

# **Endogenous formation of joint ventures with efficiency gains**

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*This paper studies stable structures of efficiency-enhancing joint ventures among symmetric firms. Efficiency gains that accrue to a joint venture are assumed to increase with its size. The socially efficient industry-wide joint venture is the stable outcome when membership of a joint venture is open to outside firms, but typically not when membership can be restricted.*

*Members of a large joint venture want to restrict membership for strategic reasons -- e.g., in order to keep rival firms' costs high. Side payments among firms do not eliminate the strategic incentives of members of a large joint venture to limit membership.*

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## 1. Introduction

This paper studies the endogenous formation of joint ventures among symmetric firms, where efficiency gains from a joint venture increase with the size of the venture. One example of a joint venture that generates efficiency gains is a *network* joint venture which links productive assets of member firms. The value of a network increases with the total number of interconnected facilities of member firms. For example, the value of an automated teller machine (ATM) network or a credit card network increases with the number of member banks.<sup>1</sup> Another example of an efficiency-enhancing business alliance is a *research* joint venture among firms with complementary research knowledge or assets. By exchanging technical know-how, by combining complementary R&D skills, or by avoiding duplication of research efforts, members of a research joint venture can develop a new technology at a lower cost than would be possible in the absence of collaboration (Grossman and Shapiro, 1986).

Because the formation of efficiency-enhancing joint ventures also frequently benefits consumers, antitrust authorities take hospitable attitudes toward joint ventures.<sup>2</sup> However, joint ventures among competing firms also raise a number of antitrust concerns, including the possibility that a joint venture could restrict competition by adopting exclusionary membership rules.<sup>3</sup> Indeed, joint-venture access rules have been the subject of a continuing controversy in antitrust (Baker, 1993; Balto, 1995; Carlton and Frankel, 1995a, 1995b; Carlton and Salop, 1996; Evans and Schmalensee, 1995; Hovenkamp, 1995; Schmalensee, 1995; and Salop, 1995).<sup>4</sup> In a recent antitrust case on exclusionary access rules, Dean Witter, the Discover Charge card issuer, alleged that the Visa joint venture caused it substantial competitive harm by denying membership. The Visa venture argued that its membership rules were necessary to protect its past investments in creating the network. It also argued that granting membership to Dean Witter would reduce competition in network services between Discover card and Visa card.<sup>5</sup>

In this paper, I attempt to shed some light on the controversy surrounding exclusionary access rules by studying stable structures of efficiency-enhancing joint ventures under two different access rules: Yi and Shin's (1997) simultaneous-move "Open Membership" rule and

Bloch's (1996) infinite-horizon sequential-move "Coalition Unanimity" rule. Under the Open Membership rule, the membership of a joint venture is open to all firms. Under the Coalition Unanimity rule, a joint venture forms if and only if all potential members agree to form the venture. Hence, the Coalition Unanimity rule allows exclusivity in membership.

I model efficiency gains of joint ventures by positing that each firm possesses one unit of a distinct "productive asset" which it can share with other member firms of the joint venture. A member of a large joint venture, with access to a large stock of assets, is assumed to have lower production costs than a member of a small joint venture. An important consequence of this assumption is that the industry-wide joint venture maximizes both consumer surplus and total social surplus: all firms' production costs are lower, and the industry output higher, under the industry-wide venture than under any other joint-venture structure.

Given that the industry-wide joint venture is the socially efficient joint-venture structure, the natural question is whether or not the industry-wide venture can be supported as a stable outcome of the interaction among profit-maximizing firms. The answer depends upon the membership rules. The industry-wide joint venture is the *unique* stable outcome under the Open Membership rule, because an outsider always benefits by joining a large joint venture.

However, the grand coalition is frequently *not* the equilibrium outcome under the Coalition Unanimity rule. While production efficiency alone calls for a widespread membership, *strategic* considerations may lead members of a large joint venture to restrict membership in their venture. By admitting a new member, the existing members gain access to just one more unit of productive assets, but the new member gains access to the total pool of assets of the venture. The new member firm therefore gains a competitive advantage against the existing members, and, as a result, the existing members may earn lower profits (despite their lower production costs). Hence, the existing members of a large joint venture may want to deny membership to outsiders if they are allowed to.

Since the reason for restricting membership lies in the disparity in cost reduction between existing members and the new member, I examine if the industry-wide joint venture can be supported as a stable outcome if unrestricted side payments among firms are feasible. Then, new members can compensate the existing members for their lost profits (for example,

through escalating membership fees). In a simple example, I show that the reduction in the existing members' profits can outweigh the increase in the new members' profits. Intuitively, the industry-wide joint venture need not maximize industry profit, because industry output is highest under the industry-wide joint venture. Hence, unrestricted side payments do not eliminate the incentives of members of a large venture to restrict membership. In contrast, the industry-wide joint venture emerges as the unique stable outcome in the Open Membership game, because an outsider can join a joint venture without the consent of its existing members. The current paper thus provides a theoretical identification of the circumstances under which mandated access to a joint venture is welfare-improving.

Before introducing the formal model, I briefly discuss related literature (other than the aforementioned antitrust literature on joint-venture access rules). First, this paper extends Bloch's (1995) analysis of stable joint ventures in oligopoly in the Coalition Unanimity game. Bloch examined the endogenous formation of efficiency-enhancing joint ventures (which he called "associations") for the case of linear demand functions and cost functions with constant returns to scale in both production and productive assets. He showed that, in the linear model, the unique equilibrium coalition structure of the Coalition Unanimity game consists of either the grand coalition or two coalitions of asymmetric sizes. I extend his analysis to general demand and cost functions. I identify conditions under which the industry-wide joint venture does and does not arise as the equilibrium outcome of the Coalition Unanimity game.

Second, in the current paper, members of a large joint venture succeed in keeping rival firms' costs high by restricting membership in their venture. Even though members of a large joint venture remain as competitors in the output market so that they lack *classical* market power to raise price by restricting their own output, the refusal to expand membership confers *exclusionary* market power to members of a large joint venture by disadvantaging rival firms. Thus, this paper can be viewed as an example of the "raising rivals' costs" literature which emphasizes the harm of exclusionary market power (Salop and Scheffman, 1983; Krattenmaker and Salop, 1986; Krattenmaker, Lande, and Salop, 1987).

The rest of the paper is organized as follows. Section 2 presents a homogeneous-good Cournot oligopoly model with productive assets. Section 3 examines the effects of joint-

venture formation on profits and consumer surplus. Section 4, the main part of the paper, analyzes equilibrium joint-venture structures under the two membership rules described above. Section 5 extends the model to symmetric product differentiation under the Cournot and the Bertrand oligopolies and shows that the main results in the previous sections continue to hold. Finally, Section 6 concludes.

## 2. Homogeneous-good Cournot oligopoly with productive assets

Consider a homogeneous-good Cournot oligopoly with inverse demand  $P(X)$ , where  $X$  is the industry output. In Cournot oligopoly, each firm chooses its output to maximize its profit, given the other firms' outputs. There are  $N$  ex-ante symmetric firms, which have access to a common production technology. Following Farrell and Shapiro (1990), I assume that a firm's cost of production depends on its stock of productive assets. Firm  $i$ 's cost function is given by  $c^i(x_i, a_i) = c(x_i, a_i)$ , where  $x_i$  is firm  $i$ 's output,  $a_i$  is firm  $i$ 's stock of productive assets, and  $c(\cdot, \cdot)$  is the common cost function. Let  $\mathbf{x} = (x_1, \dots, x_N)$  be the output vector,  $(a_1, \dots, a_N)$  be the vector of assets,  $U(\mathbf{x}) = \int_0^X P(s) ds$  be the gross consumption benefit of the bundle  $\mathbf{x}$  to the representative consumer,  $\pi^i(\mathbf{x}, a_i) = P(X)x_i - c(x_i, a_i)$  be firm  $i$ 's profits, and  $W(\mathbf{x}, \mathbf{a}) = U(\mathbf{x}) - \sum_{i=1}^N c(x_i, a_i)$  be the total social surplus. Denote partial derivatives by subscripts:  $c_x^i = c_x(x_i, a_i) = c(x_i, a_i)/x_i$ , for example. A Nash equilibrium of the Cournot oligopoly (henceforth called a Cournot equilibrium) is a vector of outputs  $(x_1, x_2, \dots, x_N)$  which satisfies the first-order condition

$$\pi^i(\mathbf{x}, a_i)/x_i = P(X) + P'(X)x_i - c_x(x_i, a_i) = 0, \text{ with equality if } x_i > 0, i = 1, \dots, N. \quad (1)$$

Until the end of Section 4, I assume that there exists a unique interior equilibrium so that the first-order condition (1) holds with equality for all firms. I make the following three assumptions on the demand and cost functions.

*Assumption 1.*  $c(x_i, x_{-i}) < 0$ ,  $c_x(x_i, x_{-i}) < 0$ , and  $c_{xx}(x_i, x_{-i}) > 0$ .

Assumption 1 states that an increase in a firm's stock of productive assets lowers both the total cost and the marginal cost of production and that the marginal cost is a weakly increasing function of output.

*Assumption 2.*  $P(X) < 0$  and  $P(X) + P'(X)x_i > 0$ ,  $i = 1, \dots, N$ .

Under Assumption 2, the inverse demand function slopes downward and outputs are strategic substitutes (Bulow, Geanakoplos, and Klemperer, 1985). Assumptions 1 and 2 jointly ensure that the Cournot equilibrium is "stable": if a firm expands output exogenously, then other firms produce less output in the new equilibrium, but total equilibrium output increases and the equilibrium price falls (Dixit, 1986; Farrell and Shapiro, 1990).

Let  $E(c^i) = c_x(x_i, x_{-i})x_i/c(x_i, x_{-i})$  be firm  $i$ 's output elasticity of the partial derivative of the cost function with respect to productive assets.

*Assumption 3.*  $2P(X) + P'(X)kx_1 - c_{xx}(x_1, x_{-1}) + (k-1)P'(X)[1 - E(c^1)] < 0$  for  $x_1 = \dots = x_k$  and  $x_1 = \dots = x_k > 0$ ,  $k = 2, \dots, N$ .

Under Assumption 3, an equal increase in productive assets for  $k$  firms with equal stocks of assets increases their equilibrium profits (see Lemma 3 below). For cost functions with  $E(c^i) < 1$  and  $c_{xx}(x_i, x_{-i}) > 0$ , Assumption 3 holds if  $2P(X) + P'(X)kx_1 < 0$ ,  $k = 2, \dots, N$ . For  $k = N$ , this last inequality states that the industry marginal revenue decreases with industry output. The conditions  $E(c^i) < 1$  and  $c_{xx}(x_i, x_{-i}) > 0$  are satisfied for cost functions of the form  $c(x_i, x_{-i}) = c(i) + \mu(i)x_i$ ,  $c(i) > 0$ ,  $c'(i) > 0$ ,  $\mu(i) > 0$  and  $\mu'(i) < 0$ , because  $E(c^i) = \mu'(i)x_i/[c(i) + \mu(i)x_i] < 1$  and  $c_{xx}(x_i, x_{-i}) = 0$ . When  $E(c^i) > 1$ , Assumption 3 becomes more restrictive as  $N$  increases, holding other things constant. For example, consider  $c(x_i, x_{-i}) = \mu(i)x_i^2$  and  $P(X) = a - X$ . Since  $E(c^i) = 2$  and  $c_{xx}(x_i, x_{-i}) = 2\mu(i)$ , Assumption 3 is satisfied if and only if  $\mu'(1) > (N-3)/2$ .

