Optimal capacity adjustment by a multiplant firm.

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Abstract

The paper studies the effect of scale economies on the optimal capacity adjustment of a mutiplant firm. It is shown that with increasing economies of scale plants are ranked in decreasing order, after which the optimal choice is to scrap the largest one. On the contrary, if there are decreasing economies of scale the optimal policy would be to wait before abandoning intermediate plants. That is, decreasing economies of scale amplify the effect of uncertainty on disinvestment and tend to increase the plant's life.

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1 Introduction

Recent literature on the real option approach to investment decisions, Pindyck (1991), Dixit (1992) and the book by Dixit and Pindyck, (1994), emphasizes the combination of irreversibility and uncertainty as a key determinant of investment dynamics. In single discrete projects or cases where capacity can be incrementally augmented, this approach highlights the role played by the value of the opportunity of waiting for more information. Because of irreversibility there is an opportunity cost of investing now rather than waiting. The firm waits to learn more about the uncertain future to reduce its operative costs. An optimal investment decision requires an expected rate of return from the project which is much higher than the NPV criterion would have indicated (i.e. long-run average cost).

Dixit (1993) adds to this result an extra option value of waiting when the investment involves choosing among mutually esclusive irreversible projects of different scale. In the output capacity-costs trade-off the firm finds it preferable to bypass currently feasible projects because in the future a different (higher) scale project will turn out to be more profitable. The purpose of this paper is to show that Dixit's intuition for investment can be extended to the case of disinvestment, for a multiplant firm. Specifically, we consider a firm endowed with n separate plants, each of which produces a different fixed output capacity flow. When the future output price is uncertain, the firm may adjust its capacity by closing some plants. However, disinvesting is costly and irreversible. Choosing which plant to abandon depends on the relationship between capacity scale and exit costs. If there are increasing economies of scale (i.e. elasticity of exit costs with respect to output capacity is lower than one), the plants are ranked in decreasing order and the optimal choice is to scrap the largest plant first. On the contrary, if there are decreasing economies of scale it may be optimal to shut down an intermediate plant. Furthermore, the output price triggering such disinvestment is lower than the trigger price when capacity can be incrementally decreased. This means that it is optimal to wait longer before committing to an irreversible disinvestment decision. Thus, decreasing economies of scale amplify the effect of uncertainty and tend to increase the plant's life.

Put together, these theoretical results partly explain the difference in the relation between firm exit and firm size that has been analysed by recent works on firms' selection processes. They challenge the already conventional wisdom of small business flexibility and their superior ability to survive in declining industries where large firms tend to fail.[1]

In the next section of the paper, we set up a simple model of a multiplant firm. In section 3, we examine the discrete capacity adjustment. In section 4, we extend our model to consider continuous capacity adjustment. Section 5 contains the conclusions.

2 The basic set up

The formal structure of the model presented consists of the following notation and assumptions.

- A1. The firm has n plants indexed by j, $j = 1, 2....n$. Each plant produces an output capacity flow X_j ranked in increasing order. Total output is given by $X = \sum_1^n X_j$.[2]

- A2. The flow cost of maintaining capacity is c per unit and there are no other operating costs. Production is constrained by capacity. Setup costs preclude addition of new plants or use of previously withdrawn plants.

- A3. The firm adjusts its capacity by deciding to abandon some plants. Scrapping is irreversible.

- A4. The firm incurs a lump-sum cost K_j to abandon plant j. This cost will be positive if disinvestment costs predominate (e.g. liquidation payments to workers or costs of restoring the site of a mine etc...,) or negative if the withdrawn plant has a sufficiently large salvage value. We consider positive exit costs as a benchmark case.

- A.5 Output price p follows a geometric Brownian motion,

$$dp_t = \alpha p_t dt + \sigma p_t dW_t \quad \text{with } \sigma > 0 \text{ and } p_0 = p. \tag{1}$$

[1] We refer here to the difference between the striking prediction by which the pattern of exit in a declining industry depends on firms size ordering: large firms are the first to reduce capacity (Ghemawat and Nalebuff, 1985, 1990), and the robust empirical result by which the selection pressure is fairly strong at the bottom of the industry size distribution of firms (smaller firms) while it appears to fall off considerably as one approaches the top of the distribution (Davies, Geroski and Vlassopoulos, 1991; Geroski, 1991).

