An Economic Model of Oil reserves Additions: Application to U.S. Data

by

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Abstract

Departing from Hotelling's assumption of fixed and known reserves, in this paper I develop an economic model of additions to proven reserves that explicitly incorporates the effects of expected resource price, cumulative reserves development, and technological progress on reserves additions. The model treats additions to proven oil reserves as output of a production process in which drilling wells is a primary input to transform some of the oil-in-place into the economic category of proven reserves. Application of the model to U.S. data for the1950-1995 period provides strong statistical support for the existence of all the three salient effects. I obtain an estimate of the price elasticity of reserves additions (absent from previous studies) which, although statistically highly significant, is small in magnitude. Using this price elasticity estimate, I show that if in the face of steady economic growth, and hence oil consumption, U.S. dependence on foreign oil is to be kept from rising in the future, *ceteris paribus*, a steady oil price increase in the range of 1.5 to 4.5 percent a year is essential.

JEL Classification: D2, H3, Q31, Q41

Key Words: reserves additions, price effect, depletion, technological progress, oil dependence

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Summary

Departing from Hotelling-type models that assume a fixed and known economic reserves size for a natural resource and focus on optimal reserves extraction, this paper has adopted the economic (as opposed to geological) concept of reserves to model the process of additions to proven reserves. It has treated reserves additions as analogous to a production process in which drilling wells act as primary input to transform some of the oil-in-place into the economic category of proven reserves. The economic model developed here explicitly incorporates the salient economic, geological, and technological effects that influence the process of additions to reserves; namely the effect of expected future price on drilling activity, the effect of cumulative reserves development on average reserves development costs, and the effect on these costs of technological progress over time. As such, the present model is also very different from the reserves estimation techniques that rely mostly on reservoirs' production history and geological/engineering principles, with little or no systematic or explicit account of economic factors.

The application of the model to U.S. data resulted in the parameter estimates that all had the "correct" signs, were statistically highly significant, and were of plausible magnitudes, thereby providing support for the effects of oil price, reserves depletion, and technological progress which underlie the model. The estimation results made it clear that increases in oil price are essential for the growth of proven reserves. In particular, they provided an estimate of the price elasticity of reserves addition, which has been absent from previous studies. Although statistically highly significant, the estimated price elasticity turns out to be quite small in magnitude both in the short run (0.11) and the log run (0.16).

The estimated price elasticity enabled us to provide an answer to the important energy policy question: What is the annual constant rate of oil price increase that is needed to keep the U.S. dependence on foreign oil from rising in the future? We noted that, depending on assumptions made about the growth rate of the economy and about the price and income elasticities of oil demand, and assuming other things remain equal, the required annual oil price increase could range from nearly 1.6 percent in the low-case scenario to about 4.5 percent in the high-case scenario.

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1. Introduction

Once again the world oil market is experiencing a crash of prices. Recalling that the last crash in 1986 was followed by a quick and dramatic decline of oil and gas drilling in the United States (within a space of nearly one year the total number of wells drilled fell by nearly one half, from 69,400 in 1985 to 34,300 in 1987) and that the level of drilling activity has been historically essential to growth of U.S. oil reserves, the current crash of oil prices naturally prompts concerns about future domestic oil and gas supplies and dependence on imported energy, should oil prices fail to make a vigorous and timely recovery.

Whether these concerns are justified or not is an important question that cannot be answered without a good understanding of the relationship between oil price changes and additions to proven reserves. And yet the existing literature does not adequately address this relationship. For example, despite considerable importance that the U.S. Department of Energy attaches to information about future oil reserves (as evidenced by its extensive reserves data collection and management efforts), it uses oil reserves estimation techniques that rely mainly on reservoirs' production history and geological and engineering principles, and fail to systematically or explicitly incorporate economic factors (see, for example, U.S. DOE/EIA (1996) and API (1992)).

Further, although there is a large body of theoretical and empirical literature that explicitly incorporates economic factors (see, for example, Epple (1985), Farrow (1985), Miller and Upton (1985), Farzin (1986), Halverson and Smith (1991), Young (1992) and Black and LaFrance (1998), among many others), their conceptual foundation rests on the seminal work of Hotelling (1931). These Hotelling-type models, while providing many valuable theoretical insights, have two main shortcomings, particularly for empirical work. First, they assume that the size of reserves are fixed and known, thus discarding the effect of oil price changes on proven reserves. Second, they are primarily concerned with the determination of oil production path based on intertemporal arbitrage of profits from *extraction* activity, thus abstracting from economic decisions regarding reserves development and discovery which are crucial to growth of oil reserves.

The notable exceptions are the insightful works of Pesaran (1990), Uhler (1979), and MacAvoy and Pindyck (1973). Pesaran estimates an integrated intertemporal optimization model of both oil exploration and extraction in the U.K. Continental Shelf, but he does not deal with additions to reserves from exertions and revisions. Uhler considers extraction and exploration decisions sequentially where first the optimal level of exploratory drilling is decided and then the optimal rate of extraction. However, like Persaran, he is not concerned with additions to proven reserves resulting from development and extensions of exiting fields. The same is also true about the theoretical model of Pindyck (1978) which simultaneously determines the optimal exploration activity and extraction rates of a non-renewable resource. On the other hand, MacAvoy and Pindyck, although primarily concerned with modeling the process of generating new discoveries, they also address the additions to reserves from extensions and revisions of existing fields, but rather unsatisfactorily. They model oil reserves extensions, in an ad hoc fashion, as a linear function of the lagged oil discoveries and the lagged number of exploratory wells drilled. Not only, they do not obtain a statistically strong fit, the coefficient for the former variable turns out to be statistically insignificant. More importantly, when they add the oil price and oil reserves depletion variables to their estimating equation, the estimated coefficient for the price variable appears with the wrong sign and that for the depletion variable turns out to be statistically highly insignificant.

