Learning-by-doing and the Development of Industrial Districts[1]

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Abstract

Marshallian districts are locales that accommodate a large number of small firms producing similar goods to be exported and benefit from the accumulation of know-how associated with workers residing there. We study the making of such districts by assuming that the cost function of a firm is a decreasing function of the total output produced in the past by the firms established in the locale. The dynamics is described by a sequence of temporary equilibria in which firms equalize profits between locales at each period. Hence changing the spatial distribution of firms affects the production history of each district. When new firms set up in a locale, they exacerbate competition on the corresponding labor market, thus leading to a wage rise that reduces the incentives for firms to locate in the most efficient district. The short-run equilibrium distribution of firms are studied as well as the long-run properties of the adjustment process.

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1 Introduction

The revival of small firms in many of the OECD countries has been characterized by different patterns of growth. In particular, the concept of *Marshallian industrial district* (MID) seems to have (re)gained the favor of many analysts and policy-makers (Goodman and Bamford, 1989; Pyke, Becattini and Senberger, 1990; OECD, 1996). Roughly speaking, a MID is defined by a locale that accommodates a large number of small firms producing similar goods for exportation and which benefits from the localized accumulation of skills associated with workers residing in this locale (Bellandi, 1989). In the words of Sforzi (1990, p. 75), a MID

> "can be defined as an organisation of the production process based on single specialised industries, carried out by concentrations made up of many small firms of similar character in particular localities achieving the advantages of large-scale production by external rather than internal economies, with social environments that feature local communities of people adhering to relatively homogeneous systems of values, and with networks of merging urban and rural settlements inside territories united by production and social links."

We will accept this definition without arguing more about what is and what is not a Marshallian district in the plethora of industrial districts one encounters in the real world. In any case, among the different kinds of existing districts, one must distinguish between at least the following two types. Some industrial districts are engaged in innovations in advanced sectors (Saxenian, 1994). However, others are involved in more traditional, labor-intensive activities, like many of those one can find in the "Third Italy". Examples include Sassuolo which is specialized in ceramic tiles, Prato is known for textiles while shoes are made in Montegranaro and wooden furniture in Nogara.

The role of MIDs in the growth of some countries is far from being negligible, at least in some countries. For example, Sforzi (1990, p. 196) observes that

> "Marshallian industrial districts experienced the fastest increase in employment both in manufacturing and total employment between 1971 and 1981, the figures being 36.7% and 37.6% respectively, far more than the other categories of local systems and the national average."

Observe also that most firms established in Italian MIDs do not correspond to decentralized units of large corporations, nor are they related to the existence of specific market niches as shown by the examples mentioned above (Amin, 1989). The same holds in other countries, thus suggesting that the Italian experience is reproducible, at least partially, in other regions of the world. Finally, it should be kept in mind that such districts existed well before the Industrial Revolution and, therefore, is not a new socio-economic structure (Hohenberg and Lees, 1985, ch. 6).

In view of the quotation above, it would be futile to aim at presenting an integrated model capturing all the distinctive features of a MID. From the geographical standpoint, a MID is an agglomeration where several Marshallian externalities are at work (Fujita and Thisse, 1996). Since Krugman (1991), modern contributions in economic geography emphasize the role of pecuniary externalities in the emergence of economic agglomerations. One example of such an externality is given by the wide array of intermediate goods and services provided in a large market which permits a fine division of labor and generates increasing returns in the aggregate (Abdel-Rahman and Fujita, 1990). In the present paper, we have chosen to concentrate on the impact of a technological externality often mentioned in the recent literature devoted to industrial districts, i.e., the *collective process of learning-by-doing* that allow local workers to improve their productivity through exchanges of information and through the use of particular schooling systems. Indeed, whatever their form, all industrial districts seem to share one basic feature, that is, *the fact that knowledge is embodied in workers living within small geographical areas and who interact together through various social processes*, such as informal discussions among workers in each firm, inter-firm mobility of skilled workers, the exchange of ideas within families or clubs, and bandwagon effects. In other words, as noticed by Bellandi (1989, p. 146)

> "personal contact within the agglomeration encourages a constant intercommunication of ideas."

It is our contention that such an externality is better suited to explain small scale and highly specialized economic agglomerations, such as MIDs, than pecuniary externality that play a more central role in the formation of large agglomerations, such as metropolitan areas. Furthermore, being local in nature, knowledge is sticky in our model because workers are supposed to be immobile. Though restrictive, it seems fair to say that this assumption fits well enough many districts in Europe where the mobility of people is known to be lower. This is especially true in several MIDs where workers have strong local roots.

The localized character of the diffusion of knowledge is a well-documented fact. For example, Jaffe *et al.* (1993) find that, in the US, citations to domestic patents are more likely to be domestic, and more likely to come from the same states and metropolitan statistical areas as the cited patents. These conclusions are strengthened by the work of Feldman (1994) and Audretsch and Feldman (1996) who observe that knowledge spillovers tend to be geographically bounded within the region where the new knowledge was created. The reader is referred to Long and Soubeyran (1996) as well as to Mai and Peng (1996) for attempts in modeling MIDs based on direct spillovers between firms.

