Price Discrimination and Price Dispersion in a Duopoly†

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Abstract

This paper analyses the problem of price discrimination in a market where consumers have heterogeneous preferences both over a horizontal parameter (brand) and a vertical one (quality). Discriminatory contracts are characterised for different market structures. It is shown that price dispersion, i.e. the observed range of prices for each class of customers, increases almost everywhere as competition is introduced in the market. The findings are discussed with reference to the UK mobile telecommunications market.

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Non-technical abstract

This paper addresses the problem of price discrimination in an oligopoly context where firms have imperfect information about consumers' characteristics. The economics of price discrimination under imperfect information is well understood under monopoly conditions but not entirely under oligopoly. The purpose of the paper is to characterise competition between two rival firms that offer discriminatory contracts. The problem is addressed in a stylised market where consumers have heterogeneous preferences both over a horizontal parameter (brand) and a vertical one (quality). The difference in consumer types over the vertical parameter gives a rationale for non-linear contracts, while the horizontal dimension is used to control for the intensity of price competition. Discriminatory contracts are characterised for different market structures: monopoly, collusive duopoly and competitive duopoly.

Matters are more complex in an oligopolistic setting that under monopoly because the presence of more than one firm gives the customer the right to buy products from different suppliers. In a competitive environment, contracts should not only perform the standard function as a screening device that gives the right incentive to customer to reveal their type, but also they should not give room to rivals. Since firms typically differ in the relative appeal to customers, I study the properties of equilibrium contracts with different intensity of competition.

In the context of the model analysed, I show that oligopolistic interaction between firms can be reduced to reformulating the participation constraints that become type-dependent, so that the solution can be thought of as a monopolist's problem with non-standard binding constraints. It is shown how there are three different mechanisms at work according to the intensity of competition. When competition from a rival is very mild, the basic discriminatory mechanism is the same one as under monopoly, and competition simply redistributes surplus. When competition becomes more intense, the presence of a rival good diminishes the screening ability of an incumbent firm, so that there are also efficiency gains. In particular, quality distortions are reduced so that prices can go up, reflecting higher product quality. Finally, any distortion is eliminated and no screening is possible when brand preferences are not too strong or consumer heterogeneity over vertical parameters is sufficiently high.

The paper also addresses two other questions: price dispersion and capacity constraints. It is shown that price dispersion, i.e. the observed range of prices for each class of customers, increases almost everywhere as competition is introduced in the market. Capacity constraints are included to study the timing of discriminatory practices. I

show that price discrimination may be introduced sooner and the transitory phase leading to the unconstrained equilibrium may last longer under competition that under monopoly.

On a more practical side, the paper seeks an explanation for the pricing patterns emerged in the mobile industry in the United Kingdom. The two separate institutional phases (legal duopoly first, then four operators in competition) correspond in fact quite neatly to different patterns of competition that are in line with the findings of the model.

1. Introduction

The purpose of this paper is to characterise competition between two rival firms that offer discriminatory contracts. Much is known about the analysis of such contracts under monopoly.[1] In practice, discriminatory practices are common in oligopolistic industries, however the economic analysis of this setting is not entirely well understood.

The rationale for discrimination stems from the fact that consumers have heterogeneous preferences over product characteristics. By designing different contracts that suit particular categories of users, a firm can expect to match better the preferences of its consumers and then extract a greater surplus from them. If a monopolist producer knew exactly the preferences of his customers (the customer 'type'), he would offer the most preferred variety to each type, and then charge a price equal to the surplus created. Obviously, the firm could not ask for an excessively high price, otherwise its potential customers would prefer not to buy. Hence the problem of the firm is relatively simple, facing only one kind of constraints, usually called 'participation' or 'individual rationality' (IR) constraints. This would be a case of first-degree price discrimination: allocations are efficient and the firm appropriates the entire surplus. However, perfect discrimination is very unlikely in practice either for legal reasons or because the firm does not observe each type, but is simply aware of its overall distribution. In a context of imperfect information, a producer faces additional constraints. Different contracts, in fact, have to be freely chosen by each consumer. In principle, a bundle designed for low types could be bought also by higher types and vice versa, and the producer must be sure that this does not happen. This is what is usually called a 'self-selection' or 'incentive compatibility' (IC) constraint. It is intuitive that the firm will be more 'cautious' with those consumers with a high willingness-to-pay. One should expect to find efficient allocations for high types, because any other variety would cause a sharp decline in the surplus they enjoy, and this would have to be compensated by a big decline in their price. On the other hand, distortions imposed to low types would have a much lower impact both on consumer surplus and on the firm's profitability. At the same time, distortions on low types can be introduced by the producer in order to make sure that high types will never decide to select a bundle different to the one designed for them.

The argument that we have just sketched in an informal way, has received a great deal of attention in the literature, following a seminal paper of Mussa and Rosen (1978) that has initiated a family of principal-agent problems illustrating the equivalence between price discrimination using quantity discounts (second-degree discrimination) and monopoly pricing of products of differing quality. They show that a monopolist offers a

[1] See Phlips (1983), Varian (1989) and Wilson (1993).

quality range that is broader than that required for efficiency (cf. also Maskin and Riley, 1983, for a general treatment). This is because by exaggerating quality differences, the firm can effectively screen different customers and discriminate between them, and it is in this respect that non-linear pricing is a particular kind of product differentiation. Efficiency, however, is achieved 'at the top', among those customers with the highest willingness to pay

In the simplest case one could think of, with just two types, one of two situations can happen. If the differences between types are very big, then any attempt to make low types buy the product would have a chain effect on contracts offered to high types. The firm is better off by reducing the size of its market by dismissing completely low types, concentrating only on high types over which it can exercise full monopoly power. In the more interesting case with type differences that are not too marked, the firm is willing to serve both categories of users. In general, its contracts have to satisfy the participation and self-selection constraints for each type. However, under a wide range of circumstances, it can be shown that only two 'standard' constraints are binding: IR for low types and IC for high types. Prices extract all the surplus from low types, while some 'informational rent' is left to high types.

Now in the oligopolistic setting, matters are more complex. The presence of more than one firm gives the customer the right to buy products from different suppliers. If firms can offer perfect substitutes, then we can expect Bertrand-type outcomes. Prices will be brought in line with costs, and customers will buy their preferred quality, paying just production costs. The efficiency properties in standard models of perfect competition are well known. However, if firms offer imperfect substitutes, then the analysis is less clear. First of all, it is unlikely that firms will decide to dismiss completely low types even when there are huge variations in the intensity of consumer preferences. A market can be left unserved by a firm only if it does not leave the possibility of profitable entry by a rival. If a market can be potentially covered by two firms, then all consumers will be served, no matter what the difference between types is. This simple observation has a consequence: a producer, even if it enjoys some advantage over its rival (think of brand preferences), cannot leave space for entry to its rival.

The notion of competition plays a central role in understanding how contracts change with respect to a discriminatory monopolist. Contracts in a competitive environment have to perform additional functions: not only they should give the right incentive to customers, but also they should not give room to rivals. Since firms typically differ in the relative appeal to customers, it is interesting to study the properties of equilibrium contracts with different intensity of competition. One implication is that the presence of a rival will have a positive impact on consumer surplus. It is not obvious, however, whether the mechanism at work is simply a transfer between buyers and sellers, or whether allocations are affected as well.

It is helpful, to fix ideas, to begin with the simple extreme of a rival that does not represent a substantial threat. The discriminatory contracts offered by an incumbent will roughly follow the same line of reasoning typical of an uncontested monopolist: the only difference is that some rent has to be left also to low types (the minimal amount such that the rival will never be able to offer a good to them in a profitable way). However, high types are not affected *directly* by the presence of an imperfect substitute: they are already enjoying a substantial informational rent, so that their participation constraint is still slack. At first sight, one could think of a monopolist 'adjusted' problem, with the two standard binding constraints: IC for high types and 'adjusted' IR for low types. Since the fundamental constraint is self-selection, the incumbent still widens the quality spectrum in order to discriminate among the consumers he faces. The basic discriminatory mechanism at work is the same one as under pure monopoly, thus competition has no impact on efficiency but simply redistributes surplus.

Turning to the more general case of firms offering goods that are not too imperfect substitutes, then the picture gets more complicated. Now high types can find appealing not only the low-type bundle, but also the rival good. The producer is then overconstrained compared to the pure monopoly case, and this has a big impact on its screening ability. The forces at work are both the presence of a potential rival good and the compatibility between the contracts offered to different consumers. The incumbent firm loses some of its screening ability, which means that the quality distortion cannot be as big as under monopoly. Competition then yields efficiency gains, but low types can also be prepared to pay a *higher* price for higher quality.

We have already said that the screening potential is completely eliminated when firms offer perfectly substitutable goods. Does this imply that efficiency is reached only in that case, or is 'sufficient' substitutability enough? We will show that the latter case is true. Efficiency is typically reached, as it is intuitive, in a region characterised by brand preferences that are not too strong, but also when brand preferences are strong *and* differences between types are substantial. This is because the willingness to pay for any good increases with the intensity of preferences. A very high type could still enjoy quite a lot of surplus even from a good that is quite distant from his ideal brand.

The problem of discriminatory contracts under imperfect competition is addressed with a model of two firms located at the endpoints of a line segment along which consumers are located. Consumers have heterogeneous preferences both over a horizontal parameter (brand) and a vertical one (quality). It is assumed that firms observe the location parameter while vertical preferences are private information. The difference in types gives a rationale for non-linear contracts, while the horizontal dimension is used to control for the intensity of price competition. It should be noted that previous work has been done on the symmetric case of unobservable horizontal parameters and observable vertical ones (Spulber, 1989; Hamilton and Thisse, 1997), while the case dealt by this paper has not been studied before, with the exception of Stole (1995).[2]

In the context of the model analysed in this paper, oligopolistic interaction between firms can be reduced to reformulating the individual rationality constraints that become type-dependent. Once the participation constraints have been modified to take account of the strategic effect deriving from competition, then the optimal solution for each firm can be thought of as a monopolist's problem with 'non-standard' binding constraints. We are able to characterise closed-form solutions that change according to preferences over brand and quality. In particular, we discuss how there are three different discriminatory mechanisms at work that define three corresponding regions according to consumers' tastes. We also address a second question of some empirical interest that is related to price dispersion, i.e. the observed range of prices for class of customers. The model that we analyse shows that, under competition, price dispersion increases almost everywhere compared to the monopoly case, despite the fact the quality range is reduced. The model is then extended to include capacity constraints. This allows us to study another interesting feature, namely the *timing* of discriminatory practices. In this case, the main finding is that price discrimination may be introduced sooner and the transitory phase leading to the unconstrained equilibrium lasts longer under competition.

The basic set up of the model is presented in section 2. The solutions to the first-best and to the monopoly case are derived in section 3. Sections 4 and 5 discuss the more complex case of a competitive duopoly, with particular reference to the effects of competition on consumer participation constraints. Section 6 analyses how the situation is affected when capacity constraints are added to the basic model. Finally, section 7 applies the main results to interpret observations from the UK mobile telecommunications market and section 8 concludes.

