Discounting and Sustainability in Integrated Assessment Models

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Abstract

Most currently employed Integrated Assessment Models are of a dynastic nature, commonly assuming a fixed relation between pure time preference, economic growth and interest rate. This rigid relation has lead to much debate on which level of discounting to adopt. Especially the quantitative results of Integrated Assessment Models have been subject to controversy because of their strong sensitivity for future discounting. Many economic analysts advocate employing a descriptive time preference, based on historic data, which usually represent an approximate efficient use of environmental resources. Others encourage assuming a prescriptive time discounting, allowing them – by taking low discount values – to model a sustainable use of environmental services. This paper argues that, although a fixed time preference relation might be convenient for economic analysis, such a supposition can be misleading. By avoiding a rigid discounting relation, the descriptive-prescriptive controversy can be avoided. It is concluded that dynastic Integrated Assessment Models are in many respects inappropriate for providing policy makers with quantitative figures about the costs of carbon dioxide emissions, and their desirable reduction levels. In contrast, Overlapping Generations models, allowing the use of a flexible discounting relation, are suitable for designing a broad class of policy instruments. With the Integrated Assessment Model ALICE 2.0, it is shown how various assumptions on demographic change and public institutions can affect the interest rate, thereby influencing the desired optimal greenhouse gas emission reductions. It is recommended that economic modelling intensifies attempting to establish both efficient and sustainable resource use. The discount rate should be treated as an endogenous variable, rather than an exogenous parameter characterising a central planner.

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1. Introduction

Over the past decade, disciplines such as environmental and energy economics have increasingly been considering large scales of time. Since establishing sustainable development has become one of the new guidelines for policy makers, and has become an intensive field of research for scientists of various fields, large time periods need to be examined, sometimes attaining the span of at least a couple of centuries. Such problems as a potential climate change, the exhaustion of natural resources, or the safe disposition of long-lived radioactive nuclear waste, compel current generations to take to a high degree into account what the results are of their actions in the far future. In the context of a large time horizon, the discount rate – a parameter playing a central role in most of economic modelling – determines to a larger extent than in research with respect to short-term questions the outcome of economic analysis.

This paper draws attention to the increased relevance to account properly for the discount rate in economic modelling. In particular, this will be done with respect to the phenomenon of global warming, which since the summits of Rio and Kyoto has reached high in the agendas of both politicians and scientists. Since a potential climate change caused by anthropogenic greenhouse gas (GHG) emissions has been recognised as a major environmental problem, a number of so-called Integrated Assessment Models (IAMs) have been developed to advice policy makers on the costs of current GHG emissions and on the desirable GHG emission reduction levels. From these IAMs, one can derive strategies with respect to the optimal levels of greenhouse gas concentrations in the atmosphere, and estimate the expected global effects of a “business as usual” carbon dioxide emission pattern. Among the score of IAMs developed since the beginning of the 1990s, some of the more well-known are CETA [Peck and Teisberg 1992], DICE [Nordhaus 1994], MERGE [Manne and Richels 1995], MERGE-II [Manne *et al.* 1995] and RICE [Nordhaus and Yang 1996].

Most IAMs presently used are of a dynastic (or Ramsey) type. They assume, some more and others less explicitly, a central planner maximising a welfare measure which aggregates utilities over different generations. The allocation of consumer goods or environmental resources is calculated from the rules for welfare distribution over these generations. Alternatively, in a so-called overlapping generations (OLG) framework, the dynamic allocation is primarily determined by the distribution of property rights over generations.[2] The analysis of many dynastic models has led to intensive debate on

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[2] OLG models exist with dynastic properties, e.g. by the assumption that a generation exists with empathy for future generations, *qualitate qua* functioning as a pseudo-central planner (see, for example, Barro 1974).

the kind of welfare functions to be used [see, for example, Azar and Sterner 1996]. In this paper, it is argued that the dynastic mechanism of welfare distribution exhibits characteristics that are unrealistic, sometimes even misleading. Its prevalent use in IAMs has led to unnecessary controversies between researchers advocating efficient resource use, on the one hand, and those who advocate sustainable resource use, on the other hand.

Discounting is commonly applied to the entire field of economics in order to convey that benefits and costs are valued less in the future than they are today. There are various reasons for attaching more value to the present than to the future. The first main reason is that economic agents are intrinsically impatient. They therefore possess a so-called “pure time preference”, attaching more value to “having now” than to “having in the future”. A second reason is that “capital is productive”, implying that one unit of a given resource now will generate more than one unit worth of this resource in the future. Consequently, an agent prefers possessing one unit of this resource now to having the foresight of possessing that unit later in time. Today, he is prepared to pay more for having one unit immediately than he would pay for having the same unit of resource at a later stage.

The dynastic IAMs commonly use a constant Rate of Pure Time Preference (RPTP) to discount future welfare levels before aggregation into a net present welfare measure. A low RPTP implies that future benefits and costs are valued relatively highly, whereas the use of a high RPTP means that relatively little value is attached to future benefits and costs. The discount rate considerably affects the calculation of the optimal carbon dioxide emissions and the corresponding emission costs. Because the role played by the discount factor in economic modelling is so important, its choice determines largely the results of analytical research, notably in the context of global warming. The precise value to be taken is subject of invariable controversy.

Koopmans [1974] points out that “discounting advances the doomsday”, implying that the less mankind cares about the future, the earlier it can expect to encounter the limits of the planet’s natural resources and amenities. It has been shown that the choice for the RPTP determines to a large extent whether or not efforts to attain an efficient resource use are compatible with a sustainable allocation of goods [Pezzey 1992, Pezzey and Withagen 1998]. The importance of discounting is intuitively clear. The environmental resource can, in the case of renewables, produce an indefinite stream of valuable services, the aggregate net present value of which directly depends on the discount rate. This potential future value has to be balanced with the immediate returns of present exhaustive resource

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Generally, however, OLG models consist of intimately linked generations without the presence of an intergenerational central welfare optimiser.

use that decreases future output. The discount rate determines which value is maximal, and thus, which use is most profitable.

Given the importance of the RPTP in the sustainability debate, many papers have been devoted to argue about its proper choice. Much of the environmental literature defends the viewpoint of employing low discount rates. The rationale behind this is that the use of high discount rates in economic modelling is generally presumed to imply policies with a negative impact on the environment. If discounting is too high, it is normally considered economically unfeasible to sustain a high quality environment. However, strictly speaking no unique relationship seems to exist between high discount rates and environmental deterioration.

As high discount rates are assumed, economic analysis teaches that the level of investment is slowed down, which decelerates economic development. Because the investments undertaken are (positively) correlated to the use of natural resources, the depletion of the latter decreases when high discount rates are assumed. Also, high discount rates may slow down investments in environmentally malign technologies.[3] The way the choice of the discount rate affects the environment, and natural resource use, is therefore not obvious in a straightforward manner, and to some extent ambiguous. This argument is often used by economists defending high discount rates. Although it is indeed not necessarily true that discount rates should be low to account for environmental and sustainability considerations, one can nevertheless consider the effects mentioned as second order. This leaves in tact, *grosso modo*, the hypothesis that environmentally favourable discount rates are low.

Some economists suggest that historic data on interest rates can be used to calibrate the level of the RPTP used in IAMs, while these in turn determine the optimal environmental resource use [Nordhaus 1994, Ch. 6]. In their opinion, time preference should be based on past evolution of the human use of resources, predominantly determined by the aim to attain efficiency. In other words, they assume that historic data determine whether a sustainable environmental resource use is economically feasible. Others strongly oppose this so called “descriptive” view, which normally leads to using relatively high values for the discount rate, and argue for a “prescriptive” approach in which one uses a low discount rate, since future generations should not be discriminated by lower welfare weights.

