Party Formation and Policy Outcomes Under Different Electoral Systems *

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Abstract

This paper provides a game-theoretic model of representative democracy with endogenous party formation. Coalition formation may occur *before* and *after* elections, and the expected payoffs from the after-election majority game affect incentives to form parties before the elections. In this way Duverger's hypothesis can be formally explained by the strategic behavior of political elites. If politicians care primarily about private benefits, the equilibrium policy outcome under a proportional electoral system coincides with the median party's position. On the other hand, with quasilinear utility, the distance from the median voter outcome may be lower with plurality rule.

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Non-technical Abstract

In most representative democracies, and in particular in parliamentary systems, there are two different kinds of *coalitional bargaining* games going on:

1. Before every election parties form as coalitions of former smaller parties, or seceeding from large parties no longer existing;

2. After every election the elected members of parliament bargain in order to agree on a majority coalition that will support the government.

Depending on how elections work, the relationship between these two stages may change significantly and this paper shows that with plurality rule the incentives to party formation before elections are so strong that we tend to observe two-party equilibria; on the other hand, with a purely proportional electoral system multipartism always prevails. It is also important to notice that different electoral systems affect public policy decisions in different ways: if politicians are purely rent-seeking then proportional representation determines an equilibrium policy closer to the one attainable in a direct democracy than with plurality rule. However, if politicians also have important ideological positions about policies, then the outcome of direct democracy may be better approximated having plurality rule.

1 Introduction

One of the main features of a representative democracy with a parliamentary system is the preeminent role that a coalition holding the majority of the votes in the Parliament plays in the determination of policy outcomes.[1] The agenda-setting power is always within the majority coalition, and the cabinet government is limited even in the realm of executive power by the need to maintain the support of the majority coalition itself. In other words, the policy outcomes of a parliamentary democracy are decided *de facto* by the majority coalition in the House(s). Thus, the legislative bargaining game determining the majority coalition is a very important one for any positive theory of the micro-political determination of economic policy (see Persson & Tabellini 1998b for the most recent account of this approach).

The players of the legislative bargaining game are usually not the individual legislators but the elected parties.[2] The number of parties represented in the Parliament and the distribution of seats among them affect, together with the ideological and political constraints on the formation of different coalitions, the negotiation process that leads to the formation of a majority coalition. But these crucial variables, i.e., the party structure and the distribution of seats, are obviously not exogenous to the institutional system:

1. For any given party structure, the distribution of seats is affected by, and varies with, the electoral system;

2. Within any given institutional system, the equilibrium party structure depends on the expected assignment of seats, and hence expected payoff, associated to each party structure.

These two relationships are very important, and have not been studied before. The introduction of the process of party formation in any game-theoretic model of representative democracy allows us to deal with such relationships and is likely to determine predictions close to reality, since party formation is one of the main strategic activities of politicians.

[1] Persson and Tabellini (1998a) and (1998b) are the first to emphasize the impact of the legislative cohesion typical of parliamentary systems on public policy, contrasting such a policy with that obtained in a presidential-congressional system, which is characterized by less legislative cohesion and a greater separation of powers. Our paper focuses exclusively on parliamentary systems, and hence legislative cohesion is assumed throughout. Laver and Shepsle (1996) provide a rich formal model of government formation, where the emphasis is on the "composition" of the cabinet rather than on legislative bargaining.

[2] In closed-list systems the electorate casts its ballot not for individual candidates but for party lists. Moreover, the elimination of secret votes in the Houses of many democracies implies more discipline, or loyalty, within parties. The party leadership can usually enforce the party line by various kinds of implicit threats.

Moreover, if voters know the true ideological position of any party facing elections, then, since no contingent contract can be signed with the voters, a party is stuck with its own ideological position. A political elite can commit to a policy platform different from its own only by forming a party with some other elite, so that the platform voters expect to be pursued by such a party is somewhere between those of the two elites. In other words, party formation gives political elites a way to “move” on the policy spectrum.

One of the main contributions in the recent formal literature on representative democracy is the work of Besley and Coate (1997). In the first stage of their three-stage game some agents choose to become candidates, in the second stage the population votes for the candidates, and in the last stage the candidate who received the most votes makes policy choices. This is a good abstract description of what happens when one decision maker has to be elected, but it is not as relevant when the object of study is the formation of a coalitional government, especially if the electoral system is proportional. In contrast to Besley and Coate, our representative democracy game allows the consideration of a variety of electoral systems, as well as all possible divisions of the electorate (into districts or constituencies) within a unified framework. For any given electoral system, the multi-stage coalition formation game introduced in this paper produces an equilibrium outcome, an equilibrium party structure, and an equilibrium government coalition.

The (*ex post*) coalition formation game involved in forming a coalitional government is played all the time, in all parliamentary democracies. Even though the party structure has been fairly stable for long periods in many countries, coalition formation before elections (*ex ante*) takes place at importasnt historical turning points, when a significant change in the rules of the overall game occurs (among which the electoral system is a crucial one), or when the political constraints or voters’ preferences are altered.[3] Obviously, not all institutional changes determine a modification in the equilibrium party structure; however, it is important to emphasize the possible role of such changes.

Besides the explicit introduction of party formation and the possibility of comparing the effects of different electoral systems on equilibrium outcomes, this paper also provides some innovations in the way the *ex post* game is treated. This game among the elected members

[3]Italy is an example where twice in the last 50 years the party structure has changed dramatically due to two different structural breaks. After World War II a major institutional mutation occurred, and the party structure altered accordingly. In 1992, following (1) a change in the electoral system (towards majoritarianism); (2) a relaxation of the constraints against forming coalitions with the former communists after 1989; and (3) a corruption scandal, which changed voters’ preferences, a huge renegotiation process on parties started, and has not ended yet. A tendency towards a reduction of the number of parties (or joint lists) can already be noted. See Laver & Shepsle (1996) for other examples.

of Parliament has been studied extensively in the literature, and most of the work on this topic uses the non-cooperative sequential bargaining model of Baron & Ferejohn (1989) as a basic tool. The model of majoritarian bargaining used in this paper has more predictive power than the other models in the literature.[4] The legislative majoritarian bargaining stage-game is modelled here as a non-cooperative *sequential demand game*, inspired by the work of Selten (1992). The Subgame Perfect Equilibrium payoff allocation of the game is consistent with the empirical finding that, within the majority coalitions or the cabinets, ministerial payoffs and private benefits (like extra-ministerial patronage) are distributed proportionally according to the relative bargaining power given by the distribution of seats. In contrast, Baron & Ferejohn's model puts too much emphasis on "proposal power," i.e., the first mover in the order of play gets a disproportionate share, which is not confirmed by experimental evidence. In our model, when politicians have lexicographic preferences, the equilibrium distribution of payoffs within the winning coalition does not depend on the order of play. Another important feature of the sequential demand game used here is that there is a unique SPE distribution of payoffs for every distribution of seats, and hence it is possible to assign an expected payoff to each and every party for every party structure. This mapping allows us to study the incentives for party formation *ex ante*. With different electoral systems the same party structure can map into different distributions of weights, and hence different distributions of payoffs. Therefore the incentives for party formation will be sensitive to the choice of the electoral system.

Duverger's hypothesis[5] is formally proved with our model. All the previous formal explanations of Duverger's law and Duverger's hypothesis refer to strategic voting.[6] This paper shows that strategic voting is *not necessary* to obtain those conjectured relationships between electoral system and incentives for party formation.

As far as the equilibrium policy outcome is concerned, the relative performance of the two systems is very sensitive to the preference orderings of politicians, and whether the "action" takes place before or after the election has repercussions on the "ability" of the various types of representative democracy to reproduce the outcome of direct democracy:

1. If politicians care primarily about private benefits (lexicographic preferences), then the proportional system determines an equilibrium policy outcome that coincides with the median party's platform (and hence, if the median party's position is the same as the

[4] See Morelli (1998a) for some robustness results on our prediction for majoritarian bargaining.

[5] This indicates the conjecture that the number of parties is always greater when the electoral system is proportional than when it is pluralitarian.

[6] See Cox (1997), Feddersen (1992), Fey (1997), Palfrey (1989).

median voter's position, the outcome of direct democracy is obtained). The proportional system allows the median party to obtain the majority vote on her preferred policy outcome because transfers cannot be used for compromises in this case.

2. If instead politicians have quasilinear utility functions, an explicit trade-off exists between the two dimensions; thus, the equilibrium outcome of a parliamentary system with a proportional electoral system is the fruit of a compromise, and is therefore bounded away from the median voter outcome.

