Options, quasi options, and the opportunity to develop a resource of environmental value

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Abstract

Faced with the choice between preserving and developing a natural resource in the presence of uncertain environmental externalities, the social return to undertaking development is stochastic. When development is irreversible and the decision-maker can defer commencement, there is value to new information that is revealed over time which reduces the uncertainty about the externalities. The assessment is therefore one of how much to develop and when to begin development.

This approach to the problem differs from conventional cost benefit analysis and optimal extraction models. The former asks the question of whether to invest a given amount and the latter asks how much is optimal to invest. In this paper a model is developed which identifies both the level and timing of investment which are socially optimal.

The solutions are found using optimal stopping techniques a single stochastic state variable, environmental cost and two control variables, timing and scale of development. Initially, the problem is solved for an omnipotent social planner, who makes the time and scale decisions. This is followed by consideration of cases in which one or both decision variables are controlled by private decision-makers. As expected, it is shown that private optimal level of investment exceeds that which is socially optimal. We find that if the social planner can control only investment timing then for levels which exceed the social optimum, it will pay to wait for environmental costs to fall below the level corresponding to the social optimum. Conversely, level of investment which are low relative to the social optimum, may never generate sufficient private returns to offset the increment to environmental cost.

The antecedents to this work are found in the environmental economics literature on quasi-option value and in the finance literature on options and investment.

Key words: Decision under uncertainty, Environmental externalities,

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1 Introduction

The opportunity to develop a resource or invest in a project which uses environmental assets can be valued in much the same way as any other investment opportunity when returns are uncertain and the timing of the investment is flexible. The irreversible losses or damage to natural resources are potentially high but generally not fully known ex-ante. If new information that reduces uncertainty becomes available over time, and the investor can defer the commencement of the project, there may be value to waiting rather than risk an uncertain outcome which cannot be undone. Old-growth forest converted for logging, mining, agriculture, housing or other commercial development cannot be returned to its original state within a reasonable planning horizon. The value of the flexibility to wait and invest under less uncertain circumstances is therefore an integral part of the project's value.

The situation described above appears in the environmental economics literature on quasi-option value. Arrow and Fisher (1974) concluded that most often, resource use involves irreversible losses and that if information about the magnitude of these losses becomes available over time, then the net benefit of development is reduced. They conclude that more conservative development is appropriate under these circumstances. Henry (1974)modeled a similar phenomenon using a contingent states framework.

In the investment literature, the consistent over-prediction of investment levels by theoretical models led McDonald and Siegal (1985, 1986), Pindyck (1988, 1991, 1993), Dixit (1994) and others to examine sources of incentives for agents to delay investment. Investment in this context amounts to incurring some level of sunk cost in exchange for a project with a given return. Either or both can be risky and it is possibility of reducing this risk is the source of incentive for agents to postpone the decision to invest. There is therefore some critical expected value of the ratio of expected returns to costs for which the project will be undertaken. Equivalently, this is a problem of investment timing. The decision-maker must be in a position to delay the commencement of the project until such time as new information provides for the maximization of expected net returns.

The purpose of this paper is to develop a model in which the social returns to an investment are uncertain because of environmental externalities which are not fully known at the time that development is begun. It is solved for both the size and timing of development. In the following section, the we pose the stochastic optimization for the development opportunity. In section three, threshold level of environmental cost which allows for development to begin is derived. It is the boundary between the period during which it is optimal to defer investment and the point at which beginning to invest is optimal. The questions of investment size and timing are considered in section four along with a comparison with the results of the purely private optimization. The final sections of the paper are devoted to discussions of extensions of the model and a comparison of this dynamic context with conventional cost-benefit analyses.

2 Decision-making when environmental costs are uncertain

When an investment project involves the development of an environmental resource, the potential public costs of lost environmental assets are likely to be at least as uncertain as other costs and returns (Fisher and Hanemann, 1987) Our starting point is the extreme case in which the private net return is known with certainty and only the environmental impacts are uncertain. Of interest is the value to waiting to begin development.