2. The model

Consumers with heterogeneous tastes buy a single unit of a certain product. They differ in the ideal brand, and this is modelled as in traditional spatial models of horizontal product differentiation. The brand space is represented by a line of unit length along which

[2] See also Martimort (1992) and Stole (1991) on the more general problem of multiprincipal incentive theory. Earlier works on third-degree price discrimination under imperfect competition include Borenstein (1985) and Holmes (1987).

consumers are distributed. Each consumer is identified with her own location d. The loss of surplus to the consumer when she buys a product which does not coincide with her ideal is dependent on the distance between the product bought and the consumer's ideal, that is the distance between the seller's and consumer's locations.

Products are also assumed to be differentiated in terms of a vertical attribute u referred to as quality. At each location there is an equal measure of two types of consumers, a high type and a low type with the former valuing a given u more than the latter. This difference is taken into account by a parameter $\theta \in \{\underline{\theta}, \bar{\theta}\}$, $\bar{\theta} \geq \underline{\theta} > 0$. Both types are uniformly distributed along the line, with mass 1 each.

When a consumer of type θ buys at a price p a product of quality u produced by a firm i located at d_i, which is then at a distance $|d - d_i|$ from her, she enjoys a net surplus:[3]

$$V = U(\theta, d, u) - p = \theta(1 - |d - d_i|)u - p.$$

Turning to the production technology, a unit of a good of quality u can be supplied at a quadratic cost: $C(u) = u^2/2$.

In principle, this set up could also be used to address the problem of second-degree price discrimination. Under monopoly conditions, in fact, the quantity-pricing problem is isomorphic to the quality-pricing problem of a monopolist (Varian, 1989). For instance, with a transformation $x = u^2/2$, preferences can be rewritten as $U(\theta, d, x) = \theta(1 - |d - d_i|)(2x)^{1/2}$, where x can be interpreted as quantity. The cost function becomes $C(x) = x$, which means that each unit is produced at a constant marginal cost equal to unity. However, we prefer to stick to the original formulation of a 'quality' problem. This is because with multiple suppliers the choice to buy only from one firm should rise endogenously in the 'quantity' problem, which would impose additional constraints.

In the remaining part of the paper we will assume that the location parameter d can be perfectly observed, while this is not the case for the preferences over the vertical attribute. However, the producer knows the distribution of θ, hence he can practice price discrimination at a given location. The horizontal parameter is introduced in this model to regulate the intensity of price competition. Its perfect observability allows us to consider the effects of 'pure' price discrimination when the producer has varying degrees of market power along the line.

3. Benchmarks: efficiency and monopoly

[3] More precisely, $V(\cdot)$ is the conditional indirect utility function of type θ and its formulation follows a tradition that goes back to Mussa and Rosen (1978). Peitz (1995) shows that it is possible to construct an associated direct utility function, despite continuity problems, so that the behaviour of discrete choice and unit demand adopted here can be derived from utility maximisation.

3.1 Efficiency

Assume that a single firm is located at the origin of the line ($d_i = 0$). The efficient allocation is the one that maximises social surplus at every point along that line:

$$\max_{\underline{u},\bar{u}} \underline{U} + \overline{U} - C(\underline{u}) - C(\bar{u}) = \underline{\theta}(1-d)\underline{u} + \bar{\theta}(1-d)\bar{u} - \frac{\underline{u}^2}{2} - \frac{\bar{u}^2}{2}$$

where the underlined variables refer to low types and overlined ones to high types. The efficient allocation is:

$$\begin{cases} \underline{u}^e = \underline{\theta}(1-d) \\ \bar{u}^e = \bar{\theta}(1-d). \end{cases}$$

Since goods of higher qualities are more expensive to produce, it turns out to be efficient to allocate them only to those consumers whose valuation of quality is sufficiently high, i.e. consumers with strong preferences over brand or over the vertical attribute. This explains why the efficient u increases with type and decreases with location.

3.2 Single-plant monopolist

Consider now the case of a monopolist operating the same plant at $d = 0$. In general, the strategy space of the monopolist would consist of a family of schedules, one for each value of d, labelled $p(u;\ d)$. Since there are only two types at each location, we can confine the attention to the special case of two contracts being offered at each d. In addition, the monopolist can also perfectly discriminate over distance (there cannot be arbitrage between consumers at different locations), so that each pair of contracts can be treated separately. As a consequence, we can drop the dependence of contracts on d for simplicity of notation. At every location, the monopolist offers pairs of contracts $(\underline{p}, \underline{u})$, $(\bar{p}, \bar{u})$ designed for the two different types that have to self-select them. His aim is to maximise total profits at d, subject to participation (IR) and incentive compatibility (IC) constraints for both types:

$$\max_{(\underline{p},\underline{u})(\bar{p},\bar{u})} \Pi = \underline{\Pi} + \overline{\Pi} = \underline{p} - \frac{\underline{u}^2}{2} + \bar{p} - \frac{\bar{u}^2}{2}$$

subject to

$$\underline{\text{IR}}:\ \underline{\theta}(1-d)\underline{u}-\underline{p}\geq 0$$

$$\overline{\text{IR}}:\ \bar{\theta}(1-d)\bar{u}-\bar{p}\geq 0$$

$$\underline{\text{IC}}:\ \underline{\theta}(1-d)\underline{u}-\underline{p}\geq \underline{\theta}(1-d)\bar{u}-\bar{p}$$

$$\overline{\text{IC}}:\ \bar{\theta}(1-d)\bar{u}-\bar{p}\geq \bar{\theta}(1-d)\underline{u}-\underline{p}$$

As it is standard in this kind of problem with adverse selection, the only binding constraints are IR for the low type and IC for the high type. Figure 1 illustrates the optimal contracts offered at a generic location *d*. Since $U(\theta, d, u) = \theta(1 - d)u$, the gross utility enjoyed by customers is represented by a ray from the origin in the (u, p) plane. $\underline{\text{IR}}$ binds which means that there is no surplus left to low types, so that the contract offered to them lies on the corresponding ray. The binding $\overline{\text{IC}}$ constraint can be rewritten as:

$$\frac{\Delta p}{\Delta u}=\frac{\bar{p}-\underline{p}}{\bar{u}-\underline{u}}=\bar{\theta}(1-d)$$

which has the same slope as the gross utility of high types. The optimal quality for high types is determined by the tangency condition of a line with the previous slope and the cost function. The monopoly equilibrium at *d* is represented by the two points L and H.

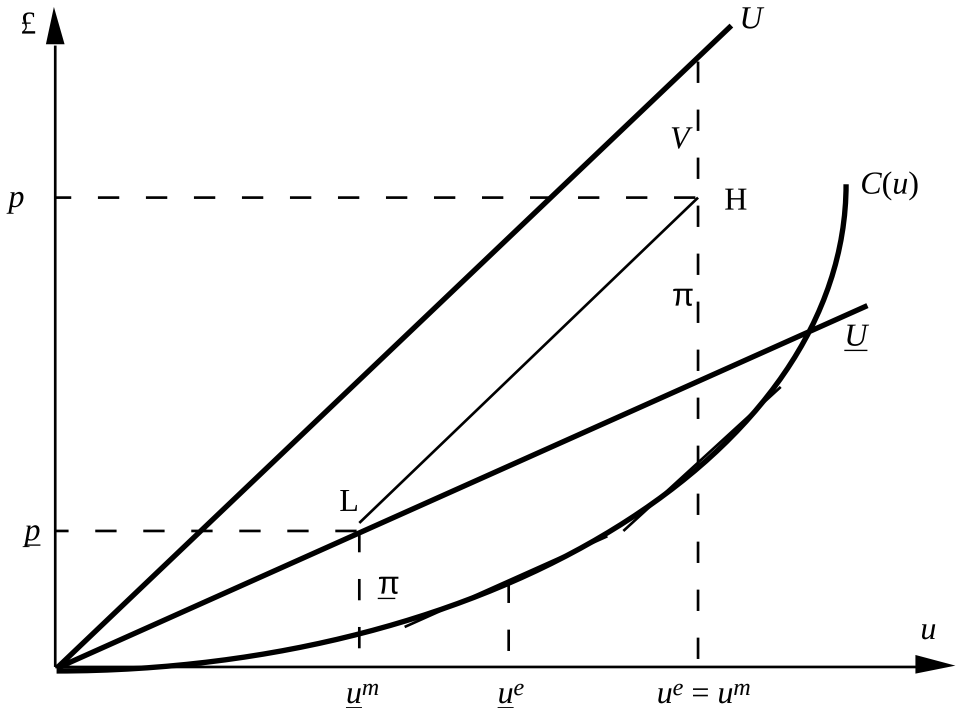

Figure 1: Monopoly - Equilibrium contracts at a given location *d*

The algebraic solution is:

$$(1)\quad \begin{cases} \underline{u}^m = (1-d)(2\underline{\theta} - \bar{\theta}) < \underline{u}^e \\ \bar{u}^m = (1-d)\bar{\theta} = \bar{u}^e \end{cases} \quad \begin{cases} \underline{p}^m = (1-d)^2(2\underline{\theta} - \bar{\theta})\underline{\theta} \\ \bar{p}^m = (1-d)^2(2\bar{\theta}^2 - 3\bar{\theta}\underline{\theta} + 2\underline{\theta}^2) > \underline{p}^m \end{cases}$$

$$\begin{cases} \underline{\Pi}^m = (1-d)^2(2\underline{\theta} - \bar{\theta})\bar{\theta}/2 \\ \bar{\Pi}^m = (1-d)^2(3/2\bar{\theta}^2 - 3\bar{\theta}\underline{\theta} + 2\underline{\theta}^2) > \underline{\Pi}^m \end{cases} \quad \begin{cases} \underline{V}^m = 0 \\ \bar{V}^m = (1-d)^2(\bar{\theta} - \underline{\theta})(2\underline{\theta} - \bar{\theta}) \end{cases}$$

The solution displays 'efficiency at the top', while the quality offered to the low type is distorted away from the efficient one. Since the fundamental constraint is self-selection, the monopolist chooses a pricing scheme that induces consumers of each quality level to prefer their own quality to any other one. The monopolist widens the quality spectrum in order to effectively discriminate among the consumers he faces. Prices extract the entire surplus from low types, while some informational rent is left to high types. Finally, the market is completely served if $(2\underline{\theta} - \bar{\theta}) \geq 0$. It will be convenient to refer to the ratio between the two type parameters, then the previous condition can be rewritten as $1 \leq k = \bar{\theta}/\underline{\theta} \leq 2$. This restriction on k says that all consumers are economically interesting for a monopolist.

While figure 1 illustrates the equilibrium values of price, quality, profit and surplus at a given location, figure 2 reports the different schedules for p, u, Π and V as functions of the distance d. While price discrimination relates to the design of contracts at each location, price dispersion is identified with the observed ranges of prices offered to different customers along the line. We will refer in the paper to two different notions of price dispersion:

- Price dispersion for a certain class of customers in a given region is defined as the difference between the highest and the lowest price observed in that region, for any given quality.[4]
- Price dispersion at a location d is defined as the difference between the highest and the lowest price observed at d.[5]

[4] As an example, in the monopoly case price dispersion for $\underline{\theta}$-types in [0, 1/2] is $\underline{p}(0)$ - $\underline{p}(1/2)$.
[5] Price dispersion at d is then $\bar{p}(d) - \underline{p}(d)$.