Notice that Assumptions 1 - 3 are jointly satisfied by weakly concave, decreasing inverse demand functions [ $P'(X) < 0$ ,  $P''(X) \leq 0$ ] and cost functions with constant returns to scale in output [ $c(x_i, y_i) = \mu(y_i)x_i$ ,  $\mu'(y_i) < 0$ ].

An inspection of the first-order condition (1) shows that any two firms that have the same stocks of productive assets produce the same level of output. Hence, these two firms earn the same level of equilibrium profits. In addition, we have the following results.

*Lemma 1.* Under Assumptions 1 and 2, in equilibrium, firm  $i$  produces more output and earns higher profits than firm  $j$  if and only if firm  $i$ 's stock of productive assets is greater than firm  $j$ 's.<sup>6</sup>

A total differentiation of firm  $i$ 's first-order condition (1) yields

$$[P'(X) + P''(X)x_i]dX + [P'(X) - c_{xx}(x_i, y_i)]dx_i - c_x(x_i, y_i)dy_i = 0, \quad (2)$$

or

$$dx_i = -\frac{c_x(x_i, y_i)}{P'(X) - c_{xx}(x_i, y_i)}dX + \frac{c_x(x_i, y_i)}{P'(X) - c_{xx}(x_i, y_i)}dy_i, \quad (3)$$

where

$$\frac{c_x(x_i, y_i)}{P'(X) - c_{xx}(x_i, y_i)} < 0 \text{ and } \frac{c_x(x_i, y_i)}{P'(X) - c_{xx}(x_i, y_i)} > 0 \quad (4)$$

under Assumptions 1 and 2. Summing (3) over  $i$  and writing  $\frac{dX}{dX} = 1$ , the output effect of changes in the stocks of productive assets is

$$dX = \frac{1}{1 + \sum_{i=1}^N \frac{c_x(x_i, y_i)}{P'(X) - c_{xx}(x_i, y_i)}} \sum_{i=1}^N \frac{c_x(x_i, y_i)}{P'(X) - c_{xx}(x_i, y_i)} dy_i \quad (5)$$

Let  $y_i = X - x_i$ . Then, by the envelope theorem, the change in firm  $i$ 's profit is given by

$$d\pi_i = P'(X)x_i dy_i - c_x(x_i, y_i)dy_i, \quad (6)$$

where the first term is the "induced output effect" and the second is the "direct cost effect" of changes in productive assets.

*Lemma 2.* Under Assumptions 1 and 2,  $x_i / c_i > 0$ ,  $x_j / c_j < 0$ ,  $X / c_i > 0$ ,  $i / c_i > 0$ , and  $j / c_j < 0$ ,  $j \neq i$ .

*Lemma 3.* Suppose that firms  $1, \dots, k$  have the same stocks of productive assets,  $k = 2, \dots, N$ . Under Assumptions 1 - 3,  $\sum_{j=1}^k i / c_j > 0$ ,  $i = 1, \dots, k$ , where  $d_j = d$  for  $j = 1, \dots, k$  and  $0$  for  $j = k+1, \dots, N$ . In particular,  $\sum_{j=1}^N i / c_j > 0$  in a symmetric equilibrium,  $i = 1, \dots, N$ , where  $d_j = d$  for  $j = 1, \dots, N$ .

*Proof.* See the Appendix.

In Lemma 2, only one firm's (firm  $i$ 's) stock of productive assets increases. Due to a lower marginal cost of production, firm  $i$  expands its output. Other firms reduce their output but total output increases and price falls. Firm  $i$  earns higher profits but other firms earn lower profits. In Lemma 3, the productive assets increase by equal amounts for a set of firms with equal stocks of assets. Lemma 3 shows that these firms' equilibrium profits increase. (The other firms earn lower equilibrium profits by Lemma 2.) In particular, starting from a symmetric equilibrium, if all firms experience an equal increase in productive assets, all firms earn higher profits if and only if Assumption 3 is satisfied (assuming Assumptions 1 - 2).<sup>7</sup> Notice that Assumption 3 is more likely to be satisfied when  $E(c^i)$  is smaller. Intuitively, production cost savings are equal to  $c(x_i, i)$ , while the losses from total output expansion are proportional to  $c_x(x_i, i)x_i$ . Hence, the smaller is  $E(c^i)$ , the more likely it is that a symmetric cost shock raises profits.

### 3. Joint-venture structures, profits and welfare



By forming a joint venture, the member firms gain access to the total pool of productive assets of members. I assume that each firm initially has one unit of productive assets so that a member of a size- $k$  joint venture has access to  $k$  "effective" units of productive assets.<sup>8</sup> This assumption implies that the efficiency gains that accrue to a joint venture increase with its size. It will be important later whether the efficiency gains increase with its size at an "accelerating" rate or "decelerating" rate -- that is, whether combined assets of firms exhibits "increasing" returns to scale ( $c_x(x_i, i) < 0, \mu(i) < 0$  for  $c(x_i, i) = \mu(i)x_i$ ) or "decreasing" returns to scale ( $c_x(x_i, i) > 0, \mu(i) > 0$  for  $c(x_i, i) = \mu(i)x_i$ ).

In order to focus on the efficiency gains from forming joint ventures, I purposely do not deal with investment decisions by venture members and, thus, the effects of membership rules on investment incentives. (See Section 5 for more discussion of this issue.)

Furthermore, I rule out the possibility that the firms use the joint venture as a vehicle to facilitate collusion in the product market.

Since the firms are *ex ante* symmetric, only the *size*, not the composition, of the joint venture matters. Thus, we can identify a joint venture by its size. Suppose that the firms form  $m$  joint ventures whose sizes are  $n_1, n_2, \dots, n_m$ , so that the resulting joint-venture structure is  $C = \{n_1, n_2, \dots, n_m\}$ ,  $\sum_{i=1}^m n_i = N$ . (For example,  $\{N\}$  is the industry-wide joint venture and  $\{1, 1, \dots, 1\}$  is the degenerate joint-venture structure where each firm forms a size-1 venture.) A member of the size- $n_i$  joint venture, in the unique Cournot equilibrium following joint-venture formation, will earn the profits of a firm with  $n_i$  units of productive assets, for all  $i = 1, \dots, m$ . Let  $\pi(n_i; C)$  denote the profits of a member of the size- $n_i$  joint venture in the joint-venture structure  $C$ .

The rest of this section ranks the per-member profits of joint ventures and then examines the effects of the formation or expansion of joint ventures on (1) consumers, (2) the profits of outsiders, and (3) the profits of insiders.

*Proposition 1.* Under Assumptions 1 and 2, a member of a large joint venture earns higher equilibrium profits than does a member of a small joint venture.

*Proposition 2.* Under Assumptions 1 and 2, the formation or merger of joint ventures increases consumer surplus but lowers equilibrium profits of members of other joint ventures.

*Proof.* Suppose that the size- $n_1$  venture, ..., the size- $n_k$  venture merge. This merger increases the effective stock of productive assets of a former member of the size- $n_i$  venture from  $n_i$  to  $\sum_{j=1}^k n_j$ , for all  $i = 1, \dots, k$ . By applying Lemma 2 in a step-by-step fashion to each of the  $\sum_{j=1}^k n_j$  firms involved in the merger, one can show that total output increases and price falls at each step. Other firms earn lower equilibrium profits. *Q.E.D.*

*Proposition 3.* Under Assumptions 1 - 3, members of a joint venture become better off if their venture merges with other ventures of equal or larger sizes.

*Proof.* Suppose that the size- $n_1$  venture, ..., the size- $n_k$  venture merge, where  $n_1 \leq \dots \leq n_k$ . Decompose the change in the effective units of productive assets into the following  $k$  steps. In step  $i$ ,  $i = 1, \dots, k-1$ , the effective stocks of assets of the former members of the size- $n_k$ , size- $n_{k-1}$ , ..., size- $n_{k-i+1}$  ventures each increase from  $n_{k-i+1}$  to  $n_{k-i}$  units. In step  $k$ , the effective stocks of assets of the merging  $\sum_{j=1}^k n_j$  firms each increase from  $n_1$  to  $\sum_{j=1}^k n_j$ . By Lemma 3, all  $k$  steps benefit the former members of the size- $n_k$  venture. *Q.E.D.*

*Proposition 4.* Under Assumptions 1 - 3, if  $n_i \leq n_j$ , a member of the size- $n_j$  joint venture becomes better off if it leaves its venture to join the size- $n_i$  venture.

Propositions 1 and 2 are direct consequences of Lemmas 1 and 2. The intuition behind Proposition 3 is simple. The members of a joint venture gain from merging with larger or equal-sized ventures, because their production costs decrease relative to other firms as well as in absolute terms. Due to the disparity in cost reduction, however, the members of larger ventures may *lose* from merging with smaller ventures. (See Example 1.) Proposition 4 can be proved in an analogous way to Proposition 3. The remaining members of the formerly size- $n_j$  venture in Proposition 4 earn lower profits, because their production costs increase both

relative to the  $n_i + 1$  firms and in absolute terms. If  $n_i > n_j$ , the existing members of the formerly size- $n_i$  venture may earn either higher or lower profits as a result of the change in the joint-venture structure because of this disparity in cost reduction.

*Proposition 5.* Under Assumptions 1 - 3, both consumer surplus and total social surplus are higher under the industry-wide joint venture than under any other joint-venture structure.

*Proof.* The result on consumer surplus follows from Proposition 2. For social surplus, first suppose that firms merge to the grand coalition (so that their costs are reduced to a new lower level) but that each firm produces its pre-merger output. Social surplus increases because production costs are reduced. Second, equalize output across firms so that each firm produces the pre-merger *average* output. This output reallocation (weakly) lowers total production cost, because marginal cost of production is (weakly) increasing in output. Third, let all firms expand their output in a symmetric way to the post-merger level. Since price is strictly above each firm's (now equal) marginal cost, this symmetric output expansion increases social surplus. *Q.E.D.*

Although total social surplus is maximized under the grand coalition, industry profits need not be. Consider the following example due to Bloch (1995).

*Example 1.*  $P(X) = a - X$  and  $c(x_i, i) = (c_0 - i)x_i$ .<sup>9</sup> The cost function exhibits constant returns to scale both in output and in productive assets. It is straightforward to show that

$$(n_i; C) = A + (N + 1)n_i - \frac{m}{j=1} n_j^2, \text{ where } A = a - c_0. \text{ }^{10} \text{ Yi (1997b) shows that } C^* =$$

$\{k^*, 1, \dots, 1\}$  maximizes industry profits, where  $k^* (N)$  is the unique positive solution to  $4A + 3(N + 1)^2[k^* + 1] - 4[N + 2][k^{*2} + (N - k^*)] = 0$  (ignoring integer constraints).