3. If the electoral system is pluralitarian the equilibrium outcome depends almost exclusively on the *ex ante* bargaining power of the political elites at the party formation stage, and the preference orderings matter much less. With respect to the proportional system, the equilibrium outcome under plurality rule is always (at least weakly) more distant from the median voter position when preferences are lexicographic, while it might well be closer in the quasilinear case.

The paper is organized as follows: in section 2 we describe the model, characterizing each stage of the game and defining formally the role of an electoral system; section 3 contains the main results for the case where politicians care primarily about private benefits; section 4 displays all the equilibrium implications of the model for the quasilinear case, and section 5 concludes. All the proofs are in the appendix.

2 The Model

2.1 Basic Assumptions

Consider a set P of "political elites" ($i = 1, 2, \ldots, p$), the initial players of the game, and assume $p \geq 3$. Each player i is characterized by a "position" on the policy space $[0, 1]$.[7] The p players form n coalitions, $(C_1, C_2, \ldots, C_n : C_k \cap C_j = \emptyset, \bigcup_{k=1}^{n} C_k = P)$. Such n coalitions are the *parties* that face elections.[8] Party i's position is denoted by θ_i and is a weighted average of the positions held by the members of the party. Formally,

$$\theta_i = \frac{\sum_{j \in C_i} \psi_j \theta_j}{\sum_{k \in C_i} \psi_k}, \tag{1}$$

[7] In this paper we consider only one dimension. The extension to a multidimensional setting will be considered in future work.

[8] Calling political elites the players at the *ex ante* stage and parties the actual players from the election stage on, is only a convenient distinction. In some contexts the initial players are actually parties themselves, and the actual players from the election stage on are joint lists; but the strategic interaction between the two stages would be exactly the same, so we will stick to the terminology in the text.

where ψ_j denotes elite j's relative *ex ante* bargaining power ($\sum_{j=1}^{p} \psi_j = 1$).[9] We assume throughout the whole paper that for every possible partition π (into n parties) of the initial players' set P, there is perfect foresight; every elite is able to solve the game in the same way. Voters (a continuum) are characterized by a distribution of preferences over the same policy dimension $[0, 1]$, and they are divided into districts (or constituencies). Knowing the *ex ante* bargaining power ψ_i and the ideological policy position θ_i of each elite i, voters know the policy position of every party that might form, and since no contingent contracts can be signed, the true position of parties is the only relevant information for voters. For any configuration (1) of the n-dimensional vector of positions and (2) of the distribution of voters' preferences, the electoral system determines a distribution of seats in the Parliament. Finally, for every distribution of seats there are expected payoffs for the n parties. These payoffs come from the majoritarian bargaining game that they have to play in order to try to enter a coalitional government.

Once the elections have determined a distribution of seats in the Parliament, the actual players of the game become the n parties, given the following *loyalty* assumption:

Assumption 1 *The parties represented in the Parliament can form coalitions but they cannot break apart. Party members in Parliament are loyal to the party leadership.*[10]

For each possible distribution of seats among parties, every party assigns the same probability distribution to the possible winning coalitions and to the possible payoff distributions. So, every party that went to the elections has, after the election results become known, an expected payoff. Before showing how those expected payoffs are determined and before showing what the derived incentives to form parties in the first place are , let us formalize the notion of an electoral system.

2.2 Mathematical Definition of Electoral Systems

There are z constituencies, or districts, and ζ seats in the Parliament. For simplicity, we make the following assumption:

Assumption 2 *The list of possible parties to vote for is the same in every constituency.*

[9] One example of a possible measure of relative *ex ante* bargaining power could be $\psi_j = w_j(p)$, the expected percentage of votes that elite j would have if $n = p$.

[10] This assumption has strong empirical support in parliamentary democracies, while it would obviously be a very strong one in presidential-congressional systems like the U.S. See Persson and Tabellini (1998a) for a clear recent discussion on this issue.

Given assumption 2, we can provide a general definition of an electoral system.

Definition 1 *An electoral system is a* 4*-touple composed of:*

1. *A set* $\mathcal{R}$*, the population of voters, of measure* $\mu_r = 1$*;*
2. *A partition of* $\mathcal{R}$ *into* z *subsets* $(R_i)_{i=1,\ldots,z}$ *(*R_i *has measure* μ_i*, with* $\sum_{i=1}^{z} \mu_i = \mu_r = 1$*);*
3. *A function* $p_j^i : R_i \longrightarrow [0,1]$ *(the fraction of votes going to party* j *in district* i*);*
4. *A function* $F_j : [0,1]^z \longrightarrow [0,1]$ *(fraction of seats for party* j*).*

Geometrically, one could also think of the domain as a rectangle, divided into z rectangles, where the area of each small rectangle represents the population of voters in a district. Voting in each district can be defined as an assignment of a fraction of votes p_j^i ($\sum_{j=1}^{n} p_j^i = 1$) for every district i and for every party j, $j = 1, \ldots, n$. The first three components of the 4-touple in definition 1 are clearly common to every electoral system, and hence in the analysis of the proportional vs. pluralitarian system we can concentrate on the last element.

2.2.1 The Proportional System

The proportional system is characterized by the function:

$$w_j = F_j(\mu) = \sum_{i=1}^{z} p_j^i \mu_i \tag{2}$$

The fraction of seats w_j is a *weight*, which determines the "bargaining endowment" of party j in any weighted majority game to be played in the Parliament after the elections. It is obtained as a weighted sum of the results of each district, where each district's weight in the overall sum depends on the relative population size.[11]

2.2.2 The Pluralitarian System

We call *pluralitarian system* a system characterized by the function:

$$w_j = F_j(g, \mu) = \sum_{i=1}^{z} g_j^i \mu_i \tag{3}$$

[11] If ζ is much bigger than n (many Parliaments have huge numbers of representatives) the fact that we consider for simplicity all possible real numbers as possible fractions of seats is not too bad an approximation, but it is indeed an approximation.

where

$$g_j^i = \begin{array}{l} 1 \text{ } \textit{if } p_j^i = \max_{h \in N} p_h^i \\ 0 \text{ } \textit{otherwise} \end{array}$$

Party j has to win in at least one district in order to have seats. The fraction of seats going to each party j depends on how many districts party j wins in, and on the relative importance of those districts.

One way to model *mixed systems* is to imagine that a fraction η of the ζ seats is assigned proportionally and the rest following the pluralitarian rule. In this way the fraction of seats obtained by party j is as follows:

$$w_j = \eta \left(\sum_{i=1}^{z} p_j^i \mu_i \right) + (1 - \eta) \left(\sum_{i=1}^{z} g_j^i \mu_i \right) \tag{4}$$

These intermediate cases are observed in the real world, together with runoffs and percentage lowerbounds. We ignore them here for simplicity. They will be introduced in future work.

2.3 The *after-elections* Majoritarian Bargaining Game

We now turn to the description of the game, starting from the legislative bargaining subgame.

Let us denote by q the quota for simple majority: $q \equiv \frac{\frac{\zeta}{2}+1}{\zeta}$. If $w_i \geq q$ for some i, obviously there is no coalition to be formed, and every decision, including the government formation, is taken by party i. The interesting case is therefore to assume

$$w_i < q \ \ \forall i.$$

A coalition of parties S is a potential majority coalition if and only if $\sum_{i \in S} w_i \geq q$. $\Omega^m(w)$ denotes the set of all minimal winning coalitions (MWC) given the vector of weights w:

$$\Omega^m(w) \equiv \left\{ S : \sum_{i \in S} w_i \geq q, \ \sum_{i \in T} w_i < q \ \forall T \subset S \right\}$$

The number of MWCs in $\Omega^m(w)$ is denoted by $m(w)$; $M^i(w) \equiv \{S \in \Omega^{m(w)} : \ i \in S\}$, and $m^i(w)$ is the cardinality of such a set. The total amount of private benefits, or rents, associated to being in office, are normalized to unity, and they are distributed in equilibrium only within the prevailing majority coalition.