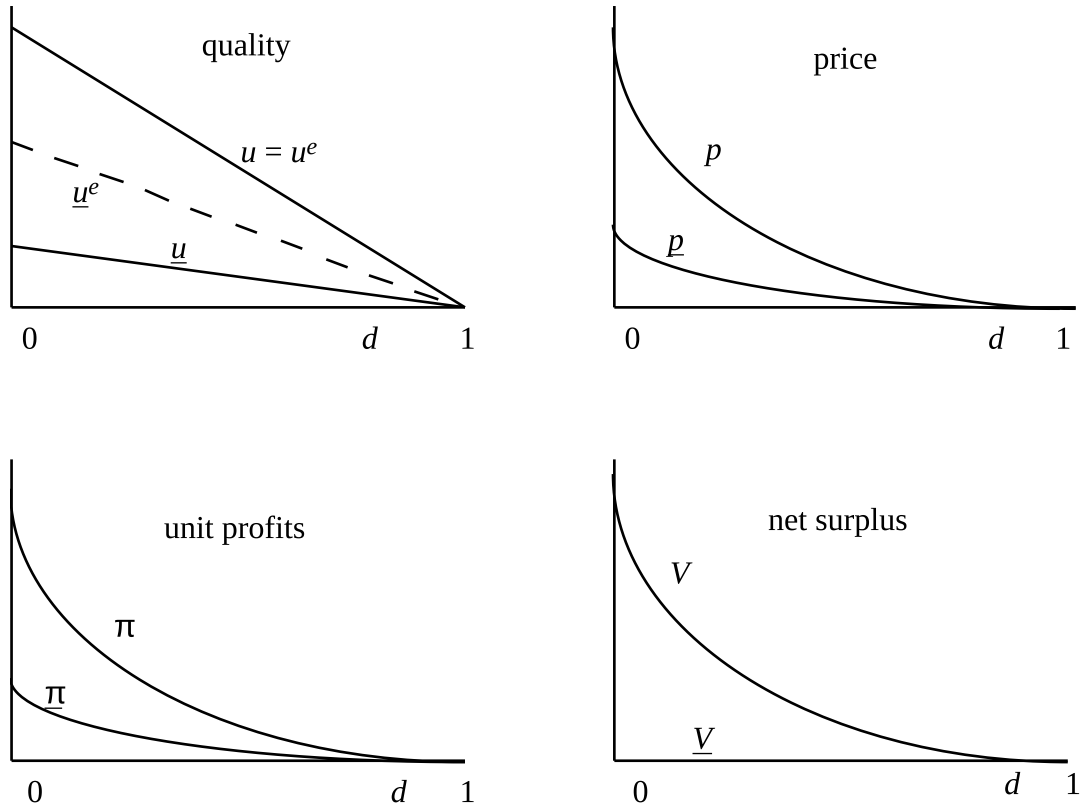

Figure 2: Monopoly - Equilibrium along the unit line

In the case $k > 2$, the monopolist would prefer not to supply low types at any location. High types would have their surplus completely extracted and the solution would be:

(1')
$$\begin{cases} \bar{u} = (1-d)\bar{\theta} = \bar{u}^e \\ \bar{p} = (1-d)^2\bar{\theta}^2 \\ \bar{\Pi} = (1-d)^2\bar{\theta}^2 / 2 \\ \bar{V} = 0 \end{cases}$$

We will show that competitive duopolists are always forced to provide goods to both types. We concentrate more on the case $1 \leq k \leq 2$ because the resulting optimal contracts are richer. In particular, a subset of such contracts can be extended for bigger differences between taste parameters so that the solution is also fully characterised for higher values of k.

3.3 Multiplant monopolist

Consider now the situation of a multiplant monopolist. Imagine there is a second plant that can be operated at the opposite end of the line ($d = 1$). This case is also coincident with the situation of two plants run by two different but perfectly collusive firms. This is introduced as a natural step towards the analysis of a competitive duopoly. The symmetric location of plants implies that contracts can be studied only in the range [0, 1/2] since contracts in [1/2, 1] are a mirror image. The monopolist's solution (1) is then still valid for locations between the origin and $d = 1/2$ which are supplied by the plant located at the origin, while the other plant provides the good to the remaining half of the market (contracts are obtained from (1) replacing d with $1 - d$).

4. Effects of competition on consumers' participation constraints

The aim of this and the following sections is to analyse the effects on quality and prices induced by the presence of a second, independent firm at $d = 1$. We will refer with a subscript 0 (respectively 1) to the variables related to the firm located at the origin of the line (respectively at the end of the line). We will drop subscripts when ambiguities do not arise. The distance between a consumer at d and firm 1 is therefore $|d - d_1| = 1 - d$. Both firms have the same technology of production and this symmetry, combined with the spatial model of preferences, immediately suggests that in equilibrium the market will be split in two equal parts. This conjecture will be confirmed as the section develops. Intuitively, consumers in [0, 1/2] will buy from firm 0 and consumers (1/2, 1] from firm 1 because each firm enjoys an advantage over the rival firm for those customers who are relatively closer. Suppose, on the contrary, that a customer at $d < 1/2$ is served by firm 1 in equilibrium. A minimum rationality requirement implies that firm 1 is making non-negative profits on that customer. By mimicking the same price-quality schedule, firm 0 can attract the customer while securing the same level of profits as firm 1. Therefore such customer cannot be served by the more distant firm.

It is crucial to note that firms are not behaving as simple monopolists in their own markets. If this was the case, the solution for firm 0 would simply be to replicate the monopolist solution (1) in [0, 1/2], but in such a case firm 1 would be able to offer a better contract to some consumers without incurring losses. Since in equilibrium firm 1 cannot supply the first half of the market, the contracts offered by firm 0 in its market must differ from those described in section 3. In particular, contracts must take into account that no profitable entry into firm 0's market by firm 1 should occur. Absence of entry does not mean absence of competition, and in fact potential entry imposes a real constraint on firm 0's actions. Firm 0 has to make offers to its customers that cannot be matched by the rival. Using an undercutting argument we can also predict that the best price-quality schedule offered by a firm to its rival's customers must follow a zero-profit condition. The existence of equilibrium contracts under more general conditions has been proven by Stole (1995) and his arguments can be translated into our model (all proofs are contained in the Annex):

Proposition 1. There exists an equilibrium in which firms split the market equally. All customers are served and the reservation value of each consumer in (0, 1/2] is greater than zero and increases in d: $V \geq d^2\theta^2/2$.

The possibility of entry gives the customer the right to buy the other firm's product under particular conditions. In a certain sense the outside option for each captive consumer is endogenised by the presence of a potential rival firm. In the monopoly case the only option left to the consumer is inaction, now the customer has the right of buying a more distant product. In the context of our model, oligopolistic interaction can be reduced to reformulating the individual rationality constraints. These take into account the net surplus that each consumer could get from a real good eventually bought from another firm. It is important to mention that the net surplus V is higher for higher types not only because they value any given quality more than lower types, but also because they would choose a higher quality variant of the distant firm's product. Once the participation constraints have been modified, then the optimal solution for each firm, within its individual market, can be thought of as a monopolist's problem as before.[6, 7]

Note that if we simply juxtapose the functions for V^1 (the net surplus that a consumer could get from the rival good) on the previous diagram obtained for the 'unadjusted' monopolist maximisation, the result is that the monopolist would lose all the low types and a fraction of the high types (those to the right of $\underline{d}$, see figure 3). Once more, this shows that the previous monopoly schedules cannot be supplied in equilibrium by firm 0. We next turn to study the new optimal allocation.

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[6] Jullien (1997) deals with the similar problem of optimal contracts offered by an uniformed principal when the agent's reservation utility is type-dependent.

[7] In principle there are multiple equilibria. Given that a firm does not sell in the rival's market, the former may impose bigger burdens on the latter, for instance offering goods priced *below* their production cost. This would force the incumbent to redesign his offers. However, any strategic-form refinement of Nash equilibria would rule out these situations.

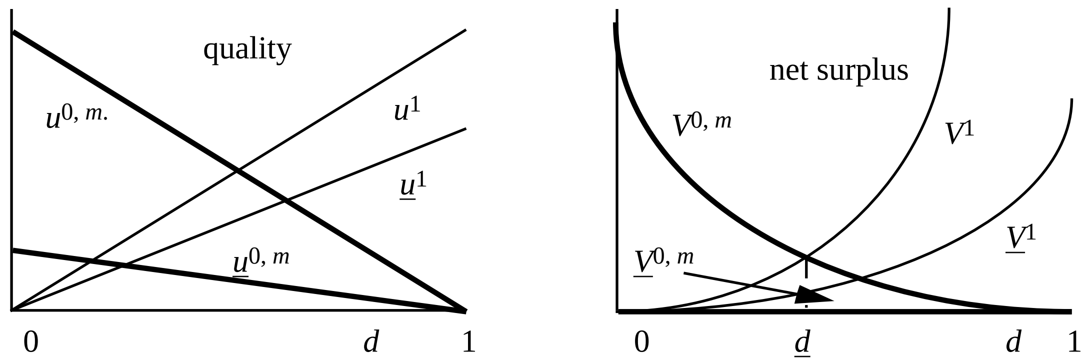

Figure 3: Quality of the rival good and reservation utility - Comparison with the monopoly case

5. Solution of the duopoly case

The previous section has developed the idea that each incumbent firm has to readjust its optimal policy when it faces a rival. In particular, no schedule offered by the rival should be able to steal customers profitably. The main implications are that both participation constraints are affected by the presence of a competing firm and that the rival good would give the type-θ customer a net surplus of $V^1 = d^2\theta^2/2$. Firm 0 has to find the optimal price-quality schedules in its market [0, 1/2] as a solution to the following maximisation problem (superscripts referring to firm 0 are omitted for simplicity of notation):

$$\max_{(\underline{p},\underline{u})\,(\overline{p},\overline{u})} \Pi = \underline{\Pi} + \overline{\Pi} = \underline{p} - \frac{\underline{u}^2}{2} + \overline{p} - \frac{\overline{u}^2}{2}$$

subject to

$$\underline{\mathrm{IR}}:\ \underline{\theta}(1-d)\underline{u} - \underline{p} \geq d^2\frac{\underline{\theta}^2}{2}$$

$$\overline{\mathrm{IR}}:\ \overline{\theta}(1-d)\overline{u} - \overline{p} \geq d^2\frac{\overline{\theta}^2}{2}$$

$$\underline{\mathrm{IC}}:\ \underline{\theta}(1-d)\underline{u} - \underline{p} \geq \underline{\theta}(1-d)\overline{u} - \overline{p}$$

$$\overline{\mathrm{IC}}:\ \overline{\theta}(1-d)\overline{u} - \overline{p} \geq \overline{\theta}(1-d)\underline{u} - \underline{p}$$

We will discuss the solution to the previous program when $1 \leq k \leq 2$, starting from the origin, $d = 0$, and then going to the right. This is because firm 0 enjoys 'power' over customers with strong brand preferences and the analysis is not complicated greatly by the change in IR constraints. Going further to the right, the intensity of competition increases because the rival good becomes better relative to firm 0's variety and new effects will play a relevant role.

In principle there could be many combinations of the four constraints in firm 0's maximisation problem but, as we show in the Annex, only three of them are plausible. Each one of these three combinations corresponds to a particular mechanism, which gives birth to an optimal solution in a certain region. We are also going to show how such regions of validity depend on the two parameters *k* and *d*.