Consider an undeveloped tract of land whose conversion to other uses is being analyzed by a local planner. In its undeveloped state, the tract may have known value from amenity, recreation as well as consumption and local systemic ecological benefits. There may also be benefits that are not yet known; for example, aquifer recharge or therapeutic uses of plants yet to be discovered. The development alternative confers commercial benefits and entails costs of construction and operation as well as the loss of known and unknown environmental benefits.

In a comprehensive valuation, this latter set of losses must also be considered as a cost of development. The size of the cost depends on the magnitude of development which permanently removes environmental and ecological assets. The costs are stochastic because the process by which assets are lost is not fully known, and future discoveries, tastes and other factors are revealed over time.

Because most environmental assets have public good characteristics (non-rivalness and non-excludability), the appropriate context is the social planner's problem. Cost-benefit analysis and optimal extraction models as part of planning require the comparison of the net present value of all costs and benefits, both public and private. For the former, the basic decision rule is that no project for which the net present value of costs exceeds the net present value of benefits is a candidate for inclusion in the planner's portfolio. The criteria for choosing among projects favors those for which the benefits exceed costs by the highest margin. In extraction models, the decision rules determine the rate at which exploitation should take place, given expectations about extraction costs, commodity prices and discount rates.

Both approaches value projects using current expectation of variables over the entire life of the enterprise. This corresponds to cases in which the decision-maker lacks the flexibility to alter plans as new information becomes available. It has been shown that, in general, this approach overestimates the project value when new information changes the net return. (Arrow and Fisher, 197; Dixit and Pindyck, 1993; Pindyck, 1991)

When the decision maker has the flexibility to use new information as it is revealed over time, the problem is to choose both the amount and timing of development in order to maximize net social benefits; that is the total of all benefits accruing to private investors and those arising from public good externalities, such as changes to environmental assets. The model below, begins by developing results of decision-making that would be undertaken by a social planner with full control over both the timing and size of development when the environmental costs of development are uncertain and dynamic. All information is assumed to be made available at no cost. As long as the net social benefits are positive, the project can be considered for inclusion in the planner's portfolio. The results contrast with the results of decisions which consider only private returns and those in which uncertainty is not explicitly considered.

All development takes place at one time and must be completed once started. There are no lags between commencement and the realization of private return. Environmental costs do change over time once the development has taken place. This is because of newly discovered uses for environmental assets and threshold effects may become apparent over time.

The model answers the question, when (if ever) is it optimal to begin investing in a project which has environmental consequences and how much (if any) development should take place. In the model, only environmental costs are uncertain. Information on these costs that is revealed over time decreases but does not eliminate uncertainty. Development can be delayed but once begun cannot be interrupted. In order to accommodate this last feature, there is no time-to-build type lag and all investment takes place at one time.

We begin by solving for the critical level of environmental cost for which some level and timing of investment is optimal. Using these results, the actual optimal size and timing of investment are solved.

The value of developing the resource is it's net private return less the public good externality of environmental impact stemming from the development. Impacts are assumed to begin from the time at which development begins and to continue indefinitely. The considered project's value is represented by:

$$F(k) = \max_{\theta, I} E_t \left(W(I)e^{-\theta r} - \int_{\theta}^{\infty} k_s e^{-sr} ds \right) \quad (1)$$

where: W is net private return; I is non-negative level of investment in development; r is social discount rate; k is incremental environmental impact in period s, and θ is the commencement time for development

Environmental impacts (both costs and benefits) are uncertain and the planner's knowledge about them changes continuously. The costs may rise or fall, depending on new previously unknown information about the natural environment. These costs rise with the level of development. That is, from time θ dk_t depends positively on the size of development. For example, the larger the area of land converted to other uses, the smaller the area from which the preservation benefits accrue. Equivalently, the more land that is developed, the greater the increment to environmental cost.