Approximately,  $k^* = \frac{(3N + 4) + \sqrt{64A/(N + 2) + (9N^2 + 8N + 16)}}{8}$ . The industry-wide joint

venture maximizes industry profits if and only if  $A > (N^3 - N^2 - 7N + 8)/4$ . Notice that  $k^* > 3N/4$ : in the industry profit-maximizing joint-venture structure, the large joint venture contains

more than three quarters of the firms and the rest of the firms form one-member ventures. Intuitively, when joint ventures of asymmetric sizes merge to the industry-wide joint venture, the production costs of members of small ventures fall by greater amounts than do the production costs of members of large ventures. Hence, it is possible that the merger to the industry-wide venture *reduces* the equilibrium outputs and profits of members of large ventures, even though the merger reduces their production costs. (For example, the merger of the size- $k$  venture and the size- $(N-k)$  venture,  $k > N/2$ , *always* reduces the equilibrium outputs and profits of members of the larger size- $k$  venture in Example 1.) Viewed another way, the merger of joint ventures has conflicting effects on industry profits: a price effect and a cost effect. The merger reduces output price, but also results in cost savings (especially for members of small ventures). If the size asymmetries among joint ventures are sufficiently large, members of small ventures do not produce much output before the merger. Therefore, the price effect dominates the cost effect, and so the merger to the industry-wide joint venture lowers industry profits. In Example 1, this occurs if and only if  $A$  is not large.

#### 4. Stable joint-venture structures

##### Rules of joint-venture formation

*Open Membership game.* Yi and Shin (1997) examine an "Open Membership" game in which membership in a coalition is open to all firms which are willing to abide by the rules of the coalition.<sup>11</sup> In this game, each firm announces an "address" simultaneously. The firms that announce the same address belong to the same coalition. Formally, each firm's action space is  $A^i = \{a_1, a_2, \dots, a_N\}$ . For each  $N$ -tuple of announcements  $a = \{a_1, a_2, \dots, a_N\} \in A^1 \times A^2 \times \dots \times A^N$ , the resulting coalition structure is  $C = \{B_1, B_2, \dots, B_m\}$ , where firms  $i$  and  $j$  belong to  $B_k$  if and only if  $a_i = a_j$  (they choose the same address). Notice that an outsider does not need the consent of the existing members of a coalition in order to join the coalition. All it has to do is simply to change its address to the one announced by members of the coalition.

*Infinite-horizon Coalition Unanimity game.* Bloch (1995, 1996) analyzes what can be called an infinite-horizon "Coalition Unanimity" game. The first firm begins the game by proposing a coalition, which forms if and only if all prospective members sequentially agree to form it. (If any potential member rejects the current proposal, the current proposal is invalidated and that firm makes a counter-proposal.) This coalition formation process continues until all firms belong to coalitions. Bloch (1996) shows that the Coalition Unanimity game yields the same stationary subgame-perfect equilibrium coalition structure as the following "Size Announcement" game: firm 1 first announces the size of its coalition  $s_1$ , and the first  $s_1$  firms form a size- $s_1$  coalition, and then firm  $s_1 + 1$  proposes  $s_2$ , and the next  $s_2$  firms form a size- $s_2$  coalition, and so on until the last firm is reached. In this game, once a coalition forms, an outsider cannot join it. Hence, the Coalition Unanimity game allows for exclusivity in membership.<sup>12</sup>

#### **Stable joint-venture structures and welfare**

*Open Membership game.*

*Proposition 6.* Under Assumptions 1 - 3, the industry-wide joint venture is the unique pure-strategy Nash equilibrium outcome of the simultaneous-move Open Membership game.

*Proof.* Consider a joint-venture structure with two or more ventures. By Proposition 4, a member of the smallest venture can increase its profit by joining a larger venture. Hence, such a joint-venture structure is not an equilibrium outcome. The industry-wide joint venture is an equilibrium outcome, because no member would be better off by changing its address and leaving the industry-wide joint venture to form a one-member venture. *Q.E.D.*

*Coalition Unanimity game.* Let  $C^u$  be the equilibrium joint-venture structure of the Coalition Unanimity game. Let  $k_0$  denote the largest value of  $k$  such that the members of a size- $(k-1)$  venture benefit by merging with a single-firm venture in any joint-venture structure.

*Proposition 7.* Suppose that  $P(X) > 0$  and that  $c(x_i, i) = \mu(i)x_i$ , with  $\mu(i) < 0$  and  $\mu(i)$  increasing in  $i$ . Then,  $k_0 = (N + 3)/2$  and  $C^u = \{k^u, N - k^u\}$ , where  $k^u = \operatorname{argmax}_{k \in \{0, \dots, N\}} \pi(k; \{k, N - k\})$ .<sup>13</sup>

*Proof.* See the Appendix.

Proposition 7 shows that the equilibrium joint-venture structure of the Coalition Unanimity game consists of either the industry-wide venture or two asymmetrically-sized ventures, for weakly concave inverse demand functions and for cost functions with constant returns to scale in output and weakly increasing returns to scale in productive assets. Notice that  $k^u$ , the size of the large coalition in  $C^u$ , is found by maximizing the *per-member* profit of the large coalition under the condition that the rest of firms form one opposing coalition. Hence, for the demand and cost functions in Proposition 7, the grand coalition is the unique equilibrium outcome of the Coalition Unanimity game if and only if  $\pi(N; \{N\}) > \pi(k; \{k, N - k\})$  for all  $k$ ,  $k_0 \leq k \leq N$ . However, this last condition is often violated. For an illustration, consider Example 1, where  $k_0 = (N + 3)/2$ . We have  $\pi(N; \{N\}) < \pi(k; \{k, N - k\})$  for all  $k > (N + 1)/2$ . Thus, the grand coalition is *not* the equilibrium outcome of the Coalition Unanimity game in Example 1. Indeed, Bloch (1995) shows that  $k^u = (3N + 1)/4$ .

Furthermore, the equilibrium joint-venture structure under the Coalition Unanimity game does *not* maximize industry profits if (but not only if)  $N \geq 9$ . For  $N \geq 9$ , the small venture in  $C^u$  has at least two members. However, the industry profit-maximizing joint-venture structure has only one venture with two or more members. Yi (1997b) shows that industry profits increase by breaking up the small joint venture in  $C^u$ . Intuitively, breaking up the small venture in  $C^u$  has three effects. First, per-unit production costs of members of the small venture increase. Second, output is shifted from members of the small venture (with high unit costs) to members of the large venture (with low unit costs). Third, total output decreases and price increases. The first effect reduces industry profits, but the second and third effects increase them. Given the significant size asymmetries of the two ventures in  $C^u$ ,

equilibrium outputs of members of the small venture are relatively small. As a result, the first effect is dominated by the second and third effects.

The next three examples illustrate the importance of returns to scale in productive assets in determining the number and sizes of equilibrium joint ventures in the Coalition Unanimity game.

*Example 2.* Consider  $P(X) = a - X$  and  $c(x_i, i) = (c_0 - i)x_i$ ,  $c_0 > 1$ . This cost function exhibits increasing returns to scale in productive assets, because  $\mu(i) = -(c_0 - 1)i^{-2} < 0$ . Hence, Proposition 7 applies: the equilibrium joint-venture structure of the Coalition Unanimity game consists of either the industry-wide venture or two ventures of asymmetric sizes. We have  $(k; \{k, N-k\}) = \left[ A + (N+1)k - \{k^{c_0+1} + (N-k)^{c_0+1} \} \right]^2$ .  $k^u$  solves  $(N+1)k^{-c_0-1} - (c_0+1)k^{-c_0} + (c_0+1)(N-k)^{-c_0-1} = 0$ . For  $c_0 = 2$ ,  $k^u = 3N^2/(4N-2)$ . For  $c_0 > 2$ , I have not been able to find an analytical solution for  $k^u$ , but simulation results show that  $k^u$  increases with  $c_0$ . For example, for  $c_0 = 10$ ,  $k^u = N$  for  $N \leq 14$  and  $k^u \approx .91N$  for a large  $N$ . For  $c_0 = 100$ ,  $k^u = N$  for  $N \leq 144$  and  $k^u \approx .99N$  for a large  $N$ . As one can expect from the simulation results,  $\lim_{c_0 \rightarrow \infty} k^u = N$ . (Suppose not. Then, from the equation that implicitly defines  $k^u$ , we have  $\lim_{c_0 \rightarrow \infty} k^u > \lim_{c_0 \rightarrow \infty} [(N+1)k^u] / [c_0+1] = N+1$ , a contradiction.)

*Example 3.* Consider  $P(X) = a - X$  and  $c(x_i, i) = (c_0 - e^{-i})x_i$ . This cost function also exhibits increasing returns to scale in productive assets:  $\mu(i) = -e^{-i} < 0$ . We have  $(k; \{k, N-k\}) = \left[ A + (N-k+1)e^k - (N-k)e^{N-k} \right]^2$ . Since  $d \left[ (N-k+1)e^k - (N-k)e^{N-k} \right] / dk = (N-k)e^k + (N-k+1)e^{N-k} > 0$  for all  $k$ ,  $1 \leq k \leq N$ , it follows that  $k^u = N$  for all  $N$ .

*Example 4.* Consider  $P(X) = a - X$  and  $c(x_i, i) = x_i/i$ . Since  $\mu(i) = 2/i^3 > 0$ , this cost function exhibits decreasing returns to scale in productive assets. We have  $(n_i; \{n_1, \dots, n_m\}) = \left[ a + m - (N+1)/n_i \right]^2$ . Define  $r_1 = 1$ , and  $r_{i+1} = I \frac{-r_i + \sqrt{r_i^2 + 4r_i(N+1)}}{2}$ ,  $i = 2, 3, \dots$ ,

where  $I(z)$  is the smallest integer which is greater than or equal to  $z$ . The Appendix shows the

following results. If a size- $r$  venture merges with a size- $r_i$  venture, the existing members of the size- $r$  venture become worse off if and only if  $r > r_{i+1}$ , independent of the joint-venture structure. Thus,  $k_0 = r_2 = I \frac{-1 + \sqrt{1 + 4(N+1)}}{2}$ . Find  $m$  such that  $\sum_{j=1}^m r_j < N < \sum_{j=1}^{m+1} r_j$ . We have  $C^u = \{N - \sum_{j=1}^{m-1} r_j, r_{m-1}, r_{m-2}, \dots, r_1\}$ . As the number of firms  $N$  increases, the number of equilibrium joint ventures increases without bound. For a reasonable  $N$ , however, the number of equilibrium joint ventures is not large. Specifically,  $C^u = \{N\}$  for  $N = 2$ ,  $C^u = \{N-1, 1\}$  for  $3 \leq N \leq 7$ ,  $C^u = \{N-r_2-1, r_2, 1\}$  for  $8 \leq N \leq 36$ ,  $C^u = \{N-r_3-r_2-1, r_3, r_2, 1\}$  for  $37 \leq N \leq 684$ , and so on.