From figure 3, it is quite clear that $\underline{\text{IR}}$ binds immediately, and this is the only change compared to the monopolist's problem. A solution can be easily found in the first region, labelled A, which runs from the origin to a certain location *d**. The solution takes into account only $\underline{\text{IR}}$ and $\overline{\text{IC}}$ binding:

$$\begin{cases} \underline{u} = \underline{u}^m \\ \bar{u} = \bar{u}^m \end{cases}$$

$$\begin{cases} \underline{p} = (1-d)^2(2\underline{\theta} - \bar{\theta})\underline{\theta} - d^2\underline{\theta}^2/2 = \underline{p}^m - d^2\underline{\theta}^2/2 \\ \bar{p} = (1-d)^2(2\bar{\theta}^2 - 3\bar{\theta}\underline{\theta} + 2\underline{\theta}^2) - d^2\underline{\theta}^2/2 = \bar{p}^m - d^2\underline{\theta}^2/2 \end{cases}$$

$$(2) \qquad \begin{cases} \underline{\Pi} = (1-d)^2(2\underline{\theta} - \bar{\theta})\bar{\theta}/2 - d^2\underline{\theta}^2/2 \\ \bar{\Pi} = (1-d)^2(3/2\bar{\theta}^2 - 3\bar{\theta}\underline{\theta} + 2\underline{\theta}^2) - d^2\underline{\theta}^2/2 \end{cases}$$

$$\begin{cases} \underline{V} = d^2\underline{\theta}^2/2 = \underline{V}^1 \\ \bar{V} = (1-d)^2(\bar{\theta} - \underline{\theta})(2\underline{\theta} - \bar{\theta}) + d^2\underline{\theta}^2/2 = \bar{V}^m + d^2\underline{\theta}^2/2 > \bar{V}^1 \end{cases}$$

thus we encounter a first region in which price dispersion increases for each type of consumers as a result of the presence of the rival firm. Notice that price dispersion increases despite the fact that the corresponding quality offered at each location is unaltered. On the other hand, price dispersion at a given location is unchanged compared to the monopoly case. The basic discriminatory mechanism at work is the same as the one discussed in section 3.2. Among the four possible constraints, only the 'standard' ones are binding. The same quality distortions result, thus competition has no impact on efficiency but simply redistributes surplus.

The effects of the alternative good on surplus are identical in absolute terms for both types of consumers but they are caused by two different reasons. Low types are offered better deals because they would otherwise change supplier. Price decreases with *d* because more distant customers have less strong brand preferences for the variety supplied at 0. In other words, it is easier for firm 0 to keep closer customers than more distant ones. High types are not directly affected by the presence of their alternative good, but they indirectly benefit through the effects on low types. After the price reduction, the low-type contract becomes appealing also for high types, therefore their price has to be reduced accordingly, in order to maintain the compatibility between contracts ($\overline{\text{IC}}$ binds).

Since at any given location high types value a product more than low types, it is easier for firm 0 to retain high types, and this explains why they are offered lower discounts in percentage terms. It also results relatively easy for the firm to separate the two classes of customers. This is true so long as the rival good does not become interesting to high types too, which is more likely for high values of $\bar{\theta}$. As a result, the extension of region A narrows as the difference in taste parameters increases.

Proposition 2. When d is in region A = [0, $d^*(k)$] discriminatory contracts are given by (2). Quality is as in the monopoly case. Price dispersion for each class of customers increases with respect to the monopoly case in the same region, while price dispersion between customers at a given location is unchanged. The width of the interval decreases with k.

When consumers located at d^* are reached, the $\overline{\text{IR}}$ constraint starts binding and the solution changes. We enter a new region, labelled B, that runs from d^* to $\hat{d}$. Three constraints are binding simultaneously and the solution results as follows:

$$(3)\quad \begin{cases} \underline{u} = \dfrac{d^2}{1-d}\dfrac{\underline{\theta}+\bar{\theta}}{2} > \underline{u}^m \\ \bar{u} = \bar{u}^m \end{cases}$$

$$\begin{cases} \underline{p} = d^2\underline{\theta}\bar{\theta}/2 = \underline{p}^m + \underline{\theta}[d^2\bar{\theta}/2 - (1-d)^2(2\underline{\theta} - \bar{\theta})] \\ \bar{p} = [(1-d)^2 - d^2/2]\bar{\theta}^2 = \bar{p}^m - [d^2\bar{\theta}^2/2 - (1-d)^2(\bar{\theta} - \underline{\theta})(2\underline{\theta} - \bar{\theta})] \end{cases}$$

$$\begin{cases} \underline{\Pi} = \dfrac{d^2}{2}[\underline{\theta}\bar{\theta} - \dfrac{d^2}{(1-d)^2}\dfrac{(\underline{\theta}+\bar{\theta})^2}{4}] \\ \bar{\Pi} = (1-2d)\bar{\theta}^2/2 \end{cases}$$

$$\begin{cases} \underline{V} = d^2\underline{\theta}^2/2 = \underline{V}^1 \\ \bar{V} = d^2\bar{\theta}^2/2 = \bar{V}^1 \end{cases}$$

In region B, firm 0 has to take into account that the rival good has also become attractive for its high-type consumers. Compared to region A, higher discounts are necessary to retain them and price dispersion for high types increases even more than before. The force at work is now the presence of a potential rival good rather than the compatibility between the contracts. In region B the firm is particularly constrained, so it first optimises on the most profitable type, and then adjusts the other contract accordingly. In order to have low types self-select the contract designed for them, firm 0 starts reducing the distortion of quality. Competition not only redistributes surplus as in region

A, but also yields efficiency gains. The quality distortion is reduced as d increases, that is for those consumers which are less sensitive to brand differences. Such consumers pay higher prices for higher quality. This also implies that price differences between types at any location are reduced. When price dispersion for a class of consumers in a given region is defined as the difference between the highest and lowest price observed in that region, then price dispersion may increase in region B also for low types. It turns out that this is true according to the ratio of vertical tastes.

Proposition 3. When d is in region B = $[d^*(k), \hat{d}(k)]$ discriminatory contracts are given by (3). High-type customers are offered the same quality as in the monopoly case and price dispersion among them increases with respect to the monopoly case. Distortions in the quality offered to low-type customers are reduced, and their prices increase with location. Price dispersion for low types increases if k is high enough ($k > 1.44$). Price dispersion between customers at a given location decreases. The width of the interval increases with k.

When the last consumer in region B is supplied, the nature of the problem changes again since the $\overline{\text{IC}}$ constraint is not binding anymore. The solution in the region close to the centre, labelled C, is the last one relevant to our model and it is characterised by:

$$(4)\quad \begin{cases} \underline{u} = \underline{u}^e \\ \bar{u} = \bar{u}^e \end{cases} \quad \begin{cases} \underline{p} = [(1-d)^2 - d^2/2]\underline{\theta}^2 = \underline{p}^m - \underline{\theta}[d^2\underline{\theta}/2 - (1-d)^2(\bar{\theta} - \underline{\theta})] \\ \bar{p} = [(1-d)^2 - d^2/2]\bar{\theta}^2 = \bar{p}^m - [d^2\bar{\theta}^2/2 - (1-d)^2(\bar{\theta} - \underline{\theta})(2\underline{\theta} - \bar{\theta})] \end{cases}$$

$$\begin{cases} \underline{\Pi} = (1-2d)\underline{\theta}^2/2 \\ \overline{\Pi} = (1-2d)\bar{\theta}^2/2 \end{cases} \quad \begin{cases} \underline{V} = \underline{V}^1 \\ \overline{V} = \overline{V}^1 \end{cases}$$

If consumer θ were to buy from the rival, she would choose $u^1 = \theta d$, enjoying a net surplus $V^1 = d^2\theta^2/2$. The difference in the quality potentially available for the two types increases approaching the centre. In practice, once the outside good is taken into account, such a difference separates the two problems and in region C firm 0 does not have to worry about the compatibility between contracts but only about both participation constraints. We saw before that in region B the quality offered to low types is increasing in d. At a certain location, which coincides with $\hat{d}$, $\underline{u}$ reaches the efficient level. After that location, the firm is not concerned by the compatibility of contracts, and it is not necessary to overshoot the efficient quality offered to low types. In the entire region C the efficient allocation is reached for all consumers. Since firm 0 still enjoys an advantage deriving from brand preferences, the price charged to customers allows for positive profits. Zero profits result only for the marginal consumers at $d = 1/2$. These are the customers who are exactly indifferent between the two brands, therefore the intensity of competition is maximal and drives away all profits. In region C we still have that price dispersion increases for both types. Since high types are offered the same quality both under monopoly and duopoly, it is clear that prices are also unambiguously lower in a more competitive environment. On the other hand, such a conclusion is not necessarily true for low types. The principal effect of competition is to reduce allocative distortions, i.e. the quality offered to low types increases. As a result, these customers are also prepared to pay a higher price and price differences between types decrease.[8] Finally, the extension of region C depends only on its left bound, since its right bound is fixed at 1/2. The left bound $\hat{d}(k)$ decreases with k, therefore region C widens as k increases.

Proposition 4. When d is in region C = $[\hat{d}(k), 1/2]$ discriminatory contracts are given by (4). Both types of customers are offered the efficient quality. Price dispersion for each class of customers increases with respect to the monopoly case in the same region, while price dispersion between customers at a given location decreases. The width of the interval increases with k.

The extension of each of the three regions that we have identified depends monotonically on k, the ratio between the taste parameters. Figure 4 draws a phase diagram that gives regions A, B and C as functions of d and k. It is useful to recapitulate the three different discriminatory mechanisms at work in each region and their dependence on location and vertical tastes. In region A, the rival good is so distant that it imposes a weak constraint on firm 0's policy. No quality adjustments are required compared to the monopoly case, rather price reductions are sufficient to outperform the rival firm. In terms of consumer tastes, region A is characterised by either strong brand preferences for firm 0's variety, and/or small differences in the evaluation of the vertical attribute. In practice, firm 0 has to offer better deals to low types, which induces price reductions also to high types in order to preserve self-selection. As d and k increase, the outside option becomes more valuable for high types either because it is closer to their ideal, or because they value quality more. In region B, the incumbent is constrained by the potential outside choices of

[8] It can be checked that the duopoly price for low types is always lower than the monopoly price in *all* region C when $k < 2^{1/2}$, and always higher when $k > 1.5$. In any case the difference between the highest and the smallest observed price increases.

both classes of consumers that still have to be induced to choose their designed contract. The self-selection problem disappears in region C, which is valid close to the centre of the line and for high enough values of k. This is because the rival goods cause a significant difference in the reservation utility for the two types. When the incumbent ensures its customers the same level of surplus as the outside option, there is not the risk that high types should try to report a type different from their own. Numerical values for the three regions are reported in the following table:

k	d^*	$\hat{d}$	*region A*	*region B*	*region C*
2	0	0.449	0	0.449	0.051
1.9	0.208	0.454	0.208	0.246	0.046
1.5	0.387	0.472	0.387	0.085	0.028
1.1	0.481	0.494	0.481	0.013	0.006
1	0.5	0.5	0.5	0	0