Several arguments exist for using low, or even zero, discount rates when evaluating future environmental damages resulting from economic activity of current generations. It is often assumed that if the discounting is low, it is possible to allow for an environment whose quality is preserved to a

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[3] On the other hand, high discount rates may as well slow down investments in environmentally benign technologies.

high degree. One of the reasons for employing long term discount rates lower than those the market reveals, is that individuals tend to assess the future with a higher discount rate than corresponds to the needs of a society as a whole. Also, the rates of return on capital investment are generally too high to be applicable for discounting the future value of the preservation of the environment. In his seminal work on intertemporal resource allocation, Ramsey [1928] uses a discount rate of zero when studying optimal social saving behaviour. Addressing the question of how much output should be used for current consumption, to yield current utility, and how much should be saved, to bring about future utility, he finds it “ethically indefensible” for a current-generation-planner to discount the utility of future generations. He thus promotes omitting the use of any discount factor. Others state that a positive, non-zero, discount rate reflects an “inadmissible myopia” [see, for example, Broome, 1992].

Most economists do not preclude the use of a discount factor, but point out that a discrepancy exists between discount rates at the private level and those at the social level. Individual preferences should not necessarily have implications for public policy, since individual discounting is often irrational from a social point of view. Therefore, it has been suggested to distinguish between two types of discount rates: a market discount rate, which reflects private decision taking, and a social discount rate, which reflects public decision taking. Individuals make both private decisions in their own interest, and public decisions, which are in the interest of society and of future generations. Sen [1982] has used a “super responsibility” argument to explain, or justify, why market rates are too high for properly accounting for the interests of future generations. Market discount rates are generated by the behaviour of individuals, while the state bares the responsibility for ensuring society’s welfare, of both current and future generations. Since the time horizon considered by the latter is much larger than that of the former, it is reasonable to assume that the discount rate in the context of state investments is different from that in the context of individual investments. Since low discount rates put more weight on future welfare, it is realistic to suppose that the state discount rate is lower than the market discount rate.

In practice, it proves difficult to bridge the gap between the “descriptors” and the “prescriptors”. In this paper, it is argued that part of the disagreement between their opposing views is due to the fact that both use a dynastic framework. Such a Ramseyan approach gives a convenient simplification of the dynamic competitive equilibrium, but it can only be used with difficulty to determine whether sustainable environmental resource use is economically feasible or not. In particular, dynastic models are not well fit for calculating optimal carbon dioxide emission reductions and supporting equilibrium prices if these emissions affect welfare levels in the long run.

The RPTP as used in a dynastic framework represents merely an auxiliary parameter, which cannot be observed in a competitive dynamic equilibrium. The discounting which typically can be observed are parameters such as the interest rate. The interest rate is a variable subject to changes over time. In a dynastic model the nature of this parameter is treated with insufficient reality, especially because the RPTP is assumed to be constant, implying that the interest rate is rigidly linked to economic growth. Using an overlapping generations (OLG) model allows for a more flexible relation between the interest rate and economic growth. Various factors can result in changes of the interest rate over time, such as local or global demographic changes[4], the implementation of sustainability policies, or a modification of the regime designed to preclude an over-exploitation of the environment.

These possible, or even likely, exogenous and endogenous changes render the extrapolation of the overall trend of historic interest rates to the future unjustified, as well as the rigid dynastic assumption inappropriate. Therefore, it is suggested that, unlike is practised in Ramsey type models, discounting should be treated endogenously. It is demonstrated that future interest rates can readily decrease below the levels predicted by the descriptive approach to dynastic models. As an alternative to these dynastic models, we discuss an OLG model providing a framework for distributing “climate endowments” over generations. Although this proposal is to some extent premature, given the small number of environmental goods over which property rights have been established so far, and given the slowness with which the set of such goods is currently expanded, a political debate about it is likely to be more concrete and bear more useful results than one on the rather abstract concept of pure discounting.

2. Discounting in dynastic models

For the purpose of generality, let us consider the case of a multiple dynasty economy. Every dynasty is characterised by a central planner’s welfare function which aggregates utilities of different generations, within a given dynasty, into one welfare measure. A commonly used welfare function aggregates utilities by weights which decline geometrically in time. It often involves a constant pure time preference. In the discrete time model, welfare can then be written as:

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[4] A global demographic change is likely to take place during the 21st century, as will be pointed out below. Also changing social security policies could have a major impact on future interest rates, e.g. those regimes implemented as a consequence of the expected ageing of the population.

$$w_{i,0} = \sum_{t=0,\ldots,\infty} \beta^t u_{i,t} \;, \tag{1}$$

where $w_{i,0}$ is the time aggregated welfare of dynasty i evaluated at t=0, the factor β^t is the time exponent of the constant utility discount factor β, and $u_{i,t}$ is the utility of dynasty i in period t or, alternatively, the sum of utilities of all members of generation t in dynasty i.[5] This form is common to many economic growth models [see, for example, Barro and Sala-i-Martin 1995], and is used in most IAMs. It can be traced back as far as Ramsey [1928].

Another commonly used form for the dynasty welfare function is the so-called maximin, or Rawlsian, welfare function. It is particularly popular among advocates of intergenerational equity. In this form, welfare is taken to be equal to the lowest utility level of all generations:

$$w_{i,0} = \min_{t=0,..,\infty} u_{i,t} \;. \tag{2}$$

Evaluated at time t, these two types of welfare functions can be represented in recurrent form, as $w_t = u_{\mathrm{t}} + \beta w_{t+1}$ and $w_t = \min\{u_{\mathrm{t}}, w_{t+1}\}$ respectively, where the subscript i is omitted for convenience. Koopmans shows that a broad class of welfare functions exists, which can be written in a recursive way. They can be expressed by the general recursive welfare relation:

$$w_t = \mathrm{H}(u_t, w_{t+1}). \tag{3}$$

In this expression, H(.,.) is the aggregator function. The aggregator functions studied by Koopmans [1960] and Koopmans *et al.* [1964] are continuous, non-decreasing, and concave in both arguments. The authors show that if welfare is to be finite, the aggregator function needs to exhibit a “time perspective”, i.e. one has to discount future utility in such a way as to arrive at a finite aggregate welfare measure. This time perspective, or discounting, appears as the contracting property of the aggregator function.

The derivative of the aggregator function with respect to its second argument, denoted by $\partial \mathrm{H}(u_t, w_{t+1})/\partial w_{t+1}$, is known as the factor of pure time preference β. Equivalently, in terms common in the IAM literature, one uses the rate of pure time preference, denoted by ρ. In general, the pure time preference factor and RPTP are time-dependent variables, denoted by ρ_{t} and β_{t} respectively. They are related by the expression:

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[5] The dynasty welfare levels $w_{i,0}$ can, on their turn, be summed into one overall society welfare measure by using certain welfare weights. These weights can be determined, for example, by using Negishi’s approach [1960].

$$\rho_t = \beta_t^{-1} - 1 = \partial H(u_t, w_{t+1})/\partial w_{t+1}^{-1} - 1, \tag{4}$$

where H(.,.) is the aggregator function. Many reasons exist for assuming this time-dependence. Lucas and Stokey [1984], as well as Epstein [1987], point out that if one presumes a constant pure time preference factor, as expressed in (1) and such that $\partial H(u_t, w_{t+1})/\partial w_{t+1} \equiv \beta$, the steady states of the economy constitute a continuum. They state that this is an unrealistic characteristic of long term economic behaviour. It is a direct result of the assumption of time-additivity, which in their opinion is a mere artefact of economic modelling. Therefore, they prefer allowing for non-constant time preferences by using a non-trivial aggregator function $H(u_t, w_{t+1})$ with $\partial H(u_t, w_{t+1})/\partial w_{t+1} = \beta_t$. Despite their arguments, the applied model makers usually adopt constant time preferences, since they provide the most convenient way of welfare aggregation.

In addition to the parameters β_t and ρ_t, both reflecting in the context of welfare aggregation discounting *vis-à-vis* pure time preferences, economic modelling requires to express discounting as a result of price depreciation of marketed goods. Often the interest rate r_t is taken to represent the price depreciation of such goods.[6] In a general model with various goods, however, every good has its own price depreciation, so that no unambiguous interest rate exists. The depreciating price of each consumer good defines its corresponding interest rate. The number of consumer goods in real life, and thereby the number of interest rates, would render economic modelling complex. For simplicity, many IAMs therefore specify one single aggregate consumer good, and identify the price depreciation of this good as the interest rate.