In Example 2, the size of the larger joint venture increases with  $\alpha$ , which is a parameter for returns to scale in productive assets. In Example 3, the returns to scale in productive assets are so large that the industry-wide joint venture is the unique equilibrium outcome. In Example 4 with decreasing returns to scale in productive assets, the number of equilibrium joint ventures increases without bound as the number of firms increases. The intuition behind these examples is straightforward. The reason why the equilibrium joint-venture structure of the Coalition Unanimity game may consist of multiple ventures is that the members of a large venture (with a large pool of productive assets) may not find it in their interest to merge with a small venture (with a small pool of productive assets). Members of a large venture may lose from the merger, because their stocks of productive assets increase by a smaller margin than do those of the members of the small venture. Hence, the profitability of the merger to the members of the large venture depends on the value of the additional assets relative to the value of the existing stock of assets -- that is, returns to scale in productive assets.

### *Membership rules and welfare*

The above analysis shows that the grand coalition is the unique equilibrium coalition structure under the Open Membership rule but typically not under the Coalition Unanimity rule. Combining this result with Proposition 5 yields the main welfare result of this paper:



*Proposition 8.* Under Assumptions 1 - 3, both consumer surplus and the total social surplus are higher under the Open Membership rule than under the Coalition Unanimity rule.

The social welfare loss from exclusivity in membership can be quite substantial. For an illustration, consider the linear model in Example 1. Let  $W(C)$  be the total social surplus under the joint-venture structure  $C$ . It is straightforward to show that  $[W(\{N\}) -$

$$W(\{3N/4, N/4\})]/W(\{N\}) = \left[ 3N \left( 16(N+2)A - 2N^3 + 9N^2 + 24N \right) \right] / \left[ 64(N+2)(A+N)^2 \right].$$

For example, for  $N = 4$  and  $A = 6$ , the loss from exclusivity is 21.5% of the maximum social surplus.<sup>14</sup>

The above welfare comparison of the two membership rules assumes that there are no side payments among firms (either within a venture or across ventures). When unrestricted side payments are feasible among firms, (through escalating membership fees, for example), a natural conjecture is that the joint-venture structure which maximizes industry profits will emerge as a stable outcome. Example 1 illustrates that the grand coalition need not maximize industry profits. Therefore, without side payments from a social planner, the Coalition Unanimity rule need not support the industry-wide joint venture as the equilibrium outcome (even with side payments across ventures as well as within ventures). The Open Membership rule, on the other hand, always supports the industry-wide joint venture even in the absence of side payments of any kind. In light of these results, it is noteworthy that the competing coalitions in the emerging High-Definition TV market in the U.S. formed the "Grand Alliance" in 1993 at the strong urging of the Federal Communications Commission (FCC). Previous to the formation of the Grand Alliance, three research joint ventures -- Zenith-AT&T, NBC-Philips-Thompson-Sarnoff, and General Instruments-MIT -- had been competing for the adoption of their own broadcasting system by the FCC. The FCC claimed that by combining the best elements of different standards, the Grand Alliance could devise a technically superior format and speed the transition to the new technology (*Wall Street Journal*, May 24, 1993).

### **Corner equilibrium**

The analysis so far has assumed that all firms produce positive output under any joint-venture structure. However, in an asymmetric joint-venture structure, it is possible that members of small ventures (which have cost disadvantages against members of large ventures) may produce no output in equilibrium. Hence, different membership rules not only affect the equilibrium joint-venture structures, but also the number of active firms. The possibility of a corner equilibrium does not change the main results of this paper. No change is necessary in Lemma 1. The only change necessary in Lemmas 2 and 3 is that strict inequalities are replaced with weak inequalities when we increase the productive assets of a firm with zero production in equilibrium. Thus, Propositions 1 - 4 continue to hold when we add the qualification "weakly" in the appropriate places. (For example, in Proposition 1, a member of a large joint venture earns *weakly* higher equilibrium profits than does a member of a small joint venture.) Proposition 5 continues to hold provided that fixed costs of production are not high. No change is necessary for Proposition 6.

Introducing a corner equilibrium does change Proposition 7 (the characterization of the equilibrium joint-venture structure in the Coalition Unanimity game), as it relies upon the interior equilibrium conditions in indispensable ways. However, the industry-wide venture is not necessarily the equilibrium joint-venture structure under the Coalition Unanimity rule. The grand coalition does not necessarily maximize industry profits, either. For an illustration, consider Example 1. Suppose that  $A < N^2 - 3N + 1$  so that the one-firm venture in  $\{N-1, 1\}$  produces zero output in equilibrium. The Appendix shows the following results: (1) the industry-wide venture is the equilibrium joint-venture structure under the Coalition Unanimity rule if and only if  $A < 1$ ; (2) the industry-wide venture yields highest industry profits for  $A < 1$ ; and (3) industry profits under  $\{N\}$  are greater than under  $\{N-1, 1\}$  if and only if  $A > N + 3$  (approximately).

## 5. Symmetrically differentiated products

### Cournot competition

Suppose that firm  $i$ 's inverse demand function is given by  $p^i(\mathbf{x}) = p(x_i, y_i)$ : firm  $i$ 's inverse demand depends only on its own output and on the sum of its competitors' outputs.<sup>15</sup> Firm  $i$ 's equilibrium output satisfies the first-order condition

$$p^i(\mathbf{x}, x_i) - x_i = p(x_i, y_i) + p_x(x_i, y_i)x_i - c_x(x_i, x_{-i}) = 0, i = 1, \dots, N. \quad (1a)$$

Define  $\bar{c}_i^C = p_x(x_i, y_i) + p_{xy}(x_i, y_i)x_i$  and  $\bar{c}_i^C = 2p_x(x_i, y_i) + p_{xx}(x_i, y_i)x_i - c_{xx}(x_i, x_{-i})$ .

*Assumption 2a.* (1)  $p_x(x_i, y_i) - p_y(x_i, y_i) < 0$ ; (2)  $\bar{c}_i^C > 0$ ; (3)  $\bar{c}_i^C - \bar{c}_i^C < 0$ ; and (4)  $U(\mathbf{x})$  is a quadratic utility function, where  $x_i = \frac{1}{N} \sum_{j=1}^N x_j$ ,  $i = 1, \dots, N$ .

*Assumption 3a.*  $2p_x^1 + [p_{xx}^1 + (k-1)p_{xy}^1]x_1 - c_{xx}^1 + (k-1)p_y^1[1 - E(c^1)] < 0$ , where  $x_1 = \dots = x_k$  and  $x_1 = \dots = x_k > 0$ , for all  $k = 2, \dots, N$ .

Under Assumption 2a, (1) a firm's inverse demand decreases in its own output and in the sum of competitors' outputs, but it is more sensitive to its own output than to a competitor's output; (2) outputs are strategic substitutes; (3) the Cournot equilibrium is stable; and (4) the representative consumer has a love-of-variety preference in the sense that she weakly prefers a balanced consumption bundle to an unbalanced one (with the same total quantity). The quadratic utility function  $U(\mathbf{x}) = aX - \frac{1}{2} \sum_{i=1}^N x_i^2 - \sum_{i=1}^N \sum_{j \neq i} x_i x_j$ ,  $0 < a < 1$ , satisfies

Assumption 2a. (Bloch (1995) examines the linear demand function  $p^i(\mathbf{x}) = a - x_i - y_i$  derived from this utility function.) When the goods are homogeneous, the last part of Assumption 2a is trivially satisfied and Assumptions 2a and 3a collapse to Assumptions 2 and 3. More generally, Assumptions 1, 2a and 3a are jointly satisfied by cost functions  $c(x_i, x_{-i}) = \mu(x_{-i})x_i$ ,  $\mu'(x_{-i}) < 0$ , and by inverse demand functions with  $p_x(x_i, y_i) - p_y(x_i, y_i) < 0$  and  $p_{xx}(x_i, y_i) + p_{xy}(x_i, y_i) > 0$  (assuming that the love-of-variety condition is satisfied). As in Section 2, a total differentiation of the first-order condition (1a) yields

$$dx_i = -\frac{C}{i}dX + \frac{C}{i}d_i, \quad (3a)$$

where

$$\frac{C}{i} - \frac{C}{i-i} < 0 \text{ and } \frac{C}{i} - \frac{c_x(x_i, i)}{i-i} > 0 \quad (4a)$$

under Assumptions 1 and 2a. Summing (3a) over  $i$  yields

$$dX = \frac{1}{1+C} \sum_{i=1}^N \frac{C}{i} d_i \text{ and } dx_i = \frac{-C}{1+C} \sum_{j=1}^N \frac{C}{j} d_j + \frac{C}{i} d_i. \quad (5a)$$

Given the similarity of these equations to the corresponding equations in Section 2, it should be clear that Lemmas 1 - 3 hold for differentiated products under Assumptions 1, 2a and 3a. Hence, Propositions 1 - 6 apply to this subsection without any change. (The *only* modification that is necessary is in the proof of Proposition 5: now the equalization of output across firms in the second step also has a consumption benefit by the last part of Assumption 2a.)

*Proposition 7a.* Suppose that  $p^i(\mathbf{x}) = a - x_i - y_i$ ,  $0 < y_i < 1$ , and that  $c(x_i, i) = \mu(i)x_i$ ,  $\mu(i) < 0$  and  $\mu'(i) > 0$ . Then,  $k_0 = (N+3)/2$  and  $C^u = \{k^u, N-k^u\}$ , where  $k^u = \operatorname{argmax}_{k \in \{k, N-k\}} k$ .

*Proof.* See the Appendix.

Proposition 7a follows logically from Proposition 7 in the current paper and Proposition 3 in Bloch (1995). Bloch shows that the unique equilibrium joint-venture structure of the Coalition Unanimity game consists of either the industry-wide venture or two asymmetrically-sized ventures for  $p^i(\mathbf{x}) = a - x_i - y_i$  and  $c(x_i, i) = (c_0 - i)x_i$ . Proposition 7a extends Bloch's result to cost functions with increasing returns to scale in productive assets.<sup>16</sup>

Finally, Proposition 8 continues to hold, because the grand coalition need not be the equilibrium joint-venture structure under the Coalition Unanimity game. For example, Bloch

(1995) shows that  $k^u = (3N - 1)/4 + 1/(2)$  for linear demand and cost functions. Hence, ignoring integer constraints,  $\{N\}$  is the equilibrium coalition structure under the Coalition Unanimity game if and only if  $2/(N + 1)$ .

### Bertrand competition

Suppose that firm  $i$ 's demand function is given by  $x^i(\mathbf{p}) = x(p_i, q_i)$ , where  $\mathbf{p} = (p_1, \dots, p_N)$  and  $q_i = \sum_{j \neq i} p_j$ ; firm  $i$ 's demand depends only on its own price and on the sum of the competitors' prices. For example, Bloch (1995) shows that inverting  $p^i(\mathbf{x}) = a - x_i - y_i$  yields  $x^i(\mathbf{p}) = a - p_i + q_i$ , where  $a/[1 + (N - 1)]$ ,  $[1 + (N - 2)]/[1 - ]/[1 + (N - 1)]$ , and  $/[1 - ]/[1 + (N - 1)]$ . Firm  $i$ 's profit is  $\pi^i(\mathbf{p}, q_i) = x(p_i, q_i)p_i - c(x(p_i, q_i), q_i)$ . The Bertrand equilibrium is a vector of prices which satisfies the first-order condition

$$\pi^i(\mathbf{p}, q_i) / p_i = x(p_i, q_i) + [p_i - c_x(x(p_i, q_i), q_i)]x_p(p_i, q_i) = 0, i = 1, \dots, N. \quad (1b)$$

Define  $B_i^B = [1 - c_{xx}(x_i, w_i)]x_p(p_i, q_i)x_q(p_i, q_i) + [p_i - c_x(x_i, w_i)]x_{pq}(p_i, q_i)$  and  $B_i^B = [2 - c_{xx}(x_i, w_i)]x_p(p_i, q_i)x_p(p_i, q_i) + [p_i - c_x(x_i, w_i)]x_{pp}(p_i, q_i)$ .