[2] For sake of simplicity, we assume that the plants, once installed, last forever.

where dW_t is the increment of a standard Wiener process satisfying the conditions $E(dW_t) = 0$, and $E(dW_t^2) = dt.$ By (1) as $E(p_t \mid p) = pe^{\alpha t}$, assuming $\alpha \leq 0$ means that the expected flow of operating profits is expected to decrease over time. However, if operating profits decrease the state variable p_t tends to zero, which represents an absorbing state for it. The price's possibly negative average rate of growth α is taken as constant over time.[3]

- A.6 The firm is risk-neutral and $\rho\,(> |\alpha|)$ is the constant discount rate.
- A.7 If the output price is zero, exit costs are not sufficiently large to prevent the firm from scrapping the plant, i.e. $cX_j > \rho K_j$.

3 Discrete capacity adjustment

Defining $V_n(p)$ as the competitive valuation of the firm with n active plants, by standard methods it is easy to show that it must satisfy the following Bellman equation (Dixit and Pindyck, 1994, pp. 179-182):

$$\frac{1}{2}\sigma^2 p^2 V_n''(p) + \alpha V_n'(p) - \rho V_n(p) + (p - c)X \ = 0 \tag{2}$$

Imposing the condition that the firm's value must be bounded when the output price becomes very large, i.e. $V_n(\infty) = 0$, a general solution of (2) will consist of two parts:

$$V_n(p) = A_n p^{-\beta} + \left(\frac{p}{\rho - \alpha} - \frac{c}{\rho}\right) X, \tag{3}$$

where A_n is a constant of integration to be determined and $-\beta$ is the negative root of the characteristic equation $\Phi(\beta) = \frac{1}{2}\sigma^2\beta^2 + (\alpha - \frac{1}{2}\sigma^2)\beta - \rho = 0$. From (3), $\left(\frac{p}{\rho-\alpha} - \frac{c}{\rho}\right) X$ indicates the expected discounted value of profit flows given by n plants that are active forever, starting from an initial price p (Harrison, 1985, p.44). On the other hand, the term $A_n p^{-\beta}$ indicates the value placed by the firm on its ability to optimally adjust its capacity in the future. In other

[3] An extension, giving a more detailed analysis of pressure that a shrinking demand may create on the firm's capacity, may be to consider an inverse demand function of type $p(X;\varepsilon) = D(X)\varepsilon$ with $D'(X) < 0$, and $d\varepsilon_t = \alpha\varepsilon_t dt + \sigma\varepsilon_t dW_t$. However, this extension is algebraically more difficult and adds few new insights.

words, it is the current value of its options (*puts*) of abandoning production by all or some of the plants in the future. Then, A_n should be positive.

Intuition suggests that a firm with n active plants will decide whether to abandon a plant, when to abandon it and which plant to abandon when demand conditions become sufficiently adverse. In particular, the firm decides to exercise the option if the price falls to a threshold $\bar{p}$ far below operating costs c. This threshold will depend on the relationship between the plants' productive capacity and their relative exit costs. We will find the value of this critical value in terms of exogenous data. To show this, let us start by supposing that the firm has decided to abandon the plant $i \in (1, 2, ...N)$. We indicate with n_{-i} the strategy of the firm with n plants apart from the i-th (we will later find the plant that looks worst at this time). Then, using the above arguments, the value of the firm with n_{-i} active plants is given by:

$$V_{n_{-i}}(p) = A_{n_{-i}}p^{-\beta} + \left(\frac{p}{\rho - \alpha} - \frac{c}{\rho}\right)(X - X_i), \tag{4}$$

where $(X - X_i) \equiv \sum_{j=1, j\neq i}^{n} X_j \equiv \sum_{1}^{n-i} X_j$. In (4), $\left(\frac{p}{\rho-\alpha} - \frac{c}{\rho}\right)(X - X_i)$ is the expected discounted value of profit flows with n_{-i} plants active and $A_{n_{-i}}p^{-\beta}$ is the value of its options to abandon more plants in the future.