In contrast to both geological/engineering models and Hotelling-type models, the present study follows these previous works (specially Pesaran (1990)) and those of Fisher (1979), Devarajan and Fisher (1982), Adelman (1990), and Farzin (1992) in adopting the economic (as opposed to geological) concept of reserves whereby the economic size of reserves of a depletable resource, instead of being assumed fixed and known, depends on expectations of future prices, reserves discovery and development costs, and the state of technology. It models additions to proven reserves as a production process in which drilling wells act as a primary input to convert some of the stock of oil-in-place into economic category of proven reserves. As such, the model attempts to incorporate the three salient economic, geological, and technological effects that influence the process of additions to proven reserves. These are, respectively, the effect of oil price on drilling activity level, the effect of reserves depletion on discovery and development costs, and the effect of technological progress on those costs.

Unlike both MacAvoy and Pindyck (1973) and Pesaran (1990) , this study does not attempt to model the process of *new field discoveries*, which entails an explicit modeling of exploratory drilling decisions under uncertainty to determine the number of wells drilled, success ratio, and discovery size per successful wells. Rather, to fill the gap left by previous works, it focuses on the task of explicitly modeling reserves additions due to extensions of existing fields and discovery of new pools in those fields; that is, the intermediate stage of developing discovered reserves into proven reserves. The model considers a typical price taking producer who forms his expectations of future prices and determines the level of his development drilling, and hence additions to proven reserves, so as to maximize the his expected profits from drilling activity. It then derives an explicit estimating equation for additions to proven reserves and estimates it for U.S. data over the period 1951-1995. In particular the model establishes a statistically significant price effect on additions to proven reserves. It yields an estimate for the price elasticity of reserves additions which is crucial to obtaining an estimate of the steady-state rate of oil price increase that is needed to prevent the U.S. oil-import dependence from rising.

The plan of the paper is as follows. Section 2 briefly discusses the main economic factors that in interaction with engineering and geological considerations influence drilling decisions and thereby the growth of reserves. Based on these considerations, Section 3 develops an economic model of additions to proven reserves. Section 4.1 discusses the method of estimation, Section 4.2 describes the data, and section 4.3 presents the estimation results. Section 5 discusses some of the economic implications of estimation results for U.S. dependence on imported oil. Summary and conclusions are given in Section 6.

2. Factors Influencing Reserves Additions

Additions to reserves are an outcome of combined geological, technological and, above all, economic variables which are hard to model. In this section I shall briefly review these while emphasizing particularly the economic factors that influence the process of reserves appreciation.

The growth of oil reserves derives basically from two broad sources. The first source is the discovery of new fields. The second source is the additions to reserves in known fields. The model developed here is not concerned with reserves additions due to new field discoveries, which are primarily the result of exploratory drilling. Rather, it concentrates on additions to reserves resulting from extensions and developments of reserves in known fields.

2.1 Reserves Extensions

Additions to oil reserves can occur as a result of extensions of existing fields and pools. Extensions are recoverable reserves that result from changes in the productive limits of known reservoirs. After the discovery of a reservoir, additional wells are normally drilled to outline the productive limits of the reservoir. In the process, more reserves may be found than initially indicated at the time of discovery. Reserves additions from this source are likely to decline rapidly as cumulative additions due to extensions increase, since a significant portion of extensions usually occurs within the first few years after the reservoir discovery.

Extensions can result from either exploratory or development well drilling. The degree of risks and returns associated with each of these modes of drilling differ substantially. Exploratory drilling usually involves few wells that are drilled beyond the geographical limits of recent discoveries in order to find new reserves or open up neglected, deeper strata in old reservoirs. With exploratory drilling, the probability of discovery is relatively small but the size of discovery can be relatively 1arge since it would be first drilling efforts in the region. On the other hand, development (or "infill") drilling involves many wells that are usually drilled in years subsequent to discovery of a reservoir in order to either reach previously unreached portions of the reservoir or to access spaces wherein the natural force of the reservoir is insufficient to mobilize the oil-in-place. In this mode of drilling, the probability of adding to existing reserves is relatively large but the expected size of additional reserves is likely to be small. Furthermore, as cumulative development drilling and hence cumulative addition to reserves increases, reserves additions resulting from new development wells is likely to decline simply because there will be less recoverable resource-in-place.

In deciding how to allocate their drilling activities between exploratory and development drilling, producers make a trade-off between expected return and expected risk, depending on their attitudes towards risk. Everything else being equal, the more risk-averse are the producers the greater would be their preference for development drilling, and vice versa. Given the producers' attitude toward risk, both the level and mode of drilling will depend on a number of economic factors among which the most important ones are likely to be the expected oil prices and drilling costs. Expectation of a lower future price is likely not only to reduce the total number of wells to be drilled, but also to give producers an incentive to shift from riskier exploratory drilling to relatively less-risky development drilling in an attempt to counterbalance expectations of low returns with relatively safer returns. In the United States, historically, most of additions to oil reserves have resulted from development drilling aiming to raise the recovery rate of oil-in-place in known fields[1].

As regards drilling costs, several important features should be noted. First, both total and marginal costs of drilling wells are likely to vary from one production district to another depending on geological characteristics of each producing district. Second, in a producing district, cost per well (average cost) may rise with the total number of wells drilled in that district within a given period of time. Third, development drilling costs have the additional feature of rising with the cumulative amount of reserves withdrawn, reflecting a shrinking size of the remaining oil-in-place as a base for reserves additions. Finally, improvements in drilling technology over time can exert a favorable effect in reducing drilling costs.