In the real world, parties share payoffs (ministers, portfolios, and other private benefits) *proportionally* to their bargaining power, which is usually related to the fraction of seats they own. In particular, Browne & Franklin (1973) show a strong empirical evidence that ministerial payoffs are usually shared proportionally to the relative weight of the members of the majority coalition, especially if the number of parties in such a coalition is large. They also show that if exceptions exist to such a rule they are in the direction of equal split (i.e., with a relatively greater share for smaller parties) when fewer parties belong to the winning coalition. In the case of only two parties in a majority coalition the tendency towards equal split is the strongest. Laver & Shoefield (1990) and Shoefield & Laver (1985) provide different accounts and data about this phenomenon, which is known, in its simplest form, as Gamson's law. The predictions obtained looking at the Subgame Perfect Equilibrium outcomes of the non-cooperative game defined below are consistent with these empirical findings, and are easy to compute. The other models of majoritarian bargaining (including the most widely used among them, i.e., Baron & Ferejohn 1989) do not yield equilibrium outcomes compatible with the above. The role of the order of play and the power of the first proposer in obtaining very large shares are overemphasized. For example, in any three-player majority game where none of them individually has more than q, the two players forming a MWC should share 50/50 the payoff of winning, as confirmed by the experimental evidence as well, while the Baron & Ferejohn model would assign 2/3 to the first proposer.[12]

The majoritarian bargaining game used here is a sequential demand game, and can be described as follows:

1. The Head of State (or Monarch) chooses the first proposer (usually a potential prime minister) and the rest of the order of play. The Head of State is assumed to choose the optimal order of play given its "super partes" preference ordering. In the case of indifference it randomizes.[13]

2. For any order of play chosen by the Head of State, players make *demands* sequentially: when the turn comes,

   (a) each player i demands a share x_i of private benefits (like a reservation price for its participation to a majority coalition), and

   (b) in addition it makes a policy proposal y_i.

   When the game arrives to a node where it is "feasible" for the player moving at such a node to form a winning coalition with a (weak) subset of the previous movers, it can choose to close such a coalition, or else it can choose to make another demand and let

[12] For experimental evidence on the fact that traditional sequential bargaining games attach too much importance to the power of being first proposer see Bolton (1991) and Ochs & Roth (1989).

[13] Randomization is usually assumed when only private benefits enter the utility function of party members, because in that case every winning coalition in the continuation equilibrium of every order of play is as good as any other for a "utilitarian" Head of State.

the game move on. Closing a winning coalition is a "feasible" option for a player only (1) if there is at least one subset of the previous movers whose total payoff demands do not exceed 1, and (2) if the parties belonging to such a subset proposed the same policy outcome.

3. Each party moves at most once: the game ends either when some player closes a coalition with some of the previous movers, or when all players have moved once, whatever comes first. If no winning coalition has been formed by then, all parties get a 0-normalized outcome (caretaker government, new elections, or similar).

As shown in the next sections, this sequential demand game has a unique Subgame Perfect Equilibrium, both when parties have lexicographic preferences and when they have a standard utility function.

2.4 Voters' Preferences and Expected Payoffs

For every distribution of seats w (endowment vector of the majoritarian bargaining game) each party i has an expected payoff $u_i(w)$.[14] The distribution of seats w among the n parties facing elections depends obviously on voters' preferences. In terms of our mathematical characterization of electoral systems, voters' preferences determine p_j^i and g_j^i. Assuming sincere voting, the knowledge of voters' preferences is sufficient to determine those mappings, still keeping n fixed. Voters have single-peaked preferences on the policy space. Voter i votes for the party j which has the closest platform:

$$\min_{j \in N} (\theta_i - \theta_j)^2.$$

The Equilibrium outcomes of the game remain equilibrium outcomes even introducing some form of strategic voting, but with strategic voting there could also be some other equilibria. We will discuss the possible changes when needed. We keep sincere voting because the emphasis of this paper is on the strategic behavior of political elites and parties at the different stages of the representative democracy game.

Notice that, since the position of a party on the political spectrum is given by the weighted average position of its components, a party C_i formed by many elites but *disconnected*, i.e., where some other party C_j holds a position in between those held by the elites forming C_i, could have the same policy position as party C_j and could therefore get the same number

[14] The term "expected" refers to the fact that the order of play might be chosen randomly by the Head of State, as long as the aggregate preference ordering is maximized.

of votes. This is why party coalitions are almost always connected. In any case, given that voters' preferences are common knowledge, every party structure π is associated to a unique (n-dimensional) vector of expected payoffs. Given this mapping, we can now move to the first stage, where the party structure is endogenously obtained.

2.5 The Party Formation Game

In our model the number n of parties in a given system must correspond to an equilibrium party structure, which we now have to define formally. Suppose that there are p political elites (finite number) who have to form parties ($n \leq p$). Each elite i has, as mentioned above, an associated $\theta_i \in [0, 1]$.

The party formation game is modeled as a *link formation* game. Aumann & Myerson (1988) used a link formation game based on Myerson's (1977) article on cooperation structures. Their game is similar to our first stage. One important difference, which does not concern the rules of the game, is that the subsequent stage game is described and solved explicitly in our model, whereas in Aumann-Myerson the individual payoffs are basically a direct exogenous function of the cooperation structure, without any modeling of what may go on among the components of the cooperation structure. Solving the subsequent stages of the game, we can attach to any party structure a vector of expected payoffs, one for each party; but one of the reasons to study the whole game is to show how this mapping depends on institutional characteristics, ideology, and voters' preferences.

Let us now analyze the modeling assumptions. A *graph* g (or cooperation structure, or network) on the players' set P is a set of p nodes and a set of *links* (non-directed segments). A link between i and j will be denoted by ij. A *component* h of a graph g is a set of nodes (all linked to one another directly or indirectly) and the set of links connecting them. A graph is divided into disjoint components, as clarified by the following formalization: denoting by $T(h)$ the set of players corresponding to the nodes of a component h of a graph g,

1. For all $i, j \in T(h) \subseteq P$, $j \neq i$, there exists a set $\{i_1, ..., i_K\} \subseteq T(h)$ such that $i_1 = i$ and $i_K = j$ and such that $i_k i_{k+1} \in h$ for all $k = 1, ..., K$;

2. $i \in T(h)$ and $j \in P \backslash T(h)$ implies that $ij \notin g$.

3. $ij \in h$ implies $ij \in g$.[15]

[15] The second part of the definition is the one that guarantees disjointness. This definition of component is taken from Currarini & Morelli (1998), a work on the efficiency of cooperation structures.

A set $T(h)$ of players, linked in a component h of a given graph, is called a *party* (denoted by C) if every player in $T(h)$ is linked to every other player in $T(h)$ *directly*. For any $h \in g$, if $T(h)$ is not a party itself (because there are some links missing), then each player in $T(h)$ is a party. Formally:

$$n(h) \equiv \begin{array}{ll} 1 & \textit{if } \forall i, j \in T(h) \ ij \in h \\ |T(h)| & \textit{otherwise.} \end{array} \tag{5}$$

$$n(g) = \sum_{h \in g} n(h)$$

and $\pi(g)$ will denote the corresponding partition of the players' set in parties, or *party structure*.

The motivation for requiring direct links among all members in order to form a party is that the loyalty of party members in the Parliament to the party's policy platform (Assumption 1) is taken for granted by the voters only if every elite constituting the party is in agreement with every other elite in it (complete subgraph). Otherwise, when some links are missing, voters are assumed to believe that the party members will not necessarily be loyal. Obviously there are many contexts in which it is very important to consider incomplete subgraphs: for example, in the literature on communication networks it is often emphasized that when links are costly the best way to connect the members of a component is to do so with the minimum possible number of links. But when the subject matter is the formation of political agreements, it is not realistic to allow some members of a party not to communicate (and hence not to agree) with one another and yet let it be perceived as a party with a unified policy platform.

The expected payoff for party C when the political elites are in a cooperation structure g can be denoted by $v(C; g)$.[16] The imputation rule assigning to elite i an expected payoff as function of the cooperation structure will then depend on the expected payoff $v(C; g)$ for the party C containing i and on ψ. The vector of *ex ante* relative bargaining powers ψ, in fact, not only determines the party position (recall (1)), but also determines the shares of private benefits within the party. If links are costly, then each elite i has to subtract from the expected payoff just described the cost $c(ij)$ for every link involving i.

We now have all the ingredients to turn to the description of the game. Suppose that at the initial node of the game the players are all singletons (g^0).[17] The players move once

[16] Since v is given only for a given set of institutions, preferences, and ideological positions, we should put all these things in the domain of the function. The way g affects the expected utility of party C_i is through the mapping that the electoral system creates from g into w, and then using $u_i(w)$. But for simplicity it is better to analyze the first-stage game for a given configuration of those features of the political environment.

[17] When a new cooperation problem arises, for example among countries, it is fair to assume that they start

each, sequentially, according to some given protocol $\rho : P \rightarrow \{1, 2, \ldots, p\}$. Each player has to choose which *arcs* (directed segments) to send. If and only if i sends an arc to j and viceversa the link ij is formed. Arcs that are not reciprocated don't count. Let us denote by a_i the action (choice of arcs) of player i; a_i^j denotes the arc sent by i to j. A strategy s_i for player i specifies an action a_i at each and every history where i has to move. Denoting by i the player in the i-th position in ρ, a history for player i is simply the set of actions $a_1, \ldots, a_{i-1}$. Every strategy profile s determines a graph $g(s)$ and the expected payoffs attached to a strategy profile s are obviously given by the expected payoffs associated to $g(s)$. Since the game is finite the set of Subgame Perfect Equilibria is non-empty, and for a given ρ backward induction implies that generically the SPE party structure is unique.

