February 26, 1998

Taxes versus quotas for a stock pollutant[1]

by Michael Hoel[2] and Larry Karp[3]

Abstract

We compare the effects of taxes and quotas for an environmental problem in which the regulator and polluter have asymmetric information about abatement costs, and the environmental damage depends on the stock of pollution. We thus extend to a dynamic framework previous studies in which environmental damages depend on the flow of pollution. As with the static analysis, taxes are more likely to dominate quotas the greater is the curvature of the abatement cost function relative to the environmental damage function. However, in the dynamic model, an increase in the discount rate, the stock decay rate, or either the regulator's or the firms' ability to make adjustments, all increase the likelihood that taxes dominate quotas. An empirical illustration of these results suggests that taxes dominate quotas for the control of greenhouse gasses.

Keywords: Pollution control, asymmetric information, taxes and quotas, stochastic control

JEL Classification numbers: H21, Q28

---

[1] We would like to thank Atle Seierstad and Jinhua Zhao for their comments on an earlier version of this model, without implicating them in any remaining errors. We thank Maureen Cropper and Todd Sandler for suggestions on data, and Steve Warmerdam for outstanding research assistance.

[2]Univ. of Oslo/ P.O. Box 1095 Blindern/ N-0317 OSLO / email: michael.hoel@econ.uio.no

[3] Department of Agriculture and Resource Economics/ 207 Giannini Hall/ Berkeley CA 94720 email: karp@are.berkeley.edu

1. Introduction

Asymmetric information plays an important role in environmental regulation. Often the polluter knows more about the abatement cost function than the regulator. In this situation, the first-best optimum can seldom be reached by using emission taxes or quotas. The first-best optimum equates the marginal abatement costs of the pollutants and the marginal environmental damage. When the regulator does not have complete knowledge of the abatement cost functions, neither an emissions tax nor an emissions quota achieve the first-best level.

Weitzman (1974) compared the expected payoff, under asymmetric information, for these two policy instruments. He assumed linear marginal costs, uncertainty only about the level of the marginal cost curves, and not their slopes, and no correlation between the uncertainty of the abatement cost and the environmental cost. Under these assumptions, an emission tax dominates a quota if and only if the marginal abatement cost curve is steeper than the marginal environmental cost curve.

Subsequent contributions to this topic fall into two categories: (a) modifying the assumptions in Weitzman's analysis[1], and (b) considering a richer class of policy tools than only an emission tax and a direct specification of the emission level[2]. More complex policies can reduce the potential loss in social welfare associated with asymmetric information about abatement costs. However, in practice, policy-makers have not used these more sophisticated methods of environmental regulation.

---

[1] See e.g. Yohe (1977), Watson and Ridker (1984), and Stavins (1996).

[2] See e.g. Dasgupta et al. (1980), Kwerel (1977), McKitrick (1997), and Roberts and Spence (1976).

Somewhat surprisingly, almost all of the literature assumes that the environmental damage comes from the *flow* of emissions.[3] However, for several important environmental problems, damages depend on the *stock*, and not the flow, of the pollution. Examples of such problems include climate change (due to atmospheric concentration of greenhouse gases), depletion of the ozone layer (due to cumulative emissions of CFCs), deterioration of soil and water quality (due to acid rain resulting from sulfur nitrogen oxides), and the pollution of rivers, lakes and oceans from emissions of organic material.

We revisit the problem originally posed by Weitzman, replacing the flow pollutant with a stock pollutant. Section 2 presents the basic model. In section 3 we analyze the case in which the entire trajectory of the emissions tax or quota must be determined at the initial time (the open-loop policy). Section 4 studies the opposite extreme, where the quota or tax can be adjusted in light of new information (the feedback policy). Section 5 provides an empirical illustration, which suggests that taxes dominate quotas for the control of greenhouse gasses.

