Taxes and Quotas for a Stock Pollutant with Multiplicative Uncertainty

Michael Hoel[1] and Larry Karp[2]

March 4, 1999

[1]University of Oslo/ P.O. Box 1095 Blindern/ N-0317 OSLO Norway/ email: michael.hoel@econ.uio.no

[2]Department of Agriculture and Resource Economics/ 207 Giannini Hall/ Berkeley CA 94720 email: karp@are.berkeley.edu

Abstract

We compare taxes and quotas when firms and the regulator have asymmetric information about abatement costs. Damages are caused by a stock pollutant. Uncertainty enters multiplicatively, i.e. it affects the slope rather than the intercept of abatement costs. We calibrate the model using cost and damage estimates of greenhouse gasses. As with additive uncertainty, taxes dominate quotas. The advantage of taxes is much greater with mulitiplicative, compared to additive uncertainy.

Key Words: Pollution control, asymmetric information, taxes and quotas, stochastic control, global warming, multiplicative disturbances

JEL Classification numbers: H21, Q28

1 Introduction

When polluters have better information than regulators about the cost of pollution abatement, social welfare depends on both the form of regulatory policy and its level. Complicated non-linear taxes (e.g. [8]) can achieve the (information-constrained) social optimum, but these types of policies are not actually used. Most actual regulatory devices rely on standards or some other form of quantity control, and to a lesser extent, linear emissions taxes. An important literature, beginning with Weitzman [22], compares social welfare under taxes and quotas when polluters and regulators have asymmetric information about abatement costs. Most subsequent contributions (e.g. [4],[10], [18], [20], [24] and [25]) to this subject study the case where the damage depends on the flow of pollution. However, several important environmental problem, e.g. global warming and acid rain, are associated with the stock rather than the flow of pollution. For these problems, the comparison of taxes and quotas requires a dynamic model.

Hoel and Karp ([7]) use a dynamic extension of Weitzman's model to compare the two policies for a stock pollutant.[1] They assume that the abatement cost is a quadratic function of emissions, that damages are a quadratic function of the stock, and that the identically, independently distributed random term alters the intercept, but not the slope, of the abatement cost function. That is, the random variable enters additively. In each period the regulator chooses the policy level, knowing the distribution but not the realization of the random variable. As with the static model, taxes are more likely to be preferred to quotas when the slope of the marginal abatement cost is large relative to the slope of marginal damages. In addition, taxes are likely to be preferred when the future is not very important, either because of a high discount rate or a high decay rate for the stock of pollution. Taxes are also likely to be preferred when the regulator uses a feedback policy rather than an open loop policy, and when new information arrives quickly (the length of a period is small). The ranking of the two policies is independent of the magnitude of uncertainty and of the current stock of pollutant.

[1]Newewll and Pizer [12] and Staring [19] also study a dynamic version of Weitzman's problem. They both restrict attention to open loop policies, whereas [7] compare open loop and feedback policies. [12] includes the generalization where the random variable is serially correlated.

Hoel and Karp calibrate the model using previous estimates of the slopes of the damage and abatement cost functions for greenhouse gasses. Even if the estimated ratio of the slopes of the two cost functions is wrong by a factor of one thousand, taxes dominate quotas for reasonable values of the other parameters. This result suggests that in fact it might be important to use taxes rather than quantity restrictions in controlling greenhouse gas emissions. There is currently considerable debate both about the extent to which emissions should be limited, and the manner in which these limits should be achieved. Thus, the conclusion that taxes are better than quotas has significant implications for policy.

Although this conclusion is robust with respect to variations in parameter values, it is conditioned on the assumption of quadratic functions and additive uncertainty. Given our lack of knowledge about actual costs and the resulting need to use a parsimonious model, the quadratic specification is reasonable. However, the assumption of additive uncertainty is made purely for its tractability. There is no reason to believe that the intercept of the abatement cost function is subject to more randomness than the slope of that function. We know that in the static model, multiplicative uncertainty can change the ranking between policies, holding fixed other parameters. In view of the importance of determining the best method of controlling greenhouse gas emission, it is worth investigating whether the conclusion in [7] survives the introduction of multiplicative uncertainty. In addition, since there are numerous other important stock pollutants, it is worth having a simple means of comparing the two policies under both additive and multiplicative uncertainty.

The next section describes and solves the linear quadratic model with multiplicative disturbances. We also discuss some of the determinants of the comparison between the two policies. The following section uses previous estimates of costs to calibrate the model. We use this calibration to compare the two policies, and then relate our results to the case of additive uncertainty.

2 The model

We define pollution emissions and costs as flows, e.g. billions of tons per unit of time or billions of dollars per unit of time. Each period lasts for h units of time and we assume that variables are constant within a period. We can think of h as being the amount of time between the arrival of new information, or the amount of time during which decisions are held fixed. Thus, h can be viewed as a measure of the flexibility of decisionmakers. The comparison between taxes and quotas depends on many parameters, including the length of each period. By explicitly including h, we are able to determine how this parameter affects the comparison.