*Assumption 2b.* (1)  $x_p(p_i, q_i) < 0$ ,  $x_q(p_i, q_i) > 0$ ,  $x_p(p_i, q_i) + (N - 1)x_q(p_i, q_i) < 0$ ; (2)  $B_i^B > 0$ ; (3)  $B_i^B + (N - 1)B_i^B < 0$ ; and (4)  $U(\mathbf{x}) = U(\mathbf{x}')$ , where  $x_i = \sum_{j=1}^N x_j / N$ ,  $i = 1, \dots, N$ .

*Assumption 3b.*  $[2 - c_{xx}^i x_p^i][x_p^i + (N - 1)x_q^i] + [p_i - c_x^i][x_p^i + (N - 1)x_{pq}^i] - (N - 1)x_q^i[1 - E(c_w^i)] < 0$  for  $p_i > c_x(x_i, q_i)$ ,  $i = 1, \dots, N$ .

Under Assumption 2b, (1) a firm's demand function decreases in its own price and increases in the sum of competitors' prices, but decreases when all firms increase prices by the same amount; (2) prices are strategic complements; (3) the Bertrand equilibrium is stable so that if a firm increases its price exogenously, all firms' prices rise in the new equilibrium (see

equations (3) - (5) below); and (4) the representative consumer prefers variety. Assumption 3b is the counterpart of Assumption 3a and ensures that a symmetric favorable cost shock to symmetric firms increases their equilibrium profits. Assumptions 1, 2b and 3b are jointly satisfied by cost functions  $c(x_i, i) = \mu(i)x_i$ ,  $\mu(i) < 0$ , and by weakly concave demand functions with  $x_p(p_i, q_i) + (N-1)x_q(p_i, q_i) \geq 0$  and  $x_{pp}(p_i, q_i) + (N-1)x_{pq}(p_i, q_i) \leq 0$ : an equal increase in all prices reduces each firm's demand and the slope of each firm's demand function.

A total differentiation of the first-order condition (1b) yields

$$dp_i = \frac{B}{i} dP - \frac{B}{i} d_i, \quad (3b)$$

where

$$P = p_j, 1/N > \frac{B}{i} \frac{\frac{i^B}{B} - \frac{i^B}{B}}{i - i} \geq 0 \text{ and } \frac{B}{i} \frac{c_x(x_i, i)x_p(p_i, q_i)}{\frac{B}{i} - \frac{B}{i}} > 0 \quad (4b)$$

under Assumptions 1 and 2b. Summing (3b) over  $i$  yields

$$dP = \frac{-1}{1 - \frac{B}{N}} \sum_{i=1}^N \frac{B}{i} d_i \text{ and } dp_i = \frac{-\frac{B}{i}}{1 - \frac{B}{N}} \sum_{j=1}^N \frac{B}{j} d_j - \frac{B}{i} d_i. \quad (5b)$$

Lemmas 1 - 3 and Propositions 1 - 6 continue to hold under the modified assumptions. (A short proof of Lemma 3 appears in the Appendix. The proof of Proposition 5 needs little modification. Since all prices are lower, and thus industry output is higher, under the industry-wide joint venture than under any other joint-venture structure, an analogous three-step proof establishes Proposition 5.)

*Proposition 7b.* Suppose that  $x^i(\mathbf{p}) = \frac{1}{N} - p_i + q_i$ ,  $0 < q_i \leq 1/[N-1]$ , and that  $c(x_i, i) = \mu(i)x_i$ ,  $\mu(i) < 0$  and  $\mu(i) \geq 0$ . Then,  $k_0 = (N+3)/2$  and  $C^u = \{k^u, N-k^u\}$ , where  $k^u = \arg\max_{k \in \{0, \dots, N\}} (k; \{k, N-k\})$ .

*Proof.* See the Appendix.

Finally, Proposition 8 continues to hold under the differentiated Bertrand oligopoly, because the grand coalition is not necessarily the equilibrium coalition structure of the Coalition Unanimity game.

## 6. Conclusion

I have studied stable structures of joint ventures among symmetric firms under the assumption that efficiency gains from a joint venture increase with its size. This assumption implies that the industry-wide joint venture is the socially efficient joint-venture structure: all firms' production costs are lowest and the industry output is highest under the industry-wide venture. Accordingly, I have focused on the stability of the industry-wide venture under alternative membership rules. The Open Membership rule sustains the industry-wide venture as the unique equilibrium outcome. In contrast, the Coalition Unanimity rule typically leads to an inefficient joint-venture structure, because members of a large joint venture restrict membership for *strategic* reasons: by denying access to the productive assets of their joint venture, members of a large venture succeed in keeping rival firms' costs high.

Furthermore, side payments among firms do not eliminate the incentives of members of a large venture to restrict access to their venture. Through a simple example, I have demonstrated that the industry-wide joint venture is not necessarily the industry profit-maximizing joint-venture structure. Hence, as long as existing members of a large joint venture can restrict membership in their venture, the grand coalition does not necessarily arise as the outcome of interfirm negotiations with unrestricted side payments.

In order to concentrate on efficiency gains that arise from pooling productive assets of members of a joint venture, I have not examined the effects of membership rules on firms' long-run investment incentives. Endogenizing investments (which determine a firm's short-run stock of productive assets) raises new interesting questions. First, if the membership of a joint venture evolves over time, open access to a joint venture may dampen investment incentives of early

member firms. Indeed, the antitrust literature on joint ventures recognizes the potential for free-riding by latecomers and endorses a membership fee structure which would require latecomers to compensate the early members for their investments (e.g., Carlton and Salop, 1996). In this context, the results of this paper suggest that member firms of a large venture have strategic incentives to demand fees from new members which are higher than necessary to recoup their investments.

Second, if joint-venture members make their long-run investments collectively in order to maximize their joint profits *after* joint-venture formation, they will reduce their investments if and only if a member firm's investment has negative effects on other member firms' joint profits. In the current paper, a firm's investment which increases its stock of productive assets reduces other firms' profits (Lemma 2), so that joint-venture formation will lead to a reduction in investment. Then, the industry investments under the industry-wide joint venture will be lower than under several competing joint ventures. This reduction in investment is analogous to the output reduction by a cartel (Deneckere and Davidson, 1985; Perry and Porter, 1985; Salant, Switzer, and Reynolds, 1983)<sup>17</sup>. Elsewhere, I investigate stable coalition structures in the context of output cartels in a linear homogeneous-good Cournot oligopoly model (Yi, 1997a). There, I show that, due to free-riding problems, the grand coalition is not a stable outcome under the Open Membership rule for  $N \geq 3$ . Bloch (1996) shows that the grand coalition is not the equilibrium outcome of the Coalition Unanimity game for  $N \geq 6$ . These results suggest that the grand coalition typically will not be a stable outcome under various membership rules when firms form coalitions in order to internalize externalities associated with investments.<sup>18</sup> In contrast, the grand coalition is a stable outcome under the Open Membership rule when symmetric firms form coalitions in order to share their existing productive assets, as in the current paper.

Finally, I have assumed symmetric firms. Extending the analysis to asymmetric firms is a nontrivial exercise, because the *composition* of a joint venture matters. However, the results of this paper suggest that big firms with large stocks of productive assets have strategic incentives to demand disproportionately higher membership fees from small firms with small stocks of productive assets. Indeed, when Sematech, a joint venture among U.S.



semiconductor manufacturers, was founded in 1988, small firms complained that its membership fee structure (1% of a member firm's semiconductor sales revenue with a minimum contribution of \$1 million and a maximum of \$15 million) discriminated against them (Irwin and Klenow, 1996). It would be interesting to conduct a formal analysis of membership fee structures of joint ventures among asymmetric firms.

## Appendix

Proofs of Lemma 3, Propositions 7, 7a, 7b, Example 4, and corner equilibrium follow.

*Proof of Lemma 3.* The first  $k$  firms have the same stock of productive assets, equal to  $x_1$ ; hence they have the same output equal to  $x_1$ , the same profit equal to  $\pi_1$ , the same  $\pi$  equal to  $\pi_1$  and the same  $\pi$  equal to  $\pi_1$ . Thus, without loss of generality, I show that the claim holds for firm 1.

$$\begin{aligned}
 \sum_{j=1}^k \frac{1}{j} &= P(X)x_1 - \sum_{j=1}^k \frac{y_1}{j} - c(x_1, \pi_1) = P(X)x_1 - \frac{k(1+\pi_1)}{1+\pi_1} - 1 - c^1 \\
 &= \frac{-P(X)x_1 - 1}{1+\pi_1} - \sum_{j=k+1}^N \frac{1}{j} - (k-1) + \frac{P(X) - c_{xx}^1}{P(X)E(c_w^1)}(1+\pi_1) \\
 &= \frac{-x_1 - 1}{(1+\pi_1)E(c_w^1)} \left[ -(k-1)P(X)E(c_w^1) + (P(X) - c_{xx}^1)(1+k-1) \right] \\
 &= \frac{-x_1 - 1}{(1+\pi_1)E(c_w^1)} \left[ 2P(X) + P(X)kx_1 + (k-1)P(X)[1 - E(c_w^1)] - c_{xx}^1 \right]
 \end{aligned}$$

$> 0$ , where the first inequality follows from  $\sum_{j=k+1}^N \frac{1}{j} > 0$  and the second inequality follows from Assumption 3. *Q.E.D.*

*Proof of Proposition 7.* Follows from the following two Lemmas.

*Lemma A1.* Suppose that Assumptions 1 - 3 hold. If  $k_0 \leq N/2$ , then  $C^u = \{k^u, N-k^u\}$ , where  $k^u = \operatorname{argmax}_{k_0 \leq k \leq N} (k; \{k, N-k\})$ .

*Proof.* Suppose that  $s_1, \dots, s_m$  are the announcements of joint-venture sizes in the stationary subgame perfect equilibrium of the Size Announcement game, and thus of the Coalition Unanimity game. The proof consists of three steps. First, I show that  $s_{m-1} \leq k_0$ . Second,  $s_{m-2} + s_m \leq k_0 + 1$ . Third,  $k_0 \leq N/2$  implies that  $m \leq 2$ .

(Step 1) Suppose that  $s_{m-1} < k_0$ . Then,  $(s_{m-1}; \{s_1, \dots, s_{m-1}, s_m\}) > (s_{m-1}; \{s_1, \dots, s_{m-1}, 1, s_{m-1}\}) < (s_{m-1}+1; \{s_1, \dots, s_{m-1}+1, s_{m-1}\})$ . The first inequality follows from Proposition 2 and the second from  $s_{m-1} + 1 \leq k_0$ . Thus, the firm announcing  $s_{m-1}$  can earn a higher payoff by instead declaring  $s_{m-1} + 1$ . (If the next announcer does not propose  $s_{m-1} - 1$ , the deviator becomes even better off.)