2.2 *Reserves Revisions*

Following the discovery of a reservoir, more accurate estimates of reserves become available in subsequent years as development drilling and production history provide new information about geological and engineering characteristics of the reservoir. Changes in oil and gas proven reserves (other than extensions) resulting from new information are referred to as revisions. They include corrections in reserves estimates due to a discrepancy between estimated and actual production in the previous year. Historically, this category of reserves additions has shown highly erratic movements, rendering its economic explanation a mute issue. Economic explanation for observed sizes of revisions is limited to the considerations that (a) the size of revisions may be associated with past levels of development drilling and production from a reservoir, although it is difficult to be certain about the direction of the association, if one exists, and (b) the errors in estimates of the prior year's reserves are likely to be positively associated with the size of prior year's reserves, and negatively with the estimates of remaining reserves in place.

The geological, engineering, and economic considerations discussed above highlight some of the key factors determining the growth of proven oil reserves. These factors will form the basis of the model presented in the next section.

3. The Model

Augmentation of oil reserves can be thought of as a production process in which the drilling of wells is the primary input and the additions to reserves is the output. To simplify the model, I abstract from risks associated with development drilling and assume that the typical producer knows the reserves yield per well drilled. This yield depends both on the current state of drilling technology and equipment and geological characteristics and production history of the reservoir, all of which in turn influence drilling cost per well. Thus, associated with any volume of reserves additions planned by the producer, there will be a certain number of new wells to be drilled and a total cost of drilling to be incurred.

Let $C(\Delta R^*_t)$ denote the total cost of adding ΔR^*_t barrels of crude oil to the existing proven reserves of an oil reservoir during period *t*. In accordance with the drilling costs characteristics discussed in the previous section, one can postulate that

$$C(\Delta R_t^*) = A(\Delta R_t^*)^{a} \left[\sum_0^{t-1} \Delta R_{t}\right]^{b} e^{-g t} \tag{1}$$

where ΔR_t^* and $\sum_0^{t-1} \Delta R_t$ are reserves addition and cumulative reserves additions in period *t,* and A, α, β, and γ are constant parameters. This specification reflects the geological and engineering experience about the behavior of the average cost of reserves additions. Namely, (I) it rises with the rate of current additions; *(ii)* it rises also with the cumulative past additions because as cumulative drilling and hence cumulative reserves additions increases, there will be less recoverable oil-in- place, thus causing the average yield of new drilling to decline. This depletion effect pushes up the average cost of developing the remaining reserves. (*iii*) Over time, technological improvements in drilling methods and equipment and in recovery techniques reduce drilling costs. More specifically, denoting by $MC((\Delta R^*_t)$ the marginal cost of adding ΔR^*_t to proven reserves, from (1) one has

$$MC(\Delta R_t^*) = A\alpha(\Delta R_t^*)^{\alpha-1} \left[\sum_0^{t-1} \Delta R_\tau\right]^{\beta} e^{-\gamma t}$$

(2)

It is plausible to expect a priori that A>0, α>1 because of diminishing returns to drilling activity , β>0 due to reserves depletion effect, and γ>0 due to technological progress effect.

It is assumed that the typical U.S. oil producer is a price taker in the oil market. Further, it is assumed that the producer acts myopically in that in each period he determines the level of his drilling activity, and hence the volume of additions to proven reserves, so as to maximize his expected profits from drilling activity in that period. The assumption of myopic producers, besides being analytically enormously convenient, is not too implausible either, at least as far as U.S. oil producers are concerned; for available empirical evidence suggests that they plan their activities based on a very short pay-back period[2]. Interestingly, for the U.K. oil producers, Pesaran's (1990) econometric results also contradicts the intertemporal model to the effect that he abandons the intertemporal framework and estimates his extraction model under the assumption of zero discount factor, which amounts to the assumption of period-by period profit maximization. Accordingly, in any period *t*, the producer determines the optimum volume of reserves additions by equating the marginal cost of adding a barrel of oil to the existing proven reserves with the expected price for that barrel in that period, i.e.

$$P_t^e = MC(\Delta R_t^*) \tag{3}$$

Given the characteristics of the marginal cost of additions, it is evident from (3) that producers' expectations of sufficiently rising price is a necessary condition for sustaining a given rate of reserves growth.

Substituting for $MC(\Delta R^*_t)$ from (2) into (3) and solving for ΔR^*_t yields

$$\Delta R_t^* = (A\alpha)^{\frac{1}{1-\alpha}} (P_t^e)^{\frac{1}{1-\alpha}} \left[\sum_0^{t-1} \Delta R_t\right]^{\frac{-\beta}{\alpha-1}} e^{\frac{\gamma}{\alpha-1}t} \tag{4}$$

which, given that $\alpha>1$, indicates explicitly that the optimal level of additions to reserves increases with the expected oil price and technological improvements in recovery methods and equipment, but declines as cumulative reserves additions rise over time.

Taking the natural logarithm, equation (4) can be more conveniently written as

$$\ln \Delta R_t^* = \frac{1}{1-\alpha}\ln(A\alpha) + \frac{1}{1-\alpha}\ln P_t^e - \frac{\beta}{1-\alpha}\sum_0^{t-1}\Delta R_t + \frac{\gamma}{\alpha-1}t + u_t \qquad (5)$$

where u_t is the random error term.

Before equation (5) can be used to estimate reserves additions, it needs to be modified both for the time lag involved in adjusting the actual level of reserves additions to the optimal level, and for the producer's expectations formation of future oil price, P^e_t .