The evolution of the environmental costs is described by the controlled diffusion process:

$$dk = \alpha(k + I1_{t>\theta})dt + \gamma k dw \quad (2)$$

where the evolution of the environmental cost increases due to the development only from the time the development actually begins. The change to environmental impact in period t depends deterministically on extant, known environmental impact k_t and the level of investment as well as stochastically on new information which is subject to the white noise process for which dw is the increment. α and γ are scalar multiples.

Prior to the commencement of investment, environmental impacts depend only on current known impact and stochastic change. During this period, the planner waits for new information which may change the expectation that total impacts are too costly to warrant new development.

To find $F(k)$ we proceed in two steps. First, the environmental cost that allows for investment of a given size is calculated. Then, the socially optimal and private contractor, investment sizes are discussed with reference to their effect on the project timing.

3 Solving for the environmental cost that allows for positive investment

There exists some level of environmental impact at which the planner will undertake development of a given size. That is, his or her expectation is that there is greater value to the development project than to additional information that might be gained by deferring the investment decision.

Assume that the optimal level of investment, I^*, and the optimal timing of investment, θ are known. The development decision, can be solved as an optimal stopping problem, in which, θ is the optimal stopping time and the environmental impact at this time, k_θ, is the single boundary between the continuation and stopping regions.

From the second term of (1), we have the discounted expected value of environmental impacts: $E\int_\theta^\infty k_s e^{-sr} ds$. In the stopping region,$s \geq \theta$, it evolves according to the dk process:

$$\text{s.t. } dk = \alpha(k + I)dt + \gamma k dw \tag{3}$$

By making appropriate substitutions of (4), the expected value of environmental impacts, is solved in terms of the parameters of the constraint, investment size and the boundary:

$$\begin{aligned} E\left(\textstyle\int_\theta^\infty e^{-rs} k_s ds | \theta, I, k_\theta\right) &= \int_\theta^\infty e^{-rs} E\left(k_s | \theta, I, k_\theta\right) \\ &= e^{-r\theta}\left(\frac{\alpha I}{r(r-\alpha)} + \frac{k_\theta}{r-\alpha}\right) = e^{-r\theta} f(k_\theta, I) \end{aligned} \tag{4}$$

To find the boundary, the problem is solved from the continuation region, that is, prior to the commencement of investment. Let the value of the project at the boundary be:

$$V = W(I^*) - f(k_\theta, I^*) \tag{5}$$

where $I*$ is the optimal investment size. The maximization is rewritten as:

$$\max_{\theta} E\left[e^{-r\theta}\left(W(I^*) - f(k_\theta, I^*)\right)\right]$$
$$\text{s.t. } dk_t = \alpha k_t dt + \gamma k_t dw,\ \ t \leq \theta \tag{6}$$

The problem, now has been reduced to identifying the stopping time or the environmental impact at the stopping time, when development begins. Two differential equations govern the solution to the planner's problem. In the continuation region the Bellman principle of optimization yields:

$$rF - 0.5\gamma^2 z^2 F'' - \alpha z F' = 0 \tag{7}$$

Additional conditions hold at the boundary, when, investment of size I^* is made the:

$$F(z) = V \ for\ z \leq k_\theta \tag{8}$$

where z is some value of environmental impact. For V, defined as the value of the project once developed and F, the value of the opportunity over all sized of environmental impact, as long as the environmental impact is smaller than the critical level, the value of the developed project is at least as large as waiting for more information.

$$F'(z) = \frac{\partial V(I^*, z)}{\partial z} \text{ for } z \leq k_\theta \tag{9}$$

and

$$-rV + \alpha z \tfrac{\partial V}{\partial z} \leq 0 \quad \text{for } z \leq k_\theta$$
$$F > V \qquad \text{for } z > k_\theta \tag{10}$$

The last condition is simply the reverse of the first, that as long as costs exceed some critical level, it is more profitable to wait, maintaining the opportunity to invest, F. The middle two conditions are necessary to ensure continuity at the boundary.