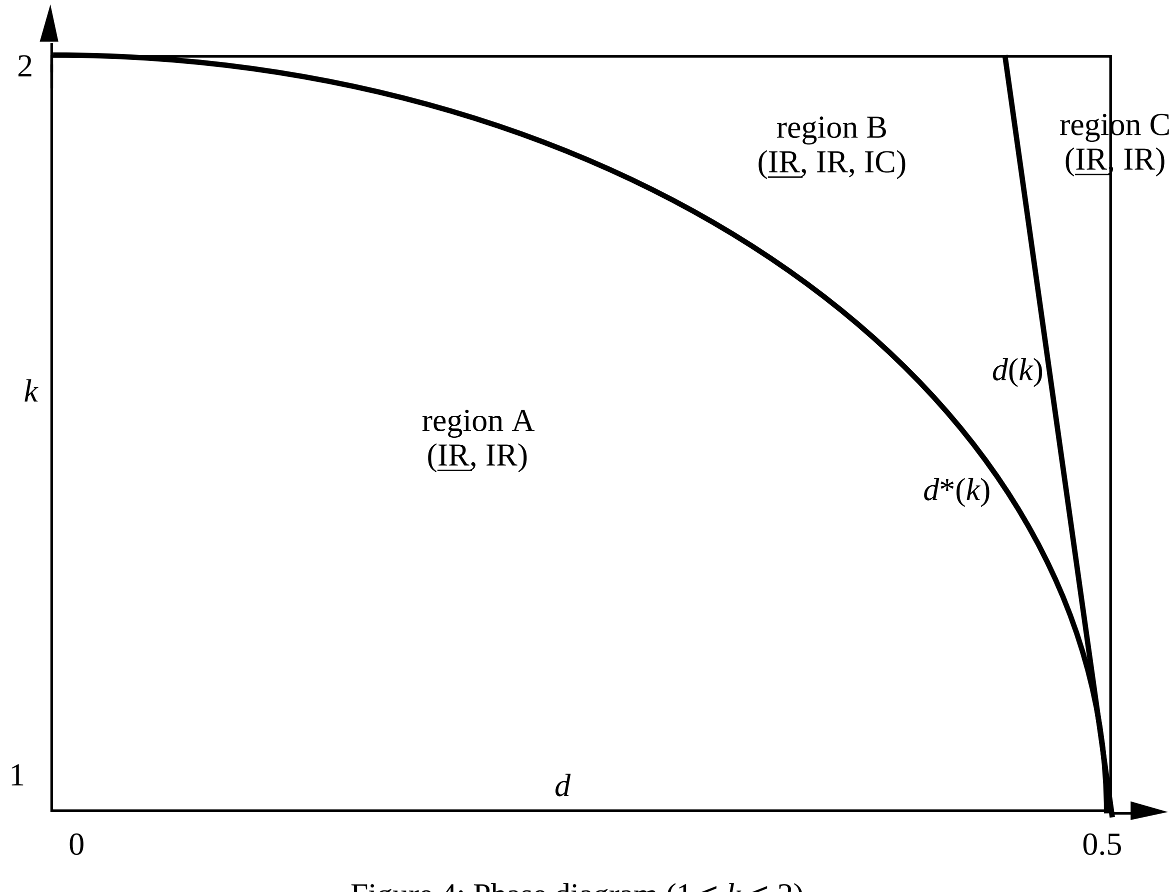

Figure 4: Phase diagram (1 ≤ k ≤ 2)

The results of Propositions 2-4 are summarised with a graphical representation in figure 5. Figure 5a shows the effect of increased competition on the quality offered by the incumbent firm at each location. As one moves from region A to region C, firm 0 sees its monopoly power being reduced and its production choice is disciplined towards efficiency. When low-type customers are not too sensitive to the rival option (region A), the incumbent still offers them inefficiently low quality as a screening device that allows him to discriminate among heterogeneous customers. When low-type consumers have weaker preferences for firm 0's brand (regions B and C), the rival good has a bigger impact because it is valued more by all consumers along the horizontal dimension and competition drives to allocative efficiency.

Figure 5b illustrates the effects on prices offered at each location to the two classes of consumers. Taken in isolation, price dispersion for each type increases in a given region compared to the monopoly case, with one possible exception (low types in region B, when k is small, which also means that region B itself is very narrow). In region A, price cuts are first needed to ensure the participation of low types, and then to satisfy self-selection of high types. Therefore discounts for high types are induced by incentive compatibility rather than the presence of their outside option. In regions B and C, the reservation utility for high types is binding everywhere, and firm 0 increases even more the discount for high types. At first sight, region B shows a somehow paradoxical result: prices for low types increase with d, and it may even be the case that they result to be higher than the monopoly price at the same location. This is consistent with the fact that firm 0 is also increasing the quality offered in the same region, therefore customers are willing to pay a premium price for a level of quality which is closer to the efficient one. In a sense, we can say that the effects of competition are more 'evident' on prices rather than quality in region A, while the reverse is true in region B. Finally, when distortions are completely eliminated, we fall in region C that shows declining prices corresponding to decreasing levels of quality.

Figure 5c shows consumer net surplus in each region. $\underline{IR}$ binds everywhere, therefore net surplus for low types coincides with the reservation utility they could derive from the rival good. This is also true for high types in region B and C, while in region A the rival good does not impose any additional constraint on top of self-selection. Since competition does not allow the incumbent to extract as much surplus as in the monopoly case, it results that net consumer surplus increases everywhere (except from the origin) under oligopoly. Also notice that at every given location, high types are always left with a bigger informational rent than low types.

Finally, figure 5d shows the profit made at each location. Not surprisingly, we find that the incumbent makes more profit on closer customers. Stronger brand preference gives an absolute advantage to firm 0 on its rival. The advantage declines as we get closer to the centre, hence competition is tougher and reaches its maximal level at $d = 1/2$. With the exception of the centre of the line, at any other location firm 0 can also extract greater profits from high types than from low types. This simple result plays a central role if firms are capacity constrained. This is studied in section 6.

We have discussed so far contracts and resulting price dispersion in each of the three regions. It is also interesting to consider *total* price dispersion for each type in the entire region [0, 1/2]. This is of some empirical relevance when data at the researcher's disposal are aggregated for class of consumers. The result is straightforward for high types and price dispersion for them increases under duopoly. For low types, on the one hand there is a downward pressure on prices due to competition, on the other hand competition also forces the producer to increase the quality supplied to low types that are therefore also willing to pay a higher price. The net effect depends on consumers' vertical preferences and in the Annex we prove the following:

Proposition 5. Total price dispersion for high types increases under competitive duopoly compared to monopoly for all values of k. Total price dispersion for low types increases under duopoly when $1 \leq k \leq 1.59$ and $1.83 \leq k \leq 2$.

To summarise, for low values of k the 'pure' price effect dominates in the central region and price dispersion is higher under duopoly because of the downward pressure on the lowest prices for type-$\underline{\theta}$ consumers with weak brand preferences ($1 \leq k \leq 1.5$). When k becomes bigger so that the price of goods sold around the centre of the line reflects the higher quality (relative to monopoly), price dispersion is still higher because of the competitive effect at d^* where the same inefficient quality as in monopoly is sold in the presence of a threat from a rival good ($1.5 < k \leq 1.59$). As k increases even more, d^* approaches the origin so the previous effect is less important and it is overcome by the low price that would be offered by a monopolist to distant low types who buy a good of inefficiently low quality ($1.59 < k < 1.83$). Finally, when k is big enough, overall price dispersion returns to increase under duopoly for a new effect deriving from the higher upper limit to the price range offered to low types, which is now determined at $\hat{d}$ ($1.83 \leq k \leq 2$).

We conclude this section with a brief discussion of the solution to the duopoly case when $k > 2$. It is easy to realise that there are only two solutions in two different intervals. Solutions (3) and (4) are still valid respectively in region B = $[0, \hat{d}(k)]$ and C = $[\hat{d}(k), 1/2]$ so that we can extend the phase of figure 2.1 for higher values of k (see figure 2.6). As far as price dispersion is concerned, the only notions that make sense are price dispersion for high types and total price dispersion. In both cases price dispersion unambiguously increases. Price dispersion between types at a given location cannot be discussed since the monopoly situation does not provide a reference case. An interesting feature is that discriminatory contracts are induced by competitive entry. A monopolist would make offers only to high types, while the presence of a rival firm obliges producers to serve their entire markets. When both types are to be supplied, then separating contracts are better than pooling ones. In this sense, when taste parameters over quality are sufficiently different, discrimination is observed *only* in a competitive environment.

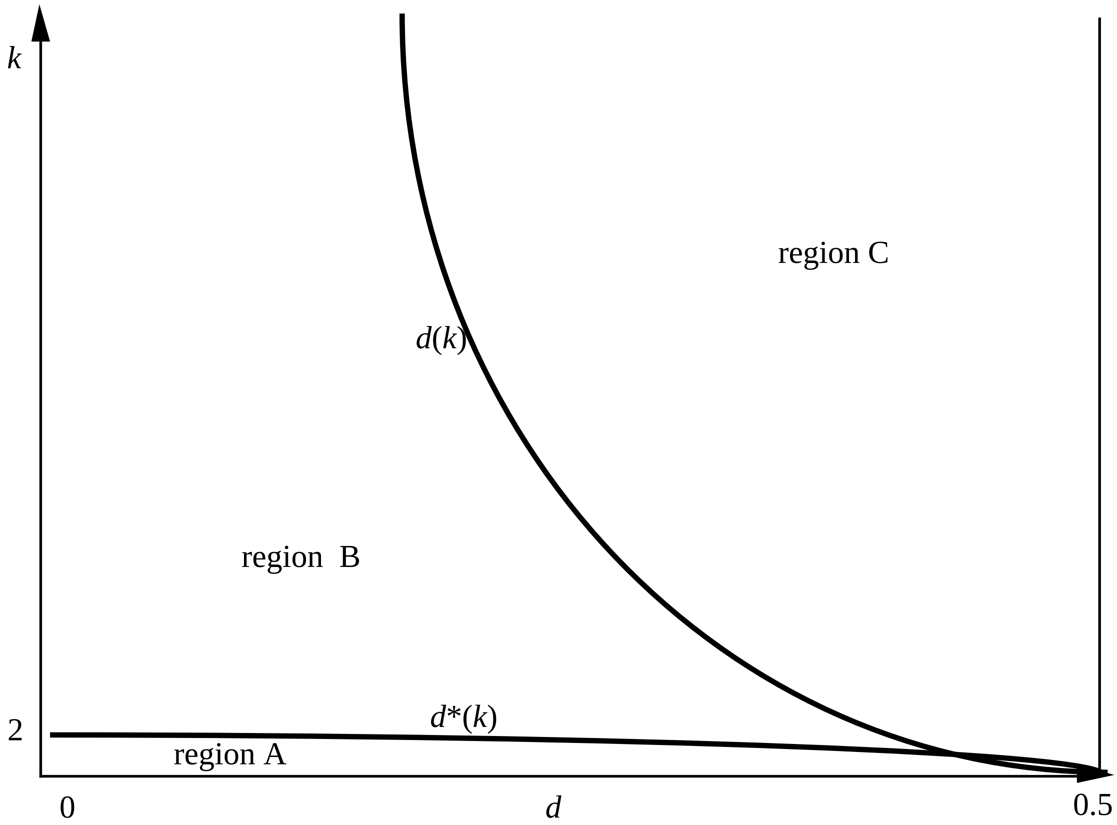

Figure 6: Phase diagram ($k \geq 1$)

6. The role of capacity constraints

The model presented in the previous sections sheds some light on price discrimination in an oligopoly. An interesting result is that price dispersion for each class of consumers increases almost in every region as one goes from monopoly to a competitive duopoly.