Specifically, we view MIDs as places where *skills are accumulated within the same population of workers*. This idea is modeled as follows: the cost function of a firm located in a district is a *decreasing function of the industry output accumulated in this district,* as in Arrow (1962) and Stokey (1986).[1] In other words, it is assumed that an individual firm's experience spills over all the units which are established in the sole district because of the social nature of the interaction which occurs among workers, an assumption implicitly made by Chipman (1970) in another context. As a consequence, firms exhibit decreasing returns in the short run but increasing returns in the long run. Such an approach is in accordance with the traditional model of *localization economies* in which all firms belonging to the same industry are positively affected by the total output of the industry at one particular location (Hoover, 1936, ch. 6). Observe also that, contrary to general beliefs, a substantial amount of knowledge used by firms turns out to be tacit and difficult to transfer from one location to another (Teece, 1977).

In our model, the economy works as follows. In each period, firms face the world price for their output and locate in order to maximize profits which are, therefore, equal across districts. Any change in the distribution of firms affects the production history of each district. Hence the equilibrium reached at the beginning of the period is destroyed when the period ends, giving some firms an incentive to move. As expected, firms are attracted by places where the stock of knowledge is "large"; this corresponds to an agglomeration force. However, the setting up of new firms in a district exacerbates competition on the local labor market by pushing up wages, thus lessening the incentive for firms to set up in districts with highly skilled workers; this is a dispersion force. As a result, the path of the economy is described by a sequence of *temporary equilibria.* It is shown that this sequence may converge toward different equilibrium spatial systems, thus making especially hard to predict the dynamics of Marshallian districts.

In order to gain more insights, we then focus on the special case of two districts and show that the long-run equilibrium exhibits an unbalanced spatial structure: more firms are steadily set up in one district. In addition, we identify necessary and sufficient conditions about the learning-by-doing process for the gap between the two districts to grow over time: more and more firms want to establish themselves in the district with the

[1] Observe that empirical studies have found "scale economies to be statistically significant but small in magnitude relative to learning-based economies" (Lieberman, 1984, p.214) in various manufacturing sectors.

larger initial stock of skills when the process of learning, first, grows fast and, then, slows down when the level of knowledge keeps rising. So we identify a condition under which Marshall (1890, p. 225)'s predictions are fulfilled:

> "history shows that a strong center of specialized industry often attracts much new shrewd energy to supplement that of native origin, and is thus able to expand and maintain its lead."

In this case, this is the initial endowment in human capital that determines the growth path of the industrial districts. However convergence may arise too. As will be seen, the features of the learning-by-doing process turn out to be essential to predict what kind of equilibrium will eventually emerge. This strongly suggests that *the regional structure of production also depends upon the social factors shaping the learning process.*

The remainder of the paper is organized as follows. The model is introduced in Section 2. In Section 3, we study the properties of a temporary equilibrium, while the long-run equilibrium is analyzed in Section 4. Finally, in Section 5 we discuss the robustness of our main results as well as some possible extensions.

2 The model and some preliminary results

Consider a set M of locales, with $i \in M = \{1, \ldots, m\}$, each endowed with a given and fixed population of workers L^i. Each worker supplies one unit of labor provided that they earn a strictly positive wage. Locale i has an initial stock of knowledge given by $S_0^i \geq 0$ which is the outcome of a historical process not analyzed here. There is a continuum of size one of (identical) entrepreneurs. An entrepreneur can create a firm by combining capital bought on the global market and labor hired in a particular locale; he then produces a homogeneous good sold to the rest of the world at a given price p_t which may vary over time. When all entrepreneurs have elected a location for period $t \geq 1$, the distribution of firms over the locales is given and denoted by the vector $(n_t^1, \ldots, n_t^i, \ldots, n_t^m)$ with $\sum_{i \in M} n_t^i = 1$.

There is an infinite number of periods $t = 1, 2, \ldots$. At the end of each period, capital is to be replaced so that entrepreneurs are free to choose a new location, while workers stay in the same locale but typically have a higher level of knowledge. Specifically, the technology is described by a labor marginal requirement which varies with the history of the district but not with the current output, and a capital requirement which depends only upon the current output. Both labor and capital are bought on perfectly competitive markets. Since labor is immobile, wages clearing local labor markets differ across locales.

Hence, our description of workers concurs with the empirical literature according to which local labor markets in industrial districts tend to be spatially-delimited and relatively self-contained in that entrepreneurs hire workers from the locale only. The implications of this assumption for our main results will be discussed in the last section. Finally, the price of capital r_t paid by an entrepreneur, called the interest rate, is assumed to be the same in all locales. Since the districts are small vis-à-vis the rest of the world, the interest rate is exogenous to the firms but may vary over time due to external market forces.