To solve the differential equation for the continuation region, a functional form is necessary. The solution of the form $F = A_1 z^{\beta_1} + A_2 z^{\beta_2}$ Is appropriate with

$$\beta_1 = \frac{(0.5\gamma^2 - \alpha) + \sqrt{(\alpha - 0.5\gamma^2)^2 + 2\gamma^2 r}}{\gamma^2}$$
$$\beta_2 = \frac{(0.5\gamma^2 - \alpha) - \sqrt{(\alpha - 0.5\gamma^2)^2 + 2\gamma^2 r}}{\gamma^2} \tag{11}$$

The coefficients A_1, A_2 are calculated by Substituting the solution for (7) to conditions (8) and (9) and setting $z = k_\theta$:

$$A_1\beta_1 k_\theta^{\beta_1 - 1} + A_2\beta_2 k_\theta^{\beta_2 - 1} = -\frac{\partial f}{\partial k_\theta}$$
$$A_1 k_\theta^{\beta_1} + A_2 k_\theta^{\beta_2} = V \tag{12}$$

and applying Kramer rule Which gives:

$$A_1 = \frac{k_\theta \frac{\partial f}{\partial k_\theta} + \beta_2 V}{k_\theta^{\beta_1}(\beta_2 - \beta_1)} \tag{13}$$
$$A_2 = \frac{k_\theta \frac{\partial f}{\partial k_\theta} + \beta_2 V}{k_\theta^{\beta_1}(\beta_1 - \beta_2)} \tag{14}$$

The critical size of environmental cost, k_θ, is calculated by substituting (5) for V into condition (10), and rearranging:

$$k_\theta = rW(I) - \frac{\alpha I}{r - \alpha} \tag{15}$$

4 Investment size and investment timing

Substituting (4) into (1), $F(k)$ becomes a function of investment, stopping time, and environmental cost at the investment time. Given the solution to the optimal investment time, the optimal investment size is calculated as: assuming that $r > \alpha$:

$$I^* = \text{argmax} E\left[e^{-\theta r}\left(W(I) - f(\theta, k_\theta, I)\right)\right]$$
$$\text{s.t. } dk = \alpha k dt + \gamma k dw \ t \leq \theta \tag{16}$$

, assuming that $r > alpha$:

To find the optimal investment given the optimal investment time and environmental cost at the time of investment we differentiate (16) with respect to I and equate to zero. For a transposable function $W(I)$, the optimal investment is given by:

$$I^* = W'^{-1}\left(\frac{\alpha}{r(r-\alpha)}\right) \tag{17}$$

Clearly, the socially optimal level of investment is smaller than the amount that a private developer would implement,$\hat{I} = W'^{-1}(0)$ since the latter does not consider the external environmental effects imposed by the investment.

To find the relationship between k_θ and investment size, we differentiate (15) with respect to I:

$$\frac{dk_\theta}{dI} = rW'(I) - \frac{\alpha}{r-\alpha} \tag{18}$$

Substituting (17) for W' indicates that k_θ reaches maximum at $I = I^*$. Thus:

$$\frac{\partial k_\theta}{\partial I} = \begin{cases} > 0 & for \quad I < I^* \\ = 0 & for \quad I = I^* \\ < 0 & for \quad I > I^* \end{cases} \tag{19}$$

k_θ, the environmental cost that allows development to begin is maximized at I^*, the socially optimal level of investment. Since it is necessary to wait for the environmental cost to fall before commencing development, the socially optimal level of investment will be undertaken before any other level of investment, (i.e. larger or smaller than the I^*)[1] Any level of investment which exceeds I^*, for example, the private optimum, causes a larger increment

[1]The necessity of waiting for k_t to fall arises for two reasons. If $k_t < k_\theta$ there would be no reason to wait to invest. Environmental cost is sufficiently small to induce the decision-maker to invest immediately.

to dk_t and thus would require a lower level of k_θ. A level of investment which is smaller than the socially optimal level, will result in private benefits that will be too small to offset the corresponding increase in environmental cost.