Lemma 1 *1. If $c(ij) = 0$ $\forall ij$ there exist multiple SPE profiles and multiple equilibrium graphs. However, one of the equilibrium graphs is always the one composed exclusively of complete subgraphs, without superfluous links.*

2. If $c(ij) > 0$ $\forall ij$, the equilibrium graph g^* is generically unique for every ρ, and contains only links leading to complete subgraphs.

3. Generically, for every ρ there exists a unique SPE party structure.

Given this Lemma (proved in the appendix), from now on we can just talk about *the equilibrium party structure*. A feature of the equilibrium party structure is that the elements of such a partition, as already mentioned at the end of the previous section, are usually "connected" parties. To see this, consider the following simple example: Let the electoral system be pluralitarian, and let $p = 3$; let one of them have the median voter position. As will be shown in the next section, some party formation must occur in this case, whatever ψ, as long as none of the three players expects to win by herself. Notice that the party composed by the elite on the left and the one on the right could never win the election against the elite in the middle; they can at most tie, but only in the measure-zero event that the two extreme parties have exactly the relative bargaining power that allows them to locate themselves exactly at the median voter position. But even in this extreme case where tying is possible, the disconnected party never belongs to an equilibrium party structure, because winning for sure is better than tying for anyone, and hence, whatever the order of play is, the middle

from "no agreement" on such a new subject, and they start building up cooperation procedures from that time on. In the case of party formation it is clearly an abstraction, since a "real" status quo would in general already display some long-standing coalitions. Before any election however it is very common to have at least some negotiations even within the long- standing coalitions about the policy platform to propose and the conditions to remain together.

player forms a party with one of the others, the one with the lower ψ. If the elite in the middle does not hold the median voter position, then in principle the disconnected party could win. But if political players care a bit about the closeness of the final policy outcome to their own favorite one, then the disconnected party would not form in equilibrium anyway, since the equivalent proposal to form a party by the middle player should always be at least as attractive. Assuming that the cost of forming a link is increasing in the distance between the two elites would capture most of these reasonings and assure connectedness for every θ. We can now generalize these and other important equilibrium features of our model, starting from the lexicographic case.

3 Equilibrium Outcomes with Lexicographic Preferences

Let us consider first the case in which politicians have lexicographic preferences: they first of all care about their share of private benefits, and then, for any given share, they would like a policy outcome as close as possible to their own. The Head of State is obviously indifferent among all the possible winning coalitions as far as private benefits are concerned. On the other hand, the probability of dissolution of a government coalition (about which all Heads of State care) is assumed to be decreasing in the "average distance" of policy positions within the majority coalition, and hence the Head of State chooses the order of play such that the average distance is minimized. In other words, denoting by r any order of play chosen by the Head of State for the majoritarian bargaining game, denoting by $S^*(r), \theta^*(r)$ the equilibrium majority coalition and the equilibrium policy outcome prevailing given r, the Head of State is assumed to choose r^* such that

$$\sum_{i \in S^*(r^*)} (\theta_i - \theta^*(r^*))^2 \leq \sum_{i \in S^*(r)} (\theta_i - \theta^*(r))^2$$

when compared with any other r. Denote by θ_m the median voter position and by $\theta_{l(\pi)}$ the position of the "median party", i.e., the party such that the coalition with all the parties on its left and the coalition with all the parties on its right are both in $M^{l(\pi)}$ (the set of feasible winning coalitions with the median party).

Proposition 1 Assume $c(ij) > 0 \ \forall ij$. For simplicity, assume also that $m^i(w(\pi)) > 0 \ \forall i, \ \forall \pi$ (no dummy players).

If the electoral system is proportional and the political elites have lexicographic preferences, then:

1. *The majority coalition* S^* *is the minimal winning coalition with the minimum "average distance" from the equilibrium policy outcome:*

$$S^* \in argmin_{S \in \Omega^m(w(\pi^*))} \sum_{i \in S} (\theta_i - \theta_{l(\pi^*)})^2; \quad (6)$$

2. *If* S^* *is unique,* $\pi^* = g^0$ $(n^* = p)$*;*

3. *The unique equilibrium policy outcome is*

$$\theta^*(r) = \theta_{l(\pi^*)} \quad \forall r; \quad (7)$$

4. *The unique equilibrium distribution of private benefits within the majority coalition depends on* $w(\pi^*)$*, and if* $p = 3$ *the two parties belonging to the majority coalition share equally the private benefits.*

The proof of the general case is in the appendix. Here we can give the intuition by discussing the simple case of $p = 3$. Consider the subgame where $n = p = 3$. Whoever is the first mover, it can only demand 1/2 of the private benefits: in fact, if it demands $1/2 + \epsilon$ the second mover can undercut, demanding $1/2 + \epsilon - \delta$ $(\delta < \epsilon)$, so that the third mover would choose to close the coalition with the second mover. Most important, the first two movers will also agree to demand $\theta_{l(\pi^*)}$ as policy outcome. To see this, suppose that one of the two extreme parties—say the one on the left—tries to demand something closer to its position when it is the first mover; if the next mover is the median party it is better off demanding her preferred policy, $\theta_{l(\pi^*)}$, since the right party would prefer that to the first mover's proposal; if instead the second mover is the right party, its demand would again be different from the first mover's choice, and in particular would be one closer to the median party's position, so that the median party, playing last, would choose the second player. For all other orders the reasoning is the same, and in all cases the median party's position is the only equilibrium outcome. Knowing this, the Head of State would always choose an order of play with the median party in one of the first two positions, because otherwise the equilibrium coalition would be an unstable one with a left party and a right party pursuing the median party's policy. If the minimum distance is the one between the left party and the median party, those two parties will be the first two movers, and viceversa. Knowing this, and given that $c(ij) > 0$, no pair of parties has incentive to form a larger party, and hence $n = 3$ is the only equilibrium number of parties.

All the results summarized in Proposition 1 deserve now some discussion.

1. There is no rigorous empirical work done on the ideological distance among the members of a majority coalition in different parliamentary systems, but some casual observations, among which the Italian experience after World War II, suggest that when the electoral system is proportional, (1) coalitions are always formed *ex post*, after the elections, and (2) they are always very homogeneous. On the other hand, reforms towards a more majoritarian electoral system force parties to make joint lists *ex ante*, and the ideological distance may be visible.[18]

2. The fact that in a representative democracy with proportional electoral system there are always a lot of parties in equilibrium is consistent with *Duverger's Hypothesis*. This result is robust to many variations of the model. If the Head of State had no discretion and could only apply the fixed rule of appointing as first mover the party with the relative majority of votes,[19] then π^* could be different from g^0, since two parties who fear the exclusion from the majority coalition may have incentive to try to obtain the relative majority and hence become first movers. However, even in this case multipartism is confirmed: n^* is always ≥ 3. S^* would not be necessarily the one of (6), but rather it would be the one with the largest of the three parties and the closer to her between the other two; but still the two parties expecting to be called to move first would refuse to form a larger party *ex ante*. If one of the two extreme parties expects the relative majority of votes and hence the role of first mover, the median party expects to be called second, regardless of who chooses the order of play, because the equilibrium outcome is the same and there could be only more instability in making a majority coalition with the opposite extreme party. If the median party is the one with the relative majority of votes when $n = 3$, then, in the very unlikely case that both the Head of State and the first mover are indifferent between having one or the other extreme party as second mover, one could think that *ex ante* the two extreme parties could have incentive to merge; but even in this case this would not happen: if the median party has a policy platform close enough to the median voter's position, the joint list of the two extreme parties would run the significant risk of actually giving the median party the absolute majority. Even with quasilinear preferences multipartism will be shown to be a robust equilibrium feature of proportional electoral systems.

3. The prediction that comes out from Proposition 1 is that countries with proportional electoral system and primarily rent-seeking politicians should display a low variance of

[18] Again, think of the center-left majority list in Italy, formed by a bunch of small and large political elites merging in the Olive Tree and by the extreme Communists.

[19] See Morelli (1998b) on the suboptimality of such a rule.

policy outcomes, and such policy outcomes should gravitate around the median voter position (assuming that the median party's position is the closest to the median voter one). Before the reform (towards a majoritarian system) of 1992, Italy had basically a pure proportional electoral system with closed lists, and the policy outcome has indeed remained quite stable, despite the many government changes.[20] ¿From the theoretical point of view, notice that if the median political elite (which is the same as the median party only if $n = p$) has a policy position that coincides with the median voter position, then our model shows how does a representative democracy with rent-seeking politicians obtain the outcome of direct democracy.