In order to capture some of the main ideas developed in the socio-economic literature on Marshallian industrial districts, we assume that *the amount of labor used by a firm is history-dependent* (or path-dependent). It is also district-specific in that the capital of skills accumulated in a district cannot be moved to another one. We would like to argue that these assumptions fit well the description of MIDs where workers are considered as immobile while their total knowledge accumulates over time through various processes of social interaction. Indeed a worker operating in a repeated production context uncovers continuously the many facets of the available techniques and gradually adjusts his behavior so as to improve his productivity over time.[2] However workers are heterogeneous because they have different skills (by nature as well as by nurture), but also because they face different experiences and have different abilities of learning. When they live in the same locale, they can share their knowledge through various types of social interactions which, in turn, rise productivity when they are combined within firms. Information and ideas have characteristics of public goods and, hence, tend to generate spillover effects. For the corresponding spillovers to work, it must be that individuals share a common environment and, therefore, reside for long enough periods of time within the same locale.[3]

The description of the many ways individuals may interact is not an easy task. However, the details of some interactions may be found in Bénabou (1996) and Montgomery (1991). In this paper, all individual interactions taking place within a district are supposed to be aggregated within a learning-by-doing process à la Arrow. Hence, though each firm faces static decreasing returns at the individual level, it enjoys dynamic increasing returns at the level of the whole district. However, a firm can benefit from this collective advantage only if it is located in the district. As discussed by Pyke *et al.* (1990) and argued in the foregoing, this is because the know-how is embodied in the workers and not in the firms.

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[2] Algorithms that seem to fit well the learning behavior of individuals have been proposed by Arthur (1994, ch. 8).

[3] Observe that Lucas (1988) similarly stresses the role of proximity in the diffusion of technological and social innovations and considers cities as the main source of economic growth and development.

Let q_t^i be the output of a firm set up in locale i at period t, w_t^i the wage prevailing in i at t, and $S_{t-1}^i = \sum_{\tau=1}^{t-1} n_\tau^i q_\tau^i$ the sum of past productions in this locale at the beginning of period t. Since all firms located in a given district face the same environment, the output across firms in a MID is equal in equilibrium and there is no need to distinguish among the corresponding firms. The cost function of a firm established in i at period t is given by the following expression:

$$C_t^i(q_t^i; w_t^i, S_{t-1}^i) = w_t^i \ell(S_{t-1}^i) q_t^i + r_t K(q_t^i). \tag{1}$$

The cost of labor, given by the first term of the RHS of (1), depends on the specificity of the district. This is because the wages w_t^i are generally different between locales, but also because the labor coefficient $\ell(S_{t-1}^i)$ accounts for the history of the district under consideration. More precisely, the labor coefficient expresses the capital of skills generated by the process of learning-by-doing during the previous periods. Formally, we assume that this coefficient is a decreasing function of the cumulative production. Observe that $\ell(S_{t-1}^i)$ is independent of the firm's current output while the history of the district is encapsulated in the output accumulated over the previous periods. Finally, it is supposed that the capital requirement of a firm $K(q_t^i)$ is the same across locales; it is strictly increasing and strictly convex, while $K(0) = K'(0) = 0$ and $K'(+\infty) = +\infty$.

The current profit of any firm established in locale i is then defined as follows:

$$\Pi_t^i(q_t^i; w_t^i, S_{t-1}^i) = p_t q_t^i - C_t^i(q_t^i; w_t^i, S_{t-1}^i). \tag{2}$$

In choosing location and output in each period t, each firms maximizes current profits. When the set of occupied districts is stable over time, this behavior is consistent with intertemporal profit maximization since capital is replaced at the end of each period.

Furthermore, since each firm has a small size, its impact on the local output is negligible. Therefore, when selecting a location, a firm ignores the change in the cumulative output and, hence, in the future production cost that will prevail in the chosen district. In other words, S_{t-1}^i acts as a technological externality for each firm established in the corresponding district. The impact of the stock of know-how on the profit level is easy to evaluate. For that, we differentiate (2) with respect to the cumulative output and use (1) in order to obtain:

$$\frac{\partial \hat{\Pi}_t^i}{\partial S_{t-1}^i} = -w_t^i \hat{q}_t^i \ell'(S_{t-1}^i) > 0$$

which shows that *the profits of firms established in a district increase with the local stock of knowledge accumulated there.*

Given w_t^i and S_{t-1}^i, maximizing (2) with respect to q_t^i yields

$$\frac{\partial \Pi_t^i}{\partial q_t^i} = p_t - w_t^i \ell(S_{t-1}^i) - r_t K'(q_t^i) \leq 0, \qquad q_t^i \frac{\partial \Pi_t^i}{\partial q_t^i} = 0, \qquad q_t^i \geq 0 \tag{3}$$

while the second-order condition is trivially satisfied. Let $\hat{q}_t^i$ be the unique solution to (3). When production in locale i is positive, this solution is identical to:

$$\hat{q}_t^i = (K')^{-1}\{[p_t - w_t^i \ell(S_{t-1}^i)]/r_t\} \tag{4}$$

which, for given r_t and p_t, turns out to depend only upon the local wage and the production history of the district under consideration. Introducing (4) into (2) leads to the value function:

$$\begin{aligned} \hat{\Pi}_t^i &= \Pi_t^i \left[\hat{q}_t^i(w_t^i, S_{t-1}^i, r_t, p_t), w_t^i, S_{t-1}^i \right] \\ &= \hat{\Pi}_t^i(w_t^i, S_{t-1}^i; r_t, p_t). \end{aligned} \tag{5}$$

