, the share of individuals willing to take the schooling test is equal to the share of C firms. Hence, it supplies $\alpha_2(\beta^*) - \alpha^*$ slots when α^* is binding and $\frac{1}{3}\sqrt{(27 - 16\beta^*)}$ slots when α^* is not binding.

In Section 4 we have shown that when Eq. (17) holds, it might be optimal for the government to set Θ above C and to create an excessive supply of graduates of H schools. Some of these graduates have relatively low non academic abilities, but firms cannot distinguish among graduates of the same school. When firms supplement the information provided by schools with a recruitment test on non academic abilities, individuals with relatively high α and relatively low β self select into L schools.

Suppose, however, that only academic abilities are known to individuals, perhaps as a result of the learning process taking place in the earlier stages of education. In this case, admission to a high quality college is a requirement to be eligible for the recruitment test provided by firms, and even individuals with relatively high α and relatively low β will take the academic admission test. Thus, when non academic ability is unknown to individuals as well as to the government and firms, there could be individuals who pass the academic test and end up working in low wage firms.

We compare the outcomes of the two alternative options available to firms (the threshold α^*chosen by the government is either binding or not) by solving the model for a given configuration of the underlying parameters and by assuming that the marginal productivity of non academic abilities, σ, is equal to 1. The results are illustrated in Table 2, where we also report for the sake of comparison the values obtained either by testing only academic abilities or by testing individual human capital.

Table 2. Testing non academic abilities by firms. $\sigma = 1$.

$\sigma = 1$	Test β α^* not binding	Test β α^* binding	Test α	Test h
β^*	1.65	1.68, 1.37	1.67	1.68, 1.37
$E[h\vert\beta \geq \beta^*]$	3.17	3.28	3.25	3.28
$E[h\vert\beta < \beta^*]$	2.94	2.87	2.89	2.87
C	0.26	0.31	0.30	0.31
α^*	1.37, 1.63	1.56, 1.87	1.58	1.56, 1.87
Y	1.57	1.71	1.68	1.71

Notes: parameters used for the solutions: $\delta = 0.6$, $b = 0.5$, $\lambda_M = 1.7$.

There are three results. First, total output is higher when the threshold α^* binds the selection of the threshold β^* by firms. This happens because, when α^* is not binding, firms focus exclusively on non academic abilities, despite the fact that academic abilities also matter for production. Figure 6 illustrates the point by considering the case $\sigma = 1$. In the figure, the two curves plot respectively non academic abilities and individual human capital as functions of α. When firms focus exclusively on β, they recruit all individuals with $\alpha \in [1.37, 1.63]$. By taking into account the outcome of the test on academic abilities, however, firms restrict their attention only to individuals with $\alpha \geq 1.56$, thus excluding those who are poorly endowed of academic abilities. When a rational government that maximizes total output realizes that the selected threshold α^* is not binding the decisions of firms, it has an incentive to increase α^* and reduce the size of the H schools sector up to the point where α^* binds the decisions of firms about β^*. By so doing, it compensates the excessive emphasis placed by firms on non academic abilities and increases total output.

Second, the expected human capital of individuals taking and passing the recruitment test $E[h|\beta \geq \beta^*]$ is higher when α^* is binding. Since expected profits are increasing in $E[h|\beta \geq \beta^*]$, firms adopting the complex technology find it more profitable to restrict the test to graduates of H schools. Third, the sequence of the two tests, by the government and by firms adopting the complex technology, reproduces the outcome of an encompassing test carried out by the government on individual human capital h because it replicates both the number of C firms and the expected human capital of recruits. This equivalence is the consequence of the fact that both the profits of C firms and total output are increasing functions of the expected human capital of the graduates of H schools, who take and pass the recruitment test.