4. Finally, the payoff distribution predicted by our model is consistent with the empirical findings of Browne and Franklin (1973) and others, and with the experimental evidence on coalitional bargaining, as discussed above and in Morelli (1998a, 1998b).

Even allowing for strategic voting, the equilibrium described here would still be an equilibrium, because voters care only about policy outcomes and here the latter is always the median party's policy. Only if the median party's position is very far away from the median voter's position, only in that case some forms of strategic voting would arise, but they would not alter the qualitative result.

Remark 1 *If participation to the political game is costly for the political elites, then in a pluralitarian system the number of parties facing elections is at most 2. (*Duverger's Law*). The equilibrium policy outcome depends on the protocol of the party formation game, and may well be far away from the median voter outcome.*

The proof of the statement above is trivial. Suppose that the members of any majority party formed *ex ante* can all benefit from participating to the elections. The players outside such a party would decide not to run though, for any small participation cost. A party not expecting to win would participate only if we introduced some uncertainty and the probability of winning was not 0. When the alternative party is close enough in votes[21] it may make sense to participate and try. For an equilibrium party structure to contain more than two parties when the electoral system is pluralitarian, the distribution of preferences must differ substantially among districts (as in UK seems to be the case, allowing the Liberal party

[20] A rigorous empirical analysis of this issue would probably require first the extension of our model to a multi-dimensional setting, and hence we have to postpone to future research the confirmation and consolidation of our comparative conjecture.

[21] See Cox (1997) for a detailed discussion of the implications of closeness.

to survive).[22] Otherwise, if the distribution of preferences is more or less the same across districts, even with uncertainty it would be easy to show that the party formation game would never lead to more than two parties. Notice therefore that strategic voting, always used to explain Duverger's Law in the literature, is not necessary. The two-party system has to prevail when the system is pluralitarian even considering just the strategic behavior of political elites. Obviously, if on top of it one allows for strategic voting as well, the tendency to a two-party system is sharpened.

Which parties get formed in the party formation stage of a representative democracy with pluralitarian elections depends on the vector ψ of *ex ante* bargaining power of the initial p players and on the order of play ρ. For example, let $p = 3$ and let the left political elite be the one with the greatest bargaining power, followed by the center one and by the right one; then, if the centrist elite is the first mover in the party formation stage, it optimally chooses to send an arc just to the right-wing elite, and the latter optimally reciprocates, because the leftists would want too large a share. The equilibrium policy outcome would then be somewhere in between the center and the right positions. Thus, if political elites care primarily about private benefits, the proportional electoral system is the only one allowing a representative democracy to mimic the results of direct democracy.

4 Equilibrium Outcomes for the Quasilinear Case

It is important now to check how do our results change when using a standard utility function, where parties care both about policy outcomes and private benefits.[23] The difficulties encountered when trying to obtain clearcut results for more than three players pushed so far most authors to analyze just the three-player case. Similarly, even though for the case of lexicographic preferences our model yields clear results for every number of players, in the quasilinear case we prefer to avoid cumbersome computations by focusing on the $p = 3$ case.

4.1 Majoritarian Bargaining with Quasilinear Preferences

Let the ideological positions of the political elites on the policy space be $0 \leq \theta_1 < \theta_2 < \theta_3 \leq 1$. Let's assume now that these three elites have the same utility function, linear in private benefits and concave in the distance between the realized policy outcome and the desired

[22] See Laver & Shepsle (1994).

[23] Most papers in the literature on legislative bargaining, including the seminal work by Baron & Ferejohn, deal only with the pure private benefits case.

one:

$$u_i = x_i + 1 - \gamma(\theta^* - \theta_i)^2 \tag{8}$$

where $\gamma \in [0,1]$ determines the weight of policy outcomes in the utility function. The total sum of transferable private benefits associated to being in a winning coalition is normalized to 1 as before. Suppose that the fractions of seats w_1, w_2, w_3 expected by the three elites if they remain three parties are such that $w_i < q\ \forall i$. Recall now that when player i moves (in the after-elections game) it makes a demand x_i and a policy proposal, which we denote by y_i. A winning coalition can be formed only if the demands are compatible and if the members have proposed the same policy outcome. The Head of State chooses the order of play that maximizes the total sum of utilities, acting as social planner.

Lemma 2 *Consider three parties involved in a majoritarian bargaining game, with policy positions* $0 \leq \theta_1 < \theta_2 < \theta_3 \leq 1$ *and with a utility function as in (8):*

1. *If party 2 (the median party) is the first mover, then the equilibrium policy outcome is* $\theta^* = \frac{\theta_1 + 2\theta_2 + \theta_3}{4}$*; if party 1 or party 3 is the first mover, then* $\theta^* = \frac{2\theta_1 + \theta_2 + \theta_3}{4}$ *and, respectively,* $\frac{2\theta_3 + \theta_1 + \theta_2}{4}$*.* θ^* *depends only on who is the first mover, and not on the rest of the protocol.*

2. *The equilibrium share of private benefits for the first proposer is less than or equal to* $\frac{1}{2}$*, and converges to* $\frac{1}{2}$ *as* γ *goes to 0.*

For example, if $\theta_1 = 0$, $\theta_2 = \frac{1}{2}$, $\theta_3 = 1$, then the equilibrium policy outcome is $\frac{1}{2}$ when party 2 is the first proposer, $\frac{3}{8}$ if 1 is the first proposer, and $\frac{5}{8}$ if 3 is the first proposer. If party 2 is the first proposer its equilibrium demand of private benefits is exactly $\frac{1}{2}$ (*equal split*), while if the first proposer is one of the other two players the equilibrium demand of the first mover is strictly less than $\frac{1}{2}$, unless $\gamma = 0$.

The majoritarian bargaining subgame can be solved, following the same backward induction technique used in the proof of Lemma 2, for any number of players, and all the qualitative results can be extended to the general case. Lemma 2 confirms, in contrast with most of the results in the literature on non-cooperative coalition formation, that the first mover never obtains more than half of the private benefits. If anything, it is the party in the middle of the policy spectrum that gets slightly more (both in terms of private benefits and in terms of policy outcome), *independently* of who is the first proposer. Now we can use this equilibrium characterization of the majoritarian bargaining subgame to analyze the impact of electoral systems on the equilibrium party structure and policy outcome.

4.2 Duverger's Hypothesis and Duverger's Law

Proposition 2 *Consider* $p = 3$ *political elites and let them have quasilinear utility functions like in (8);*

1. *If the electoral system is proportional then the equilibrium party structure is* $g^0 = 1; 2; 3;$ *(*$n = p = 3$*).*
2. *If the electoral system is pluralitarian, then* $n = 2$*.*
3. *At the after-elections stage the majority coalition* $1, 3$ *can never form in equilibrium. In the pluralitarian case, which induces coalition formation at the ex ante stage,* $\theta_2 \neq \theta_m$ *is a necessary condition for the party* $1, 3$ *to have a chance to form for some order of play of the party formation game.*

This result confirms that a proportional electoral system tends to foster multipartism, while plurality rules determine strong incentives to party formation before elections, reducing thereby the actual number of parties facing elections to two. If the number of initial players is greater than 3, then there may be some party formation going on before elections even with a proportional system, but never to the point of reaching the two-party system.

The result that "disconnected" coalitions cannot be expected to form when the system is proportional seems to be consistent with the stylized facts, and it is in contrast with Austen-Smith & Banks (1988). Only when there is heterogeneity of preferences, for example when different players have different values of γ, there may be majority coalitions with the two extreme parties, as shown by Jackson & Moselle (1998).

4.3 Equilibrium Policy Outcomes

For the lexicographic case the comparison of equilibrium policy outcomes is unambiguous, with the proportional system leading to a policy always closer to the median party's position than the pluralitarian one. In the quasilinear case the comparison is less straightforward, and depends on the relative *ex ante* bargaining power of the political elites. As in the previous section, let us use the three-player case in order to illustrate the main relationships.

Proposition 3 *If the political elites have a quasilinear utility function as in (8), the comparison of equilibrium policy outcomes is as follows:*

1. *If the cost of forming a party is increasing in the distance between the policy positions of the elites forming it and if* $\psi_1 = \psi_2 = \psi_3$ *(equal ex ante bargaining power), then*

the equilibrium policy outcome of the representative democracy model is closer to $\theta_{l(\pi^*)}$ *when the electoral system is proportional than under plurality rule, for every* γ.

2. *If* $\theta_2 = \theta_m$, *for every vector of policy positions* $\theta_1, \theta_2, \theta_3$ *there exists a lowerbound* $\underline{\psi}_2(\theta_1, \theta_2, \theta_3)$ *such that the equilibrium policy outcome of representative democracy is closer to the median voter position with a pluralitarian system than with a proportional system for every* $\psi_2 > \underline{\psi}_2$.