The firm considers abandoning the i-th plant if there is a critical level of price $\bar{p}_i$ satisfying the following *matching value condition* (5) and *smooth pasting condition* (6) (Dixit and Pindyck, 1994, pp.182-184):

$$V_n(\bar{p}_i) = V_{n_{-i}}(\bar{p}_i) - K_i \tag{5}$$

$$V'_n(\bar{p}_i) = V'_{n_{-i}}(\bar{p}_i) \tag{6}$$

We may simplify the analysis defining $G_i(p) \equiv V_n(p) - V_{n_{-i}}(p)$ as the firm's incremental value of keeping the i-th plant operative (i.e. the marginal benefit of plant i). Thus, pointing out the dependence of the coefficients A on the critical level $\bar{p}_i$, we get:

$$G_i(p; \bar{p}_i) = \Delta A_i(\bar{p}_i)p^{-\beta} + \left(\frac{p}{\rho - \alpha} - \frac{c}{\rho}\right)X_i, \tag{7}$$

where $\left(\frac{p}{\rho-\alpha} - \frac{c}{\rho}\right)X_i$ is the discounted value of profit flows coming from plant i operating forever, while the difference $\Delta A_i(\bar{p}_i) \equiv [A_n(\bar{p}_i) - A_{n_{-i}}(\bar{p}_i)]p^{-\beta}$

stands for the value of the *put* option to shut down the i-th plant in the future if the price falls below the level $\bar{p}_i$. Making use of (7) the *value matching condition* and the *smooth pasting condition* also simplify:

$$G_i(\bar{p}_i; \bar{p}_i) = -K_i, \tag{8}$$

$$G_i'(\bar{p}_i; \bar{p}_i) = 0. \tag{9}$$

The difference $\Delta A_i(\bar{p}_i)$ remains to be determined as a function of the critical level $\bar{p}_i$ by the above matching value and smooth pasting conditions. Simple calculation shows that:

$$\Delta A_i = \frac{((\rho - \alpha)\beta)\beta}{((\beta+1)\rho)^{1+\beta}} X_i^{-\beta}(cX_i - \rho K_i)^{1+\beta} > 0, \tag{10}$$

and:

$$\bar{p}_i = \frac{\beta}{\beta+1}\frac{\rho-\alpha}{\rho}(c - \rho\frac{K_i}{X_i}). \tag{11}$$

By A.7, the option to shut down is viable for the firm, i.e the exit price $\bar{p}_i \geq 0$. Moreover, as $0 < \frac{\beta}{\beta+1} < 1$, the optimal threshold $\bar{p}_i$ under uncertainty is lower than the Marshallian one with static expectations. The firm keeps alive the plant longer in the hope of recovering losses.[4]

As the aim is to choose the plant to scrap maximizing (7), we note that the choice of $\bar{p}_i$ affects $G_i(p; \bar{p}_i)$ in a very particular way. Since $p^{-\beta}$ is always positive, any change in $\bar{p}_i$ either raises or lowers the whole function $G_i(p; \bar{p}_i)$, depending on whether $\Delta A_i(\bar{p}_i)$ increases or decreases. This greatly simplifies the maximization, we should simply choose $\bar{p}_i$ to maximize $\Delta A_i(\bar{p}_i)$. Therefore, by (10), maximizing the *put* option simply means choosing among the n plants the one for which the term $X_i^{-\beta}(cX_i - \rho K_i)^{1+\beta}$ is greater.

Figures 1a and 1b illustrate the above results and how they depend on the underlying relationship between capacity and exit costs for a firm with four plants. Using Dixit's (1993) notation, we write:

$$G_i(p; \bar{p}_i) + K_i = F_0(p; \bar{p}_i) - F_i(p), \tag{12}$$

[4] With positive exit costs, if $\frac{K_j}{X_j} > \frac{c}{\rho}$ the plant is never abandoned. Comparative statics are as usual: as c increases $\bar{p}_j$ increases; if $\sigma \to 0$, $\bar{p}_j \to (c - \rho\frac{K_j}{X_j})$ and, for $\sigma > 0$, if $\frac{K_j}{X_j} \to 0$ $\frac{d\bar{p}_j}{dK_j/X_j} \to 0$. See Dixit (1989) for more details on comparative static results.

where now:

$$F_0(p;\bar{p}_i) = \Delta A_i(\bar{p}_i)p^{-\beta}, \text{ and } \quad F_i(p) = -\left(\frac{p}{\rho-\alpha} - \frac{c}{\rho}\right)X_i - K_i$$

The first term on the r.h.s. of (12), $F_0(p)$, is the value placed by the firm on the option to close plant i. When abandonment is actually carried out, the firm gains the value of the *put* option thus exercised, but loses both the expected present value of future profit flows that it would gain if the plant remained active as well as the exit cost, that is $F_i(p)$.[5] The value matching condition (8) equates the balance of these two effects and (9) determines the optimal price level for making the decision.