In practice, because of imperfect information and technical, managerial, or regulatory constraints, reserves additions adjust to the optimal level with delay. Furthermore, uncertainty in future oil prices combined with irreversiblility of drilling capital expenditures induces producers to delay the adjustment of their drilling activity levels to expectations of future prices (see Dixit and Pindyck (1994)). Since it is difficult to be certain about the exact nature of the adjustment process in oil drilling activities, and for analytical convenience, I assume a partial adjustment process. Formally,

$$\Delta R_t / \Delta R_{t-1} = (\Delta R_t^* / \Delta R_{t-1})^{\lambda}, \quad 0 \le \lambda \le 1 \qquad (6)$$

where λ is the speed of adjustment. Or, in logarithmic form

$$\ln \Delta R_t - \ln \Delta R_{t-1} = \lambda\,(\ln \Delta R_t^* - \ln \Delta R_{t-1}) \qquad (7)$$

Equation (5) gives the optimal rate of additions to reserves as a function of the expected oil price. To derive an estimating equation, it is assumed that the producer forms his expectations of future prices according to the adaptive expectations formation hypothesis.[3] That is, for any period *t*,

$$P_t^e / P_{t-1}^e = (P_{t-1} / P_{t-1}^e)^{\theta}, \quad 0 \le \theta \le 1 \qquad (8)$$

where θ is the speed of adaptation. In logarithmic form

$$\ln P_t^e - \ln P_{t-1}^e = \theta\,(\ln P_{t-1} - \ln P_{t-1}^e) \qquad (9)$$

We can now use the preceding steps to derive an equation appropriate for estimating the effect of oil price change on reserves additions.

From (9) we have

$$\ln P_t^e - (1-\theta)\ln P_{t-1}^e = \theta \ln P_{t-1} \tag{10}$$

Also, lagging (7) by one period, we obtain after some calculation

$$\ln \Delta R_t^* - (1-\theta)\ln \Delta R_{t-1}^* = \frac{1}{\lambda}\ln \Delta R_t + \frac{\lambda+\theta-2}{\lambda}\ln \Delta R_{t-1} + \frac{(1-\theta)(1-\lambda)}{\lambda}\ln \Delta R_{t-2} \tag{11}$$

Finally, lagging (5) by one period, substituting the resulting expression and (10) into (11) yields

$$\begin{aligned} \ln \Delta R_t = {} & a_0 + a_1 \ln P_{t-1} + a_2 \ln CUM_{t-1} + a_3 \ln CUM_{t-2} \\ & + a_4 \ln \Delta R_{t-1} + a_5 \ln \Delta R_{t-2} + a_6\, t + v_t \end{aligned} \tag{12}$$

where

$$CUM_{t-1} \equiv \Sigma_0^{t-1} \Delta R_t \text{ and } CUM_{t-2} \equiv \Sigma_0^{t-2} \Delta R_t \tag{12.a}$$

are cumulative reserves additions,

$$v_t = \lambda u_t - \lambda(1-\theta) u_{t-1} \tag{12.b}$$

is the reduced-form error term, and

$$\begin{aligned} a_0 &= [\lambda\theta \ln(A\alpha) + \lambda\gamma(1-\theta)]/(\alpha-1) \quad ; \quad & a_4 &= (2-\lambda-\theta) > 0 \\ a_1 &= \lambda\theta/(\alpha-1) > 0 \quad ; \quad & a_5 &= -(1-\theta)(1-\lambda) < 0 \\ a_2 &= -\lambda\beta/(\alpha-1) < 0 \quad ; \quad & a_6 &= \lambda\theta\gamma/(\alpha-1) > 0 \\ a_3 &= \lambda\beta(1-\theta)/(\alpha-1) > 0 & & \end{aligned} \tag{12.c}$$

Equation (12) gives, in logarithmic form, the actual volume of reserves additions as a function of lagged oil price, one-and two-year lagged cumulative additions, one- and two-year lagged additions, and a time trend serving as a proxy for technological progress.

4.1 <u>Estimation Method</u>

The reduced-form equation (12) is linear in seven parameters but nonlinear in the six underlying structural parameters A, α, β, γ, θ, and λ. Although the linear model is over determined, there is a nonlinear restriction in the structural model that permits exact identification of the structural parameters. This restriction is

$$a_2\, a_3\, a_4 + a_3^2 \;-\; a_5\, a_2^2 \;=\; 0 \qquad (13)$$

Solving for a_4 and rearranging, the estimating equation (12) becomes

$$\ln \Delta R_t \;=\; a_0 \;+\; a_1 \ln P_{t\text{-}1} \;+\; a_2 \;\ln\; CUM_{t\text{-}1} \;+\; a_3 \;\ln\; CUM_{t\text{-}2} \;+$$
$$(-a_3^2 \;+\; a_5\, a_2^2) / (\, a_2\, a_3)\; \ln\; \Delta R_{t\text{-}1} \;\;+\; a_5 \;\ln\; \Delta R_{t\text{-}2} \;\;+\; a_6\, t \;+\; v_t \qquad (14)$$

If the error term u_t is assumed to be white noise (i.e., $Eu_t = 0$, $Eu_tu_s = \sigma^2$ for t=s , and $Eu_tu_s =$ 0 for $t\neq s$), then, from (12.b), the composite error term v_t will exhibit autocorrelation with a first-order moving average MA(1) process; $Ev_t = 0$, $Ev_tv_t = [1+(1-\theta)^2]\lambda^2\sigma^2$, $Ev_tv_{t-1} = -\lambda^2(1-\theta)\sigma^2$, and Ev_tv_{t-I} = 0 for I >1. As a result, explanatory variables are correlated with the composite error term and the nonlinear least squares estimates of equation (14) are inconsistent.