(Step 2) Suppose that  $s_{m-2} + s_m < k_0 + 1$ . Then,  $(s_{m-2}; \{s_1, \dots, s_{m-2}, s_{m-1}, s_m\}) > (s_{m-2}; \{s_1, \dots, s_{m-2}, s_{m-1}, 1, 1, \dots, 1\}) < (s_{m-2}+1; \{s_1, \dots, s_{m-2}+1, s_{m-1}, 1, \dots, 1\}) < (s_{m-2}+2; \{s_1, \dots, s_{m-2}+2, s_{m-1}, 1, \dots, 1\}) < \dots < (s_{m-1}+s_m; \{s_1, \dots, s_{m-2}+s_m, s_{m-1}\})$ . The first inequality follows from Proposition 2 (the size- $s_m$  venture is broken into  $s_m$  size-1 ventures), and the second through last inequalities follow from successively applying the definition of  $k_0$  (since  $s_{m-2}$  is less than  $k_0$ , merging with a size-1 venture is beneficial to the size- $s_{m-2}$  venture, and so on). Thus, the firm announcing  $s_{m-2}$  can earn a higher payoff by declaring  $s_{m-2} + s_m$  instead. (If the next announcer does not propose  $s_{m-1}$ , the deviator becomes even better off by Proposition 2.)

(Step 3) Suppose that  $m > 2$ . Then,  $s_{m-2} + s_{m-1} + s_m \leq 2k_0 + 1 > N$ , a contradiction. Hence, the number of equilibrium ventures is at most two. Given that its announcement of  $k$  is followed by  $N - k$ , firm 1 chooses  $k$  which maximizes  $(k; \{k, N-k\})$  over  $k_0 \leq k \leq N$ .

*Q.E.D.*

*Lemma A2.* (1) Suppose that Assumptions 1 - 3 hold. Further suppose that  $\frac{\partial P(X)}{\partial x_i} > \frac{\partial P(X)}{\partial x_j}$  for all  $i = 1, \dots, N$  and that  $\frac{\partial P(X) - c_{xx}(x_i, x_i)}{\partial x_i} > \frac{\partial P(X) - c_{xx}(x_j, x_j)}{\partial x_j}$  for  $i < j$ . Then  $k_0 = (N+1)/2$ .

(2) Suppose that  $P(X) > 0$  and that  $c(x_i, \mu_i) = \mu_i x_i$ , with  $\mu_i < 0$  and  $\mu_i > 0$ . Then  $k_0 = (N + 3)/2$ .

*Proof.* (1) Consider a size- $k$  venture created by a merger of a size- $(k-1)$  venture (consisting of firms  $1, \dots, k-1$ ) and a size-1 venture (consisting of firm  $k$ ). This merger increases an existing member's (say firm 1's) effective stock of productive assets by one unit and the new member's by  $k - 1$  units. Consider the following infinitesimal change in the stocks of assets  $d = (d_1, \dots, d_{k-1}, (k-1)d_k, 0, \dots, 0)$ , where the first  $k - 1$  entries are  $d_j$ , the  $k$ -th entry is  $(k-1)d_k$ , and the rest 0. Then the change in a member firm's (say firm 1's) profit due to the membership expansion can be obtained by integrating  $\sum_{j=1}^k \frac{1}{x_j}$  over  $d$  from 0 to 1. For the above  $d$ , we have  $\sum_{j=1}^k \frac{1}{x_j} d_j = (k-1) \frac{1}{x_1} d_k$ , using the fact that the first  $k - 1$  firms have the same effective stock of productive assets and thus the same  $x_j$ , which is equal to  $x_1$ . These firms have the same  $\mu_j$  as well, equal to  $\mu_1$ . Hence,

$$\begin{aligned} \sum_{j=1}^k \frac{1}{x_j} d_j &= P(X) x_1 \frac{(1 + \mu_1)}{1 + \mu_1} (k-1) \frac{1}{x_1} d_k - c^1 \\ &= -P(X) x_1 \frac{(1 + \mu_1)}{1 + \mu_1} (k-1) \frac{1}{x_1} d_k + \frac{1}{1 + \mu_1} \frac{P(X) - c_{xx}(x_1, \mu_1)}{P(X) E(c^1)} \\ &= -P(X) x_1 \frac{(1 + \mu_1)}{1 + \mu_1} (k-1) \frac{1}{x_1} d_k + \frac{1}{1 + \mu_1} \sum_{j=k}^N \mu_j d_j - (k-2) \\ &> -P(X) x_1 \frac{(1 + \mu_1)}{1 + \mu_1} (N - k + 1) \frac{1}{x_1} d_k - (k-2) > 0 \text{ if and only if } k > (N + 1)/2. \end{aligned}$$

(2) Now suppose that  $P(X) > 0$  and that  $c(x_i, \mu_i) = \mu_i x_i$ , with  $\mu_i > 0$ . Then,  $E(c^i) = 1$  and  $\mu_i = 1$  for all  $i = 1, \dots, N$ . Furthermore,  $\mu_1 = \mu_2 / P(X) = \mu_k = \mu_{k+1} / P(X)$  if and only if  $\mu_1 = \mu_k$ . Making these changes in the above derivation yields  $\sum_{j=1}^k \frac{1}{x_j} d_j > 0$  if and only if  $k > (N + 3)/2$ . *Q.E.D.*

*Example 4.* Suppose that a size- $r$  venture merges with a size- $r_i$  venture. Before the merger, a member of the size- $r$  venture earns  $[(a - (N + 1)/r + m)]^2$ , where  $m$  is the number of joint ventures. After the merger, it earns  $[a - (N + 1)/(r + r_i) + m - 1]^2$ . Hence, the merger reduces the profit of a member of the size- $r$  venture if and only if  $r(r + r_i) - (N + 1)r_i > 0$ , or  $r > r_{i+1}$ .

*Lemma A3.* Suppose that  $s_1, \dots, s_i$  have been announced and that  $k$  firms are left. Let  $R(t) = \sum_{j=1}^t r_j$ . Find  $t$  such that  $R(t) < k < R(t+1)$ . Regardless of history, firm  $N - k + 1$ 's best strategy is to announce  $k - R(t-1)$ .

*Proof.* By induction on  $k$ . Suppose that  $k = 1$ . Then there is only one firm left and firm  $N$ 's only feasible strategy is to announce 1 ( $= r_1$ ). Hence, the claim is correct. Now suppose that the claim holds for  $k = 1, \dots, k-1$  and that there are  $k$  firms left,  $k \geq 2$ . If firm  $N-k+1$  proposes  $k - R(t-1)$  as prescribed, then by induction hypothesis, the following firms announce  $r_{t-1}, \dots, r_1$ . Firm  $N-k+1$  earns  $[a + i + t - (N + 1)/(k - R(t-1))]^2$ . There are three kinds of deviations firm  $N-k+1$  can consider. First, suppose that firm  $N-k+1$  instead proposes  $r < k - R(t)$ . Then, for the next announcer,  $R(t) < k - r < R(t+1)$ . By induction hypothesis, the remaining firms announce  $k - r - R(t-1), r_{t-1}, \dots, r_1$ . Firm  $N-k+1$  earns  $[a + i + t + 1 - (N + 1)/r]^2 < [a + i + t - (N + 1)/(k - R(t-1))]^2$ . (By declaring  $k - R(t-1)$  instead of  $r$ , firm  $N-k+1$  merges the size- $r$  venture (to which it belongs) and the size- $(k - r - R(t-1))$  venture. Since  $r < r_{t+1}$  and since  $k - r - R(t-1) > r_t$ , this merger is beneficial to firm  $N-k+1$ .) Second, suppose that firm  $N-k+1$  instead chooses  $r, k - R(t) < r < k - R(t-1)$ . Then, for the next announcer,  $R(t-1) < k - r < R(t)$ . By induction hypothesis, the remaining firms announce  $k - r - R(t-2), r_{t-2}, \dots, r_1$ . Firm  $N-k+1$  earns  $[a + i + t - (N + 1)/r]^2 < [a + i + t - (N + 1)/(k - R(t-1))]^2$ , because  $r < k - R(t-1)$ . Finally, suppose that firm  $N-k+1$  instead declares  $r > k - R(t-1)$ . Find  $t$  such that  $R(t-2) < k - r < R(t-1)$ ,  $t \geq 2$ . By induction hypothesis, the remaining firms announce  $k - r - R(t-3), r_{t-3}, \dots, r_1$ . Firm  $N-k+1$  earns  $[a + i + t - 1 - (N + 1)/r]^2 < [a + i + t - 1 - (N + 1)/(k - R(t-1))]^2$ , where  $r = k - R(t-2)$ . But  $< [a + i + t - (N + 1)/(k - R(t-1))]^2$ . (By declaring  $r$  instead of  $k - R(t-1)$ , firm  $N-k+1$  merges the size- $(k - R(t-1))$  venture (to which it belongs) and the size- $r_{t-1}, \dots, r_1$  ventures. Decompose this merger into  $(t - t + 1)$  steps. In step 1, the size- $(k - R(t-1))$  venture merges with the size- $r_{t-1}$  venture.

In step 2, the merged venture (whose size is  $k - R(t-2)$ ) joins with the size- $r_{t-2}$  venture, and so on until it merges with the size- $r_{t-1}$  venture. Step 1 reduces firm  $N-k+1$ 's profit, since  $k - R(t-1) > r_t$ . Step 2 further reduces firm  $N-k+1$ 's profit, since  $k - R(t-2) > r_t + r_{t-1} > r_{t-1}$ .

Similarly, the remaining steps reduce firm  $N-k+1$ 's profit.) Q.E.D.

It follows from Lemma A3 that  $C^u = \{N - \sum_{j=1}^{m-1} r_j, r_{m-1}, r_{m-2}, \dots, r_1\}$ , where  $\sum_{j=1}^m r_j < N < \sum_{j=1}^{m+1} r_j$ . Finally, when  $N$  is very large,  $r_{i+1} \approx \sqrt{r_i N}$ . Since  $r_1 = 1$ ,  $r_{i+1} \approx N^{1-(1/2)^i}$ . For any finite  $m$ ,  $\sum_{j=1}^m r_j \approx \sum_{j=1}^m N^{1-(1/2)^j} < N$ . Thus, the number of equilibrium joint ventures increases without bound when  $N$  approaches infinity.

*Corner equilibrium.* Consider  $C = \{n_1, \dots, n_m\}$ , where  $n_1 = \dots = n_{i-1} > n_i \geq \dots \geq n_m$ .