Let's assume that utility has a constant elasticity of marginal consumption, denoted by γ, and that both γ and ρ are independent of time. Let's further assume that the consumption growth rate for the single consumer good is time-dependent and denoted by g_t.[7] The price depreciation rate for this good is time-dependent, and is reflected by the interest rate r_t. Then, r_t can be expressed as consisting of a pure time preference part and a consumption growth part, satisfying:

$$(1+r_t) = (1+\rho)\,(1+\gamma g_t)\ . \tag{5}$$

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[6] One often uses the word "discounting" for the interest rate parameter r. This will not be done so in this paper for reasons of clarity.

[7] The consumption growth rate of the consumer good, in a discrete context, is expressed by $(\Delta C/\Delta t)/C - 1$, where C(t) denotes the consumption level at a given time t.

As can be seen from this expression, the interest rate obtains its time dependence purely from the consumption growth rate. Taking the linear approximation of this equation, one obtains:

$$r_t \approx \rho + \gamma g_t \,. \tag{6}$$

This relation has been given a central role in the debate on discounting [see e.g. Cline 1992]. Its direct implication is that the future interest rate decreases (increases) proportionally to a decrease (increase) in consumption growth:

$$r_{t+1} - r_t \approx \gamma\,(g_{t+1} - g_t)\,, \tag{7}$$

which follows directly from (6). This relation is a common assumption in IAMs. If one assumes that long run growth converges to a constant rate g^{LT} (where the superscript LT denotes the long term), the interest rate converges correspondingly to a level $r^{LT} = \rho + \gamma g^{LT}$. The model maker usually estimates the parameters ρ and γ by analysing historic data, after which the long term interest rate directly follows from the assumed long term economic growth.[8] The result of this approach, partly because of the limited availability of the natural resource considered, is often that the calculated interest rate path is incompatible with a sustainable economic development. Actual preferences do not appear to support sustainability and, consequently, the model presents unsustainability as the optimal solution.

Such a conclusion results from the restrictive assumption of a constant RPTP. In the more general format of the aggregator function $\mathrm{H}(u_t, w_{t+1})$, the RPTP may vary over time. Thereby, discounting is allowed to decrease if the environmental quality decreases. A direct link between historic interest rates and the economic feasibility for reaching sustainability of optimal future resource allocations does then not need to exist. Although a dynamic model with a more general aggregator function allows for a time dependent RPTP, by which its flexibility and applicability is considerably extended, it still imposes restrictions. As Ginsburgh and Keyzer [1997] point out, the dynastic model easily leads to allocations in which, from a certain point in time towards the indefinite time horizon, future generations may have to pay a debt incurred before their time of birth, a phenomenon often referred to as “slavery”.

Because the use of a descriptive pure time preference, i.e. a preference describing historic data, can lead to an unsustainable optimal solution, some researchers advocate a prescriptive approach, using a low pure time preference. In General, a prescriptive low discount rate will ensure a sustainable use of the environment and natural resources. However, advocates of the descriptive approach argue that

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[8] In fact, γ is often assumed to be unity.

using a low discount rate for environmental assessment, different from the actual discount rate observed in markets, implies an inefficient resource use. In dynastic models, efficiency and sustainability are often found to be incompatible.

3. Discounting in OLG models

Unlike the Ramsey type dynastic models, OLG models do not involve a central planner whose time preference largely affects the interest rate. Instead, the interest rate is an element of the price vector, which adjusts in order to match demand and supply of all goods in all periods. For the general OLG economy with multiple goods and multiple producers, it is impossible to predict changes of the interest rate when other exogenous parameters unfold over time. Nevertheless, under some simplifying assumptions, it is possible to relate the interest rate to a set of parameters that is large compared to that of a dynamic model.

Consider a two-generations OLG economy. At a given time period t, two generations live together, a young generation y born at the beginning of period t, and an old generation o born at $t-1$. Supply at time t consists of the endowments of the young, ω_t^y, and the old, ω_t^o, plus net production, $\mathrm{y}(p_t,\psi_t,\psi_{t+1})$. Production is a function of the prices of goods, p_t, and the prices of stocks at the beginning and end of the period considered, ψ_t and ψ_{t+1} respectively.[9] Multiple goods and stocks are allowed for, so that all prices are vectors. Both the young and the old generation demand goods for consumption, based on the prices over their life-cycles, $\mathrm{d}^y(p_t, p_{t+1})$ and $\mathrm{d}^o(p_{t-1}, p_t)$ respectively. Production and demand functions are homogeneous of degree zero in all price arguments. In a steady state, prices satisfy $1+r = p_t / p_{t+1} = \psi_t/\psi_{t+1}$, where r is the interest rate. In equilibrium, in every period, supply matches demand:

$$\mathrm{d}^o(p_{t-1}, p_t) + \mathrm{d}^y(p_t, p_{t+1}) = \omega_t^y + \omega_t^o + \mathrm{y}(p_t,\psi_t,\psi_{t+1})\ . \tag{8}$$

This equation defines a recursive relation between prices in period $t-1$, period t, and period $t+1$. It is possible that no solution to this recursive equation exists, that multiple solutions exist, or that a manifold of solutions exist (i.e. there are infinitely many solutions arbitrarily close to one another). The latter is known as indeterminacy, since it is impossible to say which price path from the manifold will be chosen. For the purpose of this paper, we will abstract from such typical OLG complexities

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[9] See Ginsburgh and Keyzer [1997, Section 8.4] for an extensive description and analysis of an OLG model along these lines.

[see Kehoe and Levine 1990], and assume that a (locally) unique equilibrium exists. At any rate, solving an OLG equilibrium is a complex task of matching demand and supply in all periods. The prices that achieve a matching between supply and demand are of an infinite, though countable, dimension. In a dynastic model, the equilibrium is found by solving a finite number of welfare maximisation problems, one for each dynasty, constrained to the budgets of the dynasties. The difference in dimensions results in a difference in complexity between OLG and dynastic models. This also explains why in the former the interest rate does not in a rather trivial way depend on only a small number of exogenous parameters.

In standard OLG models, with no intergenerational transfers, the equilibrium on the goods markets results in an equilibrium on the savings-capital market, such that life-cycle savings match the value of the capital stock. Different from a dynastic model, capital is not transferred to future generations as a bequest, but it is held by the old generation, who sells its capital stock to the young generation to pay for its pension. Both the level of private life-cycle savings and the level of capital investments have an effect on the interest rate. Thus, if some exogenous change modifies savings or investment behaviour, it affects the interest rate as well.

Figure 1 illustrates the equilibrium (E), for a general OLG economy that includes bequests and negative intergenerational transfers implied by a public debt. Life-cycle savings to pay for retirement are indicated by the curve S1, while bequests raise the saving curve to S2. Public debts decrease the saving curve to the total savings depicted by curve S3. At interest rate r*, savings S* and capital K*, cumulative savings (S3 curve) match cumulative capital investments (K curve). As depicted in this figure, capital investment is assumed to decrease with an increase in the interest rate. The rationale behind this relation is that there is little incentive to invest capital when the interest rates are high. On the other hand, savings are supposed to increase with higher levels of the interest rate. Such savings behaviour can be explained by the relatively high incentive for postponing expenses, allowing profits from saving, when these interest rates are high. From the graph one can see that if either the savings or capital curve changes, a corresponding change in the equilibrium interest rate will occur.

FIGURE 1. *The equilibrium (E) between savings (S curves) and capital investment (K curve).*

For example, if as part of a social security policy a pay-as-you-go pension system is introduced, where the currently young pay for the pensions of the currently old, the need for private life-cycle savings will decrease, and thus the interest rate increase. Generally, it is ambiguous whether the savings curves are upward sloping or downward sloping, and there is no unambiguous relation between the need for life-cycle savings and the interest rate. However, in a concise OLG model, where endowments are not too unevenly spread over the life-cycle, a decreasing need for life-cycle savings leads to an increase in the interest rate [see, for example, Blanchard and Fisher 1989, Section 3.5]. This finding demonstrates clearly that a social security policy changing the savings behaviour directly affects the interest rate.