One potential remedy is to estimate the equation using a generalized instrumental variables (GIV) procedure (nonlinear two-stage least squares in this case). GIV has two drawbacks in this context. First, there is a loss in efficiency if a consistent OLS estimator can be found. Second, choice of instruments is problematic. The only obvious instruments are additional lags of independent variables. With the limited size of the data set, use of GIV would substantially reduce the number of observations available for estimation and lead to further efficiency losses.

An alternative approach to estimation is the conditional maximum likelihood for an MA(1) process (see Hamilton (1994)). Here we need to assume that $u_t \sim (N(0, \sigma^2 I)$ and $u_0 = 0$. Defining $\varepsilon_t = \lambda\, u_t$, we have $E\varepsilon_t = 0$, $E\varepsilon_t\varepsilon_s = \lambda^2\sigma^2 = \sigma_\varepsilon^2$ for t=s, $E\varepsilon_t\varepsilon_s = 0$ for $t\neq s$, i.e., $\varepsilon_t \sim N(0, \sigma_\varepsilon^2 I)$. Using this to substitute for v_t and noting that $(1-\theta) = -\,(a_3 / a_2)$, the estimating equation (14) takes the final form of

$$\ln\Delta R_t \;=\; a_0 + a_1 \ln P_{t-1} + a_2 \ln CUM_{t-1} + a_3 \ln CUM_{t-2} + (-a_3^2 + a_5 a_2^2) / (a_2 a_3)\ \ln\Delta R_{t-1}$$
$$+\, a_5 \ln\Delta R_{t-2} \;+ a_6\, t + (a_3 / a_2)\, e_{t-1} + e_t \quad ; \quad t = 1951,\ 1952,\ \ldots,\ 1995 \qquad (15)$$

Note that t=0 corresponds to 1950, the first year in which data on all variables are available. So, $\varepsilon_{1950} = 0$, implying that $v_{1951} = \varepsilon_{1951}$, assuring that all errors in the transformed equation (15) are white noise. We estimated the six parameters (a_0 , a_1, a_2, a_3, a_5, a_6) by maximizing the log-likelihood function corresponding to equation (15), which is equivalent to minimizing the error sum of squares of the transformed nonlinear model. At convergence, the nonlinear least squares estimator of the transformed model is equivalent to the conditional maximum likelihood estimator, and thus achieves both consistency and efficiency.

4.2 Data

There are essentially two data series needed to construct the data for the estimation of equation (15) at the U.S. aggregate level: additions to reserves and prices. Reserves data for the period 1946-1976 were collected from the American Petroleum Institute (1987) and for the period 1977-1995 from U.S. DOE's Energy Information Agency (1996b)(1998a). Both sources allocate total reserves additions among revisions, extensions of existing fields, new discoveries in existing fields, and discoveries of new fields. Means, standard deviations, and percent of total proven reserves for these series are reported in Table 1. Extensions, new discoveries in existing fields, and new field discoveries all constitute additions to reserves, while revisions represent changes due to improved information of well operators about geological characteristics and the economic climate. EIA also includes a category called adjustments, which reflects differences in reserves data due to surveying a different sample of operators in each year and better information about trends for small operators (as opposed to increased information about geophysical characteristics). Since EIA's knowledge of oil operators and accuracy of reserves estimates is generally expected to improve over time, adjustments may be viewed to result from additional information. For the purpose of estimating equation (15), additions to reserves are defined as the sum of extensions, new discoveries in existing fields, and adjustments, the latter category being available only after 1976. Since the present model is primarily concerned with additions to reserves resulting from extensions and development of existing fields, the definition of reserves additions does not include new field discoveries, which are the outcome of risky exploration activities. Revisions are also excluded because they show highly erratic movements over time and because there is little economic explanation for how information about the geology of an oil field grows[4]. In contrast, we include adjustments because they reflect improved measurement of aggregate reserves additions and economic information.

For oil price, we use U.S. average domestic first purchase price[5] deflated by U.S. GDP deflator[6]. Although reserves data are available from 1946 and after, prices could only be obtained for 1949 through the present. Finally, cumulative additions to reserves are the sum of additions beginning with the first available data point. Thus, $CUM_{1946} = \Delta R_{1946}$, $CUM_{1947} = \Delta R_{1946} + \Delta R_{1947}$, etc. Means and standard deviations for ΔR_t, CUM_t, and real oil price are reported in Table 2.

4.3 Estimation Results

Nonlinear estimation was done using initial values within the theoretically admissible ranges. Convergence to the same results was achieved for two very different sets of starting values using nonlinear estimation procedures in SAS. The estimates of the parameters (a_0 , a_1, a_2, a_3, a_5, a_6), their asymptotic standards errors, and key statistics are given in Table A. The multiple correlation coefficient, R^2 , is 0.7935, the mean of the dependent variable is 6.89, and the estimated standard error of regression, *SSE*, is 1.81, all indicating a good statistical fit. All coefficients are statistically highly significant except for a_5 which is not significant. A likelihood-ratio test was also performed for the whole model, testing the joint hypothesis that all slope coefficients are zero. The calculated value of $LLR=-2 \ln (L_R/L_U)= 313.02$ far exceeded the 0.05 critical value for a χ^2 with 5 degrees of freedom. Table A also presents the estimates of the structural parameters (A, α, β, γ, θ, λ), which are calculated by using the estimates of (a_0 , a_1, a_2, a_3, a_5, a_6) in (12.c).

All of the estimates have the *a priori* expected signs and fall within the ranges of values predicted by the theory ($\alpha > 1$, $\theta < 1$, and $\lambda < 1$), thereby confirming our maintained hypotheses that the marginal cost of reserves additions increases both with the current and past cumulative rates of reserves additions, but, that, other things equal, declines over time as technology progresses.