Differentiating k_θ with respect to r indicates the affect of social time preferences as they are reflected in the discount rate, on the timing of investment.

$$\frac{dk_\theta}{dr} = W + \frac{\alpha I}{(r-\alpha)^2} > 0 \tag{20}$$

The lesser importance the society place on the future the greater future environmental costs will be tolerated, and smaller future discounted returns are required on the investment. Thus, development would take place under higher initial environmental cost, k_θ. Therefore, a small attention to future events, which means grater discount factor, results in a grater expected environmental cost.

5 A note on cost benefit analysis

The basic premise of this work is that a projects value is determined both by the timing as well as the size of investment. In contrast to conventional cost benefit analysis, it provides a functional form for the relationship among the variables which determine the value of the opportunity to invest and of the investment itself. The function F provides not only information on the optimal timing and the optimal size of investment, it can also provide results for situations in which one or both of θ or I^* are not within the decision-maker's control.

Note that:

$$F(k, \hat{I}) \leq F(k_\theta, \hat{I}) \leq F(k_\theta . I^*) \tag{21}$$

Waiting simply results in foregoing return that would be received if investment had taken place. If $k_t > k_\theta$ and there is no expectation that it will fall, then investment will never take place. Only if k_t starts out at a higher level that k_θ and decreases over time is there reason to consider waiting for the optimal timing of the investment.

The model developed in the previous sections assumed that the planner could choose how much and when to invest. In reality, however, this is rarely the case. In the absence of regulation or other instruments, these decisions are made in the private sector. In an exclusively private decision, in which no account of the environmental consequences of development is made, the project's value is farthest from the social optimum. When the planner has a greater level of control, for instance through permitting or lisencing, that can influence the timing of investment, the results are closer to the social optimum. As the size of investment approaches I^*, and k_θ rises, the project is likely to be undertaken at an earlier date and have a greater level of social benefit.

Furthermore, note that boundary between the continuation and the stopping regions, or the investment and preservation regions is not affected by the size of uncertainty that is depicted by the coefficient γ in (2). The reason to this is that when we carry a conventional cost benefit analysis, we consider a sum of the costs and the benefits. An expected value of such sum does not allow the variance of the cost to effect timing of the investment. It does, however, affect the size of the social net benefit from investment in the appropriate time. Thus, if the project is to be weighted against some government funds, for example, the grater the uncertainty around the environmental cost, the smaller the expected net benefit of the project is, and there is more ground for rejecting the entire project.

6 Extensions

6.1 Investment over the planning cycle

The preceding model made the assumption that the development project was completed in a single stage and that the planning process involved a single decision. It is more realistic to think in terms of projects whose completion takes several stages and which may or may not have return associated with intermediate stages. Most mining projects for example begin with a preliminary exploration stage to determine the characteristics and quality of the reserves. when extraction takes place, individual reserves are exhausted in sequence. All but the first stage will generate income and decisions as to whether to continue are revised based on information on new discoveries as to the quality of reserves, commodity prices and so on. A planning decision of this sort is most appropriately modeled as a series of optimal stopping problems, each with possibly more than one continuation and stopping region. In addition, account must be taken of the fact that the life of the project may exceed the life of the planning regime under which it was started.