2. The Model

We use a discrete time formulation, in which each stage lasts for h units of time; units of time are arbitrary. The parameter h is the maximum of two numbers: the number of units of time between realizations of a random variable that affects firms' marginal benefits, and the number of units of time that must elapse before a firm can change its decision. Thus, h is a measure of the frequency with which new information arrives, or the frequency with which

---

[3] The exception is Staring's (1995) thesis, which studies the open loop ("non-flexible") model in section 3, but does not consider the case where policies can be revised in the light of new information.

firms are able to use new information. Suppose, for example, that units of time are months. If marginal benefits change randomly every six months, but firms are able to change their output only at yearly intervals, then h = 12. In this case, firms' limited flexibility is a constraint. If marginal benefits change randomly every six months but firms are able to change their output every three months, h = 6. Firms have no incentive to make changes unless they obtain new information. To avoid an uninteresting taxonomy, we hereafter assume that the two numbers are equal, and interpret h as both a measure of firms' flexibility and the frequency of arrival of new information.

Let $x(t)$ be the constant flow of pollutant during the stage beginning at time t, so x(t)h is the contribution to the stock during that stage. The stock, $S(t)$, decays at a constant rate δ, so $\Delta \equiv e^{-\delta h}$ is the fraction of the stock remaining in the next stage, in the absence of additional pollution. With additional pollution x(t)h, the stock at time t+h is

$$S(t+h) = \Delta S(t) + x(t)h \ . \tag{1}$$

An increase in pollution is equivalent to a decrease in abatement, and thus provides a cost savings. This flow of cost savings, i.e. the benefit, is given by the quadratic function $B(x,\theta)$:

$$B = B\big(x(t), \theta(t)\big) = f + \big(a+\theta(t)\big)\, x(t) - \frac{b}{2}\, x(t)^2 \tag{2}$$

where f, a and b are positive parameters, and $\theta(t)$ is the realization of a random variable. The random variable θ(t) is independently and identically distributed, with mean 0 and variance

$\sigma^2(h)$. As in Weitzman's model, uncertainty affects the level but not the slope of marginal benefits. Since B is a rate, the total benefit obtained in the stage beginning at time t is $B(x(t),\theta(t))h$. Our formulation ignores discounting and decay within a period.

We can think of θ as being the sum of many iid random variables, so that the variance of θ depends on h. For example, the level of marginal benefits depends on the accumulation of many small events that occur over the length of a period. At the beginning of a period the firm observes the sum of the random events that occurred during the previous period. With this interpretation, $\sigma^2(h)$ is proportional to h: $\sigma^2 = kh$, where k is a constant. Our analysis does not depend on the precise form of $\sigma^2(h)$, but we recognize that there is a relation between the value of σ^2 and h. In order to consider limiting cases when $h \to 0$, we adopt

Assumption 1: σ^2 is of the same order of magnitude as h or smaller, i.e. $\sigma^2(h) \sim 0(h)$.

The regulator chooses either a quantity restriction or a tax. In the former case the regulator chooses an upper limit on x(t), and in the latter case he chooses a tax, p(t), per unit of pollution. In the next two sections we compare the two policies under different assumptions about the regulator's flexibility. Firms are competitive and myopic. They observe the realization of $\theta(t)$ and then make their decision at time t. The regulator does not observe the value of $\theta(t)$.[4]

We assume that if the regulator uses a quota, it is always binding. This assumption requires that the support of θ be sufficiently small. Thus, when the regulator chooses quantity restrictions, $dS \equiv S(t+h) - S(t)$ is nonstochastic, and the expectation of the flow of benefits is f

[4] With taxes, the regulator can infer the value of $\theta(t)$ after observing the firm's current output, but given the assumption that θ is iid, this inference provides no information on future values of θ.

+ ax - bx^2/2 (since Eθ = 0). With quotas, both dS and E{B(·)} are independent of σ.

If the regulator uses a tax, p(t), firms choose x to maximize [B(·) - p(t)x]. The first order condition is a + θ(t) - bx(t) = p(t), which we rewrite as

$$x(t) = \frac{a-p(t)}{b} + \frac{\theta(t)}{b} \equiv z(t) + \frac{\theta(t)}{b} . \tag{3}$$

Choosing *p*(*t*) is equivalent to choosing *z*(*t*), the expectation of the flow of pollution. Hereafter, we treat a regulator who uses taxes as choosing z(t). From (2) and (3), the *regulator's* expectation of benefits at stage t is

$$E\big(B(x(t),\theta(t))\big)h = E\left[B\left(z(t) + \frac{\theta(t)}{b},\theta(t)\right)\right]h = \left(f + az(t) - \frac{b}{2}z(t)^2 + \frac{\sigma^2}{2b}\right)h \tag{4}$$

where the expectation is taken at any time s < t. Using equation (3) and (1), the variance of dS is $(\sigma h/b)^2$.