This function gives the maximum profit a firm can earn if it chooses to locate in district i; it will be used in the next section to describe the arbitrage made by entrepreneurs between locales. Using (3), we can rewrite the value function (5) as follows:

$$\hat{\Pi}_t^i = r_t \lambda(\hat{q}_t^i) \tag{6}$$

where

$$\lambda(q) \equiv K'(q)q - K(q). \tag{7}$$

The expression (6) will be central in the determination of the equilibrium distribution of firms across active districts.

3 Temporary equilibrium between districts in the short run

At period $t = 0$, no firms are located and each locale i has an initial stock of knowledge $S_0^i \geq 0$. At each period $t \geq 1$, firms set up in a locale so as to maximize their profits. Everything else being equal, firms are attracted by the district with the highest stock of know-how in order to enjoy the highest possible productivity of labor. During this process, wages are determined in each local labor market and we will see that a large number of firms in a district leads to a higher local wage. Hence the current equilibrium distribution of firms is the outcome of the interplay between these centrifugal and centripetal forces. The former is history-dependent while the latter is given by the immobility of the labor force in each locale. In the equilibrium prevailing at $t \geq 1$, firms' profits are equal between districts.

Assuming positive production, and hence positive wage, full employment in district i means that:

$$n_t^i \ell(S_{t-1}^i)\hat{q}_t^i = L^i.$$

This condition implies that the number of firms in district i is given by:

$$n_t^i = L^i/\hat{q}_t^i \ell(S_{t-1}^i). \tag{8}$$

A district is said to be *active* in period t if it accommodates firms whose outputs are strictly positive and paying strictly positive wages. From (3), we obtain:

$$w_t^i = \frac{p_t - r_t K'(\hat{q}_t^i)}{\ell(S_{t-1}^i)} > 0 \tag{9}$$

which holds if and only if

$$p_t/r_t > K'(\hat{q}_t^i).$$

Given the properties of K', the equation $p_t/r_t = K'(q)$ has a unique solution given by:

$$q_t^* = (K')^{-1}(p_t/r_t). \tag{10}$$

Since $K(q)$ is strictly convex, it is then readily verified that w_t^i is positive if and only if:

$$\hat{q}_t^i < q_t^*.$$

Stated differently, a district is active in the short run if and only if the actual number of firms given by (8) exceeds some minimum size:

$$n_t^i > \underline{n}_t^i \equiv L^i/\ell(S_{t-1}^i)q_t^*.$$

We are now equipped to study the equilibration of profits between districts in order to determine the distribution of firms during period $t \geq 1$. Let I_t be the set of districts where firms are established. Observe that this set is endogenously determined in each period. Differentiating the function λ given by (7) with respect to q and using the strict convexity of K shows that λ is strictly increasing. Therefore, (6) and the equilibrium condition of equal profits between active districts imply that $r_t\lambda(\hat{q}_t^i) = r_t\lambda(q_t^j)$ for all $i, j \in I_t$. Hence the firms' equilibrium output is the same regardless of the active district where they are set up:

$$\hat{q}_t^i = \hat{q}_t, \qquad i \in I_t. \tag{11}$$

In other words, *firms produce the same output in all active districts despite of different labor requirements generated by different histories.* Given the equilibrium wages, this equality is realized through the relocation of firms between locales. Furthermore, (11) implies that variations in the interest rate do not affect the equality of production levels between districts, even though the common output level varies with r_t as shown by (4). However (11) does not mean that the *total* output is equal across districts since the number of firms varies from one district to another.

To simplify notation, we set $v(S) = 1/\ell(S)$ where v is strictly increasing. Since the total mass of firms is one, it follows from (8) and (11) that the equilibrium output of a firm in any district $i \in I_t$ is given by

$$\hat{q}_t = \hat{q}_t(I_t) = \sum_{i \in I_t} L^i v(S^i_{t-1}). \tag{12}$$

It is worth noting that a district may be active in a certain period but not in the next one; and vice versa. Accordingly, the production of firms depends not only upon the stocks of knowledge but also on the current set of active districts. Therefore, the evolution of the productive system implies a process of *creation* and *destruction* of industrial districts, through the endogenous determination of I_t, in an environment which is not stochastic.

Replacing (12) in (8) yields the equilibrium distribution of firms corresponding to I_t:

$$n^i_t = n^i_t(I_t) = \frac{L^i v(S^i_{t-1})}{\sum_{j \in I_t} L^j v(S^j_{t-1})}, \qquad i \in I_t. \tag{13}$$

As shown by (13), *the higher the cumulative output in an active district* (or the higher the number of workers living in this district), *the higher the number of firms which choose to locate there.* And, consequently, the lower the number of firms which lie in the other districts. Furthermore, the equilibrium distribution of firms depends on the history of *all* the districts active during the current period. In other words, it is not sufficient to consider districts in isolation to determine their production; *they are parts of a more general system in which they are embedded* (Saxenian, 1994).