Even though the *ex ante* bargaining power of the political elites is exogenous in this paper, one could argue that, if $\theta_2 = \theta_m$, a very high relative bargaining power for the median party is very likely, since it is basically the one deciding which party to form and the others compete to be with her. Therefore, in contrast with the prediction in the lexicographic case, Proposition 3 suggests that if parties care about policy outcomes and if the bargaining power of the median political elite is high enough, convergence towards the median voter outcome occurs with a pluralitarian system.

5 Concluding Remarks

As parties play a crucial role in the determination of public policies, economists should care about the determinants of party formation and party strategies. This is the first paper where the incentives to party formation before elections and the strategic coalition formation after elections are clearly distinguished and where the party structure is one of the equilibrium outcomes. The process of party formation has never been directly introduced in the game-theoretic representation of parliamentary systems. Baron (1989, 1991) studied self-enforcing party-like behavior, but made no explicit mention of when and why the number and size of parties change.

The second important contribution of this paper is the attempt to provide a formal model of representative democracy that applies to *every* electoral system. Besley and Coate (1997) introduced a simple model of representative democracy that is specific to systems using the first-past-the-post rule, while Baron and Diermeier (1998) provide a dynamic model of parliamentary systems valid only with proportional elections. Since these two models are very different from one another, the results cannot be compared. None of them can be extended to consider both electoral systems, nor to the explicit treatment of party formation.

Our model yields precise predictions about policy outcomes, majority coalitions, and equilibrium party structure, for every electoral system. The comparison among institutions hinted by our paper is therefore particularly relevant for European countries and for any country considering a transition or a reform in the electoral institutions.

Consistently with empirical and experimental evidence, our simple model of majoritarian bargaining in Parliament does not yield a disproportionate payoff share for the first proposer, hence reducing the impact of the order of play on payoff distribution. With this realistic modification of the way legislative bargaining is usually modeled, and using a simple version of a link formation game for the party formation stage, the model should also appear very tractable and flexible. In fact, while almost all the papers in the literature have results only for the three-player case, most of our results are already generalized, or easily generalizable, to any number of initial players. Besides these modeling innovations, the main results of the paper concern the relative performance of the two extreme kinds of electoral system in terms of party structure and policy outcomes:

1. We have shown that Duverger's hypothesis and Duverger's law can be viewed also as outcomes of the strategic behavior of political elites, rather than just as the result of strategic voting.

2. Since the equilibrium policy outcome when the elections are proportional is decided within a coalition that is formed after elections, such an outcome is sensitive, as any other outcome of any bargaining game, to the preference orderings of the parties at the bargaining table. On the other hand, when the system is pluralitarian the majority party forms before elections, and the policy platform of such a party depends on the relative *ex ante* bargaining power of the elites forming it. Because of this different sensitivity to political preferences in the two systems, the distance from the median voter outcome turns out to be smaller with proportional elections if preferences are lexicographic, but it may be smaller in the pluralitarian case when a more general utility function determines more compromises at the legislative bargaining stage.

If reproducing the outcome of direct democracy is a valuable feature, then in a system with proportional elections we should paradoxically hope that politicians care primarily about their private benefits and rent-seeking activities.

Among the many extensions of this paper that one can think of, the most important are the consideration of multidimensional policy spaces and the introduction of a dynamic setting. The interaction of office-holding and reelection motivations with the functioning of different institutions would probably yield some important results on the stability of coalitions, parties, and policies in different constitutions. Using the framework proposed here, another possible extension is the introduction of the runoff, the percentage lowerbound, the division into districts (relevant when voters have different preferences in different regions) and other institutional complexities, in order to obtain some more comparative results and to provide tools for constitutional designers. Finally, some new results could be derived on policy convergence: in fact, in the absence of contingent contracts and commitment power, convergence of the usual Downsian kind is less likely than that obtained through party formation.

Appendix

Proof of Lemma 1.

Claim 1 *A necessary condition for a graph g with an incomplete component h to be part of an equilibrium, is that there must be no $C(h) \subset T(h)$ such that, given ψ, the expected payoff $v(i;g)$ $\forall i \in C(h)$ is lower than the expected payoff if they formed party $C(h)$.*

The proof of Claim 1 is simple: if such $C(h) \subset T(h)$ existed, then the first player in $C(h)$ according to ρ among those who send arcs (reciprocated) to players in $T(h) \setminus C(h)$ can deviate by not sending those arcs, and everybody else in $C(h)$ would optimally do the same in the continuation game. Using this Claim, let us now turn to the three parts of the Lemma.

1. Since the first-stage game is finite, existence of a SPE is not a problem. If $c(ij) = 0$ $\forall ij$, it is obvious that many strategy profiles and graphs can be part of an equilibrium, as there are many profiles and graphs that lead to the same party structure π with the same 0-cost. In particular, we can easily show that if g' with superfluous links is a SPE graph, then the graph g^* without superfluous links (with all complete subgraphs) such that $\pi(g') = \pi(g^*)$ is a SPE graph as well. The following argument suffices: call $h \in g' = g(s')$ the incomplete subgraph of g', i.e., the component of g' where some direct links are missing (but not all); the graph g^* can be obtained through a strategy profile s^* that differs from s' for the fact that i does not send arcs to any $j \in T(h)$, $\forall i \in T(h)$, at any history;[24] but if s' is a SPE, then s^* must be a SPE as well, since given Claim 1 the elimination of those arcs does not alter the payoff perspectives of any player.

2. Suppose now that $c(ij) > 0$ $\forall ij$. In this case, consider any s such that $g(s)$ includes an incomplete component h with some superfluous links. Such a strategy profile s cannot be a SPE. To see this, take the first player in ρ among those who send reciprocated arcs in h according to s—call this player i; if i deviates by not sending any arc to any $j \in T(h)$, this is a profitable deviation (given the positive cost of each link) and no player has interest, nor a chance given Claim 1, to interfere.

[24] s^* could differ from s' also for some other arcs here and there, and as long as links don't change the results are unchanged.

3. Having shown that superfluous links can exist in equilibrium only when $c(ij) = 0$ for some ij, the last thing to show is that the party structure is, in any case, generically unique. In fact, the only cases in which this finite game can have multiple equilibrium party structures is when some player is indifferent between reciprocating the arc of a player or sending it to another one, or when one player is indifferent between sending an arc to i or to j; but these cases of indifference are of measure zero, since they can occur only for a finite number of values of ψ and θ in $\Delta^{p-1} \times [0,1]^p$, i.e., in the cross product of the simplex of bargaining power with the p-dimensional space of policy positions. This generic uniqueness holds for every ρ. **QED**

Proof of Proposition 1. Suppose $n^* = p \geq 3$ and let's first analyze the majoritarian bargaining stage outcomes.

Whatever the order of play r, the MWC $S^*(r)$ formed by the first players in r prevails, and the unique SPE payoff distribution is

$$x_i^*(w) = \frac{w_i'}{q'},\ \forall i \in S^*$$

where the weights' vector w' and the quota q' are the equivalent homogeneous representation of w, q.[25] If w happens to be homogeneous itself, then the share of each party in the majority coalition is exactly the ratio between the number of votes it owns and the majority quota (proportionality). In the case of $n = 3$ $\frac{w_i'}{q'} = \frac{1}{2}$. Morelli (1998a) proves all the above, and provides other cooperative and non-cooperative models all leading to the same prediction.[26]

The most important thing to show here is that for every r the unique policy outcome is $\theta_{l(\pi^*)}$. Suppose first that the median party is the first mover. In this case it demands the equilibrium share $\frac{w'_{l(\pi^*)}}{q'}$ and its own policy platform. In equilibrium all the subsequent players (at least up to the point where a MWC S can be closed) have to demand their proportional share as well, and they are better off agreeing with the median party on the median party's policy position. To see this, suppose instead that some player i in S wants to deviate demanding $\theta \neq \theta_{l(\pi^*)}$; this cannot be a profitable deviation, and leads to the exclusion of party i from the prevailing coalition, because in the continuation game the other

[25] For example, if $n = 4$ and $w = \frac{3}{7}, \frac{2}{7}, \frac{1}{7}, \frac{1}{7}$, one MWC has 5 votes and the others have 4, and hence w would not belong to a homogeneous representation. But the vector $w' = \frac{2}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}$ is equivalent to w, in the sense that the relative bargaining power of the players is unchanged, and is homogeneous. See Peleg (1968) for a precise definition of homogeneous representation of majority games, and see Morelli (1998a, 1998b) for more details and examples on the relevance and robustness of our prediction on payoff distribution in coalitional bargaining.