The main message of Table 2 is that the emphasis placed by schools on academic abilities does not reduce economic efficiency, provided that firms add to the information supplied by schools their own testing of non academic abilities. This result, however, is not general but depends on the distribution of academic and non academic abilities in the population. To see why, consider Panel (c) in Figure 4. In this case, an encompassing test of individual human capital allocates to H schools both individuals with low α and high β and individuals with high α and β. The former group is excluded from H schools by a test based only on academic abilities, that selects individuals with $\alpha \geq \alpha^*$[27]. A sequential testing strategy cannot replicate the outcome of the encompassing test on h because of two reasons: a) when firms ignore the information provided by schools, they focus their testing on non academic abilities, thus ignoring the important contribution of academic abilities; b) when firms restrict their testing to graduates of H schools, they automatically exclude from the recruitment test the group of individuals with low α and high β, who are admitted to H schools by an encompassing test.

Therefore, we find again that additional information on the empirical distribution of abilities in the population is necessary before selecting the most appropriate (output maximizing) admission policy.

To conclude this section, we stress that the equivalence between an encompassing test run by government led schools and a sequential testing strategy also relies on the assumption that the design and implementation of tests is not costly both for the government and for firms. Given that the purpose of

[27] With the configuration of parameters used in Tables 1 and 2 and assuming that $\sigma = 1$, the test on academic ability yields $\alpha^* = 1.625$ and $Y = 1.911$. The test based on individual human capital admits to H schools individuals with $\alpha \leq 1.083$ and $\alpha \geq 1.690$ and yields $Y = 1.945$.

the paper is to provide a simple benchmark model for the discussion of admission tests in the presence of multiple abilities that are poorly correlated in the population, we feel that this assumption is justified. Clearly, when monetary costs are introduced into the model, whether total output net of the resources spent to design and implement the tests is higher or lower with the sequential testing strategy or with the encompassing admission strategy depends on the costs faced both by the government and by firms.

6 Conclusions

Concern is often voiced about the fact that most admission tests to higher education are designed to measure academic abilities and potential success in school rather than in the labor market. Since economic performance is affected both by cognitive abilities and by non academic abilities, admission procedures based only upon cognitive traits could generate an inefficient allocation of resources.

In this paper, we have used a simple signalling model that includes both the demand and the supply of skills to compare the economic performance of alternative admission tests when individuals are endowed with both academic and non academic abilities and both abilities matter for individual productivity. We have shown that a necessary condition for an encompassing admission test based on both abilities to yield a more efficient allocation of talents than a standard test based only on cognitive abilities is that individual human capital be a non monotone function of academic abilities. This condition depends both on the distribution of abilities in the population and on the contribution of each ability to individual human capital and productivity.

We have also shown that the outcome of an encompassing test can be replicated by a sequential testing strategy that consists of two stages: in the first stage, government run schools test academic abilities and in the second stage firms test non academic abilities on the sub-sample of graduates of high quality schools. The equivalence between these alternative admission policies depends, however, on the empirical distribution of abilities in the population.

We conclude that the empirical observation that both academic and non academic abilities matter for job performance is not sufficient to warrant more balanced admission criteria to higher education. Additional information on the empirical distribution of abilities and on their contribution to individual productivity is required before choosing the most appropriate selection criteria. Our results suggest that worries about the excessive emphasis placed by schools on academic abilities in their admission procedures could be exaggerated, either because a more balanced admission procedure does no better or because its outcome can also be achieved by a sequential strategy that involves both the government and employers.

7 Appendix

We prove that, when Eq. (17) holds and $C > \Theta$, total output can be increased by reducing α^* and by raising Θ to the point where $C = \Theta$.

First, when $C > \Theta$ the number of C firms is given by

$$C = \frac{(1-\delta)\left[\frac{\Theta}{C}E(h|H) + \left(1-\frac{\Theta}{C}\right)E(h|L)\right] - (1-\delta b)}{\lambda_M} \tag{A2.1}$$

Total differentiation of Eq. (A2.1) with respect to C and α^* yields that

$$sign\frac{\partial C}{\partial \alpha^*} = sign\frac{\partial\left[\frac{\Theta}{C}E(h|H) + \left(1-\frac{\Theta}{C}\right)E(h|L)\right]}{\partial \alpha^*} \tag{A2.2}$$