A potential cause for a major change in future interest rates is the global demographic transformation, which is likely to take place during the next century. It is expected that life expectancy will increase by about 20 years over that period [World Bank 1994], while the productive lifetime will not raise proportionally. The answer to the question whether such an alteration in demography will increase or decrease the long run interest rate is ambivalent. In case of a pay-as-you-go system, the ageing will increase transfers from the young to the old generation. This could induce a diminishing savings behaviour, which, as can be deduced from Figure 1, leads to an increase of the interest rate [see Auerbach *et al*. 1989]. If instead of a pay-as-you-go system a fully funded system is implemented, with every generation paying for its own pension, ageing will increase the need for life-cycle savings in order to allow paying for a longer retirement period. According to Figure 1, this shifts the savings curve upwards, which shifts the equilibrium (E) to smaller values of the interest rate.[10] The effects can be substantial, as will be demonstrated below.[11]

A substantial change in interest rates in an OLG framework can also originate from a varying public savings behaviour. In the simple two-generation one-good pure exchange model by Gale [1973], it is shown that a forward transfer of income to future generations decreases the interest rate. Suppose a classical economy, where ideally dynamic efficiency is guaranteed. If a transfer is not used by future generations to increase their income, but enters the economy as so-called fiat money, the economy will converge to a “monetary” steady state with a “golden rule” interest rate, i.e. an interest rate equal to consumption growth.

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[10] As above, it is assumed that the endowments are not too unevenly distributed over the life-cycle.

[11] In Gerlagh [1998, Section 3.3.1] it is shown, by a simple numerical example, that a process of ageing, reflecting the expected demographic change in the next century, is capable of shifting a dynamically efficient equilibrium path with an interest rate of some 5 per cent per year, to a dynamically inefficient equilibrium in which the interest rate has dropped to about 0.5 per cent per year. This is substantially below the expected long term economic growth rate.

Howarth and Norgaard [1992] extend an OLG model with a natural resource, and show that the intergenerational distribution of property rights over the resource affects the interest rate. However, their model has only three periods. Gerlagh [1998a, Theorem 4.6] goes further and extends the result to an infinite horizon OLG economy with an environmental resource which has amenity value. He shows that a policy which sets aside part of the value of the environmental resource for future generations - which can be interpreted as a transfer of income to those future generations - reduces the interest rate. More importantly, Gerlagh shows that the decrease in interest rate ensures that the welfare of future generations exceeds a reference level, set by a sustainability policy that protects the environment by the use of strict environmental measures. Thereby, the intergenerational distribution of property rights over the environmental resource, via the use of a trust fund, is effectively used to let the discount rate endogenously decrease to a sustainable level.

4. Numerical illustration

To illustrate the arguments given above, we use the OLG integrated assessment model ALICE 2.0, applied to the issue of climate change.[12] ALICE is calibrated to replicate the so-called Business as Usual IS92a-scenario [IPCC 1992] for the case that climate change is not internalised in the economy. A comprehensive and detailed description of version 2.0 of ALICE can be found in Gerlagh [1998a, e.g. Section 5.2]. A brief specification is given in the appendix to this paper. ALICE 2.0 distinguishes periods of 20 years, the first of which corresponds to the interval 2000-2020. In every period a new generation is born. To start with, each generation lives for three periods, including one adolescent period and two adult periods. The model extends the existing OLG-IAMs [see, for example, Stephan *et al.* 1997 and Howarth 1998] in two ways.

First, it includes a rise in global life expectancy, or ageing, which is supposed to take place in the 21st century, i.e. during the first five intervals. It is represented by a transition from generations living two adult periods to generations living three adult periods. The single adolescent period is supposed to remain unchanged in this transition. Secondly, it includes a redistribution mechanism for the entire value of one particular environmental resource: clean air. The latter is considered clean when it has not been subjected to excessive carbon dioxide emissions. Redistribution is not restricted to sharing the revenues of emission taxes between the currently old and young, but involves more broadly a redistribution between future and present generations.

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[12] ALICE is the acronym for Applied Long-term Integrated Competitive Equilibrium model.

Whereas the existing OLG studies do not add much new valuable insight with respect to the findings of a typical dynastic model, the extensions implemented in the OLG model presented in this paper alter its findings considerably. Previously, this lack of new insight provoked doubt on the usefulness of OLG models with respect to Ramsey type models. Our analysis, on the contrary, suggests that OLG models might have a significant added value. The conclusions from earlier OLG-IAMs can largely be explained by their assumption of stationary structural economic variables such as demography. However, if a realistic transition of these parameters is taken into account, such as in ALICE 2.0, the OLG model is found to show substantially different resource allocations, compared to a model in which such a transition is absent. The conclusion of our analysis is therefore that the additional two elements, the expected demographic transition during the 21st century, and the intergenerational distribution of property rights over a natural resource, have far-reaching consequences for the optimal use of that resource.[13] Since the model is generic in its set-up, the results are expected to extend to environmental resources other than clean, carbon-dioxide-poor, air.

Below, a number of figures are presented. Figure 2 depicts the interest rate, defined for the years between two periods starting at 2020. Figures 3 and 4 present the flow variables CO_2 emission price and net CO_2 emission, each describing a flow during the periods 2000-2020, 2020-2040 and-so-forth. In these graphs, the periods are identified with the corresponding central years, 2010, 2030, et cetera. Figure 5 and 6 illustrate the state variables CO_2 concentration and global mean temperature, defined for the years between two periods: 2000, 2020, et cetera.

Four scenarios are specified to show how the demographic change and the intergenerational distribution of property rights over the environmental resource affect the results of our OLG analysis. The scenarios are summarised in Table 1. The first scenario, labelled I, assumes that the environmental resource is given, or “grandfathered”, to the first generation. This scenario abstracts from any increase in life expectancy. The second scenario, labelled II, includes the above described demographic change, in addition to the grandfathering of the environmental resource to the first generation. Since grandfathering implies that future generations will have to pay if they want to enjoy a clean environment, scenarios I and II may collectively be denominated “Victims Pay”. The third scenario, labelled III, assumes that the property rights over the environmental resource are equally

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[13] It is worth underlining the different character of these two additional elements: whereas the ageing of the population is a mere exogenous fact, choosing between alternatives of how to distribute property rights of an environmental resource over generations is a matter of policy making.

distributed over all generations by the use of a trust fund [Gerlagh 1998a, Section 4.3.3], while abstracting from the demographic change of an ageing population. The fourth scenario, labelled IV, includes the demographic change of ageing, in addition to the equal distribution of the environmental resource over all generations via a trust fund. Sharing the property rights over the resource implies that the polluters will have to buy the permits to emit; the scenario is therefore labelled "Polluters Pay".

[Insert Table 1 here]

Figure 2 shows the interest rate evolution for each of the four scenarios. Three results require special attention. First, comparing the interest rates between scenarios I+II and III+IV in 2020, we see that grandfathering the environmental resource implies initially an interest rate of close to 2 per cent per year higher than in the trust fund case. This can be explained as follows. If the resource is capitalised and becomes private property (scenarios I+II), this implies an increase of the total value of the capital stock, and thus an upwards shift of the K-curve in Figure 1. The equilibrium E shifts right-upwards, causing an increase in the interest rate. However, if the environmental resource is treated as public property (scenarios III+IV), the public savings curve increases as well, by the same value. Combining the shifts in both the capital and savings curve in this case, the equilibrium E shifts overall vertically upwards, leaving the interest rate invariant to the Business as Usual scenario to which the model is calibrated.

Secondly, the intergenerational distribution of the property rights over the environmental resource has a major impact on the long term value of the interest rate. Sharing the property rights with future generations, instead of grandfathering it, decreases the interest rate from 7.3 to 2.9 per cent per year in 2200, when the population is not assumed to age. Including the demographic change, we find that sharing the property rights decreases the interest rate from 2.5 to 0.5 per cent in 2200.

Thirdly, comparing the scenarios with and without ageing, one concludes that ageing decreases the future interest rate substantially, from 7.3 to 2.5 per cent in 2200 under grandfathering, and from 2.9 to 0.5 per cent in 2200 if property rights are shared with future generations. The figure reveals the importance of both demographic changes and environmental policies for the long term evolution of the interest rate.