This is consistent with observations in some industries.[9] It may also explain some of the data at our disposal for the UK cellular industry, which are described in section 7. It is certainly true that the range of observed prices in the industry has sharply widened after the entry of new PCN operators. Despite this encouraging result, the model does not necessarily describe another major empirical observation, namely tariff proliferation. Incumbent firms have introduced tariff packages aimed at *new* categories of users as competition increased. These users were not supplied before, and in that respect tariff proliferation represents another aspect of discrimination.

The solution of the model shows that when $k > 2$ low types are supplied only in a competitive environment. On the other hand, when $1 \leq k \leq 2$, there are as (infinitely) many contracts in a monopoly (or collusive duopoly) as in a competitive duopoly. The entire population of customers is supplied, and in this sense there is the same discrimination in all market structures, even if the contracts are indeed affected by competition. It is not therefore the case that a segment, which is not supplied by a monopolist, happens to be served in a more competitive environment. This is not surprising with our formulation since the restriction on $k \leq 2$ implies that all types are economically interesting for a monopolist. So we cannot expect to find an equilibrium with tariff proliferation in a more competitive market.

In the description of the industry contained in section 7, it also emerges that firms are heavily constrained by their capacity. In principle a firm may be willing to serve its entire potential market, and discriminate among different types, in practice this may not be feasible if the firm cannot supply everybody. In such a case, a producer will probably decide to serve only some users, especially those with the highest willingness to pay. Capacity can be increased over time, and this has constantly been observed too, so that the time dimension becomes extremely relevant. We may ask a related but different question to the problem of price dispersion and price discrimination. How do price dispersion and price discrimination change in time as capacity constraints are relaxed? What is the influence of the degree of competition between firms? In this section we seek an answer using a very simple modification of the basic model. We emphasise that our framework does not allow us to obtain more dispersion in the long run, that is when capacity constraints are not binding, rather we can analyse the timing of price dispersion and discrimination. In particular, we are interested in understanding when a monopolist would start discriminating among his customers compared to a similar problem faced by a competitive firm.

[9] Borenstein and Rose (1994) document large price dispersion in the US airline market as competition on airline routes increases. Shepard (1991) finds that price differentials in gasoline retailing are not cost-driven but derive from price discrimination in a multifirm market.

The set up of the model is as in section 2. When a monopolist is extremely constrained in its supply decisions, solution (1) is not feasible and it may be the case that only high types are served. In this situation, there would be no incentive compatibility complications. High types would see their surplus completely extracted and the solution would be given by (1'), also for small values of k. The profit expressions are recalled for convenience:

$$(1)\qquad \begin{cases} \underline{\Pi} = (1-d)^2(2\underline{\theta} - \overline{\theta})\overline{\theta}/2 \\ \overline{\Pi} = (1-d)^2(3/2\,\overline{\theta}^2 - 3\overline{\theta}\underline{\theta} + 2\underline{\theta}^2) > \underline{\Pi} \end{cases} \qquad\qquad (1')\qquad \overline{\Pi} = (1-d)^2\overline{\theta}^2/2$$

Simple comparison of unit profits in (1) and (1') shows that it is never in the monopolist's interest to serve only high types when there are no capacity problems. The unconstrained firm can always make more profits at a given location by selling to the two types rather than extracting more profit from high types while excluding low types from consumption. On the other hand, a very constrained firm would certainly start to supply only high types.

The firm increases its capacity continuously over time and we assume it does so linearly and at a unit rate. Thus in a unit period the firm can build capacity to supply a unit mass of consumers. Of course, this is a very simplistic way to model capacity constraints, but we are not interested here in the investment decision, rather we take such decision for granted and study the implications of capacity constraints on pricing strategies and their timing.

As before, we first study the case of a monopolist with a single plant located at the origin. Then we turn to the case of two plants at the opposite ends of the line, when they are operated by a single firm (or two colluding ones) and by two rival firms. We will mainly discuss the more interesting case with $1 \le k \le 2$.

The solution of the monopoly problem is found by the comparison between equations (1) and (1'). We will refer with a superscript s (for 'single') when only a single type is supplied at a location, so that (1') applies, and with a superscript b (for 'both') when both types are served and (1) is valid. As the first increment of capacity becomes operating, the firm supplies only high types, i.e. those consumers that give the highest possible profit at single location. Comparisons of the profit functions show that, at any given location (except from d = 1), the following inequalities hold:

$$\overline{\Pi}^b(d) + \underline{\Pi}^b(d) > \overline{\Pi}^s(d) > \overline{\Pi}^b(d) > \underline{\Pi}^b(d).$$

Also note that all profit functions are strictly decreasing in d. As time goes by, more capacity becomes available and more distant customers are served. The profit made on 'single' high types decreases with d, until it may become profitable to serve low types at the origin. It is important to observe that the firm is not simply comparing the profit made on a distant 'single' high type with the profit made at the origin on a low type. In fact, when a low type is served, a chain effect results in a lower price being offered to the high type at the same location. This loss made on existing customers of high type delays the introduction of 'both' contracts, so we can say that discrimination appears relatively late in time because of self-selection. In any case, discrimination is introduced before the entire class of high types is served. Low types start to be supplied when the profit made at the origin, net of the loss on the existing high type due to incentive compatibility, equals the monopoly profit on a single high type at a more distant location d_1 (fig. 7a):

(5) $$\overline{\Pi}^{s}(d_1) = \underline{\Pi}^{b}(0) - [\overline{\Pi}^{s}(0) - \overline{\Pi}^{b}(0)].$$

In our formulation there is a 1:1 correspondence between distance and time, so we can also say that discrimination appears at $t_1 = d_1$. Since d_1 is relevant to the timing of the introduction of price discrimination, it is useful to discuss it more closely. After substituting into (5) the values of the profit function, it results that d_1 depends on k in the following way:

$$d_1 = 2(1\text{-}1/k).$$

The restriction $1 \leq k \leq 2$, implies $0 \leq d_1 \leq 1$, i.e. discrimination is delayed as k increases. When the vertical preferences of high types are relatively stronger than low types, it is convenient for a constrained monopolist to continue to supply them at more distant locations for some time, both because high types enjoy the product despite being distant and because the first discriminatory offer at zero causes a considerable loss from the existing customer. (As a limit case, when $k > 2$ the monopolist would never supply low types, which is equivalent to say that discrimination is infinitely delayed.)

After t_1 both customers are served simultaneously as capacity increases. Customers are served at different rates that can be further characterised. The net profit made at a location where both types are supplied must equal the net profit made at a location where only high types are served. Denoting by d_b and d_s respectively the previous locations, it must then be $\overline{\Pi}^{s}(d_s) = \underline{\Pi}^{b}(d_b) - [\overline{\Pi}^{s}(d_b) - \overline{\Pi}^{b}(d_b)]$. After substituting the relevant expressions into the previous equation, one can find that low types are supplied at a rate $\underline{r} = k/2$ and high types at a rate $\bar{r} = 1 - k/2 < \underline{r}$. Both types are then served until t_2 = 2 when the entire market is covered. From that time onwards, price discrimination and price dispersion remain constant at the unconstrained monopoly level, as given by equation (1). Figure 7b illustrates the pattern of subscription and discrimination over time, while 7c sketches price dispersion for each class in the initial phase. (Figure 7c is drawn taking into consideration that at time $t_1 < t < t_2$ locations $d_b = tk/2 - k + 1$ and $d_s = t + k - tk/2 - 1$ are reached simultaneously.)

We now turn to the case of a monopolist with two plants at the opposite ends of the line (or a collusive single-plant duopoly). Each plant starts producing and supplying its closest customers of high type. If d_1 > 1/2, then each plant will reach the centre of the line at time $t_1 = 1/2 < d_1$ and discrimination will appear exactly at 1/2, starting from customers closest to the plants. On the other hand, if d_1 < 1/2, then contracts to low types are offered as in the single-plant monopoly case until $t_1 = d_1$, then each plant starts supplying both types at different rates until the high type at the centre of the market is served (this happens at $t_1' = t_1 + (1/2 - t_1)/\bar{r}_1 = (3-2k)/(2-k)$). After t_1' the remaining customers, i.e. low types, are served at a unit rate until t_2 = 1 when each plant supplies its entire share of the market. Price dispersion remains constant after that time, that is the static unconstrained equilibrium is reached in half of the time compared to the single-plant equilibrium. The transition phase to the unconstrained equilibrium lasts for a shorter period for a simple reason. A monopolist is interested in supplying all the market, provided that he has sufficient capacity. When he can operate two plants he will be able to reach the desired unconstrained equilibrium much faster. In terms of vertical preferences, in a collusive duopoly discrimination appears at t_1 = 1/2 when $4/3 \le k \le 2$ and at $t_1 = d_1 \le 1/2$ when $1 \le k \le 4/3$. Figure 8 depicts the collusive duopoly case.

Our final discussion concerns the case of a competitive duopoly, whose equilibrium in absence of capacity constraints is described by equations (2), (3) and (4). The transitory phase is more complicated than before. There is an intermediate constrained period that does not have an equilibrium in pure strategies. There is an equilibrium in mixed strategies where firms post contracts at every location with some probability. The full analysis is beyond the scope of this work and we illustrate here only the salient characteristics that are sufficient to characterise the timing of discrimination.

(A). At the very beginning, each firm behaves as a monopolist. Capacity constraints are so binding that there is no possible interaction between firms that are serving each own neighbourhood. Then, there are two cases that arise according to the value taken by d_1, which influences the time after which a single-plant monopolist would start to discriminate.

(B1). If d_1 > 1/2, then discrimination starts when the centre of the line is reached, that is at t_1 = 1/2. At that time both firms starts supplying the closest low types and, at the same time, they reduce the price of high types both in the neighbourhood of the origin *and* in the neighbourhood of the centre. Increased dispersion at the origin derives from incentive compatibility as before. Price cuts around the centre arise because of competitive interaction. In particular, at $t_1 = 1/2$ there is a sudden downward jump in the high-type prices at different locations whose monopoly 'single' profit could be attractive for the rival firm located at the opposite end (note that we do not reach instantaneously the zero profit condition at the centre). As some incremental capacity becomes available, prices decrease both near to the origin (where both types are served) and near to the centre (high types only). The reason is again that close to the origin we observe the effects on existing customers that self-select their offer, while close to the centre there cannot be excessive expected profits that would otherwise attract the rival. The segment that benefits from firm competition widens while the bound on profits at those locations lowers. At some point in time this mechanism ends, and the remaining low types are gradually supplied while still ensuring no profitable entry in each own market, until $t_2 = 1$ and all customers are served for the first time.

(B2). If $d_1 < 1/2$, then in the first period nothing changes compared to monopoly or collusive duopoly. Low types start being served at $t_1 = d_1 < 1/2$, after that time each firm starts supplying both types until the high type at the centre of the market is served. After this moment, the remaining low-type customers are supplied, while discounts are gradually offered also to existing high-types around the centre, for the same reason as before, i.e. there should not be profitable opportunities left to the rival. At $t_2 = 1$, all customers are served for the first time.