(Case 1)  $A < 1$ . I show that members of all joint ventures other than the largest ones produce zero output in equilibrium. The proof is by induction. First, a member of the size- $n_m$  venture produces no output in equilibrium if and only if  $A + n_m - \sum_j (n_j - n_m) \leq 0$ , where the summation runs only over active firms. We have  $A + n_m - \sum_j (n_j - n_m) n_j = A + n_m - n_1 = A - 1 \leq 0$ . Next, suppose that the size- $n_j$  venture shuts down,  $j = i+1, \dots, m$ . A member of the size- $n_i$  venture produces no output in equilibrium, because  $A + n_i - \sum_{j=1}^i (n_j - n_i) n_j = A + n_i - n_1 = A - 1 \leq 0$ . Now, if  $i = 2$  so that  $n_1 > n_2$ , the members of the size- $n_1$  venture each earn  $[A + n_1]^2 / [n_1 + 1]^2$ . Since  $A < 1$ , they become better off by admitting an outsider firm into their venture until the industry-wide venture is formed. If  $i \geq 3$ , the merger of the size- $n_1$  venture through the size- $n_{i-1}$  ventures increases the profits of their members (Proposition 3). In both cases, we have  $(N; \{N\}) > (n_j; C)$  for all  $j = 1, \dots, m$  and all  $C$ . Thus, the industry-wide venture is the unique equilibrium outcome of the Coalition Unanimity game. It also yields the highest industry profits.

(Case 2)  $1 < A \leq N^2 - 3N + 1$ . Let  $\pi(C)$  denote the industry profit under the coalition structure  $C$ . Since the size-1 venture produces no output in  $\{N-1, 1\}$ , we have  $\pi(\{N-1, 1\}) = (N-1)[A + (N-1)]^2 / N^2$ . Since  $\pi(\{N\}) = N[A + N]^2 / (N+1)^2$ , we have  $\pi(\{N\}) > \pi(\{N-1, 1\})$  if and only if  $A < \frac{2N^2 - 1 + \sqrt{N^6 + 2N^5 + N^4 - 3N^3 - N^2 + N}}{N^2 - 1} = N + 3$ .

*Proof of Proposition 7a.* Follows from combining the following Lemma with Lemma A1.

*Lemma A2a.* (1) Suppose that Assumptions 1, 2a and 3a hold. Further suppose that (a)  $p_{xy}(x_i, y_i) = 0$ ; (b)  $E(c^i) = 1$  for all  $i = 1, \dots, N$ ; and (c)  $\frac{C}{i} \frac{2(1 + \frac{C}{i}) - (N-1) \frac{C}{i}}{(N-1) \frac{C}{i}} = \frac{C}{j}$  for

$i = j$ . Then  $k_0 = (N + 1)/2$ .

(2) Suppose that  $p^i(\mathbf{x}) = a - x_i - y_i$ ,  $0 < a < 1$ , and that  $c(x_i, y_i) = \mu(i)x_i$ ,  $\mu(i) < 0$  and  $\mu(i) > 0$ . Then  $k_0 = (N + 3)/2$ .

*Proof.* (1) As in Lemma A2, consider  $d = (d_1, \dots, d_k, (k-1)d_{k+1}, 0, \dots, 0)$ .

$$\begin{aligned} \frac{\partial}{\partial x_1} \ln P_y^1 &= (P_y^1)^{-1} \frac{\partial P_y^1}{\partial x_1} = \sum_{j=1}^k \frac{y_j}{x_1} - c(x_1, y_1) \\ &= -(P_y^1)^{-1} \frac{\partial}{\partial x_1} \left[ \frac{(1 + \frac{C}{1})}{1 + \frac{C}{1}} (k-1) \left( \frac{C}{1} + \frac{C}{k} \right) + \frac{C}{1} + \frac{\frac{C}{1} \left[ \frac{C}{1} - \frac{C}{1} \right]}{P_y(x_1, y_1) E(c^1)} \right] \\ &\quad - \frac{(P_y^1)^{-1} \left[ 1 + \frac{C}{1} \right] x_1 \left[ \left[ 1 + \frac{C}{1} - (k-1) \frac{C}{1} \right] \frac{C}{1} - (k-1) \frac{C}{1} \frac{C}{k} \right]}{1 + \frac{C}{1}} \end{aligned}$$

0 if and only if  $k = (N + 1)/2$ . The first inequality follows from  $\left[ \frac{C}{1} - \frac{C}{1} \right] / P_y^1 = 1 / \frac{C}{1}$  and from  $E(c^1) = 1$ . The second inequality follows from condition (c).

(2) For  $p^i(\mathbf{x}) = a - x_i - y_i$  and  $c(x_i, y_i) = \mu(i)x_i$ , we have  $\left[ \frac{C}{1} - \frac{C}{1} \right] / P_y^1 = 1 / \frac{C}{1}$ ,  $E(c^1) = 1$  and  $\frac{C}{1} = N \frac{C}{1}$ . And  $\mu(i) > 0$  implies that  $\frac{C}{1} = \frac{C}{k}$  if and only if  $1 = k$ . Making these changes in the above derivation shows that  $\frac{\partial}{\partial x_1} \ln P_y^1 = 0$  if (but not only if)  $k = (N + 3)/2$ . *Q.E.D.*

Notice the similarities and differences between Lemmas A2 and A2a. While the first part of Lemma A2 *technically* extends to differentiated products, the condition that  $\frac{C}{i} = (N - 3)/(N + 1)$  may be violated. For example, consider  $p^i(\mathbf{x}) = a - x_i - y_i$  and  $c(x_i, y_i) = \mu(i)x_i$ .

$\frac{C}{i} = \frac{1}{[2 - ]}$  approaches 0 as  $\frac{1}{i}$  approaches 0, thereby violating the above condition. Hence, Lemma A2a relies on new conditions. The condition that  $p_{xy}(x_i, y_i) = 0$  in Lemma A2a corresponds to  $P(X) = 0$  in Lemma A2. Unfortunately, the last condition in the first part of Lemma A2a depends on  $\frac{C}{i}$ , the sum of *all* firms' responsiveness parameters. As a result, I have not been able to obtain general sufficient conditions on the demand and cost functions which satisfy the last condition in the first part of Lemma A2a. An exception occurs for linear demand functions, as reported in the second part of Lemma A2a.

*Proof of Lemma 3 for the differentiated Bertrand oligopoly.*

$$\begin{aligned} \frac{1}{j} \frac{d}{dx} \left[ \frac{1}{j} \right] &= [p_1 - c_x^1] x_q^1 \frac{1}{j} \frac{d}{dx} \left[ \frac{q_1}{j} - c(x_1, 1) \right] \\ &= \frac{[p_1 - c_x^1] \frac{B}{1 - \frac{B}{i}} E(c_w^1)}{[p_1 - c_x^1] \frac{B}{1 - \frac{B}{i}} E(c_w^1)} - x_q^1 E(c_w^1) \left\{ (k-1) + \frac{N}{j=k+1} \frac{B}{j} \right\} + \left( \frac{B}{i} - \frac{B}{i} \right) \left\{ 1 - k \frac{B}{1} - \frac{N}{j=k+1} \frac{B}{j} \right\} \\ &= \frac{[p_1 - c_x^1] \frac{B}{1 - \frac{B}{i}} E(c_w^1)}{[p_1 - c_x^1] \frac{B}{1 - \frac{B}{i}} E(c_w^1)} \frac{k}{N} \left[ -x_q^1 E(c_w^1) (N-1) - \left[ \frac{B}{i} + (N-1) \frac{B}{i} \right] \right] \end{aligned}$$

$> 0$  under Assumption 3b. The first inequality follows from  $\frac{B}{j} < 1/N$  for  $j = k+1, \dots, N$ .

*Q.E.D.*

*Proof of Proposition 7b.* Follows from combining the following Lemma with Lemma A1.

*Lemma A2b.* (1) Suppose that Assumptions 1, 2b and 3b hold. Further suppose that (a)

$x_{pq}(p_i, q_i) = 0$ ; (b)  $E(c^i) = 1$  for all  $i = 1, \dots, N$ ; and (c)

$$\frac{B}{i} \frac{2(1 - \frac{B}{i})(1 + \frac{B}{i}) - (N-1) \frac{B}{i} (1 + \frac{B}{i})}{(N-1) \frac{B}{i} (1 - \frac{B}{i})} > \frac{B}{j} \text{ for } i < j. \text{ Then } k_0 = (N+1)/2.$$

(2) Suppose that  $x^i(\mathbf{p}) = \frac{1}{N} - p_i + q_i$ ,  $0 < \frac{1}{N} < [N-1]$ , and that  $c(x_i, i) = \mu(i)x_i$ ,  $\mu(i) < 0$  and  $\mu(i) = 0$ . Then  $k_0 = (N+3)/2$ .

*Proof.* (1) For  $d = (d_1, \dots, d_k, (k-1)d_{k+1}, 0, \dots, 0)$ , we have

$$\sum_{j=1}^k \frac{1}{j} = [p_1 - c_x^1] x_q^1 - \frac{(k-1)(1 - \frac{B}{1})(\frac{B}{1} + \frac{B}{k})}{[1 - \frac{B}{1}]} + \frac{B}{1} + \frac{(\frac{B}{1} - \frac{B}{1}) \frac{B}{1}}{E(c_w^1)}$$

$$\frac{[p_1 - c_x^1] x_q^1}{[1 - \frac{B}{1}]} \left[ -(k-1)(1 - \frac{B}{1}) \frac{B}{k} + \left\{ (1 - \frac{B}{1}) - (k-1)(1 - \frac{B}{1}) + (1 - \frac{B}{1}) / \frac{B}{1} \right\} \frac{B}{1} \right]$$

0 if and only if  $k \leq (N+1)/2$ . The first inequality follows from  $E(c_w^1) \geq 1$  and from  $(\frac{B}{1} - \frac{B}{1}) / x_q(p_1, q_1) \geq 1 / \frac{B}{1}$ . The second inequality follows from the last condition stated in Lemma A2b.

(2) Analogous to Lemma A2a. *Q.E.D.*



## References

- Areeda, P. "Essential Facilities: An Epithet In Need Of Limiting Principles." *Antitrust Law Journal*, Vol. 58 (1989), pp. 841-853.
- Baker, D.I. "Compulsory Access to Network Joint Ventures Under the Sherman Act: Rules or Roulette?" *Utah Law Review*, Vol. 1993 (1993), pp. 999-1133.
- Balto, D. "Access Demands to Payment Systems Joint Ventures." *Harvard Journal of Law and Public Policy*, Vol 18 (1995), pp. 623-669.
- Bloch, F. "Endogenous Structures of Association in Oligopolies." *Rand Journal of Economics*, Vol. 26 (1995), pp. 537-556.
- \_\_\_\_\_. "Sequential Formation of Coalitions with Fixed Payoff Division." *Games and Economic Behavior* Vol. 14 (1996), pp. 90-123.
- Brander, J.A. and Spencer, B.J. "Strategic Commitment with R&D: the Symmetric Case." *Bell Journal of Economics* Vol. 14 (1983), pp. 225-235.
- Bulow, J.I., Geanakoplos, J.D., and Klemperer, P.D. "Multimarket Oligopoly: Strategic Substitutes and Complements." *Journal of Political Economy*, Vol. 93 (1985), pp. 488-511.
- Carlton, D.W. and Frankel, A.S. "The Antitrust Economics of Credit Card Networks." *Antitrust Law Journal*, Vol. 63 (1995a), pp. 643-668.
- \_\_\_\_\_ and \_\_\_\_\_. "The Antitrust Economics of Credit Card Networks: Reply to Evans and Schmalensee Comments." *Antitrust Law Journal*, Vol. 63 (1995b), pp. 903-915.