FIGURE 1. *Interest rate plotted against time, for the period 2000-2200.*

For evaluating the consequences of different interest rate evolutions on climate change, we assume that CO_2 emissions decrease by one per cent point for each 4 US$/tC increase in the emission price.[14] This value is an average of the numbers found in the literature, ranging from 1 to 6 US$/tC [Cline 1992, Ch. 4]. This implies that a 100 per cent decrease in CO_2 emissions corresponds to a price of 400 US$/tC. We abstract from endogenous technological improvement, as well as from transition costs associated with a shift towards a backstop energy technology producing no CO_2 emissions.

Not considering ageing, grandfathering leads to low emission prices, reaching 100 US$/tC in 2200 (see Figure 3). If we include ageing, the interest rate decreases, and the carbon emission price slowly increases from nearly zero in 2000 to 100 US$/tC in 2100 and 400 US$/tC in 2200. Scenario I can be considered as being the closest to the business as usual, or benchmark, scenario. The CO_2 emissions in this scenario (see Figure 4) are roughly stable on the time scale considered. This approximately stable emission level is mainly explained by an autonomous increase in energy efficiency and an autonomous expansion of backstop technologies, assumed in the benchmark scenario. The relatively small reduction in emissions close to the year 2200 is induced by the expected emission price increase around that date for this scenario. It should be understood, however, that these estimates are dependent on the benchmark assumptions made. Chakravorty *et al*. [1997], for example, assume an autonomous decline of fossil fuel energy use before 2100 because of competitive solar energy supply.

FIGURE 2. *CO_2 emission price, for the period 2000-2200.*

Let's compare the two grandfathering scenarios, in which the inclusion of ageing provokes a decrease in the interest rate. In the early periods, net emissions of scenario II exceed slightly the levels of scenario I, because of the ageing of the population: the resulting increase in life-cycle savings and the associated increase in the man-made capital stock produces more carbon dioxide than in an economy without ageing. In the 22nd century, however, the small effect of this demographic change is dominated by the increasing difference in the carbon dioxide price of these two scenarios. The steady rise in emission prices in scenario II outweighs the increase in emissions resulting from economic growth. This leads to a rapid decrease of emissions in scenario II, whereas in scenario I only a modest

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[14] From here, we will interchange the terminology "emission costs" and "emission prices", since they are considered to have the same value. Whereas the former is the assumed monetary value of the costs emissions provoke, the latter is the price one will likely need to pay in the future in order to be permitted to emit carbon dioxide.

emission reduction takes place at the end of the 22nd century. In scenario II, cumulative emissions from today up to the year 2100 amount to about 1450 GtC, which is comparable with the corresponding IPCC IS92a scenario [IPCC 1992]. By the year 2200, net emissions will have attained the level of zero.

FIGURE 3. *Net CO_2 emission, for the period 2000-2200.*

Altering the analysis from a “victims pay” to a “polluters pay” perspective, by the introduction of a trust fund, substantially shifts upwards the carbon dioxide emission price curve. In the case of the ageing scenario, the increase is already visible in the first period, whereas it becomes apparent in the second half of the 21st century for the no ageing case. The inclusion of ageing has a considerable effect on the emission price evolution. If no ageing is considered, the trust fund emission price remains neatly between the two grandfathering scenarios, over the entire time lapse studied. This applies as well to the emission levels, in as far as the 22nd century is considered. If ageing is included in the calculations (scenario IV), the emission price increases rapidly to 400 US$/tC in less than a century. By assumption, this price level implies a 100 per cent reduction of net CO_2 emissions. Scenario IV does not limit the expansion of net emissions in the medium term (up to 2030), leaving present generations the possibility to adapt and to develop alternative energy sources. After 2050 net emissions decrease rapidly, and by 2080 a complete substitution of fossil fuel energy carriers by carbon free energy sources has taken place.

FIGURE 4. *CO_2 concentration, for the period 2000-2200.*

As can be seen from a comparison between Figures 3 and 4, CO_2 concentration levels in the atmosphere lag behind CO_2 emission patterns. In turn, an inspection of Figure 6, depicting the global average temperature increase, relative to the pre-industrial level, over the interval 2000-2200, shows that the mean temperature lags behind the CO_2 concentration level. For the processes behind these lagging phenomena, resulting from the supposed “geobiochemical” dynamics of the CO_2 emission and absorption system, we refer to the appendix of this paper. Figure 4 and 6 show that, regarding the carbon dioxide concentration and average global temperature increase, the first three scenarios nearly coincide during the forthcoming 150 years. Only scenario IV, in which not only future generations are given a share of the environmental resource value, but in which also the expected demographic change occurring in the 21st century is accounted for, succeeds to start reducing temperatures slightly in the 22nd century. The environmental policy of scenario IV endogenously adjusts the interest rate to a sustainable level.

FIGURE 5. *Global mean temperature increase, relative to the pre-industrial level, for the period 2000-2200.*

5. Conclusion

The figures presented in this paper illustrate the importance of taking into account two phenomena, which may affect future interest rates but are for methodological reasons often neglected in dynastic models. The first is the expected modification in life expectancy of the world population. This change is implemented in our OLG model in an exogenous way. The second is the need for possessing an instrument designed for preserving the natural environment, such as grandfathering or the establishment of a trust fund. This is accounted for endogenously.

Our purpose has been not so much to present yet another set of numbers, adding to the many results which have been published already on the subject of global warming and greenhouse gas mitigation. Our main message is to emphasise that, in general, a cautious use and interpretation of results on this matter is required. It is shown that the results are highly sensitive for the value of the discount parameter used. For economic modelling it is recommended to employ a discount parameter subject to changes over time. Using a fixed interest rate relation is undesirable.

Discounting is of major importance to the sustainability issue. A high discount rate is generally considered to render sustaining a high quality environment difficult. A low discount rate, on the contrary, could make a sustainable resource use economically feasible. In the standard dynastic model with time-additive discounted welfare functions, and with a constant RPTP, the future interest rate depends on a limited number of parameters. These parameters are usually calibrated on historic data. With this descriptive approach, many economists advocate that it is possible to determine whether sustainability can be realised or not. The dynastic model, however, over-simplifies the analysis. It is questionable whether a dynastic model should be used for predicting future interest rates, since welfare is not necessarily distributed as if there were a central planner. It is not unrealistic to assume that future interest rates decrease more than can be explained via the dynastic model equations (5) and (7).

Alternatively, in OLG models, the discounting is related to many variables in the economy that may change over time. It is pointed out that demographic changes and the specification of property rights over environmental resources have an affect on discounting. Also changes in social security, such as a modification of the retirement system (not explicitly considered in this paper, although indirectly treated via the assumption of an ageing population), are likely to affect future discounting. These changes may lead to future interest rates that are considerably lower than those encountered in the past. The OLG model ALICE used for our analysis shows that a variation of the environmental public policy affects the interest rate. It is possible to specify property rights over environmental resources and distribute these over generations in such a way that the discount rate can endogenously adjust to its sustainable level.

The way in which applied dynastic IAMs are used for environmental policy calculations suggests that historic data on interest rates determine whether sustainability is economically feasible or not. The dynastic models often suggest that a choice for sustainability is at contrast with efficiency. Clearly, this thinking misleads the responsibility of present policy makers. By the assumption of a flexible discounting relation, an OLG analysis reveals that instruments bringing forward an environmental resource use being both efficient and sustainable, can exist. If a policy maker wishes to establish a combination of both efficiency and sustainability, the use of OLG models can provide insights which can be attained with difficulty from dynastic models.

Appendix. Concise model description of ALICE 2.0

The OLG model ALICE 2.0 is based on its predecessor ALICE 1.2. The calibration of the latter is extensively described in [Gerlagh 1998b]. Version 2.0 extends version 1.2 with respect to the specification of climate change processes, as described in [Gerlagh 1998a, Section 5.2]. The model distinguishes discrete time steps, $t \in T = \{1,\ldots,\infty\}$, representing periods of 20 years each. The first step corresponds to the interval 2000-2020. In every interval a new generation is born. The model only describes the adult part of the life-cycle, i.e. from the age of 20 onwards. This implies that a two-period life represents an individual who reaches the age of 60, and a three-period life represents an individual reaching the age of 80. Consumption of children, in the age between 0 and 20, is accounted for by consumption of their parents. A generation is called young when its members have an age between 20 and 40; middle aged are those individuals between 40 and 60, and old is the generation with members between 60 and 80.