(C). At $t_2 = 1$, all types are supplied and prices are either equal to the collusive case, or lower around the centre. But, as further capacity becomes available, price dispersion continues to increase with time. This process stops when firms become virtually unconstrained, that is at $t_3 = 2$ and they both have enough capacity to serve the entire population so that the competitive duopoly prices are reached.

We can now compare the different cases analysed in this section (the findings are summarised in table 1 below). If we compare the single-plant monopoly with competitive duopoly, the main result is that discrimination (tariff proliferation) appears first under duopoly when $k > 4/3$, while when $k < 4/3$ it appears at the same time. In words, when vertical tastes are sufficiently heterogeneous, a constrained monopolist is happy to delay the introduction of tariffs designed for low types because this allows him to make some higher temporary profits on 'distant' high types. This option is not available to competitive firms, since they cannot compete profitably in the rival market where the opponent firm enjoys an absolute advantage deriving from brand preferences. The transitory phase ends in both cases at $t_2 = 2$, so, in a slightly different perspective and with an abuse of terminology, we can also state that the 'dynamics' of price discrimination and proliferation last for a longer period in a competitive duopoly.

Turning to the comparison of a colluding duopoly with a competitive one, we have shown how discrimination appears at the same time, but the static unconstrained equilibrium is reached later in time under competition. Therefore also in this case, the 'dynamics' of price competition and discrimination are more persistent, even if there is a somehow different mechanism. The explanation lies in the fact that excess capacity is harmful for competitive firms. It is the ability of serving rival customers that imposes a real threat on each producer and this causes price cuts or reduced screening ability, as we have illustrated in section 5. The strongest rivalry arises when both producers are able to supply the entire market of mass 2. Before they reach that point, firms still have some market power, so they are not forced to rush to the maximal level of price competition. Rather they do it gradually, and price dispersion increases time after time. As capacity becomes available, prices are continuously readjusted downwards and firms exploit their capability of having transitory profits higher than the unconstrained situation. In this sense, the transitory phase lasts as long as firms can in an attempt to restraint themselves from being too aggressive.

A point that is worth to mention is related to the pattern of price cuts. In the collusive case, price cuts appear first at the origin, and they are caused only by incentive compatibility, which is required to sustain discrimination. This is also observed under competition, but, in addition, there are also cuts around the centre because of competitive interaction on consumers with weak brand preference. As soon as the centre is reached, prices always change in that area until the unconstrained equilibrium is reached, while in the collusive case a discount is offered in the centre only at the very last moment.

case $k > 4/3$ ($d_1 > 1/2$)

time:	0	1/2	t_1	1	2
monopoly					
collusive duopoly					
competitive duopoly					

case $k < 4/3$ ($d_1 < 1/2$)

time:	0	t_1	1/2	1	2
monopoly					
collusive duopoly					
competitive duopoly					

only high types supplied | both types supplied (constrained) | both types supplied (unconstrained)

Table 1: Timing of discrimination and lenght of the constrained phase

7. Relevance to the UK mobile communications industry

In this section we would like to discuss the practical relevance of the results, with particular reference to the UK mobile communications market. We borrow from the evidence discussed by Valletti and Cave (1998). The UK cellular industry is interesting for several reasons. First of all, it is a relatively young industry and few studies have addressed its characteristics despite the impressive expansion observed in many countries. In second place, because of regulatory intervention, it is possible to find in the industry two phases, one of a legal duopoly and a subsequent more competitive one. Although in general a higher number of firms in a market does not imply tougher competition, the particular experience of the UK cellular market strongly suggests that incumbents were colluding during the duopoly phase, while new entry probably ended the period of collusion. This sort of experiment allows us to compare the effects of a higher degree of competition on firms' strategies.

From 1985 to 1993 Cellnet and Vodafone were the only two operators in the market and the period was marked by very limited forms of price competition, no tariff innovation and very high rates of return for both network operators, supporting the hypothesis of collusive behaviour. The break-up of collusive practices is related to the entry of Mercury One-2-One and Orange in 1993. The range of observed prices in the industry has sharply widened after the entry of the new operators and the number of basic tariffs has increased to almost 20 (tariffs are highly non linear, typically they involve two-part schemes or equivalent forms of quantity discounts). It is also documented that the tariff for some consumers (heavy business users on the analogue network) has remained unchanged for a long time despite the more competitive environment, while other categories of users have benefited from lower prices and new tariffs. The following table reports the monthly expenditure and the discounts received on the previous period (if any) by some typical users on the incumbents' networks:

Usage	Time	Network	Before 10/92	10/92	10/93	4/95	4/96
H	P	D	n.a.	n.a.	£ 100	-	-20%
H	O	D	n.a.	n.a.	£ 78	-	-20%
M	P	D	n.a.	n.a.	n.a.	£ 50	-20%
M	O	D	n.a.	n.a.	n.a.	£ 40	-30%
H	P	A	£ 112	-	-10%	-	-
H	O	A	£ 84	-	-7%	-	-
L	P	A	n.a.	£ 40	-15%	-	-10%
L	O	A	n.a.	£ 33	-15%	-	-12%

H: high user (300 min/month), M: medium user (100'), L: low user (50')
P: peak calls only, O: peak and off peak, A: analogue network, D: digital

The industry has also been also affected by various degrees of capacity constraints. Capacity has increased over time, so that the time dimension becomes extremely relevant and the model presented here can be helpful to interpret the timing of discriminatory offers. The relaxation of capacity constraints in a linear fashion is certainly very simplistic. However, this approximation can be justified by regulatory intervention (there is a coverage requirement that forces the operators to invest incrementally in a minimum number of cells; the Government has also released additional frequency bands) and technological advances (more efficient use of the available spectrum).

In summary, incumbents have first introduced (high) business tariffs, and then they have tried to discriminate among different categories of users. Price cuts have been extremely selective, with the tariff for some consumers almost unchanged despite the more competitive environment. A proliferation of tariff packages has occurred as the UK cellular industry became more competitive (entry of new operators) and as additional capacity became available. Prices have been cut in a selective way (more consistent discounts for low users). In practice, the dimensions of vertical product differentiation are multiple: usage, coverage and reliability of getting through are probably the most important. At the same time horizontal parameters include peak/off-peak bands as well as brand preferences, or the time to the end of the contract with an operator. However, we hope that the mechanisms described by the model could provide useful insights to the problem of oligopolistic price discrimination, which is explained just in terms of horizontal (brand) and vertical (quality) preferences.

8. Concluding remarks

This paper has analysed price discrimination and price dispersion in a duopoly game. Optimal contracts have been characterised and it has been shown that contracts change according to taste parameters over brand and quality. Because of horizontal product differentiation, each firm enjoys an advantage over its closer customers and therefore captures them. Despite this fact, firms behave as if the rival firm was offering the best possible deal to its own customers. This results in consumers' participation constraints being type-dependent.

Some of the findings of this paper can be summarised as follows:

• When vertical preferences are not too different ($1 \leq k \leq 2$), there are three different discriminatory mechanisms at work that define three corresponding regions according to consumers' tastes. In the region where horizontal preferences dominate, price dispersion increases with respect to monopoly, without any change in the corresponding quality. As brand preferences become weaker and/or differences between customers are more marked, quality distortions are reduced gradually until they are eliminated.

• There is an entire range of vertical taste parameters ($k > 2$) which support discriminatory contracts *only* in a competitive environment.

• As competition is introduced in the market, price dispersion, always increases for high types. For low types, it increases almost everywhere despite the fact the quality range is reduced.

• An extension of the model with capacity constraints has also been studied to address the problem of timing of discriminatory contracts. We have found that price discrimination is introduced sooner and the transitory phase leading to the unconstrained equilibrium lasts longer under competition.

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Annex: Proofs

Proof of Proposition 1

First we show that firm 1 has to offer contracts in [0, 1/2]. Suppose firm 1 does not offer any contract. Then the outside option for customers is zero as in the monopolist case and the solution is still given by (1). But any contract $(p_1,\ u_1)$ offered by firm 1 to customers at d with $u_1{}^2/2 < p_1 < \underline{\theta} d u_1$ will attract all low types while making strictly positive profits, which cannot be true in equilibrium. Next we prove the second part of the proposition, using a simple undercutting argument. Start from a candidate equilibrium in which firm 1 offers $p_1 = C(u_1)$ and firm 0 maximises its profits taking as given the rival schedule. As it is shown in the following proofs, then firm 0's maximisation takes into account that at least one of the IR constraints is binding. If firm 1 prices below cost then it attracts customers but it would also make losses which cannot be optimal. If it prices above, then at least one of the IR constraints is relaxed and firm 0 could slightly increase its price without losing customers, but then there would be room for firm 1's profitable entry. If firm 1 offers contracts $(u^2/2,\ u)$ to customers in firm 0's market, then consumer θ would choose u^1 = argmax $\theta[1 - (1 - d)]u - u^2/2 = \theta d$, enjoying a net surplus $V^1 = d^2\theta^2/2$. This is the minimum level of utility that firm 0 has to provide to its customers and replaces the previous IR condition.

Proof of Propositions 2-4

We discuss here the solution to firm 0's problem under duopoly. We rewrite for convenience the four constraints that have to be satisfied:

$$\underline{\text{IR}}:\ \underline{\theta}(1-d)\underline{u} - \underline{p} \geq d^2 \frac{\underline{\theta}^2}{2}$$

$$\overline{\text{IR}}:\ \overline{\theta}(1-d)\overline{u} - \overline{p} \geq d^2 \frac{\overline{\theta}^2}{2}$$

$$\underline{\text{IC}}:\ \underline{\theta}(1-d)\underline{u} - \underline{p} \geq \underline{\theta}(1-d)\overline{u} - \overline{p}$$

$$\overline{\text{IC}}:\ \overline{\theta}(1-d)\overline{u} - \overline{p} \geq \overline{\theta}(1-d)\underline{u} - \underline{p}$$

We first assume that $\underline{\text{IR}}$, $\overline{\text{IR}}$ and $\overline{\text{IC}}$ all bind, then we will verify that $\underline{\text{IC}}$ holds. The firm has in principle four choice variables, but the three binding and independent constraints allow only for one degree of freedom. The firm exploits its remaining decision choice concentrating on high types that are particularly sensitive to the vertical parameter u. Contracts are:

$$\begin{cases} \underline{u} = \dfrac{d^2}{1-d}\dfrac{\underline{\theta}+\bar{\theta}}{2} \\ \underline{p} = d^2\dfrac{\underline{\theta}\bar{\theta}}{2} \end{cases}$$

$$\begin{cases} \bar{u} = (1-d)\bar{\theta} \\ \bar{p} = \bar{\theta}^2[(1-d)^2 - d^2/2] \end{cases}$$

To show that we have found an equilibrium, we still have to check the $\underline{\text{IC}}$ constraint:

$$\underline{\text{IC}}\text{: } d^2\frac{\underline{\theta}^2}{2} > \bar{\theta}\underline{\theta}(1-d)^2 - \bar{\theta}^2(1-d)^2 + d^2\frac{\bar{\theta}^2}{2}$$