- \_\_\_\_\_ and Salop, S.C. "You Keep on Knocking But You Can't Come In: Evaluating Restrictions on Access to Input Joint Ventures." *Harvard Journal of Law and Technology*, Vol. 9 (1996), pp. 319-352.
- Carnevale, M.L. "HDTV Bidders Agree to Merge Their Systems." *Wall Street Journal*, May 24, 1993.
- Deneckere, R. and Davidson, C. "Incentives to Form Coalitions with Bertrand Competition." *Rand Journal of Economics*, Vol. 16 (1985), pp. 473-486.
- Dixit, A. "Comparative Statics for Oligopoly." *International Economic Review*, Vol. 27 (1986), pp. 107-122.
- Economides, N. and White, L.J. "One-Way Networks, Two-Way Networks, Compatibility, and Antitrust." In D. Gabel and D. Weiman, eds., *Opening Networks to Competition: The Regulation and Pricing of Access*. Boston: Kluwer Academic Press, 1997.
- Evans D.S. and Schmalensee, R. "Economic Aspects of Payment Card Systems and Antitrust Policy Toward Joint Ventures," *Antitrust Law Journal*, Vol. 63 (1995), pp. 861-901.
- Farrell, J. and Shapiro, C. "Asset Ownership and Market Structure in Oligopoly." *Rand Journal of Economics*, Vol. 21 (1990), pp. 275-292.
- Federal Trade Commission. *Anticipating the 21st Century: Competition Policy in the New High-Tech, Global Market Place*. Washington: Federal Trade Commission, 1996.
- Grossman, G.M. and Shapiro, C. "Research Joint Ventures: An Antitrust Analysis." *Journal of Law, Economics, and Organization*, Vol. 2 (1986), pp. 315-337.

Hovenkamp, H. "Exclusivity Rules in Network Joint Ventures." Statement submitted to FTC Hearings on Global Competition/ High-Tech Innovation (1995).

Irwin, D.A. and Klenow, P.J. "High-Tech R&D Subsidies: Estimating the Effects of Sematech." *Journal of International Economics*, Vol. 40 (1996), pp. 323-344.

*Journal of Economic Perspectives*. Symposium on "Network Externalities." *Journal of Economic Perspectives*, Vol. 8 (1994), pp. 93-150.

Katz, M.L. "An Analysis of Cooperative Research and Development." *Rand Journal of Economics*, Vol. 17 (1986), pp. 527-543.

Krattenmaker, T.G. and Salop, S.C. "Anticompetitive Exclusion: Raising Rivals' Costs To Achieve Power over Price." *Yale Law Journal*, Vol. 96 (1986), pp. 209-293.

\_\_\_\_\_, Lande, R. and Salop, S.C. "Monopoly Power and Market Power in Antitrust Law," *Georgetown University Law Review*, Vol. 76 (1987), pp. 241-269.

Perry, M.K. and Porter, R.H. "Oligopoly and the Incentive for Horizontal Merger." *American Economic Review*, Vol. 75 (1985), pp. 219-227.

Salant, S.W., Switzer, S. and Reynolds, R.J. "Losses from Horizontal Mergers: The Effects of an Exogenous Change in Industry Structure on Cournot-Nash Equilibrium." *Quarterly Journal of Economics*, Vol. 93 (1983), pp. 185-199.

Salop, S.C. "Exclusionary Access Rules in Standards and Network Joint Ventures." Statement submitted to FTC Hearings on Global Competition/High-Tech Innovation (1995).

\_\_\_\_\_ and Scheffman, D.T. "Raising Rivals' Costs." *American Economic Review Papers and Proceedings*, Vol. 73 (1983), pp. 267-271.

Schmalensee, R. "On Antitrust Issues Related to Networks." Statement submitted to FTC Hearings on Global Competition/High-Tech Innovation (1995).

Seade, J. "Profitable Cost Increases and the Shifting of Taxation: Equilibrium Responses of Markets in Oligopoly." Unpublished manuscript, Warwick University (1985).

Werden, G. "The Law and Economics of the Essential Facilities Doctrine." *St. Louis University Law Review*, Vol. 32 (1988), pp. 432-80.

Yi, S-S. "Stable Coalition Structures with Externalities." *Games and Economic Behavior*, Vol. 20 (1997a), pp. 201-237.

\_\_\_\_\_. "Industry-Profit Maximizing Joint-Venture Structure in a Linear Cournot Oligopoly." Unpublished Manuscript, Dartmouth College (1997b).

\_\_\_\_\_ and Shin, H. "Endogenous Formation of Research Coalitions with Spillovers." Unpublished Manuscript, Dartmouth College, revised (1997).

## Footnotes

<sup>1</sup> For general overviews of network externalities and network joint ventures, see the Symposium on Network Externalities in the *Journal of Economic Perspectives* (1994) and Economides and White (1997).

<sup>2</sup> For example, a recent report by the Federal Trade Commission (FTC) on competition policy recognizes that many collaborative agreements among competitors are procompetitive (FTC, 1996, p. 6).

<sup>3</sup> Other concerns include the possibility that a joint venture could serve as a vehicle for collusion in the input market and/or in the output market. See Carlton and Salop (1996) for a detailed discussion.

<sup>4</sup> Some of these references are written statements of the participants in the FTC hearing on competition policy. Both written statements by the participants and complete transcripts of the hearing are available on the FTC's homepage <<http://www.ftc.gov/opp/global.htm>>. The controversy on access rules goes back to the "essential facilities" doctrine which mandates open access to a joint venture which provides an input essential for competition (*United States v. Terminal R.R. Ass'n of St. Louis*, 224 U.S. 383 (1912) and *Associated Press v. United States*, 326 U.S. 1 (1944)). See Areeda (1989) and Werden (1988) for more discussion of this doctrine.

<sup>5</sup> *SCFC ILC, Inc. v. Visa USA, Inc.*, 36 F.3d 958 (10th Cir. 1994). The Bylaws of the Visa venture state that Visa membership is unavailable to any bank that issues "competitive" cards, which include American Express and Discover but not MasterCard. The district court reached a verdict for Dean Witter. The Tenth Circuit reversed, holding that the Visa venture did not have the market power to inflict substantial harm to Dean Witter. Carlton and Salop (1996) argue that the Tenth Circuit erred by considering the *individual* market power of member firms of the Visa venture, rather than the *collective* exclusionary market power of the venture. For detailed discussions of this case, see Carlton and Frankel (1995a, 1995b), Carlton and Salop (1996), and Evans and Schmalensee (1995).

<sup>6</sup> Formal proofs of Lemmas 1, 2 and Propositions 1, 4 are available on the author's homepage <<http://www.dartmouth.edu/~ssyi>> or upon request from the author.

<sup>7</sup> Lemma 3 is not limited to the case of productive assets. More generally, under Assumptions 1 - 3, a favorable symmetric cost shock (such as a reduction in excise taxes, in import tariffs, or in input prices) to a subset of symmetric firms increases their profits. Previously, Seade (1985) showed this result for the special case of an industry-wide symmetric cost shock at a symmetric Cournot equilibrium under the assumption of constant returns to scale in output.

<sup>8</sup> In general, productive assets of firms will overlap with each other so that the combined assets of a joint venture are less than the sum of the assets of its members. This potential overlap in productive assets of venture members will not change the analysis, as long as each firm owns at least one productive asset that is unique.

<sup>9</sup> To be precise,  $c(x_i, i) = \max[0, (c_0 - i)x_i]$ , as the unit production cost cannot be negative. Thus, for Assumption 1 to hold, we need  $c_0 \geq N$ . I assume that this is the case. If  $c_0 < N$ , an increase in productive assets ceases to reduce production costs after some point. See Bloch (1995) for a detailed analysis of this type of decreasing returns to scale in productive assets.

<sup>10</sup> For notational simplicity, profit is multiplied by  $(N + 1)^2$ . This has no effect on the analysis. The same comment applies to Examples 2 - 4 (but not to the subsection on corner equilibrium in Section 4). The interior equilibrium condition is satisfied if  $\pi(1; \{N-1, 1\}) > 0$ , which in turn holds if and only if  $A > N^2 - 3N + 1$ . For Examples 2, 3, and 4 below, this condition is satisfied if and only if, respectively,  $A > (N - 1)^2 - N$ ,  $A > (N - 1)e^{N-1} - Ne$ , and  $a > N - 1$ .

<sup>11</sup> In the context of this paper, the "rule" of a joint venture is that members agree to pool their productive assets in order to reduce their production costs but to compete against each other in the output market.

<sup>12</sup> Of course, the Coalition Unanimity rule is not the only coalition formation rule which allows for exclusivity in membership. Earlier versions of this paper examined other exclusionary membership rules and showed that the main conclusion of this paper continued to hold: members of a large joint venture have strategic incentives to limit membership in order to keep rival firms' costs high.

<sup>13</sup> Proposition 7 also applies to cost functions with increasing returns to scale in production of the following form:  $c(x_i, i) = (i) + \mu(i)x_i$ , with  $(i) > 0$ ,  $\mu(i) < 0$  and  $\mu'(i) < 0$ . The only modification necessary to the proof of Proposition 7 (the second part of Lemma A2) is that  $E(c^i)$  is less than or equal to 1, instead of being equal to 1.

<sup>14</sup> The social surplus is multiplied by  $(N + 1)^2$ . The equilibrium joint-venture structure under the Coalition Unanimity game is approximated to  $\{3N/4, N/4\}$ . But the results for  $N = 4$  are exact.

<sup>15</sup> The assumption that the *composition* of competitors' outputs does not affect firm  $i$ 's demand is equivalent to the assumption that  $p^i(\mathbf{x})/x_j = p^i(\mathbf{x})/x_k$ ,  $i \neq j \neq k$ : an increase in firm  $j$ 's output the same effect on firm  $i$ 's inverse demand as an increase in firm  $k$ 's output does for any  $\mathbf{x}$ .

<sup>16</sup> But Proposition 7a uses the same linear demand function as in Bloch (1995). The proof of Proposition 7a in the Appendix reports conditions on  $\frac{C}{i}$  and  $\frac{C}{i}$  under which Bloch's results are extended. However, these conditions are sufficiently complex that it is hard to interpret

them for nonlinear demand functions. See the remarks following the proof of Proposition 7a in the Appendix for details.

<sup>17</sup> An important difference is that the reduction in investment does not necessarily reduce total social surplus, as it is possible that firms overinvest from a social welfare point of view (Brander and Spencer, 1983, Proposition 5).

<sup>18</sup> In a more general setting, joint-venture formation may increase member firms' total investments if there are synergy effects from joint investments or if investments create positive spillovers to other firms. Whether or not joint-venture formation will result in a net increase in investments depends on the strength of the synergy effects and the research spillovers (e.g., Katz, 1986). For high spillovers, Yi and Shin (1997) show that the grand coalition is the socially efficient structure as in the current paper. However, due to free-riding problems, the grand coalition is rarely an equilibrium outcome under various membership rules.