For each plant, the firm is able to valuate $F_i(p)$ which is represented by a negative straight line. So, to maximize the *put* option, we need only to calculate the tangencies of $F_0(p)$ with each of the straight lines $F_i(p)$ and select the one with the highest ΔA_i. $\bar{p}_i$ is also determined by the tangency. According to the relationship between capacity and exit costs two possible configurations may emerge:

- (a) **Larger plants have higher than proportionate exit costs.**
- (b) **Larger plants have less than proportionate exit costs.**

In (a) the straight lines intersect in the positive ortant so that their upper envelope form a decreasing convex function. In this case the tangency may occur in each of the four plants. In fig.1a this occurs for $i = 3$. If the price is above $\bar{p}_3$, the firm finds it optimal to wait and scraps plant 3 when the price falls to this level. Obviously, if only plant 1 were active, it would have been abandoned at a higher threshold price given by the tangency of $F_0(p)$ with a lower constant ΔA and $F_1(p)$. However, in the multiplant case, plant 3 is better placed in the cost-capacity space (K, X) so that the firm finds it convenient to keep plant 1 active and wait for the price to fall sufficiently low to withdraw plant 3.

In (b), on the other hand, the straight lines intersect only in the negative ortant. The plants are ranked in decreasing order starting from the largest one, i.e. plant 4 in fig. 1b. This would be the first to be scrapped.

$\ll$ Figures 1a and 1b about here $\gg$

[5] When the firm decides to scrap the plant the price is so low that $\left(\frac{p}{\rho-\alpha} - \frac{c}{\rho}\right) < 0$.

4 Continuous capacity adjustment

To get more precise results on the trade-off between exit costs and capacity, we may generalize the maximization of $X_i^{-\beta}(cX_i - \rho K_i)^{1+\beta}$ allowing the firm to adjust its size continuously. Suppose that the capacity continuum is indexed by $X \leq \bar{X}$ and the corresponding exit costs are given by the "cost function" $K = C(X)$. Here we refer to a firm endowed with a perfectly divisible plant, as though it consisted of so many *Lego*© bricks, of maximum size $\bar{X}$. The firm has to decide which plant in terms of how many bricks to scrap, taking account of the relationship between exit costs and capacity given by C.

For an interior optimum the first order condition requires:

$$cX^* - \rho C(X^*) + (1+\beta)\rho(C(X^*) - XC'(X^*)) = 0, \tag{13}$$

or rearranging:

$$\frac{X^* C'(X^*)}{C(X^*)} = \frac{\beta}{1+\beta} + \frac{cX^*}{(1+\beta)\rho C(X^*)}.$$

From (13), a necessary condition for an optimal solution is a cost elasticity $\frac{XC'(X)}{C(X)} > \frac{\beta}{1+\beta} > 1$.[6] This elasticity is greater than one when the average costs $\frac{C(X)}{X}$ are increasing. Thus, larger plants have higher than proportionate exit costs or there are decreasing economies of scale (case (a) above). When this condition does not hold, that is if larger plants have lower than proportionate exit costs or there are increasing (or constant) economies of scale (case (b) above), the optimum would be a corner solution on the extreme left-hand side. This corresponds to an optimal policy of waiting to scrap the largest plant in the spectrum of output capacity, i.e. to scrap the entire firm, $X^* = \bar{X}$. [7]

[6] It goes without saying that if exit costs are negative and no operating costs are present, condition (13) reduces to: $\frac{XC'(X)}{C(X)} = \frac{\beta}{1+\beta} > 1$. This is analogous, for exit, to the one proposed by Dixit(1993). For a firm that can incrementally contract its capacity see also Dixit and Pindyck (1998).

[7] It is worthwhile, however, to note that although the presence of decreasing economies of scale is a necessary condition for (13) to hold, it is not suffcient. By the second-order condition $c - \rho C'(X^*) - (1+\beta)\rho X C''(X^*) \leq 0$, after some manipulation, the cost elasticity must also satisfy $\frac{X^* C'(X^*)}{C(X^*)} \leq 1 + \frac{(1+\beta)}{\beta} \frac{(X^*)^2 C''(X^*)}{C(X^*)}$. Then, for an interior optimum we also need $C''(X^*) > 0$.