Assume an investment of size $\tilde{I}$ was carried when the environmental cost were $k_{\tilde{\theta}}$. In the model of the previous section the project would be begun and completed in period θ and there would be no further decisions. If, however, the project would have required several stages or if subsequently, reason were found to enlarge the initial project and the later decisions were being made by a different planner than the first, then the evolution of the environmental impact is described by the process: $dk = \alpha(k + \tilde{I})dt + \gamma k dw$. Let the second and subsequent amounts of investment be given by $I_2, I_3 ....$

The task now is to find the investment timing $\theta_2, theta_3, ...$ that will maximize social net benefit from period, i on. Calculating the expected environmental impact of the new investment equation (4) becomes:

$$\begin{aligned} E\left(\int_{\theta_i}^{\infty} e^{-rs} k_s ds | \theta_i, I, k_{\theta_i}\right) &= \int_{\theta_i}^{\infty} e^{-rs} E\left(k_s | \theta_i, I, k_\theta\right) \\ &= e^{-r\theta_i}\left(\frac{\alpha(\tilde{I} + I_i)}{r(r-\alpha)} + \frac{k_{\theta_i}}{r-\alpha}\right) = e^{-r\theta_i} f(k_{\theta_i}, I) \end{aligned} \tag{22}$$

and the environmental cost that entails such investment, k_{θ_i}, is:

$$k_{\theta_i} = rW(I_i) - \frac{\alpha(\tilde{I} + I_i)}{r - \alpha} \tag{23}$$

For positive I_2 of the same size as $\tilde{I}$, commencement will take place for lower environmental cost. That is to say that $k_{\tilde{\theta}} > k_{\theta_i}$. Thus, before further investment is made, environmental cost should further be reduced. It is not sufficient, that they reach the former $k_{\tilde{\theta}}$.

This formant of sequential decision making much more closely reflects reality than the standard optimization in which information over the entire sequence of investments is taken as given and the development plan set accordingly. In fact, in some cases, the latter is impossible to obtain a priori. Such is the case in the mining project prior to initial exploration because of the relatively high degree of uncertainty surrounding the quality of the reserve. Similarly, if little is known about environmental impacts, as is often the case, the sequential treatment is the only appropriate approach.

6.2 Global change

The model in section (4) is easily adopted to explain in part the foot dragging governments seem to take when investments to reduce global worming is advocated by environmentalists and some economists. Consider the minimizing total cost over time that are combined of cost of investment as are reflected in increased cost of production etc., W(I), and environmental cost k. the evolution of the environmental cost is said to be reduced by the investment in cleaner technology in the following way:

$$dk = \alpha(k - I)dt + \gamma k dw \tag{24}$$

Solving the minimization in much of the same way constructed above, $f(k_\theta, I)$ is calculated to be: $f(k_\theta, I) = \frac{k_\theta}{r-\alpha} - \frac{\alpha I}{r(r-\alpha)}$. Thus, k_θ is:

$$k_\theta = rW(I) + \frac{\alpha I}{r - \alpha} \tag{25}$$

and since W(I) is a cost function in this presentation of the problem we find that:

$$\frac{\partial k_\theta}{\partial I} > 0 \tag{26}$$

So that when only small investment is selected it would be optimal to carry it in the near future when environmental costs are still small. However, larger investment should not take place until a larger environmental cost is seen. Since the environmental costs are uncertain in their nature, and the investment needed to reduce them is deterministic, it is optimal to wait till the time when the actual large environmental cost are here, before investing heavily in their reduction.

7 Conclusion

This paper investigates the timing and size of investment that maximize uncertain social returns. The social returns are uncertain because of environmental externalities which are not fully known at the time that development takes place. When environmental costs are uncertain and change over time, and development is irreversal after it is initiated, there is an economic value attached to waiting for new information to come up before development takes place. If, in addition, the social planner is able to determine the size of the investment, it should affect its timing.

When environmental cost are reduced over time and the decision-maker is able to defer the commencement of investment, it is optimal to wait until a critical maximum level of costs is reached before investing. The maximum level of environmental cost that allow for any other level of investment that is less than the social optimum is smaller. Therefore, an investment that is not socially optimal would have to be deffered further. This result calls for a lower level of investment than that which results from a private decision. It also differs from the conventional cost benefit analysis result, even when environmental externalities are taken into account. The value of new information that is absent in conventional cost benefit analysis, adds an economic value to the opportunity to delay investment to a future date until a reduction in the environmental cost associated to the development, is detected.

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