Each generation is denoted by the date t on which it starts consumption: it then enters the model. The generation denoted t is born at time t-1.[15] Generations are of different size, denoted by n_i, with i the first interval in which the generation consumes. Furthermore, the life-cycle lengths of generations are not identical. A demographic change is specified to represent increasing life expectancy, modelled as a transition from a lifecycle of two periods to one of three periods. This transition is supposed to take place during the 21st century, that is during the first five intervals considered in the model. In the first interval, only a young and middle-aged cohort coexist, without the presence of an old aged generation. The middle aged in the first interval die at the end of that interval. Twenty per cent of the young in the first interval, i.e. the middle aged in the second interval, live for a third period. Hence, in the third interval there is a small group of old consumers. Of the young generation in the second interval, i.e. the middle aged of the third interval, forty per cent live three periods, while the others live only for two periods. Life expectancy continues to increase linearly, until all members of the generation which is young in the 2080-2100 interval live through a lifecycle of three periods. The life expectancy transition is then completed.

Let now $n_{i,t}$ denote the size of generation i in interval t, so that $n_{i,i}$ denotes the size of generation i when it starts its consumption. We assume that no member of a generation dies before the start of the second period of its life: $n_{t,t+1} = n_{t,t} = n_t$, for all t. The increase in life expectancy implies that the number of people living the full three periods increases linearly: $n_{1,3} = 0.2\, n_{1,2}$, $n_{2,4} = 0.4\, n_{2,3}$, and so forth, until $n_{5,7} = n_{5,6}$. The time evolution of the size of a generation is defined recursively according to a logistic growth curve:

$$n_{t+1} = (a^n - (a^n - 1)(n_t / \overline{\mathrm{n}}))n_t \,, \tag{9}$$

where a^n is the growth factor, and $\overline{\mathrm{n}}$ the maximal size of a cohort; n_t is assumed to be small with respect to $\overline{\mathrm{n}}$. In interval t, the population size N_t is given by:

$$N_t = n_{t+1,t} + n_{t,t} + n_{t-1,t} + n_{t\ 2,t} \,, \tag{10}$$

where $n_{t+1,t}$ represents the children who enter the model, by starting to consume, in the next interval. The other variables represent the young, the middle aged and the old, respectively. The population at the beginning of an interval can be thought of as the average of the population in the previous and coming interval:

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[15] All quantities referring to a particular generation are indexed by the subscript t. Alternatively, the label i is used where convenient.

$$\tilde{N}_t = \tfrac{1}{2}(N_{t-1} + N_t)\,. \tag{11}$$

The series $\tilde{N}_t$, for $t = 1,\ldots,\infty$, depends on the variables a^n, and $\bar{n}$ via formulas (9) and (10). It also depends on the size of the generation which starts consumption in 1960 and dies at the start of the model in 2000. These parameters are calibrated such that the series approximates World Bank data on world population [World Bank 1994]. The results are shown in Table 2. The table also shows the modelled increase in life expectancy in ALICE, which reasonably captures the main characteristics of the World Bank forecasts on life expectancy. To obtain the numbers of world population, we calibrated the parameters for two cases: with and without ageing.

[Insert Table 2 here]

Generations maximise their lifetime utility $U(C_i,B)$, derived from rival consumption of the consumer good during the life-cycle, $C_i=(C_{i,t},C_{i,t+1},C_{i,t+2})$, and non-rival consumption of the resource amenity, for convenience referred to as “environmental services”, $B=(B_t,B_{t+1},B_{t+2})$. We omit the subscript i in the consumption of the resource amenity to stress that the amenity consumption is the same for all generations. The consumption behaviour of generations of which not all members live three periods is based on some further assumptions. For any member of a generation, until the beginning of the third period, only the probability of living three periods is known. At the beginning of the third period, any member is either alive or not. Each member is supposed to maximise expected life-time utility. There are no non-intended bequests to future generations resulting from the uncertainty in lifetime [Hurd 1989], because there is an intra-generational life-insurance company to which all members of a generation pay their savings in the second period of life. The insurance company repays the savings to the living members of the generation in the third period. Under this condition, the generation can be described by one representative consumer who maximises aggregate utility subject to one budget constraint. His utility function U(.) is a nested CES function of the general form:

$$U(C_i\,,B) = \Big[\sum_{t=i,\ldots,i+2} \sigma^{t-i} n_{i,t} (C_{i,t}^{a} B_t^{1-a})^{\frac{\rho-1}{\rho}}\Big]^{\frac{\rho}{\rho-1}}, \tag{12}$$

with a constant expenditure share $a = 0.9$ for the consumer good, a share of $1 - a = 0.1$ for the resource amenity, and an intertemporal elasticity of substitution $\rho = 0.67$. The consumers’ time preference factor σ is set equal to unity. Notice that taking different values for σ will affect the numerical results in a similar way as described before, that is, if we decrease the consumers' time preference factor σ, the S1-curve in Figure 1 will decrease, and the equilibrium E shifts to the right, increasing the interest rate. However, the consumers' time preference is not the only factor determining the interest rate: for the long term evolution, other elements such as ageing are at least of equal importance.

The generations maximise utility subject to the budget constraint:

$$max\ \{\ \mathrm{U}(C_i\ , B)\ \mid \sum_{t=i,\ldots,i+2} (p_t\, C_{i,t} + \varphi_{i,t} B_t\ ) \leq \sum_{t=i,\ldots,i+2} (w_t\, L_{i,t} + H_{i,t})\ \}, \tag{13}$$

where p_t, w_t denote the prices of the rival consumer good and labour, respectively, $\varphi_{i,t}$ are the so-called Lindahl prices for generation i in period t for non-rival consumption of the resource amenity, $L_{i,t}$ denotes the labour endowment and $H_{i,t}$ is the income which generation i receives in period t as its share in the value of the environmental resource.

ALICE 2.0 includes a simple production sector for a man-made consumer good, consisting of one private firm. It uses labour units $L_t = L_{t-2,t} + L_{t-1,t} + L_{t,t}$, emission units E_t and a man-made capital stock K_t as production factors. The capital stock is itself produced by this production sector: we therefore assume that the capital stock is made up of the consumer good, and that it has to be replaced after use in one period. The production structure can thus be represented by:

$$Y_t + K_{t+1} = \mathrm{h}_t(K_t\, , E_t\, , L_t\,)\ , \tag{14}$$

where $\mathrm{h}_t(.,.,.)$ is a continuous, differentiable, linearly homogeneous, and concave production function. In addition, it is strictly increasing in the first and third argument, and has a bounded derivative for the second argument, $0 < \partial\ \mathrm{h}_t(.,0,.)\ /\ \partial\ E_t < \infty$. This latter qualification implies that the emission units are valuable, but not essential for production. The subscript t is maintained to allow for technological innovation. More specifically, capital and labour are combined in a Cobb-Douglas aggregate, and the productivity of this aggregate good is supposed to be a quadratic function of the emission intensity. This is done in such a way that the optimal emission levels decrease by 1 per cent for every 4 US$/tC price increase of the emission units, and that the maximum emission levels follow the IS92a scenario [IPCC 1992].

The climate change issue is represented through an "environmental firm". Peck and Teisberg [1992] and Nordhaus [1994] have much contributed to the development of stylised economic IAMs by providing highly simplified representations of biogeochemical interactions to make these applicable in macro-economic models. The typical simplified aggregate representation that has thereafter evolved links emissions to concentrations, concentrations to temperatures, and temperatures to damages. We follow this literature.