The firm's flow of pollution in the absence of regulation is x^*. If its actual level of pollution is x, its abatement is $x^* - x$. The firm's abatement costs in the absence of uncertainty are $\frac{b(x^*-x)^2}{2}$, which we rewrite as $-(f + ax - \frac{b}{2}x^2)$. We introduce multiplicative uncertainty by replacing the parameter b with $\frac{b}{\theta}$ and then writing abatement costs as $-(f + ax - \frac{b}{2\theta}x^2)$.[2] The random variable θ is independently and identically distributed. At the beginning of the period the firm, but not the regulator, observes the current value of θ. The first and second moments of θ are $E\theta = \bar{\theta}$ and $E\theta^2 = \gamma$, so $var(\theta) = \sigma^2 = \gamma - \bar{\theta}^2$. The regulator knows the probability distribution of θ.

If the regulator chooses a tax p per unit of pollution, the firm minimizes the sum of abatement cost and tax payments. The first order condition to the firm's cost minimization problem implies that the flow of pollution is

$$x = \left(\frac{a-p}{b}\right)\theta \equiv z\theta, \tag{1}$$

which uses the definition $z = \frac{a-p}{b}$. We can think of the regulator as choosing the variable z rather than the tax p. Thus, under a tax, the flow of pollution in any period is a random variable. When the regulator chooses a quota, x, we assume that it is binding. With a quota, the flow of pollution in a period is deterministic.

If we normalize by setting $\bar{\theta} = 1$, then $var(\theta) = \gamma - 1$ With this normalization, z is the expected flow of pollution when the regulator uses the tax $p = a - bz$. We provide the formulae for general values of $\bar{\theta}$ and then specialize to $\bar{\theta} = 1$.

The flow of damages resulting from the stock of pollution, S, is $cS + \frac{g}{2}S^2$. The firm ignores these costs, but the regulator cares about them. The flow

[2] In his reply to Malcomson[10], Weitzman [23] models multiplicative disturbances by having the random variable divide rather than multiply the slope of marginal costs. We adopt this formulation because it leads to a slight simplification in the derivations. Of course the two formulations are equivalent, since one is obtained from the other merely by re-defining the random variable. An (equivalent) alternative, replacing $\frac{bx^2}{\theta}$ with $(b+\theta)\,x^2$, can also be obtained by redefining variables.

of payoff (the negative of abatement costs minus damages) for the regulator is

$$(f + ax - \frac{b}{2\theta}x^2) - \left(cS + \frac{g}{2}S^2\right). \tag{2}$$

Using (1)and (2), the regulator's expected flow of payoff in a period if she uses a tax (T) is

$$\lambda\left(t;T\right) \equiv \lambda\left(z_t, S_t;T\right) = f + \left(az_t - \frac{b}{2}z_t^2\right)\overline{\theta} - \left(cS_t + \frac{g}{2}S_t^2\right). \tag{3}$$

If the regulator uses a quota (Q) her expected flow of payoff in a period is

$$\lambda\left(t;Q\right) \equiv \lambda\left(x_t, S_t;Q\right) = f + ax_t - \frac{b}{2}x_t^2 E\left(\frac{1}{\theta}\right) - \left(cS_t + \frac{g}{2}S_t^2\right). \tag{4}$$

Since each period lasts for h units of time, and since all variables are assumed to be constant within a period, the regulator's expected payoff for a period is $\lambda(t;i)h$, for $i = T, Q$ (taxes, quotas).

The equation of motion for the stock of pollutant, S, is

$$S_{t+h} = \Delta S_t + x_t h \tag{5}$$

where the fraction of stock that persists until the next period is $\Delta = e^{-\delta h}$; δ is the continuous time decay rate.

With a discount factor $\beta = e^{-rh}$ and an initial value of the stock S_0, the present discounted value of the regulator's payoff is

$$J\left(S_0;i\right) = \max E\sum_{t=0}^{\infty}\beta^t\lambda\left(t;i\right)h$$

subject to (5), for $i = T, Q$.

Under quotas, the regulator has a standard deterministic linear-quadratic problem. The term $E\left(\frac{1}{\theta}\right)$ enters the payoff as a constant which multiplies b (see 4); uncertainty has no other effect on the problem. Under taxes, the regulator has a stochastic control problem with multiplicative disturbances. In this case, the "Principle of Certainty Equivalence" does not apply: the state contingent optimal control rule depends on both the mean and the variance of θ.[3] However, the control rule is still linear, and the value function is still quadratic, (as is the case for both the deterministic problem and the problem with additive uncertainty). Thus, we can use standard methods to solve the control problem under taxes. We first solve this problem, and then use our result to obtain the solution to the problem under quotas.