[26] These results on payoff distribution were obtained assuming $u_i = x_i$, i.e., that legislators cared only about private benefits. But obviously they extend to the lexicographic case.

players keep asking their proportional share and nobody demands θ because it is bound to lose against $\theta_{l(\pi^*)}$. Now suppose that some other player i, different from the median party, is the first mover: is there an equilibrium where it can propose θ closer than $\theta_{l(\pi^*)}$ to θ_i and where such θ is the equilibrium outcome? The answer is no. In fact, suppose without loss of generality that θ_i is to the left of $\theta_{l(\pi^*)}$ ($\theta_i < \theta_{l(\pi^*)}$); then, the first in r who has a position $\theta_j \geq \theta_{l(\pi^*)}$ has a profitable deviation by demanding $\theta_{l(\pi^*)}$, because all the subsequent players with a position on the same side of the median party's position would do the same optimally. So, the first mover and everybody else can be in the prevailing MWC only if it demands the median party's policy.

Given all the above, the Head of State chooses the order of play so

that the members of the S^* defined in (6) are in the first positions, but the way this subset of players is ordered does not matter for the outcome.[27] Moreover, if S^* (from (6)) is unique, the members of S^* know in advance that they will be in the prevailing MWC, and hence none of them have incentives to merge with anybody else *ex ante*. By the same token, the players who know they will not belong to the majority coalition because they have a position too distant from the median party do not want to waste resources forming a party with any other party either. Thus, if S^* is unique, $n^* = p$ is the only equilibrium number of parties. The only case in which some party formation can occur under a proportional system is when S^* is not unique, i.e., when it is not certain which set of players the Head of State will let play first.

QED

Proof of Lemma 2. Consider first the case in which party 1 is the first mover, and suppose first that party 2 is chosen as second mover. Party 1 maximizes u_1 with respect to x_1, y_1 subject to

$$u_2(x_1, y_1) \geq \max_{x_2, y_2} u_2(x_2, y_2) \text{ S.T. } u_3(x_2, y_2) \geq u_3(x_1, y_1)$$

The constrained max for player2 can be written as:

$$\max_{x_2, y_2} L_2 \equiv x_2 + 1 - \gamma(y_2 - \theta_2)^2 + \lambda[(1 - x_2) + 1 - \gamma(\theta_3 - y_2)^2 - (1 - x_1 + 1 - \gamma(\theta_3 - y_1)^2)].$$

$$\frac{\partial L_2}{\partial x_2} = 0 \;\rightarrow\; \lambda = 1;$$

Using this, the second FOC yields:

$$\frac{\partial L_2}{\partial y_2} = 0 = -4\gamma y_2 + 2\gamma\theta_2 + 2\gamma\theta_3 \rightarrow\; y_2^* = \frac{\theta_2 + \theta_3}{2};$$

[27] In the Baron & Ferejohn type of models the first proposer has instead a special role and an excessive power.

Substituting $\lambda = 1$ and $y_2 = \frac{\theta_2+\theta_3}{2}$ in the third FOC we have:

$$\frac{\partial L_2}{\partial \lambda} = 0 \rightarrow \; x_2^* = x_1 + \gamma(\theta_3 - y_1)^2 - \gamma\frac{(\theta_3-\theta_2)^2}{4}.$$

Thus, the constrained max for player2 is

$$u_2(x_2^*, y_2^*) = x_1 + \gamma(\theta_3 - y_1)^2 - \gamma\frac{(\theta_3-\theta_2)^2}{2} + 1.$$

Then player1 has to choose x_1, y_1 to maximize

$$\begin{aligned} L_1 \equiv & \quad u_1(x_1, y_1) + \beta[u_2(x_1, y_1) - u_2(x_2^*, y_2^*)] \\ = & \quad x_1 + 1 - \gamma(y_1 - \theta_1)^2 + \beta[1 - 2x_1 - \gamma(\theta_2 - y_1)^2 - \gamma((\theta_3 - y_1)^2 - \tfrac{(\theta_3-\theta_2)^2}{2})] \end{aligned}$$

$$\frac{\partial L_1}{\partial x_1} = 0 \rightarrow \; \beta = 1/2;$$

$\frac{\partial L_1}{\partial y_1} = 0$ implies

$$\theta^*(1,2,3) = y_1^* = \frac{2\theta_1 + \theta_2 + \theta_3}{4}; \tag{9}$$

$\frac{\partial L_1}{\partial \beta} = 0$ implies

$$\begin{aligned} x_1^* &= \tfrac{1}{2} - \gamma\left[\tfrac{(3\theta_2-2\theta_1-\theta_3)^2+(3\theta_3-2\theta_1-\theta_2)^2}{32} - \tfrac{(\theta_3-\theta_2)^2}{4}\right] \\ &= \tfrac{1}{2} - \gamma\tfrac{[\theta_2^2+4\theta_1^2+\theta_3^2-4\theta_1\theta_2-4\theta_1\theta_3+2\theta_3\theta_2]}{16} \\ &= \tfrac{1}{2} - \gamma\tfrac{[(\theta_2+\theta_3)-2\theta_1]^2}{16} \leq \tfrac{1}{2} \end{aligned} \tag{10}$$

So, $\lim_{\gamma\rightarrow 0} x_1^* = \frac{1}{2}$.

If player3 moves second, instead of player2, the procedure is obviously the same. The constraint for player1 would be

$$u_3(x_1, y_1) \geq \max_{x_3, y_3} u_3(x_3, y_3) \text{ S.T. } u_2(x_3, y_3) \geq u_2(x_1, y_1)$$

The constrained max for player3 can be written as:

$$\max_{x_3,y_3} L_3 \equiv x_3 + 1 - \gamma(y_3 - \theta_3)^2 + \lambda[1 - x_3 + 1 - \gamma(\theta_2 - y_3)^2 - (1 - x_1 + 1 - \gamma(\theta_2 - y_1)^2)]$$

$$\frac{\partial L_3}{\partial x_3} = 0 \;\rightarrow\; \lambda = 1;$$

$$\frac{\partial L_3}{\partial y_3} = 0 \rightarrow \; y_3^* = \frac{\theta_2 + \theta_3}{2};$$

$$\frac{\partial L_3}{\partial \lambda} = 0 \rightarrow \; x_3^* = x_1 + \gamma(\theta_2 - y_1)^2 - \gamma\frac{(\theta_3-\theta_2)^2}{4}.$$

Thus, the constrained max for player3 is

$$u_3(x_3^*, y_3^*) = x_1 + \gamma(\theta_2 - y_1)^2 - \gamma\frac{(\theta_3 - \theta_2)^2}{2} + 1.$$

Then player1 has to choose x_1, y_1 to maximize

$$\begin{aligned} L_1 \equiv & \quad u_1(x_1, y_1) + \beta[u_3(x_1, y_1) - u_3(x_3^*, y_3^*)] \\ = & \quad x_1 + 1 - \gamma(y_1 - \theta_1)^2 + \beta[1 - 2x_1 - \gamma(y_1 - \theta_3)^2 - \gamma(\theta_2 - y_1)^2 + \gamma\tfrac{(\theta_3-\theta_2)^2}{2}] \end{aligned}$$

$$\frac{\partial L_1}{\partial x_1} = 0 \rightarrow \ \beta = 1/2;$$

$\frac{\partial L_1}{\partial y_1} = 0$ implies $\theta^*(1,3,2) = \theta^*(1,2,3)$ as obtained in (9); $\frac{\partial L_1}{\partial \beta} = 0$ implies (10). So, if 1 is the first mover, then the policy outcome will be certainly the one given in (9), no matter what partner player1 has.