Finally, it remains to describe how wages clear the local labor markets in I_t for a given equilibrium distribution of firms. Using (9) and introducing the equilibrium output given by (12) in K' gives the equilibrium wage prevailing in each active district as a function of the stocks of skills:

$$w^i_t = w^i_t(I_t) = \left[p_t - r_t K' \left(\sum_{j \in I_t} L^j v(S^j_{t-1}) \right) \right] v(S^i_{t-1}) > 0, \qquad i \in I_t. \tag{14}$$

Hence the wage prevailing in a district depends on *the history of all currently active districts.* In particular, the wage in district i falls with the size of the labor force available there, but also with the supply of labor in other active districts. Since S^i_{t-1} appears both in the numerator and denominator of (14), the relationship between w^i_t and S^i_{t-1} is not necessarily monotone. For example, if S^i_{t-1} is small enough the corresponding wage first rises and then falls as long as the productivity of labor becomes sufficiently large in all districts.

In order to shed more light on the mechanism of competition for labor in each district, we rewrite (14) as a function of the local number of firms. Plugging (13) in K' inside (14) yields

$$w^i_t = \left[p_t - r_t K'\left(\frac{L^i v(S^i_{t-1})}{n^i_t}\right)\right] v(S^i_{t-1}).$$

It is then immediate to check that *the local wage increases with the number of firms established in the corresponding district.* If more firms are located in the district during the previous periods, the productivity of the local labor force is higher, thus making this district more attractive. This agglomeration force is counterbalanced by the dispersion force generated by the wage decreases that arise in the districts where the moving firms were established.

As seen above, all districts may not be active. It is easy to check that the set I_t of active districts is such that:

$$\hat{q}_t(I_t) < q^*_t \tag{15}$$

which, given (10), is equivalent to:

$$p_t/r_t > K'\left[\sum_{i\in I_t} L^i v(S^i_t)\right]. \tag{16}$$

We need below a sufficient condition for all districts to be active in all periods. When $I_t = M$, (17) holds provided that $\hat{q}_t(M) < q^*_t$ for all t. Since labor is required even when the stock of knowledge is large, it is reasonable to assume that v is bounded above; let $v^\infty < \infty$ be the upper bound of v. Hence a sufficient condition for all districts to be active in all periods is that:

$$v^\infty \sum_{i\in M} L^i < q^*_t.$$

The developments above leads to the following proposition.

Proposition 1 *Assume that*

$$p_t/r_t > K'\left(v^\infty \sum_{i\in M} L^i\right) \tag{17}$$

holds for all t. Then, in each period, there exists a unique temporary equilibrium. Furthermore, this equilibrium is such that each district is active at each period and the temporary equilibrium is given by (12), (13) and (14).

Proof. Given (17), the equilibrium wage at the current period is positive in each district. Furthermore, for each period, the expressions (12), (13) and (14) are the unique solutions to the equilibrium conditions. Existence then follows by construction. □

Clearly, condition (17) is likely to hold when the population of workers is not too large, when the price of the output is large and the interest rate low.

The distribution of firms at a given period affects the current production levels and, therefore, the stocks of know-how that will prevail at the subsequent period. This generates new forces that induce the relocation of some firms until a new equilibrium is reached. Indeed, the new stocks of know-how change the profit values and destroy the equilibrium just realized. This equilibrium has therefore the nature of a *temporary equilibrium.* As a consequence, the state of the economy changes over time and its path is studied in the next section.

4 The rise and fall of industrial districts

In this section we study the dynamics generated by the accumulation of knowledge. Our research strategy is to analyze the behavior of the cumulative production in each district from any period t to the next. If i is an active district in period t, the total equilibrium output $\hat{Q}_t^i = n_t^i \hat{q}_t^i$ in this district is given by:

$$\hat{Q}_t^i = L^i v(S_{t-1}^i), \qquad i \in I_t \tag{18}$$

so that the new stock of know-how at the end of period t is equal to

$$S_t^i = S_{t-1}^i + L^i v(S_{t-1}^i), \qquad i \in I_t. \tag{19}$$

Since this equation involves only the local stock of know-how, the history of a district which is active in *each* period is driven by the process of learning-by-doing within the district only. Note also that the stock of know-how given by (19) is strictly increasing as long as the corresponding district is active. However, if the district is idle in period t, the stock of knowledge no longer rises:

$$S_t^i = S_{t-1}^i, \qquad i \notin I_t$$

so that, despite the direct independence of S_t^i with respect to S_{t-1}^j for all $j \neq i$, the dynamics of the whole system of locales is interdependent through the endogenous determination of the set I_t. In addition, changes in the world price of the good and/or the interest rate affect the destruction/creation of districts via the modification of the configuration I_t. This observation is important because it shows that *external factors* (such as fashion changes, new differentiated products or monetary shocks) *may have a dramatic impact on the dynamics of Marshallian industrial districts*, thus confirming the volatility of this production system stressed in the empirical literature. In particular, according to the value of the relative price p_t/r_t, production is either concentrated into a small number of districts (possibly one) or distributed among many of them (possibly all).