We begin by "guessing" that the value function under taxes is quadratic: $J(S;T) = \rho_0 + \rho_1 S + \frac{\rho}{2} S^2$ for some parameters ρ_0, ρ_1, ρ_2. Using this guess, we can write the regulator's dynamic programming equation under taxes as:

$$\begin{aligned} \rho_0 + \rho_1 S + \frac{\rho_2}{2} S^2 &= \max_z \{ \lambda(z, S; T) h \\ &\quad + \beta \underset{\theta}{E} \left[\rho_0 + \rho_1 (\Delta S + z\theta h) + \frac{\rho_2}{2} (\Delta S + z\theta h)^2 \right] \} & (6) \\ &= \max_z \left\{ \alpha_0 + \alpha_1 \overline{\theta} h z + \frac{\alpha_2 h}{2} z^2 \right\}, & (7) \end{aligned}$$

which uses the definitions

$$\begin{aligned} \alpha_0 &= \left(f - cS - \frac{gS^2}{2} \right) h + \beta \left(\rho_0 + \rho_1 \Delta S + \frac{\rho_2}{2} (\Delta S)^2 \right) \\ \alpha_1 &= a + \beta (\rho_1 + \rho_2 \Delta S) \\ \alpha_2 &= \beta \rho_2 h \gamma - b\overline{\theta}. \end{aligned}$$

The optimal control rule is

$$z^* = -\frac{\alpha_1 \overline{\theta}}{\alpha_2}. \tag{8}$$

Substituting equation (8) into (7) and equating coefficients of powers of S gives the equations for ρ_i.

$$\begin{aligned} \rho_2 &= -gh + \beta \rho_2 \Delta^2 - \frac{(\beta \rho_2 \Delta)^2 h \overline{\theta}^2}{\beta \rho_2 h \gamma - b\overline{\theta}} & (9) \\ \rho_1 &= -ch + \beta \rho_1 \Delta - \frac{(a + \beta \rho_1) \rho_2 \Delta h \overline{\theta}^2}{\beta \rho_2 h \gamma - b\overline{\theta}} & (10) \\ \rho_0 &= fh + \beta \rho_0 - \frac{(a + \beta \rho_1)^2 h \overline{\theta}^2}{2 (\beta \rho_2 h \gamma - b\overline{\theta})}. & (11) \end{aligned}$$

[3] Under additive uncertainty, the optimal levels of x and z, for a given level of S, are the same. This "certainty equivalance" does not hold under multiplicative disturbances.

Given that the value function is bounded above and that it is quadratic, it must be the case that $\rho_2 < 0$. We first obtain the negative root of equation (9) and then solve the linear equations (10) and (11) recursively.

We obtain the solution to the problem under quotas by specializing the above solution under taxes. Using the normalization $\bar{\theta} = 1$, we simply replace $\bar{\theta}$ and γ by 1 and replace b by $bE\left(\frac{1}{\theta}\right)$ in the equations for ρ_i and in the definitions for α_i above, to obtain the solution under quotas.

The model with additive uncertainty [7] is simpler, and provides a useful basis for comparison. Under additive uncertainty, we write abatement costs as $-\left(f + [a + \theta]\, x - \frac{b}{2}x^2\right)$. We solve the optimization problem using the same procedure as above, and obtain a system of equations that correspond to equations (9) - (11). In that system of equations the second moment (γ) appears only in the equation for ρ_0. This feature enables us to obtain a closed form comparison of the value functions under taxes and quotas. With multiplicative uncertainty all the parameters ρ_i depend on γ under taxes; all of these parameters depend on $E\left(\frac{1}{\theta}\right)$ under quotas. However, (with multiplicative uncertainty) the parameters γ (under taxes) and $E\left(\frac{1}{\theta}\right)$ (under quotas) enter the equations for ρ_i in different ways, precluding a simple comparison of the payoffs under the two policies. Nevertheless, we can identify similarities in the comparisons under additive and multiplicative uncertainty.

With additive uncertainty, we know that taxes are more likely to dominate quotas when the convexity of the abatement cost function is large, relative to the convexity of the damage function, i.e. when b is large relative to g. This ranking is also likely to hold under multiplicative uncertainty, and the intuition is essentially the same. The value of the regulator's program is the sum of the expected value in the current period, $\lambda(t; i)h$, and the expected continuation payoff, $\beta J(S_{t+h}; i)$ (equation 6). Taxes and quotas affect these two terms differently, and the extent of the difference depends on the magnitude of b and g.

The following thought experiment helps in understanding this difference: hold future policies constant, and compare the payoff under a quota in the current period and under a tax that produces the same expected flow of pollution (i.e., let $x = z$ in the current period). Since we are holding future policies constant, the function that gives the continuation payoff, $J(S_{t+h})$, (but not it's argument S_{t+h}) is independent of the current policy. We saw that in going from taxes to quotas, the parameter b is replaced by $bE\left(\frac{1}{\theta}\right) > b$. Consequently, the current expected flow of benefits under a quota x is less than the expected flow of benefits under a tax that generates the same expected level of emissions: That is, if $z = x$, then the expectation of current benefits is greater under taxes. The larger is b, and the larger is $E\left(\frac{1}{\theta}\right)$, the greater is the difference in expected benefits. Consequently, taxes tend to be preferred when b or $E\left(\frac{1}{\theta}\right)$ are large.