The system with H and L schools survives only if $E(h|H) - E(h|L) > 0$, that implies the following restriction on α^*

$$\alpha^* < \alpha^{'} = 1.5 + \frac{0.67}{\sigma} \tag{A2.3}$$

The computation of the partial derivative on the right hand side of Eq. (A2.2) yields

$$sign\frac{\partial\left[\frac{\Theta}{C}E(h|H) + \left(1-\frac{\Theta}{C}\right)E(h|L)\right]}{\partial \alpha^*} = -\ sign\,(0.5 + 2.625\sigma - 1.5\alpha^*\sigma)$$

A sufficient condition for the above expression to be negative is

$$\alpha^* < \alpha_2 = \frac{0.5 + 2.625\sigma}{1.5\sigma} \tag{A2.4}$$

It can be shown that there are two possibilities: a) $\alpha^{'} < \alpha_2$; b) $\alpha^{'} > \alpha_2$. In the former case (A2.4) is clearly satisfied. The latter case occurs if $\sigma < 1.3467$. Since $\frac{\partial \alpha_2}{\partial \sigma} < 0$ and $\alpha_2 \cong 2$ when $\sigma < 1.3467$, Eq. (A2.4) is satisfied in this case too. In either case the sign of $\frac{\partial C}{\partial \alpha^*}$ is negative.

Next maximize total output $Y = \Theta E(h|H) + (C - \Theta)E(h|L) + 1 - C$ to obtain

$$\frac{\partial Y}{\partial \alpha^*} = \frac{\partial C}{\partial \alpha^*}\frac{E(h|L) - 1}{1 - C} - [0.5 + 2.625\sigma - 1.5\sigma\alpha^*] \tag{A2.5}$$

The first component of the right hand side is negative because $E(h|L) - 1 > 0$; the second component is also negative because of the argument above. $\square$

References

[1] Acemoglu, D. and Pischke, S. (1998), The Structure of Wages and Investment in General Training, *NBER Working Paper n. 6357.*

[2] Arrow, K. (1993), Excellence and Equity in Higher Education, *Education Economics*, 1, 1, 5-12.

[3] Ashenfelter, O. and Rouse, C., (1999), *The Payoff to Education*, Princeton University.

[4] Betts, J. (1998), The Impact of Educational Standards on the Level and Distribution of Earnings, *The American Economic Review*, 266-275.

[5] Bishop, J. (1992), The impact of academic competencies on wages, unemployment and job performance, *Carnegie Rochester Conference Series on Public Policy*, 37, 127-194.

[6] Blackburn, M. and Neumark, D. (1993), Omitted Ability Bias and the Increase in the Return to Schooling, *Journal of Labor Economics, 11, 3, 521-544.*

[7] Brewer, D., Eide, E. and Ehrenberg, R., (1997), Does it pay to attend an elite private college? Cross cohort evidence on the effects of college quality on earnings, *NBER Working Paper no. 5613.*

[8] Brunello, G. and Ishikawa, T. (1999), Elite Schools, High Tech Jobs and Economic Welfare, *Journal of Public Economics*, 72, 3, 395-419.

[9] Card, D. and Krueger, A. (1992), Does School Quality Matter? Returns to Education and the Characteristics of Public Schools in the United States, *Journal of Political Economy*, 100, 1, 1-40.

[10] Caselli, F., (1999), Technological Revolutions, *The American Economic Review*, 89, 78-102.

[11] Cawley, J., Conneely, K., Heckman, J. and Vytlacil, E., (1996), Measuring the Effects of Cognitive Ability, *NBER Working Paper* no. 5645.

[12] Cook, P. and Frank, R., The Growing Concentration of Top Students in Elite Schools, in Clotfelter, C. and Rothschild, M. (eds.) (1993), *Studies in the Supply and Demand in Higher Education*, Chicago University Press.

[13] Costrell, R. (1994), A Simple Model of Educational Standards, *The American Economic Review*, 956-971.