In order to gain more insights about the dynamics of the productive system, we consider throughout the remaining of the paper the special case where $p_t = p$ and $r_t = r$ for all t. Let q^* be the solution of (11) which is now independent of t. The sequence of active districts can then be constructed as follows. A configuration I_t is said to be *feasible* if (15) holds. In period 1, several configurations I_1 may be found such that:

$$\hat{q}_1(I_1) < q^*.$$

Given (6), firms' profits increase with the output level so that profits are highest for the feasible configuration associated with the highest output. Accordingly, all feasible configurations in period 1 can be ranked in terms of profitability. We assume that firms are able to build such a ranking and to coordinate on the feasible configuration yielding the highest profits. Let I_1^* be the resulting configuration (or one of them when there are multiple equilibria). During period 1, production occurs only in the locales belonging to I_1^* so that the stocks of knowledge rise in these locales only.

In period 2, firms select among the feasible configurations I_2 such that:

$$\hat{q}_2(I_2) < q^*$$

the configuration I_2^* yielding highest profits. For our purpose, it is worth noting that $I_1^* \cup I$, with I included in $M - I_1^*$, can never be a feasible configuration in period 2. Indeed, by construction of I_1^*, it must be that:

$$q^* < \hat{q}_1(I_1^* \cup I) < \hat{q}_2(I_1^* \cup I), \qquad I \subseteq M - I_1^*$$

so that the necessary condition for $I_1^* \cup I$ to be feasible is violated. It is then clear that the configuration of active districts cannot expand from one period to the next. This does not imply, however, that I_2^* is included in I_1^*. If some districts of I_1^* may stop being active, it may happen that I_2^* contains districts which were not active in period 1. It is possible to state some stability results, however.

Denote by $_t$ the set of feasible configurations at period $t \geq 1$ (that is, all the configurations I_t for which (15) holds).

Proposition 2 *For all $t \geq 1$, we have:*

(i) $_{t+1} \subseteq _t$*;*

(ii) $_{t+1} = _t$ *if and only if* $I_{t+1}^* = I_t^*$*;*

(iii) $_{t+1} \subset _t$ *if and only if* $I_{t+1}^* \neq I_t^*$*.*

Proof.

(i) The inclusion follows immediately from the fact that $I_t \notin _t$ implies $I_t \notin _{t+1}$ since (15) remains violated.

(ii) Suppose that $_{t+1} = _t$. Consequently, I_t^* is still feasible in $t+1$. Lemma 1 in Appendix then implies that:

$$\hat{q}_{t+1}(I) \leq \hat{q}_{t+1}(I_t^*) < q^*$$

for all $I \in _{t+1}$ so that I_t^* is the equilibrium configuration in $t+1$. Suppose now that $I_{t+1}^* = I_t^*$. By definition of I_t^*, we have:

$$\hat{q}_t(I) \leq \hat{q}_t(I_t^*), \qquad \text{for all } I \in _t.$$

Lemma 2 in Appendix implies that $\hat{q}_{t+1}(I) \leq \hat{q}_{t+1}(I_t^*)$. Hence, we have:

$$\hat{q}_{t+1}(I) < \hat{q}_t(I_t^*) < q^*, \qquad \text{for all } I \in _t.$$

In other words, any feasible configuration in period t is still feasible in period $t+1$. Part (i) therefore leads to $_t = _{t+1}$.

(iii) This follows immediately from parts (i) and (ii). □

In words, the feasible configurations define a sequence of *nested* sets, a result suggestive of a tendency toward agglomeration. Furthermore, the *stability* of the set of feasible configurations over time is equivalent to that of the equilibrium one.

It follows from Propositions 1 and 2 that a sufficient condition for I_t^* to be selected in each period $t \geq 1$ is given by (17) where M is replaced by I_t^*:

$$v^\infty \sum_{i \in I_t^*} L^i < q^*. \tag{20}$$

Our purpose is now to study the comparative evolution of active MIDs within such a stable configuration. In this case, current profit maximization is equivalent to intertemporal profit maximization since the life span of capital is one period. For simplicity, we restrict ourselves to the case of *two* locales initially identical except that the initial stocks of knowledge differ with $S_0^1 > S_0^2$. In order to stress the role of the learning process, we work again with the labor coefficient $\ell(x)$.