With taxes, uncertainty affects both the expectation of the flow of benefits and the variance of dS. The model with taxes has two obvious but important characteristics, which we summarize in

Remark: (1a) The effect of uncertainty on both the expectation of single-stage benefits ($\sigma^2 h/2b$) *and on the variance of dS* ($\sigma^2 h^2/b^2$) *is independent of z, and thus of the tax. (1b) The ratio of the two effects of uncertainty per unit of time depends on the length of the stage.*

Remark (1a) follows from the linear-quadratic structure with additive errors. Remark (1b) is useful for understanding the importance of *h* in comparing the two policies. We can re-state this remark using the definition $\Gamma \equiv (\sigma^2/2b)/(\sigma^2 h/b^2) = b/2h$; Γ is the ratio of the effects of θ on the current payoff and on the variance of dS. This ratio depends on h, the number of units of time between the arrival of new information (which equals the number of units of time between changes in firms' decisions). The parameter Γ is, of course, independent of units of time[5] and it is also independent of $\sigma^2(h)$.

The flow of damages during the stage beginning at time t is

$$D = D\big(S(t)\big) = cS(t) + \frac{g}{2}S(t)^2 \tag{5}$$

where c and g are positive parameters. The total damage during the stage is D(S(t))h. The instantaneous discount rate is r, so the discount factor is $\beta \equiv e^{-rh}$. We now have all of the elements of the model. The regulator's payoff is the present discounted value of the expectation of the stream of benefits minus costs, $[B(x,\theta) - D(S)]h$, where the equation of motion is given by (1).

With quantity restrictions, where the evolution of the state is nonstochastic and the expected payoff within each period is also independent of σ^2, the *regulator's* problem consists

[5] For example, suppose that the length of a stage is one year. If we let one unit of time equal one year, then h = 1. If we change units of time to equal months, but hold constant the length of a stage, then h = 12. However, since we have measured benefits as a function of the *flow* of pollution, we need to replace b with 12b. Thus, changing units of time from years to months does not change Γ.

of the payoff

$$\sum_{i=0}^{\infty} \beta^i \left[f + ax(t+ih) - \frac{b}{2}x(t+ih)^2 - cS(t+ih) - \frac{g}{2}S(t+ih)^2 \right] h \tag{6}$$

and the constraint, equation (1). With taxes, the regulator wants to maximize

$$E\sum_{i=0}^{\infty} \beta^i \left[f + \frac{\sigma^2}{2b} + az(t+ih) - \frac{b}{2}z(t+ih)^2 - cS(t+ih) - \frac{g}{2}S(t+ih)^2 \right] h \tag{7}$$

subject to the constraint

$$S(t+h) = \Delta S(t) + z(t)h + \frac{\theta(t)h}{b} . \tag{8}$$

We obtain equation (8) by using equations (1) and (3). In taking expectations of the payoff we use Remark 1a and the assumption that θ is iid to replace $E\{\theta^2/2b\}$ by $\sigma^2/2b$. The expectation in equation (7) is taken with respect to the evolution of S.

The nature of the problem changes fundamentally as h changes, as Remark 1b suggests. The smaller is h, the more frequently firms adjust their decisions. Since damages are convex in S, uncertainty about the evolution of S increases expected damages (by Jensen's inequality) and reduces the regulator's expected payoff. However, uncertainty increases the expected flow of benefits because each firm's output is positively correlated with its marginal benefits (equation (3)) under taxes. The parameter Γ provides a measure of the effect of uncertainty on expected benefits relative to its effect on expected damages *per unit of time*.

The expected payoff under quotas is independent of the amount of uncertainty (σ). Therefore, the ranking of taxes and quotas depends on the ratio between the benefit and the cost caused by uncertainty under taxes. Since Γ is decreasing in h, the more frequently firms adjust their decisions (i.e., the more frequently they receive new information about marginal benefits), the greater is the expected benefit relative to the expected cost caused by uncertainty. This ratio of benefits to costs (measured by Γ) becomes infinite as h becomes small. Consequently, for sufficiently small h, taxes dominate quotas, as we show in the next two sections.