Noting that the adoptive expectations hypothesis (9) can be written equivalently in the form of a Koyck lag-distribution model, i.e., $\ln P_t^e = \theta \sum_{i=1}^{\infty} (1-\theta)^{i-1} \ln P_{t-i}$, the value of 0.27 obtained for the adaptive parameter, θ, suggests that in forming their expectations of future oil price, producers do not simply rely on current price but attach rather significant weights to very recent prices too. This may partly reflect oil producers' perception of oil price volatility (stemming, for example, from political uncertainties in the oil exporting nations and uncertainties in domestic regulatory policies) which cautions them against extrapolating current prices. On the other hand, the rather large estimated value of 0.92 obtained for the estimate of the adjustment parameter, λ, suggests rapid adjustment of actual to optimal reserves additions. This may in turn reflect the relatively much less risk and much smaller capital expenditures involves in developing the existing reservoirs or finding new pools in existing fields than would be the case for new fields discoveries.

Of particular interest is the impact of actual oil price on additions to reserves. The coefficient of the price variable not only has the correct positive sign, it is also statistically well determined (the associated *t*-ratio is 2.28). In fact, from equation (12) the short-run and the long-run price elasticities are

$$\eta_{SR} = a_1$$
$$\eta_{LR} = a_1 / (1 - a_2 - a_3 - a_4 - a_5) \qquad (16)$$

where η_{LR} is calculated by holding ΔR constant at its steady state value and using (12.a).

Inserting the estimates of the coefficients in (16), and recalling from (13) that the implied value of a_4 is $a_4=(-a_3^2 + a_5 a_2^2)/(a_2 a_3)$ =0.79388, we obtain η_{SR} = 0.11 and η_{LR}= 0.1627 . Accordingly, in magnitude, the impact of oil price on reserves additions is quite small and is fully born out in a very short period of time; the five-year price elasticity is calculated to be $\eta_{5\text{-}y}$ =0.1620.

Table A: Estimate of the Reserves Addition Function, 1951-1995

Coefficient	Estimated Value	Standard Errors	Structural Parameter	Value
a_0	4.89023	1.10263	A	1.16904 E-20
a_1	0.10911	0.04795	α	3.36118
a_2	-1.48456	0.12458	β	3.78107
a_3	1.07201	0.05567	γ	0.05269
a_5	-0.05182	0.15050	θ	0.27789
a_6	0.00574	0.00292	λ	0.92822

R^2 =0.7935, SEE=1.810, $(MSE)^{½}$= 0.2134, LLR =313.02> $\chi^2_{0.99}(5)$=15.1
Dep. Mean=6.8912.

Substituting the estimates of the coefficients in equation (15) yields the following reserves additions function

$$\ln \Delta R_t = 4.89023 + 0.10911 \ln P_{t-1} - 1.48456 \ln CUM_{t-1} + 1.07201 \ln CUM_{t-2} + 0.79388 \ln \Delta R_{t-1} - 0.05182 \ln \Delta R_{t-2} + 0.00574\, t - 0.72166\, \mathrm{e}_{t-1} + \mathrm{e}_t$$

$$(t=1, 2,...,T, \epsilon_{0=0}) \qquad (17)$$

Besides the price impact, two other important features of the reserves additions function (17) are worth emphasizing. First, as seen from the coefficient of CUM_{t-1} variable, there is a statistically highly significant and quite large negative effect of depletion on reserves addition. However, the positive (but smaller) coefficient of $CUM_{t-2,}$ indicates that, all else equal, the negative depletion effect *diminishes* as possibilities of additions to reserves are exhausted and hence the rate of reserves additions declines eventually to zero. In fact, recalling that $CUM_{t-1} = CUM_{t-2} + \Delta R_{t-1}$, the absolute value of the elasticity of reserves addition with respect to cumulative additions declines from its short-run value of 1.48 to the long-run steady-state value of zero. Second, despite this, the negative depletion effect is always large enough to outweigh the positive effect of technological progress, and thereby, barring adequate increases in oil price, causing reserves additions to decline.

5. <u>**Economic Implication of the Price Impact**</u>

The econometric results presented in the previous section make clear a simple but important point: oil price increases are essential for the growth of proven reserves. The importance of this point is better appreciated once we note that according to the estimated reserves addition function (17), even if one assumes that the exogenous technological progress will continue steadily over time and also ignores the negative depletion effect, then with oil price remaining *unchanged* at its 1995 level, the cumulative additions to reserves (due purely to technological progress) will be only about 8% over 10 years. On the other hand, ignoring the technological progress but allowing for the negative depletion effect, a 1% steady annual increase in oil price brings about 44% cumulative reserves additions over the same 10-year period. This raises a question the answer to which can provide valuable insight about the role of oil price in shaping the dynamics of U.S. dependence on imported oil. Namely, *What is the annual constant real rate of oil price increase that is needed to keep the share of oil imports in total oil consumption, the reserves-production ratio, and hence the reserves-consumption ratio, constant over time?* It turns out that the price elasticity of reserves additions, 0_{LR}, estimated from equation (17) is indeed a key element in answering this question.