It follows from (8) that $S_0^1 > S_0^2$ implies $n_1^1 > n_1^2$. This means that the stock of know-how increases more in district 1 than in district 2. This remains true for any period so that we have $n_1^1 > n_1^2$ for all t: *the district with the larger initial stock of knowledge has always more firms that the other.*[4]

However this difference in size might hold under convergence or divergence between the two districts. We now want to find under which conditions the relocation process of firms from district 1 to 2 is monotone, that is, n_t^1/n_t^2 increases over time when $S_0^1 > S_0^2$ and $L_0^1 = L_0^2 = L$. For that, we study the behavior of the ratio n_t^2/n_t^1 which is by (8) equivalent to the behavior of $\ell(S_{t-1}^1)/\ell(S_{t-1}^2)$. Using (13), we have:

$$n_t^2/n_t^1 = \ell(S_{t-1}^1)/\ell(S_{t-1}^2) > n_{t+1}^2/n_{t+1}^1 = \ell(S_t^1)/\ell(S_t^2)$$

which holds if and only if:

$$\frac{\ell(S_t^1)}{\ell(S_{t-1}^1)} < \frac{\ell(S_t^2)}{\ell(S_{t-1}^2)}. \tag{21}$$

Set $x = S_{t-1}^1$ ($x' = S_{t-1}^2$) and $y = S_t^1$ ($y' = S_t^2$). Then, (18) implies that $y = x + L/\ell(x)$ and $y' = x' + L/\ell(x')$. Define

$$F(x) \equiv \frac{\ell(y)}{\ell(x)} = \frac{\ell[x + L/\ell(x)]}{\ell(x)}$$

so that (21) amounts to $F(x) < F(x')$. Since $x > x'$, this inequality means that F must be decreasing. Differentiating F yields:

$$sign\ F'(x) = sign\ \left[\frac{\ell(x)}{\ell'(x)} - \frac{\ell(y)}{\ell'(y)} - \frac{L}{\ell(x)}\right].$$

Set also $f(x) = x + \ell(x)/\ell'(x)$ so that

$$sign\ F'(x) = sign\ [f(x) - f(y)].$$

Since $y > x$, a necessary and sufficient condition for the sign of F' to be negative is that f be strictly increasing. A simple calculation shows that this condition is equivalent to:

$$2(\ell')^2 - \ell\ell'' > 0. \tag{22}$$

[4] Note that this result is true for any two active districts so that district sizes are ranked according to the relative value of their initial stocks.

Hence, concentration rises in district 1 while the number of firms in district 2 decreases over time if and only if (22) is satisfied. Simultaneously, wages increase monotonically in locale 2 while it falls in locale 1. This implies that district 1 “expands” while district 2 “shrinks” over time both in size and wealth. As a consequence, the divergence between districts becomes sharper and sharper even though both districts remain active. It is worth noting that condition (22) means that function $\ell(x)$ cannot be “too” convex. This seems to be consistent with empirical studies conducted in manufacturing sectors where the log-linear form with a coefficient slightly smaller than one turns out to be fairly robust (Lieberman, 1984). Though evidence is missing for industrial districts, it would seem reasonable to expect a larger coefficient because of the denser nature of interactions within MIDs. Clearly, more work is called for here in order to have a sharper view of the dynamics of MIDs.

We have shown the following result:

Proposition 3 *Assume an economy with two initially identical districts expect for the initial stocks of know-how and such that (20) holds for* M*. Then, if* $2(\ell')^2-\ell'\ell'' > 0$ *(resp.* < 0*), the equilibrium path of the economy involves an increasing (resp. decreasing) mass of firms in the district with the larger initial advantage.*

The interest of this proposition is to show that the process of relocation is always monotone. There is divergence or convergence according to the strength of the learning-by-doing process. Put differently, *the spatial properties of the long run equilibrium depend less on history* (here the initial conditions) *than on the nature of the social process that allows workers to improve their know-how over time.* When condition (22) holds, a marginal advantage in the initial stock of knowledge suffices to generate a growing imbalance between the two locales which are otherwise identical. Though the two districts might be initially very similar, the learning-by-doing process leads to a rising concentration of production and wealth in the district with the (possibly small) initial advantage. Despite the fact that competition is fiercer in the local labor market, one district becomes larger and larger than the other. This is because the pace of learning is slow, thus preventing the lagging district to catch up with the leading one. On the other hand, when (22) is not satisfied, even when the initial advantage of one district is large, the small district catches up with the big one.

Thus, history still matters in that the initial conditions determine which district is the bigger one, but *history has no influence on the type of dynamics governing the evolution of districts.* All in all, the growth of the system of districts depends on both the history and the social institutions used by workers to increase their productivity. Observe, finally, that there is no cross-migration of firms: once a firm has decided to move to district 1 (or 2), it is profitable for this firm to stay put in all subsequent periods.

5 Concluding remarks

Though we have used a very simple model, the analysis above shows that a learning-by-doing process à la Arrow has allowed us to capture some of the main feature of MIDs. In particular, we can describe the way the distribution of activities among districts changes over time: the economy converges toward a balanced/unbalanced spatial structure, depending on the way workers interact to build their stock of knowledge.