As the emissions of CO_2 account for the main anthropogenic contribution to the greenhouse effect, we focus on the carbon cycle and its relation to climate change. Let us assume that the atmospheric GHG accumulation can be represented in an abstract way by a "linear box" model in which one distinguishes between five separate spheres each having different properties with respect to carbon dioxide absorption [Maier-Reimer and Hasselman, 1987]. It is assumed that the CO_2 emitted is distributed over the five boxes in amounts corresponding to shares a_i (i = 1,…,5) of total emissions. Within each box, the CO_2 concentration exponentially adjusts to its natural level at an annual adjustment rate $1/\tau_i$. The parameter τ_i is the so-called *e*-folding time or turnover time. It is the expected period that a CO_2 gas particle remains in box *i*, or in other words, the time after which the CO_2 concentration has decreased to 1/*e* its initial value. Let M_t^i denote the total amount of anthropogenic GHGs in box *i* at the beginning of period *t*, as a resultant of both anthropogenic GHG emissions into the atmosphere and natural dissipation of these GHGs from the atmosphere, and let E_t be the anthropogenic emissions during period *t*. We then have the following relation:

$$M_{t+1}^i = e^{-N\tau_i} M_t^i + a_i E_t \ , \tag{15}$$

where $\Sigma_i a_i = 1$ and N is the period length in the discrete time model.[16]

The accumulation of GHGs causes an increase of the equilibrium global mean temperature. For CO_2, the temperature increase is expected to be of approximate logarithmic nature:

$$T_t^{eq} = \overline{\mathrm{T}} \ \ {}^2\log(1+\sum_i M_t^i / \overline{\mathrm{M}}) \ , \tag{16}$$

where T_t^{eq} is the long term equilibrium temperature, as a function of the sum over all boxes of the anthropogenic GHG levels at time *t*, $\Sigma_i M_t^i$; $\overline{\mathrm{T}}$ is the benchmark equilibrium temperature increase

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[16] Maier-Reimer and Hasselman [1987] have estimated the parameters a_i and τ_i for the linear 5-box model. Their findings conclude, remarkably, that 14 per cent of emissions remain in the atmosphere for an infinite horizon (τ_i is infinite), implying that the absorption capacity of the biogeochemical system is exhaustible. Furthermore, 24 % of emissions have an *e*-folding time of about 300 years, 32 % of about 80 years, 21 % of about 20 years, and 9 % of less than 2 years. Note that the boxes are merely a result of econometric modelling and do not correspond to physical entities.

associated with a doubling of total accumulated anthropogenic GHGs in the atmosphere, and $\overline{\mathrm{M}}$ is the natural level of (non-anthropogenic) GHGs in the atmosphere (see [IPCC 1992][17]).

The earth surface and atmosphere have a certain warmth capacity. Therefore, the temperature slowly adjusts to a long term equilibrium level:

$$T_{t+1} = e^{-N\varepsilon} T_t + (1 - e^{-N\varepsilon}) T_{t+1}^{eq}, \tag{17}$$

where ε is the annual adjustment rate, which is set to 2 per cent per year following Peck and Teisberg [1992].

The above equations relate to complex biogeochemical relationships. Although present-day knowledge of these relationships is somewhat limited, the approximations made are based on a reasonable understanding of the physical processes involved. For calculating the impacts of climate change, however, or even for establishing some “best guess” scenario, our knowledge is usually reckoned to be insufficient. In general, it is assumed that damages caused by climate change outweigh its benefits. The lack of knowledge, however, is unmistakably revealed by sensitivity analyses, e.g. carried out by varying the so-called damage functions. These damage functions are supposed to provide a reduced form for the many complex damages associated with climate change, such as the loss of coastal zones due to sea level rise, the loss of biodiversity, an increased frequency of vector borne diseases such as malaria, and the occurrence of various extreme weather events. Some damage functions take the global temperature as argument, others take the rate of increase of global temperature as argument. Some damage functions are quadratic, others are of a higher or lower order (see, for example, [Tol 1995]). This lack of understanding is recognised by the IPCC [1996a, Section 6.2.13]. Nevertheless, the IPCC establishes several overall damage estimates [IPCC 1996a, Table 6.4]. The use of the IPCC figures for our analysis does not mean that we consider them unconditionally reliable, but reflects our wish to maintain compatibility with currently used IAMs. Typically, present IAMs include a reduced damage function h(.). The damage parameter, denoted by D_t, is usually a function of actual temperature increase, such that $D_t = \mathrm{h}(T_t)$. It is often expressed in monetary units or as percentage of GDP.

The IAMs have different ways of incorporating damage function, either by subtracting damages from production or consumption, or directly from utility. This practice is to some extent misleading, because environmental damage might be better understood as a decrease in the quality or quantity of an environmental capacity (or “environmental function”), than as a reduction in the flow of man-made

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[17] See also this reference for a list of various GHGs and their potential contribution to global warming.

goods. Environmental degeneration can lead to a reduction in both biomass and biodiversity, with the former measuring the quantity and the latter the quality of the environmental function considered [IPCC 1996b, Section 1.3.2]. A decrease in biodiversity constitutes a change in environmental quality, as opposed to environmental quantity. In a reduced model, this can in our opinion be best described via a non-rival amenity value [Gerlagh 1998a, Section 5.2.3]. Therefore, we use a non-rival good labelled “environmental services”, whose level is given by:

$$B_{t+1} \leq e^{-Nd} B_t + (1 - e^{-Nd})(1 - \mathrm{f}(T_{t+1})), \tag{18}$$

where d is the annual adjustment rate, and f(.) is the reduced form function describing long term losses of environmental services. This function is increasing on its positive domain, and f(0) = 0 (see [Tol 1995] for various possible reduced form functions). In Alice 2.0, f(.) is taken to be quadratic; $\mathrm{f}(T_{t+1}) = 1$ for T_{t+1} equal to a temperature increase of 6 C relative to the pre-industrial level. Hence, for a temperature increase of 3 C (the usual benchmark case for the year 2100), the long term resource amenity decreases by 25 per cent, amounting to a welfare loss equivalent to 3 per cent of GDP according to equation (12).

If one considers the biogeochemical system as a firm, the latter can be interpreted as “producing” emission units E_t and environmental services B_t.[18] The biogeochemical system can also be seen as a renewable resource, which deteriorates because of GHG emissions. This deterioration constitutes the core of the sustainability problem [Clark 1997]. By definition, the resource income shares should sum to the total value of the resource, that is:

$$\sum_{i=0,\ldots,\infty} H_i = \sum_{t=1,\ldots,\infty} (p_t^e E_t + p_t^b B_t). \tag{19}$$

where p_t^e represents the price of emission units, $p_t^b = \varphi_{t-2,t} + \varphi_{t-1,t} + \varphi_{t,t}$ the price of environmental services, while $H_i = H_{i,i} + H_{i,i+1} + H_{i,i+2}$ is the aggregate resource share received by generation i over its life-cycle.

In the grandfathering scenario, the environmental resource is given to the first generation, $t = 0$, which receives all present and future revenues as income:

$$H_0 = \sum_{t=1,\ldots,\infty} (p_t^e E_t + p_t^b B_t), \tag{20}$$

$$H_t = 0 \quad \text{for } t=1,2,\ldots, \tag{21}$$

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[18] As environmental service one could consider, for example, clean air or biodiversity.

In the trust fund scenario, the basic idea is that all generations receive a claim that is equal to the value of the maximum environmental services level, corresponding to $B_t = 1$ and $E_t = 0$:

$$H_{i,t} = \varphi_{i,t} \,. \tag{22}$$

However, if all generations together receive the value of the potential output, as stated in this equation, it is not clear whether this distributes the entire actual value of the environmental resource. Sharing the value of the environmental resource should also satisfy equation (19). Now, let us treat the environment such that it resembles a profit-maximising firm.[19] The value of the actual profit maximising output exceeds the value of the potential output, which is equal to the value of the claims received by all generations (see equation (22)). As an ad-hoc rule, the surplus between potential and actual output is given to the first generation,

$$H_0 = \varphi_{0,1} + \sum_{t=1,\ldots,\infty} (p_t^e E_t + p_t^b B_t - \sum_{i=t-2,\ldots,t} \varphi_{i,t}) \,, \tag{23}$$

so that (19) is satisfied.

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[19] Profit maximising is only feasible if the resource production set is convex. This is, in general, not the case where increased pollution leads to diminishing marginal environmental changes, as implied by the logarithmic temperature function (16). Fortunately, the convex damage function f(.) in (18) offsets the non-convexity, and a numerical analysis [Gerlagh, Section 5.2.4] shows that the overall production set is locally convex, so that optimisation under restrictions can be applied.