To see this, let R_t, C_t, Q_t, and M_t denote, respectively, the levels of proven oil reserves, consumption, production and imports at time t, then noting that $C_t = Q_t + M_t$, one has

$$\frac{R_t}{C_t} = \frac{R_t}{Q_t} . \frac{Q_t}{C_t} = \frac{R_t}{Q_t} \left(1 - \frac{M_t}{C_t}\right) \tag{18}$$

In 1995, the share of oil imports in total consumption was about 58 percent (M/C=0.58) and the proven reserves-production ratio was about 10 years (R/Q=10), implying that the then available proven reserves could support the 1995 level of consumption for only 4.2 years.[7]

Suppose, as an energy policy objective, we wish to keep the ratios R_t /C_t and R_t /Q_t at a constant level (say, at their 1995 values), so that the share of imports in consumption M_t /C_t remains constant over time, then we need

$$\frac{\dot{R}_t}{R_t} = \frac{\dot{C}_t}{C_t} \tag{19}$$

Next, let us for simplicity assume that the long -run oil consumption function is of the constant elasticity form of $C_t = C(Y_t, P_t) = D\, Y_t^{a}\, P_t^{-b}$, where $D>0$ is a constant, Y_t is the aggregate income (GDP), P_t is oil price, and $a>0$ and $b>0$ are constant income and price elasticities of oil demand, then

$$\frac{\dot{C}_t}{C_t} = a\frac{\dot{Y}_t}{Y_t} - b\frac{\dot{P}_t}{P_t} \tag{20}$$

Assuming a constant long-run GDP growth rate, i.e., $\dot{Y}_t / Y_t = g$, and focusing, as the question on hand requires, on a constant rate of oil price increase, i.e., $\dot{P}_t / P_t = d$, equation (20) reduces to

$$\frac{\dot{C}_t}{C_t} = a g - b d \tag{20'}$$

Now, by definition, the change in proven reserves equals reserves additions less production, i.e. $\dot{R}_t \equiv \Delta R_t - Q_t$, so that, dividing through by R_t,

$$\frac{\dot{R}_t}{R_t} = \frac{\Delta R_t}{R_t} - \frac{Q_t}{R_t} \tag{21}$$

Substituting from (21) and (20') into (19), we have

$$\frac{\dot{R}_t}{R_t} = \frac{\dot{C}_t}{C_t} = ag - b\delta = \frac{\Delta R_t}{R_t} - \frac{Q_t}{R_t} \tag{22}$$

Since Q_t/R_t is assumed to remain constant, it follows from (22) that $\Delta R_t / R_t$ must also be constant, which in turn implies that

$$\frac{\Delta \dot{R}_t}{\Delta R_t} = \frac{\dot{R}_t}{R_t} = ag - b\delta \tag{23}$$

Finally, letting η denote the price elasticity of reserves additions, we have $\Delta \dot{R}_t / \Delta R_t = \eta \, \dot{P}_t / P_t = \eta\delta$, which by submitting in (23) and solving for δ, yields

$$\frac{\dot{P}_t}{P_t} = \delta = \frac{ag}{b + \eta} \tag{24}$$

Equation (24) gives the constant real rate of oil price increase that we were seeking in answer to the question posed at the beginning of this section. The required rate of oil price increase varies directly with the GDP growth rate and the income elasticity of oil demand and inversely with the price elasticities of reserves addition and oil demand. One may think of this rate as a premium that needs to be paid in order to prevent U.S. dependence on foreign oil from rising in the future in the face of steady rates of economic growth and hence oil consumption. Using our estimate of the long-run price elasticity of U.S. reserves additions (η=0.162), Table B presents the magnitude of the required rate of oil price increase calculated for different stipulated values of long-run GDP growth rate (g=0.02, g=0.05), oil demand price elasticity (b=0.50, b=.75) and income elasticity (a=0.75, a=1.0). The required price increase will be as low as 1.6 percent a year in the low-case scenario specified by an expected steady economic growth rate of 2 percent a year and an oil demand characterized by a high price elasticity of 0.75 and a low income elasticity of 0.75. In the opposite (high-case) scenario, characterized by a high rate of economic growth (3 percent a year) and a high oil demand (price elasticity of 0.5 and income elasticity of 1.0), the needed rate of oil price increase rises to about 4.5 percent a year.

Even under this latter scenario, the required rate of price increase seems modest, or at least not unduly large; it implies a doubling of the real oil price in 15 years [8]. On the other hand, if oil price is kept *constant* at its 1995 level and, for reserves additions, reliance is made exclusively on the exogenous technological progress rate of 0.57 percent a year, and even if the negative reserves depletion effect is ignored, then with a steady 3 percent growth rate in oil consumption (resulting, for example, from an oil demand income elasticity of 1 and a GDP growth rate of 3 percent per year), the share of oil import in consumption is calculated to rise by nearly 1.5 percent a year, implying that it would rise from its 1995 level of 58 percent to over 72 percent in the same 15 years time[9]. Of course, allowing for the negative depletion effect, the implied degree of foreign oil dependence will most likely be significantly higher than that.

$$\dot{P}_t / P_t = d = \frac{a\,g}{b + h}$$

Table B: (in %), O=0.162

	b=0.5		*b=0.75*	
	g=0.02	*g=0.03*	*g=0.02*	*g=0.03*
a=1.0	*3.62*	***4.53***	*2.19*	*3.30*
a=0.75	*2.26*	*3.40*	***1.64***	*2.47*

6. Conclusion

Departing from Hotelling-type models that assume a fixed and known economic reserves size for a natural resource and focus on optimal reserves extraction, this paper has adopted the economic (as opposed to geological) concept of reserves to model the process of additions to proven reserves. It has treated reserves additions as analogous to a production process in which drilling wells act as primary input to transform some of the oil-in-place into the economic category of proven reserves. The economic model developed here explicitly incorporates the salient economic, geological, and technological effects that influence the process of additions to reserves; namely the effect of expected future price on drilling activity, the effect of cumulative reserves development on average reserves development costs, and the effect on these costs of technological progress over time. As such, the present model is also very different from the reserves estimation techniques that rely mostly on reservoirs' production history and geological/engineering principles, with little or no systematic or explicit account of economic factors.