Consider the case in which the Head of State chooses the order $2,1,3$ (the case of $2,3,1$ is symmetric). Player2 chooses x_2, y_2 to maximize u_2 subject to

$$u_1(x_2, y_2) \geq \max_{x_1, y_1} u_1(x_1, y_1) \text{ S.T. } u_3(x_1, y_1) \geq u_3(x_2, y_2)$$

$$\max_{x_1, y_1} L_1 \equiv x_1 + 1 - \gamma(y_1 - \theta_1)^2 + \lambda[1 - x_1 + 1 - \gamma(\theta_3 - y_1)^2 - (1 - x_2 + 1 - \gamma(\theta_3 - y_2)^2)]$$

$$\frac{\partial L_1}{\partial x_1} = 0 \rightarrow \ \lambda = 1;$$

$\frac{\partial L_1}{\partial y_1} = 0$ implies

$$y_1^* = \frac{\theta_1 + \theta_3}{2};$$

$\frac{\partial L_1}{\partial \lambda} = 0$ implies

$$x_1^* = x_2 + \gamma(\theta_3 - y_2)^2 - \gamma\frac{(\theta_3 - \theta_1)^2}{4}.$$

$$u_1(x_1^*, y_1^*) = x_2 + \gamma(\theta_3 - y_2)^2 - \gamma\frac{(\theta_3 - \theta_1)^2}{2} + 1.$$

$$\max_{x_2, y_2} L_2 \equiv x_2 + 1 - \gamma(y_2 - \theta_2)^2 + \beta[1 - 2x_2 - \gamma(y_2 - \theta_1)^2 - \gamma(\theta_3 - y_2)^2 + \gamma\frac{(\theta_3 - \theta_1)^2}{2}]$$

$$\frac{\partial L_2}{\partial x_2} = 0 \rightarrow \ \beta = 1/2;$$

$\frac{\partial L_2}{\partial y_2} = 0$ implies

$$\theta^*(2,1,3) = y_2^* = \frac{\theta_1 + 2\theta_2 + \theta_3}{4}; \tag{11}$$

$\frac{\partial L_2}{\partial \beta} = 0$ implies

$$x_2^* = \frac{1}{2} - \gamma \frac{((\theta_1 + \theta_3) - 2\theta_2)^2}{16}. \tag{12}$$

If $\theta_1 + \theta_3 = 2\theta_2$ (which happens for example when $\theta_2 = \frac{1}{2}$ and the other two positions are equidistant from θ_2) then $x_2^* = \frac{1}{2}$ for every γ; otherwise it is less than that and converges to $1/2$ as γ goes to 0. If the protocol is $2, 3, 1$ instead of $2, 1, 3$ the expressions for x_2^* and y_2^* are the same, just inverting the indices for 1 and 3. Similarly to the case where 1 is the first mover, party 2 is indifferent between 1 and 3 as second mover, while the social planner is not: this is because the sum of utilities is in general different.[28]

QED

Proof of Proposition 2.

1. Suppose $n = p = 3$. If party 1 is the first proposer we know already from Lemma 2, since $x_1^* < \frac{1}{2}$, that $u_1(x_1^*, y_1^*) < \frac{3}{2}$. In particular,

$$u_1(x_1^*, y_1^*) = \frac{3}{2} - \gamma \frac{((\theta_2 + \theta_3) - 2\theta_1)^2}{8}$$

If 2 was the second mover, it would belong to the prevailing MWC, and its indirect utility would be

$$u_2(x_1^*, y_1^*) = \frac{3}{2} + \gamma \frac{\theta_2(\theta_1 + \theta_3 - \theta_2) - \theta_1\theta_3}{2}$$

$$u_1(x_1^*, y_1^*) + u_2(x_1^*, y_1^*) = 3 + \gamma \frac{4\theta_2(\theta_1 + \theta_3 - \theta_2) - 4\theta_1\theta_3 - ((\theta_2 + \theta_3) - 2\theta_1)^2}{8}$$

If instead the first proposer is party 2, then

$$u_2(x_2^*, y_2^*) = \frac{3}{2} - \gamma \frac{((\theta_1 + \theta_3) - 2\theta_2)^2}{8};$$

and party 1 as second mover would obtain

$$u_1(x_2^*, y_2^*) = \frac{3}{2} - \gamma \frac{(\theta_1^2 + \theta_2\theta_3 - \theta_1\theta_2 - \theta_1\theta_3)}{2};$$

$$u_2(x_2^*, y_2^*) + u_1(x_2^*, y_2^*) = 3 - \gamma \frac{[(\theta_1 - \theta_3)^2 + (2\theta_2 - 2\theta_1)^2]}{8}.$$

Comparing the two total sums we have $u_1(x_1^*, y_1^*) + u_2(x_1^*, y_1^*) > u_2(x_2^*, y_2^*) + u_1(x_2^*, y_2^*)$ if and only if

$$4\theta_2(\theta_1 + \theta_3 - \theta_2) - 4\theta_1\theta_3 - ((\theta_2 + \theta_3) - 2\theta_1)^2 + (\theta_1 - \theta_3)^2 + (2\theta_2 - 2\theta_1)^2 > 0$$

[28] If there is a cost of keeping coalitions and such cost is increasing in the distance of ideological positions of the members, then the first mover would not be indifferent either.

or,

$$2\theta_2\theta_3 - \theta_2^2 + \theta_1^2 - 2\theta_1\theta_3 > 0$$

Adding and subtracting θ_3^2 this becomes

$$-(\theta_3 - \theta_2)^2 + (\theta_3 - \theta_1)^2 > 0$$

which is always true because $\theta_2 > \theta_1$. All this implies that between orders $1, 2, 3$ and $2, 1, 3$ the social planner prefers $1, 2, 3$. Symmetrically, one could show following the same steps that the social planner prefers $3, 2, 1$ to $2, 3, 1$. Finally, $1, 3, 2$ and $3, 1, 2$ can be proved in pretty much the same way to be strictly dominated by all the four options above. Thus, the social planner will always choose to place party 2 second in the order. The choice between order $1, 2, 3$ and $3, 2, 1$ obviously depends on the distances $\theta_2 - \theta_1$ and $\theta_3 - \theta_2$. The Head of State chooses $1, 2, 3$ over $3, 2, 1$ iff

$$\theta_2 < \frac{\theta_3 + \theta_1}{2}.$$

As a result of all the above, party 2 knows that it will belong to the MWC in any case. Whoever between 1 and 3 has the closest position to θ_2 knows it will be the first mover and that will belong to the MWC for sure as well. Then, since two players are not willing to form a majority party with anybody *ex ante*, the equilibrium party structure is the three-party system.

2. If the system is pluralitarian, on the contrary, $n = 3$ cannot be an equilibrium number of parties: in fact, if $n = 3$ then all the seats would go to the party with the most votes and hence the other two players have incentive to merge. If $\theta_2 = \theta_m$ then, depending on the exogenous order of play of the first stage game, either 1 forms a party with 2 or 2 forms a party with 3. The party 1,3 does not form because at most it would have probability 1/2 of winning, while the other two possible pairs lead to victory with probability 1. If $\theta_2 \neq \theta_m$ then there are parameters' values such that party 1,3 could form in equilibrium, but only if, in addition to having probability 1 of winning, $c(13)$ is not greater than $c(12)$ nor than $c(23)$.

QED

Proof of Proposition 3. Assume, without loss of generality, that $\theta_2 - \theta_1 < \theta_3 - \theta_2$, so that the equilibrium order of play of the legislative bargaining game is, as shown in the proof of Proposition 2, $1, 2, 3$.[29]

[29] This happens if the proportional system allows the three parties to reach that stage, each of them with $w_i < q$.

1. Suppose first that $\psi_1 = \psi_2 = \psi_3$. In this case, recalling the assumption that the cost of forming a party is higher when the distance between the members is greater, the equilibrium policy outcome with a pluralitarian electoral system is either $\frac{\theta_1+\theta_2}{2}$ or $\frac{\theta_2+\theta_3}{2}$. On the other hand, we know from Lemma 2 that the equilibrium outcome with a proportional system is $\frac{2\theta_1+\theta_2+\theta_3}{4}$. With a proportional system the distance from the median party's position is therefore $\frac{3\theta_2-2\theta_1-\theta_3}{4}$. If the outcome with a pluralitarian system is $\frac{\theta_1+\theta_2}{2}$, the distance from the median party's position is $\frac{\theta_2-\theta_1}{2}$ that is always greater than $\frac{3\theta_2-2\theta_1-\theta_3}{4}$. If the outcome is $\frac{\theta_2+\theta_3}{2}$ the assumption that $\theta_2 - \theta_1 < \theta_3 - \theta_2$ once again guarantees that the distance from the median party's position is lower in the proportional case.

2. If $\theta_2 = \theta_m$, the disconnected party $1, 3$ can never form in equilibrium, and hence the equilibrium outcome under a pluralitarian system is either $\frac{\psi_1\theta_1+\psi_2\theta_2}{\psi_1+\psi_2}$ or $\frac{\psi_3\theta_3+\psi_2\theta_2}{\psi_3+\psi_2}$.[30] The distance between the equilibrium outcome of a representative democracy using a pluralitarian electoral system and that of direct democracy tends to 0 as ψ_2 goes to 1, recalling that $\psi_1+\psi_2+\psi_3 = 1$. Thus, if $\theta_2 \neq \frac{2\theta_1+\theta_3}{3}$, which implies that the equilibrium outcome under a proportional system does not coincide with θ_m, there must exist $\underline{\psi}_2$ such that $\forall \psi_2 > \underline{\psi}_2$ the pluralitarian system leads to an outcome closer to the median voter outcome than under a proportional system.

QED

[30] The equilibrium outcome is the former for sure if $\psi_1 < \psi_3$ and $c(12) \leq c(23)$.

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