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TABLE 1. *Scenario specification.*

	No ageing included	Ageing included
"Victims Pay" Environmental resource endowed to first generation: Grandfathering	Scenario I	Scenario II
"Polluters Pay" Environmental resource shared with future generations: Trust fund	Scenario III	Scenario IV

TABLE 2. *Population and Life-expectancy in ALICE.*

	1960	1980	2000	2020	2040	2060	2080	2100	2200
Population WB[1]	n.a.	n.a.	6.1	7.7	9.0	9.9	10.6	11.0	n.a.
Population ALICE 2.0 (ageing)[2]	n.a.	n.a.	6.1	7.7	9.0	10.0	10.7	11.1	11.1
Population ALICE 2.0 (no ageing)[2]	n.a.	n.a.	6.1	7.6	9.0	10.0	10.6	11.0	11.3
Life expectancy at birth WB[1]	n.a.	63.5	67.4	71.2	74.7	77.9	80.3	82.6	n.a.
Life expectancy at birth ALICE 2.0 [3]	60.0	64.0	68.0	72.0	76.0	80.0	80.0	80.0	80.0

1. Population in billion people, life-expectancy in years, calculated from [World Bank 1994].
2. The population at the beginning of period *t* is taken to be the average population of periods *t*-1 and *t*. In its turn, the population in period *t* includes the children that enter the model one period later.
3. With ageing.

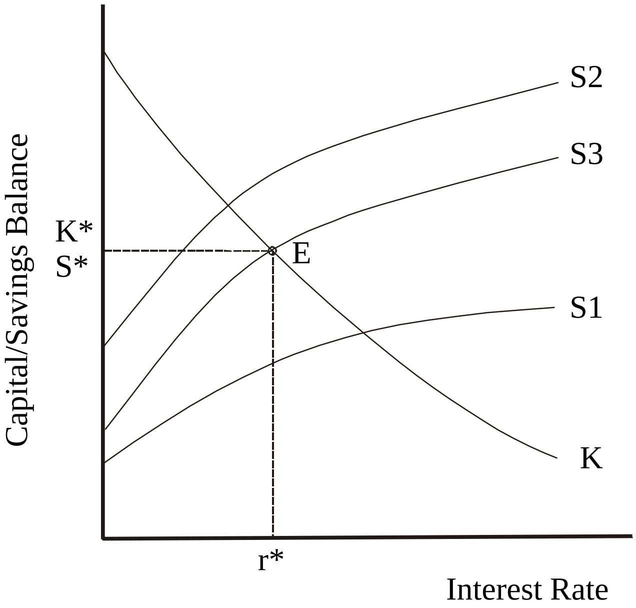

FIGURE 1. *The equilibrium (E) between savings (S curves) and capital investment (K curve).*

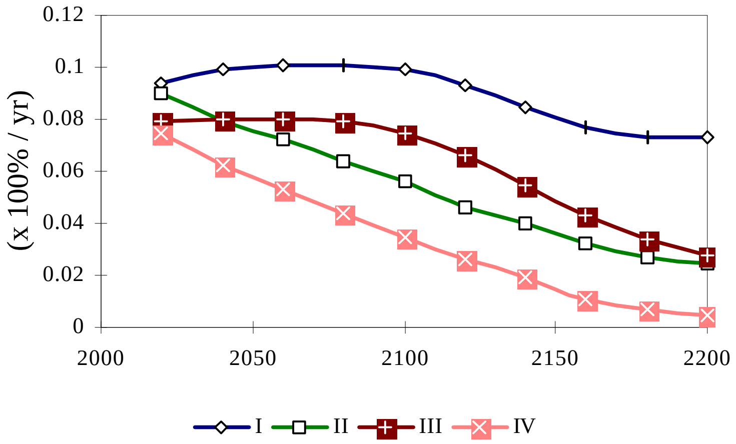

FIGURE 1. *Interest rate plotted against time, for the period 2000-2200.*

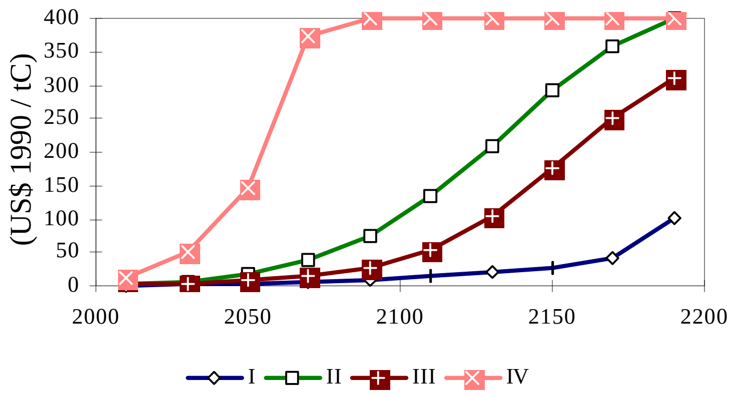

FIGURE 2. *CO_2 emission price, for the period 2000-2200.*

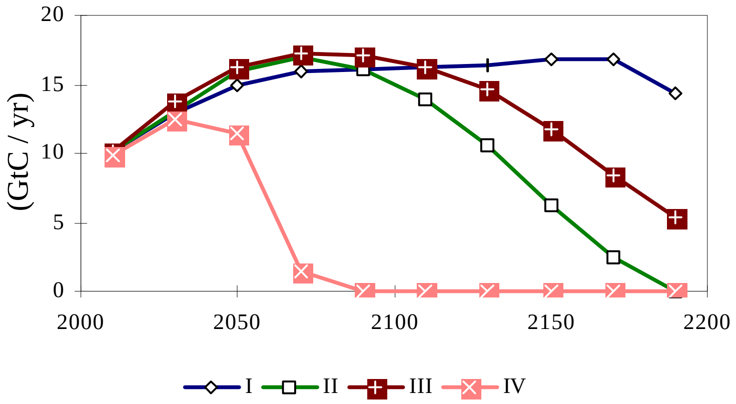

FIGURE 3. *Net CO_2 emission, for the period 2000-2200.*

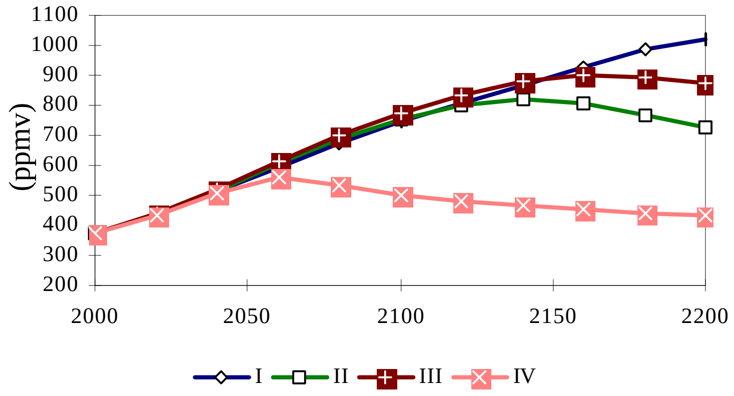

FIGURE 4. *CO_2 concentration, for the period 2000-2200.*

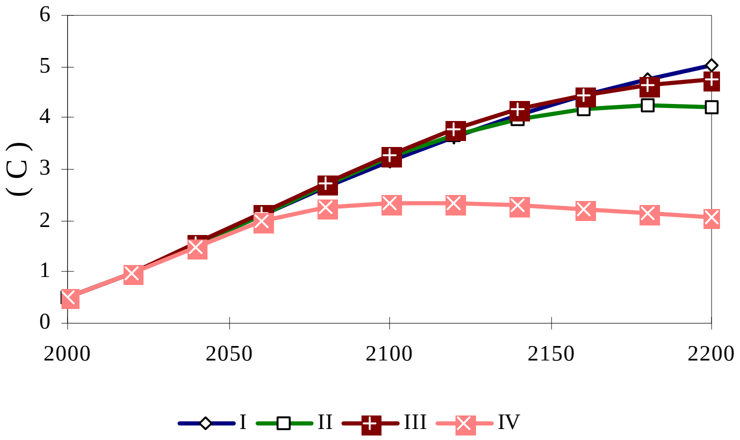

FIGURE 5. *Global mean temperature increase, relative to the pre-industrial level, for the period 2000-2200.*