However, when a quota is replaced by a tax, the current level of emissions becomes stochastic; x is replaced by the stochastic variable $z\theta = x\theta$, so the level of the stock in the next period is stochastic. In view of the concavity of the value function ($\rho_2 < 0$) and Jensen's inequality, the expectation of the value function is lower under uncertainty. The concavity of the value function is increasing in the parameter g: the absolute value of the negative root of equation (9) is an increasing function of g under either taxes or quotas. Thus, a larger value of g increases the concavity of the value function and tends to make the regulator prefer quotas. For a given degree of concavity of the value function, the loss in future welfare from using a tax rather than a quota is an increasing function of the variance of θ. Thus, a larger variance (larger γ) tends to make the regulator prefer the quota.

In other words, given the current level of S and holding fixed future policies, a tax (relative to a quota) in the current period leads to higher current expected benefits but a lower expectation of future benefits. The future becomes more important relative to current benefits (and thus the quota tends to be preferred) if either the discount factor β or the retention factor Δ are larger. Thus, larger values of β or Δ make it more likely that a quota is preferred.

This thought experiment helps to provide intuition for the comparison of policies, but it does not provide the basis for a proof because future policies are in fact different under taxes and quotas. Consideration of limiting cases shows that the intuition described above is certainly correct over some range of parameter space. For example, if $g = 0$ then $\rho_2 = 0$ (under both policies). In this case it is easy to show that taxes are preferred to quotas regardless of the other parameter values. If $b = 0$ the firm's decision rule, equation (1), is not defined. However, for b arbitrarily close to 0, the firm's decision rule is defined and we can compare the payoffs by considering their limiting values. For $x = z$ and $b = 0$ the single period payoffs under taxes and quotas are equal (equations (3) and (4)). However, the evolution of the stock is stochastic under taxes and remains deterministic under quotas. Thus for $b = 0$ the only difference between the two policies is that quotas enable the regulator to exactly control the evolution of the state, whereas taxes enable the regulator to choose only the mean of the evolution of the state. In this case, we expect the payoff to be higher under quotas. In view of the continuity of all the functions that define ρ_i, we conclude that taxes dominate quotas for sufficiently small g, and the ranking is reversed for sufficiently small b.

Taxes dominate quotas if $\beta = 0$, since in this case the regulator is unconcerned with the future. The future is also relatively unimportant if $\Delta = 0$. In this case the stock in period $t+1$ is $x_t h$ and we are back to a static model. The values of ρ_1 and ρ_2 are the same under taxes and quotas, so a comparison of the policies requires only a comparison of the values of ρ_0. It is straightforward to show that for $\Delta = 0$ taxes dominate quotas if and only if

$$\frac{E\frac{1}{\theta} - 1}{var(\theta)} > \beta h^2 \frac{g}{b}. \tag{12}$$

Here, taxes are preferred to quotas if: $\frac{g}{b}$ is small, the future is discounted heavily (β is small) or it is possible to make adjustments quickly (h is small).

The left side of (12) shows that the type of uncertainty as well as its magnitude affects the comparison. For a fixed value of $E\frac{1}{\theta}$, taxes are more likely to be preferred if the magnitude of uncertainty, measured by $var(\theta)$, is small. However, for a given distribution function, $E\frac{1}{\theta} - 1$ varies with the variance of θ. The left side of equation (12) is not necessarily monotonic in $var(\theta)$.

3 An Application to Global Warming

The previous section shows how to compare the benefits of taxes and quotas. Under the assumption that uncertainty about marginal abatement costs is additive, [7] compares these policies as a means of controlling global warming. With additive uncertainty we need estimates of the ratio $\frac{g}{b}$ and of the parameters r,δ, and h. Even if the largest available estimate of $\frac{g}{b}$ understates the true ratio by a factor of 1000, taxes dominate quotas for reasonable values of $r, \delta$and h. The robustness of the comparison suggests that in fact taxes are likely to yield a higher payoff than quotas. Here we want to determine if this conclusion holds when the slope of abatement costs is uncertain. With multiplicative disturbances we also need estimates of the intercepts a and c and information about $var(\theta)$ and $E\frac{1}{\theta}$

In order to obtain estimates of the parameters of the damage and abatement cost functions, we use estimates of the absolute levels of damages and abatement costs. Our unit of time is years and we set $h = 1$, so one period equals one year. We measure costs in billions of 1990 dollars, and the stock of carbon in billions of tons. The estimated stock in 1990 was 800 (billion tons) [6], [17],and the estimated Gross World Product (GWP) was 2,200 (billion dollars)[11], [16].