Before turning to the comparison of taxes and quantity restrictions, we note that if damages depend on the flow of pollution rather than the stock (e.g., $D(x) = cx + gx^2/2$), our model is equivalent to Weitzman's. For that model, Weitzman shows that taxes are preferred to quotas if and only if $1 > g/b$. In our model (with stock dependent damages) the parameters g and b have different units, so their ratio is not unit-free (unlike the number 1). Therefore g/b cannot provide a criterion for ranking policies in the dynamic model. Nevertheless, the ranking is related to this ratio.

3. Non-Flexible Tax or Quota

In this section we consider the case in which the trajectories of the tax and the quota must be determined at $t = 0$. Here we fix the amount of firms' flexibility, parameterized by h, and assume that the regulator has no flexibility. Once he has chosen his policy trajectory, it is written in stone.[6]

---

[6] The regulator is able to announce a time-varying policy, and in that sense does have flexibility. We define the inflexible regulator as one who has to choose a trajectory at time 0 and then follow it. We could specialize our model to consider the "inflexible" regulator as

We noted that when quotas are used, the expectation of the payoff is independent of σ and the evolution of S is nonstochastic. We define Q(S(0)) as the maximized value of the payoff in (6), subject to (1). We define T(S(0);σ) as the maximized value of the payoff in (7), subject to (8) under an open-loop policy. When there is no uncertainty, the two policies obviously have the same payoff: Q(S(0)) ≡ T(S(0);0). We will also use the following

Remark 2: The optimal quota trajectory $\{x_i^*\}_{i=0}^{\infty}$ *is identical to the expected pollution trajectory under optimal taxes,* $\{z_i^*\}_{i=0}^{\infty}$ *(i.e.,* $x_i^* = z_i^*$ *for all i).*

This fact is another consequence of the linear-quadratic structure with additive errors.

We calculate expected damages under a tax by using equation (8) to obtain

$$\begin{aligned} S(ih) &= \Delta^i S(o) + h\left[z((i-1)h) + \Delta z((i-2)h) + \ldots + \Delta^{i-1} z(0)\right] \\ &+ \frac{h}{b}\left[\theta((i-1)h) + \Delta\theta((i-2)h) + \ldots + \Delta^{i-1}\theta(0)\right] . \end{aligned} \tag{9}$$

In view of the quadratic form of damages, we can write the expectation of damages as

$$ED(S(ih)) = D(ES(ih)) + \frac{g}{2}\left[E(S(ih))^2 - (ES(ih))^2\right] . \tag{10}$$

Equations (9) and (10) imply

$$ED(S(ih)) = D(ES(ih)) + \frac{gh^2\sigma^2}{2b^2}\,\frac{1 - \Delta^{2i}}{1 - \Delta^2} . \tag{11}$$

We already noted that uncertainty increases the single period expected benefits (when

---

one who must choose a constant policy, but this would not change our results.

a tax is used) by the amount $h\sigma^2/2b$ (see equation (7)). Using equations (7) and (11), and Remark 2, we can write T(S(0);σ) - Q(S(0)) as

$$T(S(0);\sigma) - Q(S(0)) = \sum_{i=0}^{\infty} \beta^i \left[\frac{\sigma^2 h}{2b} - \frac{gh^2\sigma^2}{2b^2} \frac{1-\Delta^{2i}}{1-\Delta^2} h \right] \tag{12}$$

which we simplify to obtain

$$T(S(0);\sigma) - Q(S(0)) = \frac{\sigma^2 h}{(1-\beta)2b} \left(1 - \frac{gh^2\beta}{b(1-\beta\Delta^2)} \right). \tag{13}$$

Equation (13) implies that an emission tax is superior to a quota (i.e., T[S(0)] > Q[(S(0))] if and only if $g/b < \phi_1$, where

$$\phi_1 \equiv \frac{1-\beta\Delta^2}{\beta h^2} = \frac{(1-\beta\Delta^2)/h}{\beta h} . \tag{14}$$