In our model, workers living in inactive districts are supposed to stay put. This assumption may be relaxed by allowing them to migrate toward either the rest of the world or the active districts. In the former case, our results remain basically the same. In the latter, workers' migrations strengthen the attractive power of the active districts and exacerbates the agglomeration process since now both firms and workers tend to concentrate in a smaller number of locales. Furthermore, there is no geographical diffusion of knowledge in our framework. This is because the know-how is individual-specific and because workers are immobile. The immobility of workers is indeed a strong dispersion force which is stressed by most analysts of MIDs. Finally, instead of assuming that the world price is given, the model could be extended to the case of Cournot competition on the world market, but the temporary equilibrium conditions of Section 3 would then be given by inequalities expressing that no firms want to leave or enter a district in the short run.

There are several possible extensions which are worth studying. First, many authors insist on the combination of economic and social factors in the making of MIDs. Here we have assumed a simple social process that ignores many facets of the problem. The model would gain by integrating more socio-economic variables. In particular, one should attempt to model individuals "embedded" in a close-knit community of workers who monitors one another's behavior closely (Granovetter, 1985). One possible way to approach this problem is to consider the sharing of knowledge among workers as an informal coinsurance mechanism, using for example the framework developed by Coate and Ravaillon (1993) but to show here that knowledge sharing can be a substitute to migration.

Second, we have seen that the shape of the learning-by-doing process is critical for the geographical distribution of firms. But we ignore the rules that might govern the aggregation of individual learning processes and, hence, we do not know how individual behavior may lead to functions that may or may not satisfy (22). Third, the process of creation/destruction of districts should be accompanied by a similar process regarding firms. In each period, the number of firms could be determined through a free entry/exit zero-profit condition. Fourth, the model could be reformulated within the framework of an overlapping generation structure in order to emphasize the role of institutions, such as families and schools, stressed by Becattini (1990) and others in the process of accumulation of knowledge. Last, one open question abound MIDs is their ability to adjust in a world characterized by changes and turbulences. For that, our model should become a building-block of a more general model where competition with the rest of the world is explicitly considered. It would also be interesting to pursue the characterization of industrial districts in order to be able to compare them to metropolitan areas as alternative production systems.

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Appendix

Lemma 1 *If* $\hat{q}_t(I) \leq \hat{q}_t(I_t^*)$*, then* $\hat{q}_{t+1}(I) \leq \hat{q}_{t+1}(I_t^*)$*.*

Proof. Let $x_t^i \equiv L^i v(S_{t-1}^i)$. For any configuration I, we set:

$$\begin{array}{lcl} A_t & = & \sum_{i \in I \cap I_t^*} x_t^i \\ B_t & = & \sum_{i \in I - I_t^*} x_t^i \\ A_t^* & = & \sum_{i \in I \cap I_t^*} x_t^i \\ B_t^* & = & \sum_{i \in I_t^* - I} x_t^i \end{array}$$

while similar expressions hold for period $t+1$ in which x_t^i is replaced by $x_{t+1}^i \equiv L_i v(S_t^i)$. We then have:

$$\begin{array}{lcl} \hat{q}_t(I) & = & A_t + B_t \\ \hat{q}_{t+1}(I) & = & A_{t+1} + B_{t+1} \\ \hat{q}_t(I_t^*) & = & A_t^* + B_t^* \\ \hat{q}_{t+1}(I_t^*) & = & A_{t+1}^* + B_{t+1}^* \end{array}$$

where $A_t = A_t^*, A_{t+1} = A_{t+1}^*$ by construction and $B_t = B_{t+1}$ since $S_t^i = S_{t-1}^i$ for all $i \in B_t$. Accordingly, for any feasible configuration I, $\hat{q}_t(I) \leq \hat{q}_t(I_t^*)$ implies that $B_t \leq B_t^*$. In addition, we have $\hat{q}_{t+1}(I) = A_{t+1}^* + B_t$. Since the stocks of knowledge rise in each locale of $I_t^* - I$, we get $B_t^* \leq B_{t+1}^*$ so that

$$\hat{q}_{t+1}(I) \leq A_{t+1}^* + B_t^* \leq A_{t+1}^* + B_{t+1}^* = \hat{q}_{t+1}(I_t^*).$$

□

Lemma 2 *If* $I_t^* \in {}_{t+1}$*, then* $I_{t+1}^* = I_t^*$*.*

Proof. By definition, I_t^* is such that:

$$\hat{q}_t(I) \leq \hat{q}_t(I_t^*) < q^* \qquad \text{for all } I \in {}_t.$$

Then, Lemma 1 implies that:

$$\hat{q}_{t+1}(I) \leq \hat{q}_{t+1}(I_t^*) \qquad \text{for all } I \in {}_t.$$

Furthermore, we have $\hat{q}_{t+1}(I_t^*) < q^*$ since $I_t^* \in {}_{t+1}$. Consequently,

$$\hat{q}_{t+1}(I) \leq \hat{q}_{t+1}(I_t^*) < q^* \qquad \text{for all } I \in {}_t.$$

Since ${}_{t+1} \subseteq {}_t$ by part (i) of Proposition 2, we get:

$$\hat{q}_{t+1}(I) \leq \hat{q}_{t+1}(I_t^*) \qquad \text{for all } I \in {}_{t+1}$$

so that $I_{t+1}^* = I_t^*$. □