Another way to see the same result is to refer to the interpretation given by equation (3). As long as the firm's option value to decrementally abandon the (dX)th unit of its capacity is given by (10) and $dX^{-\beta}(cdX - \rho C'(X)dX)^{1+\beta} \equiv (c - \rho C'(X))^{1+\beta}dX$, we are able to recover A by integration. That is:

$$A(X) = \int_0^X dA = \frac{((\rho-\alpha)\beta)\beta}{((\beta+1)\rho)^{1+\beta}} \int_0^X (c - \rho C'(x))^{1+\beta} dx \tag{14}$$

If the economies of scale were constant instead of decreasing, the value of future capacity adjustment would be larger the larger the size of the firm, i.e. $\max[A(X)] = A(\bar{X})$. On the contrary, if the economies of scale decrease sufficiently rapidly the value placed by the firm on its options to reduce capacity in the future does not concern its entire size, i.e. $\max[A(X)] = A(\hat{X})$ where $X^* \leq \hat{X} \leq \bar{X}$.

Assuming that an interior optimum exists, let us now return to the aspect that interests us most, the fact that the relation between output scale and exit costs indicates an extra-option value of waiting. To highlight this option value we compare our case with the textbook case where capacity can be incrementally increased/decreased and all units of capital share the same exit costs (Dixit and Pindyck, 1994 ch.11). Expanding in a Taylor series the cost function and ignoring terms of order greater than second yields $C(X) \cong C'(0)X + \frac{1}{2}C''(0)X^2$, with $C'(0) > 0$ and $C''(0) \geq 0$. Substituting this expression in (11) and (13) we obtain:

$$\bar{p}(X^*) = \frac{\beta}{\beta+1}\frac{\rho-\alpha}{\rho}(c - \rho C'(0)) - \frac{\beta}{\beta+1}\frac{(\rho-\alpha)}{2}C''(0)X^*, \tag{15}$$

and X^* is given by:

$$(c - \rho C'(0))X^* - \frac{(2+\beta)}{2}\rho C''(0)(X^*)^2 = 0. \tag{16}$$

With $C''(0) = 0$, the model is equal to the textbook one. $C'(0) = \frac{K}{X}$ indicates the exit costs per unit of capital installed and the price triggering disinvestment becomes independent from the firm's scale. When the price falls to $\bar{p}_{ces} = \frac{\beta}{\beta+1}\frac{\rho-\alpha}{\rho}(c - \rho C'(0))$ the firm finds it optimal to disinvest, condition (16) determines how many units of capital should be abandoned. However, as all units have the same exit costs the optimal decision is of binary type comparing the smallest and the largest plants. In particular, as $c - \rho C'(0) > 0$ then $X^* = \bar{X}$, and the entire firm will be scrapped.

When $C''(0) > 0$ the firm may find it optimal to scrap only a part of its productive capacity, so that $X^* = \frac{2(c-\rho C'(0))}{(2+\beta)\rho C''(0)} \leq \bar{X}$. Furthermore, as the direct value of an incremental unit of capacity is constant but the value of the put option on that unit decreases $\bar{p}(X)$ also decreases. That is, the price that triggers such disinvestment is lower than the trigger price under constant economies of scale:

$$\bar{p}_{des} = \frac{1+\beta}{2+\beta}\left[\frac{\beta}{\beta+1}\frac{\rho-\alpha}{\rho}(c-\rho C'(0))\right] \equiv \frac{1+\beta}{2+\beta}\bar{p}_{ces} \tag{17}$$

Thus, decreasing economies of scale hasten the effect of uncertainty and increase the plants' life, contrary to the conventional constant economies of scale textbook case. Finally, a larger σ lowers β and then both $\frac{\beta}{\beta+1}$ and $\frac{1+\beta}{2+\beta}$. By (17), this means that greater uncertainty makes it optimal to wait until a lower threshold price is reached and then, by (16), to disinvest in a larger plant. To illustrate this effect and to show how it depends on the instantaneous variance of output price, we presents a numerical solution, normalizing the operating and exit costs at $c = 1$ and $C'(0) = 10$ respectively. Yet, $\rho = 0.05$, $\rho - \alpha = 0.04$, and for the standard deviation of the output price $\sigma = 0,\ 0.2,\ 0.3$ (at annual rates). Given this parameter values we obtain:

	$\sigma =$	0.0	0.2	0.3
$-\beta$		0.000	1.3508	0. 7346
$\frac{\beta}{\beta+1}$		0.000	0.5746	0. 4235
$\frac{1+\beta}{2+\beta}$		0.500	0. 7016	0. 6343
$\bar{p}_{ces}$		0.500	0. 2298	0. 1694
$\bar{p}_{des}$		0.250	0. 1612	0. 1074

Thus the simple NPV rule, which says that the firm should disinvest as long as the output price falls short of the variable cost corrected for the marginal abandonment cost, $c - \rho C'(0) = 0.5$, is in error. For this set of parameter values and with constant economies of scale, the price threshold for disinvestment will fall dramatically to 0.2298 and 0. 1694 with volatility 0.2 and 0.3 respectively. Furthermore, we observe that with decreasing economies of scale the threshold will decrease even further, i.e. 0.1612 and 0.1074 with volatility 0.2. and 0.3 respectively.