For small h, $\beta \approx 1-rh$ and $\beta\,\Delta^2 \approx 1-(r+2\delta)h$. Using these approximations and equation (14), we obtain an approximation for ϕ_1, which we denote as ϕ_1^a ("a" for approximation):

$$\phi_1 \approx \phi_1^a \equiv \frac{r+2\delta}{h} . \tag{15}$$

From (14) or (15) we see that taxes are more likely to dominate quotas the larger the discount or depreciation rate, or the smaller is the slope of marginal damages relative to marginal benefits. An increase in the firms' flexibility, i.e. the frequency with which they receive and act on new information about marginal costs (1/h), also makes the use of taxes more attractive. Thus, provided that h > 0, taxes are certainly preferred to quotas when h is sufficiently small. The criterion for ranking the policies is independent of the initial stock, *S(0)*.

Under Assumption 1, the amount by which taxes are preferred to quotas approaches 0 as $h \to 0$. In the limit as firms are able to respond infinitely frequently, taxes and quotas are equivalent. In order to establish this, we can use the approximations of β and $\beta\Delta^2$ to rewrite equation (13) as

$$T(S(0);\sigma) - Q(S(0)) \approx \frac{\sigma^2(h)}{2br}\left(1 - \frac{gh(1-rh)}{b(r+2\delta)}\right). \tag{13'}$$

Provided that σ^2 is of the same order of magnitude as h (or smaller), $T(S(0);\sigma) - Q(S(0))$ converges to 0 as $h \to 0$. In the limit, taxes and quotas are equivalent, although for sufficiently small but positive h taxes are preferred to quotas.

4. A Flexible Tax or Quota

This section studies the case where the regulator has the same degree of flexibility as firms. At the beginning of each stage, the regulator observes the current value of the stock, but not the current realization of θ, and chooses the current policy level. Here the regulator uses a feedback rule. The previous section considered the case where the regulator had to commit to a tax or quota trajectory at the initial time, i.e., he used an open-loop policy. The two cases thus represent two extreme assumptions about the regulator's flexibility.

The regulator's increased flexibility has no value if he uses a quota. Equation (1) shows that under a quota the development of the stock is nonstochastic, so the regulator learns nothing from observing it.[7] Therefore, nothing is gained by postponing the decision of $x(t)$ until t, rather than choosing the entire emission path at the initial time: the open-loop and feedback policies are identical and give the same payoff. In addition, the random variable θ does not affect the expectation of the current payoff. (The payoff in equation (6) is independent of σ^2.) The value of the regulator's program under a flexible quota is Q(S), defined in the previous section.

If the policy is an emission tax, the flow of pollution and thus the evolution of the stock is stochastic (equation 8). In this case, flexibility in setting the tax increases the regulator's payoff. The optimal emission tax at any time depends on the stock of the pollutant at that time. When the tax path must be chosen at time $t = 0$, it is not possible to achieve the first best outcome, because at time 0 the regulator is not certain of the stock at time $t > 0$. Denoting the optimal value of regulator's program under a flexible tax as $\tilde{T}(S;\sigma)$, it is clear that $\tilde{T}(S;\sigma) \geq T(S;\sigma)$. In general, the inequality is strict. When there is no ambiguity, we suppress the second argument of $T(\cdot)$ and $\tilde{T}(\cdot)$.

The flexible regulator is more likely to prefer taxes over quotas. There exist parameters such that the regulator would prefer a quota rather than a tax under an open loop policy, but allowing the regulator to use a feedback policy reverses the ranking. This

---

[7] Since, by assumption, the quota is binding in every state, the regulator does not even learn past values of θ. In any case, because θ is iid, information on past realizations would be worthless.

conclusion holds because we know that when $g/b > \phi_1$, the "inflexible regulator" prefers the quota, i.e. Q(S) > T(S), and that in general $\tilde{T}(S) > T(S)$. Therefore, for some parameter values $\tilde{T}(S) > Q(S) > T(S)$. The rest of this section derives the criterion for a tax to be preferred to a quota when policies are flexible.