The application of the model to U.S. data resulted in the parameter estimates that all had the "correct" signs, were statistically highly significant, and were of plausible magnitudes, thereby providing support for the effects of oil price, reserves depletion, and technological progress which underlie the model. The estimation results made it clear that increases in oil price are essential for the growth of proven reserves. In particular, they provided an estimate of the price elasticity of reserves addition, which has been absent from previous studies. Although statistically highly significant, the estimated price elasticity turns out to be quite small in magnitude both in the short-run (0.11) and the log-run (0.16).

The estimated price elasticity enabled us to provide an answer to the important energy policy question: What is the annual constant rate of oil price increase that is needed to keep the U.S. dependence on foreign oil from rising in the future? We noted that, depending on assumptions made about the growth rate of the economy and about the price and income elasticities of oil demand, and assuming other things remain equal, the required annual oil price increase could range from nearly 1.6 percent in the low-case scenario to about 4.5 percent in the high-case scenario.

The need for steady oil price increases (of course, at an appropriate rate) may seem to conflict the popular belief that low oil prices stimulate, or even are necessary for, a steady growth of U.S. economy. This popular belief, however, is based on inaccurate understanding of the relationship between oil price and economic growth. For one thing, it confuses the cost of economic adjustment to a *sudden* and *sharp* price increase with the effect of an anticipated and slow price increase on economic activity. As is now well understood, for the United States, the latter effect is likely to be negligible because of a very small share of energy in GDP (about 3 percent) and significant inputs substitution possibilities in the production of aggregate output (see, for example, the early work of Hogan and Manne (1979) among many others). More importantly, the papular belief stems from a myopic view in that it ignores the costs that depressed oil prices impose on the economy in the form of raising its dependence on foreign (and riskier) oil in the long run. From this perspective, the oil price increases computed here may be interpreted as a price that needs to be paid in order to enhance reserves additions and thereby prevent foreign oil dependence from rising in the face of a steady growth of the economy and hence oil consumption.

The model presented here can be improved considerably both in theoretical and empirical respects. It can be extended to explicitly incorporate the risky activity of new oil field discoveries. It would then be interesting to examine how the element of risk combined with irreversibility of exploration capital expenditures may affect the price elasticity of reserves additions from this source. Intuitively, risk and irreversibility should lower the price elasticity; but this effect must be contrasted with the opposing effect of higher returns from new fields discoveries. It would also be interesting to estimate the present model at a desegregated level for major oil producing states or oil fields to

Table 1

Summary of Reserves Data

Millions of U.S. Barrels

	1949 through 1995			1977 through 1995		
Series	**Mean**	**Standard Deviation**	**Share of Total Proved Reserves**	**Mean**	**Standard Deviation**	**Share of Total Proved Reserves**
Revisions	1,137	547	3.84%	1,174	641	4.24%
Extensions	861	510	2.91%	508	134	1.84%
New Discoveries in Existing Fields	169	71	0.57%	139	60	0.50%
New Fields	376	1,416	1.27%	137	83	0.49%
Adjustments	NA	NA	NA	209	148	0.76%
Total Reserves (Beginning of Year)	29,632	3,673	100.00%	27,668	2,949	100.00%

Note: Revisions and adjustments can be either positive or negative.

provide both further statistical tests of the model and insight into cross-states differences in the price elasticity of reserves additions.

Table 2
Summary Statistics for Regression Variables (1949-1995)

Series	**Units**	**Mean**	**Standard Deviation**
Additions to Reserves (ΔR_t)	Millions of Barrels	1,114.30	511.39
Cumulative Additions (CUM_t)	Millions of Barrels	36,514.00	14,025.00
Real Price (P_t)	1982 Dollars Per Barrel	12.20	5.99

Endnotes:

1. This is in contrast to additions to gas reserves which have come primarily from exploration wells, with development drilling playing only a minor role. For this reason, for oil, the overall effect of expected lower prices on reserves additions may not be as severe as it may be for natural gas. The opposite case is likely to hold with regard to the effect of expected higher prices on reserves additions.

2. Farzin (1986) obtains an estimate of 33.4% for the real, before-tax discount rate used by U.S. oil producers, and an estimate of less than five years for their extraction time-horizon.

3. Strong statistical support for the adaptive price expectations formation hypothesis has been obtained by Pesaran (1990) and Farzin (1986) for, respectively, U.K. and U.S. oil producers.

4. The formal reserves measurement models and techniques involve only geological and well production data; economic considerations enter only as subjective revisions to the model. Thus, it is assumed that the information content of revisions is mostly non-economic.

5. To be strictly consistent with the theoretical model developed here, oil price should measure the price of a barrel of proven reserves in the ground, or its *in situ* price. However, since reserves prices are generally unobservable, the well-head first purchase price is used as proxy. This is reasonable, noting that in principle the competitive well-head price reflects the sum of the reserves price and the unit production cost, and that, for the U.S. as a whole, and in real terms, the average production cost seems to have remained more or less constant (API (1993)), thus rendering the two prices highly correlated.

6. Real prices are indexed to 1982 U.S. dollars, using GDP deflator from *Economic Report of the President*, 1997.

7. Calculations are based on data from U.S.DOE/EIA(1998b).

8. Note that this is partly due to the price elasticity of reserves addition, for in the absence of the price effect on reserves additions (i.e., if η=0) the required rates of oil price increase will be higher; 2 percent in the low-case scenario and 6 percent in the high case.

9. Obviously, it is always possible to draw heavily on existing proven reserves to increase the domestic oil production rate and hence lower the imports share in consumption. However, such a policy, will not be sustainable. Therefore, the calculations made here assume that the net change in reserves remains constant over time, i.e., $\dot{R}_t \equiv \Delta R_t - Q_t = \text{constant}$, or $\Delta \dot{R}_t = \dot{Q}_t$, implying that domestic production rate is adjusted by the extent of the change in reserves additions.

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