The inequality holds because we can rewrite the previous condition as:

$$(1-d)^2\bar{\theta} - d^2\frac{\bar{\theta}+\underline{\theta}}{2} > 0 \text{ for all } 0 \le d \le 1/2.$$

Finally, the Lagrangian multipliers of the three binding constraints are:

$$\lambda(\underline{\text{IR}}) = \frac{(1-d)^2 k - d^2(k+1)/2}{(1-d)^2(k-1)}$$

$$\lambda(\overline{\text{IR}}) = \frac{-(1-d)^2(2-k) + d^2(k+1)/2}{(1-d)^2(k-1)}$$

$$\lambda(\overline{\text{IC}}) = \frac{(1-d)^2 - d^2(k+1)/2}{(1-d)^2(k-1)}$$

The last two expressions are monotonic in *d* and have a single root in (0, 1/2). Let d^* and $\hat{d}$ respectively be the locations that make $\lambda(\overline{\text{IR}}) = 0$ and $\lambda(\overline{\text{IC}}) = 0$:

(A1)

$$d^* = \frac{-2(2-k) + \sqrt{2(k+1)(2-k)}}{3(k-1)}, \; 1 < k \le 2$$

$$\hat{d} = \frac{-2 + \sqrt{2(k+1)}}{k-1}, \; k > 1$$

• $\lambda(\underline{\text{IR}})$ is always non-negative for $0 \le d \le 1/2$ and $1 \le k \le 2$.
• For small values of *d*, $\lambda(\overline{\text{IR}})$ is negative. The corresponding constraint is therefore not binding and the maximisation problem *sub* $\underline{\text{IR}}$ and $\overline{\text{IC}}$ gives solution (2) in the text. The interval of validity is denoted as region A = [0, d^*].

• For intermediate values of d, the three multipliers are positive. The contracts are optimal, and coincide with solution (3) in the text. The interval of validity is denoted as region B = $[d^*, \hat{d}]$.
• For high values of d, $\lambda(\overline{\mathrm{IC}})$ is negative. The corresponding constraint is therefore not binding and the maximisation problem *sub* $\underline{\mathrm{IR}}$ and $\overline{\mathrm{IR}}$ gives solution (4) in the text. The interval of validity is denoted as region C = $[\hat{d}, 1/2]$.
By differentiating the relevant conditions, we can discuss how boundaries change with k:

$$\frac{\partial d^*}{\partial k} = -\frac{(1-d^*)^2 + d^{*2}/2}{2(1-d^*)(2-k) + d^*(k+1)} < 0$$

$$\frac{\partial \hat{d}}{\partial k} = -\frac{\hat{d}^2}{2(1-\hat{d}) + \hat{d}(k+1)} < 0$$

$$\frac{\partial(\hat{d} - d^*)}{\partial k} = \frac{4}{3(k-1)^2} + \frac{5-k-3(3+k)\sqrt{2(1+k)}}{3(k-1)^2\sqrt{2(1+k)(2-k)}} > 0 .$$

It is worthy discussing the value taken by positive Lagrangian multipliers in the three regions. As one would expect, in region A, $\lambda(\underline{\mathrm{IR}}) = 2$ and $\lambda(\overline{\mathrm{IC}}) = 1$, as in the monopoly case. If low types are given 1 unit less of surplus, the increase of the firm's profit function is double because of the chain effect from contracts compatibility. On the other hand, the relaxation of self-selection has a 1:1 effect on profits. Region B shows an intermediate case with $1 \le \lambda(\underline{\mathrm{IR}}) \le 2$, $0 \le \lambda(\overline{\mathrm{IR}}) \le 1$, $1 \ge \lambda(\overline{\mathrm{IC}}) \ge 0$. The multipliers of the constraints in region C are $\lambda(\underline{\mathrm{IR}}) = \lambda(\overline{\mathrm{IR}}) = 1$: the two contracts are independent and relaxation of one constraint does not have any indirect effect on the profit function.
Few other comments can be made. Note that there cannot be a solution with only $\overline{\mathrm{IR}}$ and $\overline{\mathrm{IC}}$ binding. One would obtain a pooling solution

$$\begin{cases} \bar{u} = \underline{u} = \bar{\theta}(1-d) \\ \bar{p} = \underline{p} = \bar{\theta}^2(1-d)^2 - d^2\bar{\theta}^2/2 \end{cases}$$

that does not satisfy $\underline{\mathrm{IR}}$: the rival good would be chosen by low types since $\underline{V}^0(d) < \underline{V}^1(d)$. Too much surplus would be extracted from low-type consumers that will not accept the offer of firm 0, in contradiction to Proposition 1.
Also note that region C cannot be adjacent to region A. The intuition is that a sudden upward jump in $\underline{u}$ reduces the gap between qualities and this makes the contract designed

for low types attractive also for consumers with stronger preferences over vertical parameters.

We can also give a graphical representation of the various equilibria. Here, we would like to draw a picture similar to figure 1, valid for the monopoly case, adding the new constraints arising from the rival good. The net utility V^1, derived by consuming the more distant rival good, can be read by the difference between the gross utility θu and its production cost, given that each type consumes the efficient quality of the distant variety. Since region A is close to the origin, the utility functions U^0 for both types in the (p, u) space are quite steep, in addition the reservation utility for each type is rather small. From figure 9 it can be seen how only $\underline{\text{IR}}$ is affected while the outside option for the high type does not play any role. As d increases, both U^0's rotate clockwise while U^1's anti-clockwise until the rent left to the high type is equal to the rival option. Region A is also affected similarly by an increase in k. This can be seen holding $\underline{\theta}$ constant at a given location. As $\bar{\theta}$ increases the outside option acquires a bigger weight until it becomes binding. Similar figures could also be drawn for those consumers located in regions B and C.

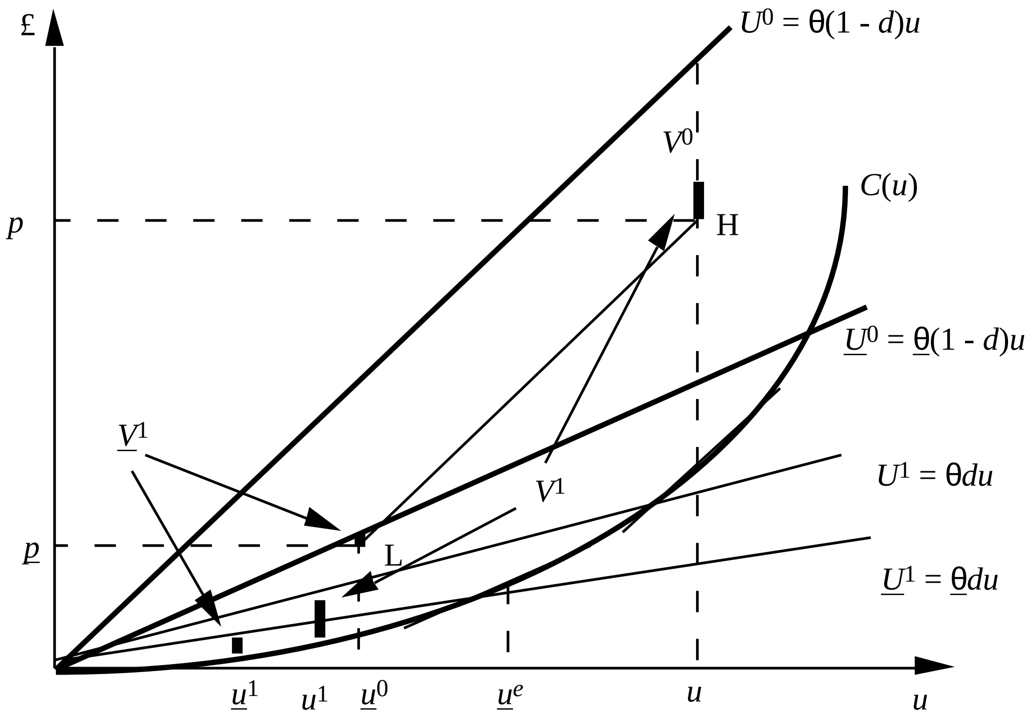

Figure 9: Duopoly - Equilibrium contracts at a given location in region A ($\underline{\text{IR}}$ and $\overline{\text{IC}}$ bind)

Using expressions (1)-(4), one can prove the results on price dispersion at a given location:

· Region A: $\Delta p = \bar{p} - \underline{p} = \bar{p}^m - \underline{p}^m = \Delta p^m$

· Region B: $\Delta p = \Delta p^m - d^2 \underline{\theta}(\bar{\theta} - \underline{\theta})/2 - (1-d)^2(2\underline{\theta} - \bar{\theta})^2 < \Delta p^m$

· Region C: $\Delta p = \Delta p^m - d^2(\bar{\theta}^2 - \underline{\theta}^2)/2 - (1-d)^2(\bar{\theta} - \underline{\theta})^2 < \Delta p^m$

Price dispersion for each class of customers results directly from equations (2)-(4). The only non-trivial case is price dispersion for low types in region B. It increases compared to monopoly (Proposition 3) when:

$$\left|\frac{\underline{\theta}\bar{\theta}}{2}(d^{*2} - \hat{d}^2)\right| > \underline{\theta}(2\underline{\theta} - \bar{\theta})[(1-d^*)^2 - (1-\hat{d})^2]$$

which, after substitutions, is satisfied when $k > 1.44$. Notice that when price dispersion decreases (i.e. for small k), the interval corresponding to region B becomes very small.

Proof of Proposition 5

In the monopoly case, the schedule of prices offered to low types is strictly decreasing in d and total price dispersion is given by $\underline{p}^m(0)$ - $\underline{p}^m(1/2)$. When we turn to the duopoly case, the price at the origin is unaltered, while close to the centre prices reflect two contrasting effects. In particular, when k is low, the difference between the monopolist distorted quality and the duopolist efficient one is modest, therefore it is likely that the 'pure' price effects prevails. It can be checked that when $k < 3/2$ the price at the centre of the line under duopoly is lower than the monopoly price at the same location, hence price dispersion increases in a competitive duopoly. When $k > 3/2$, at the centre the lowest price under duopoly is greater than the lowest price observed in a monopoly, still we cannot conclude that price dispersion is diminished. In fact, in the duopoly case prices for low types are not monotonic in d. The lowest price overall may not be at 1/2, rather at d^* where a very low-quality good is sold. Similarly the highest price overall may not be at 0, rather at $\hat{d}$ where the good is of a relatively high quality. It can be checked that $\underline{p}(d^*) < \underline{p}(1/2)$ implies $d^{*2} < 1/(4k)$ which is always satisfied when $k > 1.5$. The condition $\underline{p}(\hat{d}) > \underline{p}(0)$ implies $\hat{d}^2 > 2(2 - k)/k$ which is satisfied when $k > 1.81$. Therefore, in the range $1.5 \leq k \leq 2$ that remains to be discussed, overall price dispersion for low types under duopoly is given by $[\underline{p}(0) - \underline{p}(d^*)]$ when $k < 1.81$ and $[\underline{p}(\hat{d}) - \underline{p}(d^*)]$ when $k > 1.81$. After substitution of equations (1) and (3) for $\underline{p}(\cdot)$ and (A1) for the boundary conditions, one gets two inequalities in k whose numerical solution gives Proposition 5.