5 Conclusions

The policy implications of the model are worth discussing. Firms with different scale economies respond differently in terms of capacity adjustment. While with increasing economies of scale they respond by abandoning the largest installed plant as soon as possible, with decreasing economies of scale they may tend to increase the plants' life. Moreover, the volatility of the economic environment is an important impediment to disinvestment. This means that policies aimed at restructuring an industry facing a shrinking demand by granting subsidies for scrapping the oldest plants, have smaller effects on firms with decreasing economies of scale than on firms with increasing economies of scale. On the contrary, policies designed to reduce volatility have relatively more effect on firms with decreasing economies of scale than on firms with increasing economies of scale.

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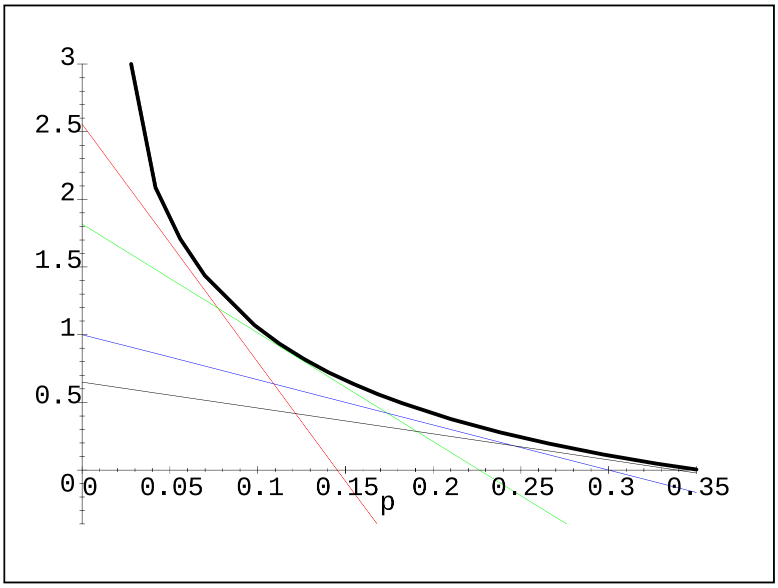

Fig.1a, Put Option Value, $F_0(p;\bar{p}_i)$, and $F_i(p)$, $i = 1, 2, 3, 4$.

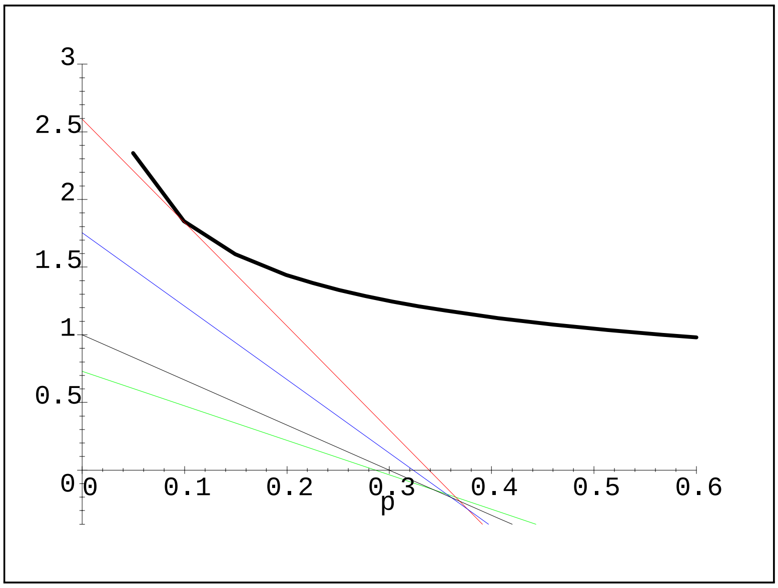

Fig.1b, Put Option Value, $F_0(p;\bar{p}_i)$, and $F_i(p)$, $i = 1, 2, 3, 4$