We assume that the cost of a stock higher than 800 is $\frac{g(S-800)^2}{2}$, so that the parameter c is given by $c = -800g$. We use the parameter ϕ to denote the annual percentage reduction in GWP due to doubling the world atmospheric stock of carbon The parameters ϕ and g satisfy

$$\frac{\phi 22000}{100} = \frac{g800^2}{2} \Longrightarrow g = 6.875 \times 10^{-4}\phi. \tag{13}$$

A high estimate for the annual cost resulting from a doubling of the stock of carbon is 400, implying $\phi = 1.8$; many other estimates are approximately half of that magnitude.[4] We consider three damage functions that correspond to three values of ϕ: $\phi = 1$ (a conservative damage estimate); $\phi = 5$ (a high damage estimate) and $\phi = 30$ (an extremely high estimate). For the conservative estimate, $g = 6.875 \times 10^{-4}$ and for the extremely high estimate, $g = 2.0625 \times 10^{-2}$.

There are a range of estimates of annual emissions in 1990. We adopt a "moderate" estimate of 6 billion tons per year ([11], [16],). There also exist a range of estimates of the absolute costs of reducing emissions. These estimates vary according to country and time period. It is cheaper to reduce emissions slowly, because of the lower adjustment costs (assuming that these are convex in the rate of adjustment) and because of technological improvements. A "moderate" estimate is that a 50% reduction in emissions leads to a 1% loss in GWP, or 220 billion 1990 dollars [13].[5] If we assume that decreasing emissions (x) below 6 results in abatement costs of $\frac{b}{2}(6 - x)^2$, the moderate estimate (1% loss of GWP due to a 50% reduction of emissions) implies that $b = 48.9$, $a = 293.3$.

[4] For a range of estimates see:[1], [2], [3], [5], [6], [9], [13], [14], [15] [16], [21].

[5] This estimated cost refers to a reduction that is phased in over decades, not an instantaneous reduction. Obviously, our model ignores adjustment costs. If we included adjustment costs, firms would solve a dynamic optimization problem, rather than a sequence of static problems, and the regulator's problem would be changed accordingly.

The conservative estimates for damages and for abatement costs imply that the ratio $\frac{g}{b} = 1.4062 \times 10^{-5}$. This ratio is critical in ranking taxes and quotas. In the previous section we explained why quotas are more likely to be preferred when the ratio is large. Since the ratio has little intrinsic economic interpretation, we perform sensitivity studies by varying the parameter ϕ (the percentage loss in GWP due to doubling the stock of carbon from 800 to 1600). Equation (12) shows that g (and thus the ratio $\frac{g}{b}$) is proportional to ϕ.

To compare the two policies we also need assumptions about the random variable. We have two free parameters, $E\frac{1}{\theta}$ and $var(\theta)$. In order to reduce the dimensionality of parameter space, we assume that θ is uniformly distributed with support $[1-\epsilon, 1+\epsilon]$, i.e.

$$\theta \sim U\left[1-\epsilon, 1+\epsilon\right]. \tag{14}$$

This distribution implies[6]

$$E\theta = 1, \quad E\theta^2 \equiv \gamma = \frac{3+\epsilon^2}{3}, \quad var(\theta) = \frac{\epsilon^2}{3} \tag{15}$$

$$E\frac{1}{\theta} = \frac{1}{2\epsilon}\left[\ln\left(\frac{1}{1-\epsilon}\right) - \ln\left(\frac{1}{1+\epsilon}\right)\right] = \frac{\ln(1+\epsilon) - \ln(1-\epsilon)}{2\epsilon}. \tag{16}$$

The thought experiment described in the previous section suggests that taxes are more likely to dominate quotas when $E\frac{1}{\theta}$ is large and when $var(\theta)$ is small. For the limiting case $\Delta = 0$, we saw (equation (12)) that the comparison between the policies depends on the ratio of $E\frac{1}{\theta} - 1$ and the variance. Using equations (15) and (16) we can write this ratio as

$$\Phi(\epsilon) \equiv \frac{E\frac{1}{\theta} - 1}{var(\theta)} = \frac{3\left[\ln(1+\epsilon) - \ln(1-\epsilon) - 2\epsilon\right]}{2\epsilon^3}. \tag{17}$$

Using L'Hospital's rule, we find that $\Phi(\epsilon)$ approaches ∞ as $\epsilon \to 0$. Since the function is bounded for $\epsilon > 0$, it must be decreasing over some range.

[6]In defining costs, damages, and emissions as flows, we have made the parameters a, b, c, g, r, and δ independent of the length of a period, h. However, the parameter ϵ is clearly not independent of h. A reasonable (but not necessary) assumption is that ϵ^2 is of the same order as h.