[14] Currie, J. and Thomas, D., (1995), Race, Children's Cognitive Achievement and the Bell Curve, *NBER Working Paper* no. 5240.

[15] Eicher, T. (1996), Interaction between Endogenous Human Capital and Technological Change, *Review of Economic Studies*, 63, 127-144.

[16] Epple, D. and Romano, R. (1998), Competition between Private and Public Schools, Vouchers and Peer Group Effects, *The American Economic Review*, 33-60.

[17] Evans, W. and Schwab, R. (1995), Finishing High School and Starting College: Do Catholic Schools Make a Difference?, *The Quarterly Journal of Economics*, 941-974.

[18] Fernandez, R. (1998), Education and Borrowing Constraints: Tests vs Proces, *NBER Working Paper 6588.*

[19] Green, A., Wolf A. and Leney, T. (1999), *Convergence and Divergence in European education and training systems*, Bedford Way Papers, Institute of Education, University of London, 1999.

[20] Hanushek, E., (1986), The Economics of Schooling: Production and Efficiency in Public Schools, *Journal of Economic Literature*, 24, 3, 1141-77.

[21] Heckman, J., (1999), Policies to Foster Human Capital, *NBER Working Paper 7288.*

[22] Heckman, J., Lochner, L., and Taber, C., (1998), General Equilibrium Cost Benefit Analysis of Education and Tax Policies, *NBER Working Paper* 6881.

[23] Herrnstein, R. and Murray, C., (1994), *The Bell Curve: Intelligence and Class Structure in American Life*, The Free Press, New York.

[24] Kremer, M. and Maskin, E., (1997), Segregation by Skills and the Rise of Inequality, *NBER Working Paper no. 5718.*

[25] Legree, P., Pifer,M. and Grafton, F., (1996), Correlations among cognitive abilities are lower for higher ability groups, *Intelligence*, 23, 1, 45-57.

[26] Lindbeck, A. and Snower, D. (1999), Multi-Task Learning and the Reorganization of Work, *IZA Discussion Paper* 39.

[27] Malcomson, J. (1997), Contracts, hold ups and labor markets, in *Journal of Economic Literature*, 35, 1916-57.

[28] Mayberry, P. and Carey, N. (1997), The effect of aptitude andexperience on mechanical job performance, *Educational and Psychological Measurement*, 57, 1, 131-149.

[29] OECD (1997), *Literary Skills for the Knowledge Society*, Paris.

[30] Redding, S. (1996), The Low Skill, Low Quality Trap: Strategic Complementarities between Human Capital and R&D, *The Economic Journal*, 106, 458-470.

[31] Schwartz, R., Donnelly, M., Nash, P. and Young, B., (1991), Interrelationships between knowledge and performance based testing in a surgery clerkship, *Academic Medicine*, 66, 9, 19-21.

[32] Spence, M. (1973), Job Market Signalling, *The Quarterly Journal of Economics*, 77, 355-379.

[33] Stiglitz, J. (1975), The Theory of Screening, Education, and the Distribution of Income, *The American Economic Review*, 283-300.

[34] Truxillo, D., Bennett, S., Collins, M. (1998), College education and police job performance: A ten-year study, *Public Personnel Management*, 27, 2, 269-280.

[35] Venti, S. and Wise, D. , (1982), Test Scores, Educational Opportunities and Individual Choice, *Journal of Public Economics*, 18, 35-63.

[36] Ventsel, E.S., (1983), *Teorija Verojatnostej*, Nauka, Moscow.

[37] Weiss, A. (1983), A Sorting cum Learning Model of Education, *Journal of Political Economy*, 91, 3, 420-442.

[38] Willis, R. and Rosen, S., (1979), Education and Self Selection, *Journal of Political Economy*, 87, 2, S7-S36.

[39] Winston, G. (1999), Subsidies, Hierarchy and Peers: The Awkward Economics of Higher Education, *Journal of Economic Perspectives*, 13-36.

[40] Wise, D., (1975), Academic Achievement and Job Performance, *The American Economic Review*, 350-366.