However, the function is not analytic at $\epsilon = 0$, and its behavior is difficult to analyze. We therefore use Maple to obtain it's graph, and find that for non-negligible values of ϵ $(\epsilon > .001)$ $\Phi(\epsilon)$ is strictly increasing. Consequently, if Δ is small and the random variable is uniformly distributed with a non-negligible variance, an increase in the variance makes taxes more likely to dominate quotas.

Our estimate of $\delta = .005$ ([6], [14])[7] implies $\Delta = .995\,01$ when $h = 1$. For greenhouse gasses Δ is not small, and therefore we need a complete solution in order to compare the policies. We know that the comparison also depends on the stock size. We assume that the continuous discount rate is $r = .03$.

Figures 1 - 3 graph the difference between the present discounted value of payoff under taxes and quotas $(J(S;T) - J(S;Q))$ for S in the interval $[800, 2000]$. We use nine combinations of parameter values[8]: $\phi \varepsilon (1, 5, 30)$ and $\epsilon \varepsilon (0.2, 0.4, 0.6)$. The value $\epsilon = 0.2$ implies a standard deviation (which equals the coefficient of variation) of 0.149, and $\epsilon = 0.6$ implies a standard deviation of 0.258.

For the conservative estimates ($\phi = 1$ and $\epsilon = .2$), when $S = 800$ taxes dominate quotas by about 450 (billion 1990 dollars – see Figure 1), approximately 2% of GWP, or twice the estimate of the annual loss in GWP due to a 50% reduction in annual emissions.[9] This difference decreases slightly with the stock size. Tripling the value of ϵ leads to nearly an eight-fold increase in the advantage of taxes when $S = 800$. For the high estimate of damages ($\phi = 5$) taxes still dominate quotas (Figure 2). As the analysis of the previous section suggests, this difference decreases when damages are larger. In addition, the difference becomes more sensitive to the stock size. For the extremely high estimate of damages ($\phi = 30$) taxes continue to dominate quotas for moderate stock levels, but by a smaller amount. However, for stock levels between 1350 and 2000, quotas yield a higher payoff.

Table 1 reports the expected steady state under taxes (the first entry) and the steady state under quotas (the second entry) for the nine sets of parameter values. The expected steady state under taxes is always larger

[7]This estimate may be low, thus biasing our results in favor of quotas. Other estimates suggest a value of $\delta = .0083$ [17].

[8]The other parameter values are: $h = 1, r = .03, \delta = .005, b = 48.9$, $a = 293.3$ (which correspond to the estimate that a 50% reducition in emissions leads to a 1% fall in GWP).

[9]Note that this amount is the difference in the present discounted value of the stream of payoffs under the two policies. With a yearly discount factor of $e^{-.03} = .970\,45$, a present discounted value of 450 implies an annual flow of 13.3.

Table 1: (Expected) steady states under taxes and quotas

	$\epsilon = .2$		$\epsilon = .4$		$\epsilon = .6$	
$\phi = 1$	1173,	1158	1173,	1112	1173,	1025
$\phi = 5$	1087,	1076	1087,	1043	1087,	978
$\phi = 30$	914,	909	913,	898	913,	874

than under quotas. In all cases, the (expected) steady state decreases with the severity of damages and with the magnitude of uncertainty. Increasing ϵ leads to a slight fall in the steady state under quotas, but a scarcely perceptible fall under taxes.[10] Since we have no estimates of the actual magnitude of ϵ, this insensitivity is encouraging. Not surprisingly, increasing damages (ϕ) decreases the (expected) steady state.

Perhaps the most striking feature of Table 1 is the similarity of the steady states. The largest number (1173) is only 34% larger than the smallest number (874), despite a considerable range in parameters (300% for ϵ and 3000% for ϕ). If we think that $\phi = 5$ and $\epsilon = 0.4$ are reasonable upper bounds for the parameters, and $\phi = 1$, $\epsilon = 0.2$ are reasonable lower bounds, the results suggest that a target steady state of carbon stock between 1045 and 1175 is optimal. In other words, we should attempt to keep the stock below 150% of its 1990 level. The optimal steady states under severe damages ($\phi = 30$) are well below the level (1350) beyond which quotas dominate taxes. Thus, Table 1 reinforces the conclusion that taxes are a better policy instrument than quotas.

In the absence of regulation, $x = 6$ and the steady state is $S = 1203$. This non-intervention steady state is approximately 2.5% higher than the largest steady state in Table 1, and 37% higher than the smallest steady state. Whether regulation leads to a large change in the outcome depends on the regulator's beliefs about the severity of damages.

Given our parameter assumptions, it is obvious that for any $S \geq 800$, the optimal values of both z and x must be less than 6.[11] For example, when $\epsilon = .2$ and $\phi = 1$, the optimal control rule under taxes is $z = 6.2528 - 3.43x10^{-4}S$. For $S = 800$, the optimal expected level of emissions is $z = 5.98$. At the

[10] We rounded to whole numbers, so the slight decrease of the expected steady state under taxes, due to an increase in ϵ, is usually not perceptible.

[11] For $S \geq 800$ an increase in stock increases damages. Since abatement costs are quadratic and are minimized at $z = 6$, under taxes, and at a value of $x < 6$ under quotas, the optimal values of both z and x must be smaller than 6.

Table 2: Payoff difference under taxes and quotas with additive uncertainty

	$\epsilon = .2$	$\epsilon = .4$	$\epsilon = .6$
$\phi = 1$	4.6132×10^{-3}	1.8452×10^{-2}	4.1516×10^{-2}
$\phi = 5$	4.6071×10^{-3}	1.8428×10^{-2}	4.1462×10^{-2}
$\phi = 30$	4.575×10^{-3}	1.8299×10^{-2}	4.1174×10^{-2}

steady state $S = 1173$, the optimal expected flow of pollution is 5.85. The corresponding levels under the optimal quota are slightly smaller ($x = 5.9$ for $S = 800$ and $x = 5.78$ for the steady state $S = 1158$.) Increasing damages (ϕ) or uncertainty (ϵ) leads to lower optimal levels of emissions. These results are driven by the assumption that abatement costs are large relative to the stock-related damages.[12]

For purposes of comparison, we consider the magnitude of the difference in the value functions under taxes and quotas, when the uncertainty is additive (rather than multiplicative). That is, suppose that abatement costs are $-\left(f + (a + \theta^*)\,x - \frac{b}{2}x^2\right)$, where θ^* is a zero mean, i.i.d. random variable with variance $var(\theta^*)$. Using equation (18) in [7], the difference between the present discounted value of payoff under taxes and quotas is

$$\frac{var(\theta^*)h\left(1 + \frac{\beta\rho_2}{b}\right)}{(1-\beta)2b}. \tag{18}$$

The parameter ρ_2 is the negative root of (9) with $\gamma = 1$. (Equivalently, ρ_2 is the negative root of equation (17) in [7]). With additive uncertainty the difference is independent of stock size, increasing in the magnitude of uncertainty, and decreasing in the ratio $\frac{g}{b}$. If θ and θ^* have the same distribution, so that $var(\theta^*) = \frac{\epsilon^2}{3}$, then the differences in payoff under taxes and quotas for additive uncertainty, for the nine pairs of values of (ϵ, ϕ), are given in Table 2.

However, allowing θ and θ^* to have the same distribution *does not provide a fair comparison.* Under a tax z, the variance in the additional pollution

[12] The calculations that we report are intended to help in assessing the relative merits of taxes and quotas. The fact that our stationary model does not take into account changes in abatement technology, population and income, means that we understate the true optimal level of abatement (overstate the optimal levels of x and z). The fact that we ignore adjustment costs tends to cause upward biases in transitional abatement (understate the optimal levels of x and z), but has no effect on steady state optimal values.

in a period is $(zh)^2 var(\theta)$ when uncertainty is multiplicative, and the variance is $h^2 var(\theta^*)$ when uncertainty is additive. Therefore, if we want the magnitude of uncertainty to be of roughly the same order in the two cases, it makes more sense to set $var(\theta^*) = z^2 var(\theta)$. The equilibrium z is a (decreasing) function of S, but since $z < 6$ for any $S \geq 800$, we obtain an upper bound for the difference in the value functions under additive uncertainty by multiplying the numbers in Table 2 by 6^2. For example at $\epsilon = .2, \phi = 1$, this upper bound is .166 08, or 166 million 1990 dollars. Table 1 shows that for all sets of parameter values, taxes dominate quotas. However, the difference is extremely small. We noted that under the conservative parameter values with multiplicative disturbances, taxes dominate quotas by 450 billion dollars. These results suggest that even if the assumptions of additive and multiplicative uncertainty lead to the same qualitative result (a preference for taxes) there is likely to be a large quantitative difference. Under the conservative parameter values, the ratio of these differences is $\frac{450}{.166} = 2711.$.

4 Conclusion

There has been great interest in the effect of economic activity on stocks of greenhouse gasses, and in the relation between these stocks and global warming. There is a growing consensus that limiting the stock of greenhouse gasses is important to human welfare, but there has been little research on the best means of achieving such a limit. A large body of literature compares taxes and quotas in the presence of asymmetric information between regulators and firms, but assumes that damages are related to emissions rather than stocks. This literature is therefore not directly applicable to the problem of controlling greenhouse gasses. Previous research that examined the relative merits of the two policies for stock pollutants assumes that the random variable affects the intercept but not the slope of abatement costs. We extend this literature by allowing the random variable to enter multiplicatively.

We provide closed form expressions for the difference in value functions under the two policies. As with additive uncertainty, taxes tend to dominate quotas when: (i) the slope of the abatement cost is large relative to the slope of the damage function, (ii) stock effects are unimportant because of a high discount rate or a high decay rate, or (iii) new information arrives at short intervals (h is small). However, with multiplicative disturbances, the magnitude of uncertainty and the size of the stock also affect the comparison. The Principle of Certainty Equivalence does not apply, so the expectation of the optimal level of emissions under the two policies differs.

We use previous estimates of the magnitude of abatement cost and stock damage to calibrate a quadratic model. The results support the conclusion that taxes dominate quotas. Quotas may dominate if damages are extremely high (much higher than any estimates suggest), but even then only when the stock is substantially above the highest estimates of optimal steady state levels. With additive uncertainty, taxes dominate quotas but the difference in expected payoffs is negligible. For the same range of parameter values, but multiplicative uncertainty, the difference in the present discounted value of the stream of payoffs is significant; it may amount to several percent of 1990 Gross World Product.

References

[1] Barns, D.W., J.A. Edmonds, and J.M. Reilly (1993) "The Edmonds-Reilly-Barns Model" in The Cost Cutting Carbon Emission: Results from Global Models, OECD, Paris.

[2] Bruce, J.P., H. Lee, and E.F. Haites, eds. (1996), Climate Change 1995: Economic and Social Dimensions of Climate Change IPCC, Contribution of Working Group III to the Second Assessment Report of the Intergovernmental Panel on Climate Change, Cambridge University Press.

[3] Cline, W.R. (1992), The Economics of Global Warming, Institute for International Economics, Washington DC.

[4] Dasgupta, P., Hammond, P. and Maskin, E. (1980), "On imperfect information and optimal pollution control", Review of Economic Studies 47, 857-60.

[5] Fankhauser, S. (1995), Valuing Climate Change. The Economics of the Greenhouse, Earthscan, London.

[6] Falk, I., and R. Mendelsohn (1993), "The Economics of Controlling Stock Pollutants: An Efficient Strategy for Greenhouse Gases," Journal of Environmental and Economic Management 25, 76-88.

[7] Hoel, M. and L Karp (1998) "Taxes Versus Quotas for a Stock Pollutant" Working Paper 29.98 Fondazione Eni Enrico Mattei.

[8] Kwerel, E. (1977), "To tell the truth: imperfect information and optimal pollution control", Review of economic studies 44, 595-601.

[9] Maddison, D. (1995), "A Cost-Benefit Analysis of Slowing Climate Change," Energy Policy 23, 337-346.

[10] Malcomson, J. (1978), "Prices vs. Quantities: A Critical Note on the Use of Approximations" Review of Economic Studies , 203-207.

[11] Manne, A.S. (1993) "Global 2100: Alternative Scenarios for Reducing Carbon Emissions" in The Cost Cutting Carbon Emission: Results from Global Models, OECD, Paris.

[12] Newell, R.G. and W.A Pizer (1998) “Stock Externality Regulation Under Uncertainty” Resources for the Future Working Paper.

[13] Nordhaus, W.D. (1991), “To Slow or Not to Slow: The Economics of the Greenhouse Effect,” The Economic Journal 101, 920-937.

[14] Nordhaus, W.D. (1993), “Rolling the ’DICE’: An optimal transition path for controlling greenhouse gases,” Resource and Energy Economics 15, 27-50.

[15] Norhaus, W.D. (1994) “Expert Opinion on Climate Change” American Scientist 82, 45 - 51.

[16] OECD (1992), Global Warming: The Benefits of Emission Abatement, OECD, Paris.

[17] Reilly, J.M. (1992), “Climate-Change Damage and the Trace-Gas-Index Issue,” in Reilly, J.M. and M. Anderson, eds. Economic Issues in Global Climate Change: Agriculture, Forestry, and Natural Resources.

[18] Roberts, M. and Spence, A.M. (1976), “Effluent charges and licenses under uncertainty”, Journal of Public Economics 5, 193-208.

[19] Staring, K. (1995), “Regulering av forurensing under usikkerhet - en dynamisk modell” (Regulation of pollution under uncertainty - a dynamic model). Master degree thesis, Department of Economics, University of Oslo.

[20] Stavins, R.N. (1996), “Correlated uncertainty and policy instrument choice”, Journal of Environmental Economics and Management 30, 218-32.

[21] Tol, R.S.J. “The Damage Costs of Climate Change: Towards More Comprehensive Calculations” Environmental and Resource Economics 5, 353-374.

[22] Weitzman, M. (1974), “Prices versus quantities”, Review of Economic Studies 41, 477-91.

[23] Weitzman, M (1978) “Reply to ‘Prices vs. Quantities: a Critical Note on the Use of Approximations”’ Review of Economic Studies pp 209 - 211.

[24] Watson, W.D. and Ridker, R.G. (1984), "Losses from effluent taxes and quotas under uncertainty", Journal of Environmental Economics and Management 11, 310-26.

[25] Yohe, G.W. (1977), "Comparisons of price and quantity controls: A survey", Journal of Comparative Economics 1, 213-33.