We note that for $\sigma = 0$, $\tilde{T}(S;\sigma) \equiv T(S;\sigma) \equiv Q(S)$. We use dynamic programming to determine the function $\tilde{T}(S;\sigma)$, and then show how this function is related to Q(S) for $\sigma > 0$. The single period expected payoff in equation (7) is λh, with $\lambda \equiv [f + az - bz^2/2 + \sigma^2/2b - cS - gS^2/2]$. Using this definition of λ and equation (8), we write the Dynamic Programming Equation (DPE) under the flexible tax as

$$\tilde{T}(S) = \max_{z} \lambda(z,s)h + \beta E_{\theta}\tilde{T}\left(\Delta S + zh + \frac{\theta h}{b}\right) \tag{16}$$

The method of solving the DPE (16) is standard, so we merely sketch the steps. We know that the value function is quadratic: $\tilde{T}(S) = \rho_o + \rho_1 S + \rho_2 S^2/2$. We substitute this "trial solution" into equation (16), and use the first order condition to find the optimal control rule as a function of the state S, the known parameters, and the unknown parameters ρ_i. Substituting this control rule into (16) gives the maximized DPE, which is a quadratic equation in S. Equating coefficients of S^0, S and S^2 (in the maximized DPE) gives expressions for the equilibrium values of ρ_i. The equilibrium values of ρ_2 and ρ_0 satisfy

$$\rho_2 = \left(\frac{(\Delta\beta\rho_2)^2}{b - \beta h\rho_2} - g\right)h + \beta\Delta^2\rho_2 \tag{17}$$

and

$$\rho_0 = \left[f + \frac{(a + \beta\rho_1)^2}{2(b - \beta\rho_2 h)} \right] h + \beta\rho_0 + \frac{\sigma^2 h}{2b}\left(1 + \frac{\beta\rho_2 h}{b}\right). \qquad (18)$$

The parameter σ affects the constant term $(f + \sigma^2/2b)h$ in the expected payoff flow and the variance of the additive error $(h\sigma/b)^2$ in the stock evolution. Therefore, in view of well-known properties of the linear-quadratic control problem with additive errors, σ affects only the equilibrium value of ρ_o; the values of ρ_1 and ρ_2 are independent of σ. In addition, from equation (18), σ interacts with ρ_2 but not with ρ_1. Consequently, in order to determine how σ affects the value function, do not need to know the value of ρ_1, and we therefore do not include the equation that determines that parameter.

From (18) we see that quotas are preferred to flexible taxes if and only if the equilibrium value of ρ_2, which we denote as $\hat{\rho}_2$ (= $\tilde{T}''$), is less than $-b/\beta h$. In Weitzman's static problem, the ranking of taxes and quotas depends on the curvature of the benefits function (b) relative to the curvature of the damage function (g). In the dynamic problem with stock pollution, the ranking depends on the curvature of the benefits function relative to the curvature of the value function (ρ_2).

Rearranging equation (17) and dividing by h, we can write $\hat{\rho}_2$ as the unique negative root of $m(\rho_2) = 0$, where

$$m(\rho_2) \equiv \beta\rho_2^2 + \left(g\beta h - \frac{b(1 - \beta\Delta^2)}{h}\right)\rho_2 - gb \ . \qquad (19)$$

We want to know whether $\hat{\rho}_2$ is greater or less then $-b/\beta h$. Since $m(0) < 0$, $m(\hat{\rho}_2) = 0$, and $m'(\hat{\rho}_2) < 0$, we know that $\hat{\rho}_2 < -b/\beta h$ if and only if $m(-b/\beta h) < 0$. (See figure 1.)
Using equation (19) to evaluate $m(-b/\beta h)$ gives

$$m\left(-\frac{b}{\beta h}\right) = 2b^2\left[\frac{2 - \beta\Delta^2}{2\beta h^2} - \frac{g}{b}\right]. \tag{20}$$

Equation (20) and the previous remarks imply that quotas are preferred to taxes if and only if g/b exceeds a critical value, denoted ϕ_2, given by

$$\phi_2 \equiv \left(\frac{2-\beta\Delta^2}{2(1-\beta\Delta^2)}\right)\left(\frac{1-\beta\Delta^2}{\beta h^2}\right) \equiv \gamma\phi_1 \tag{21}$$

with $\gamma \equiv (2 - \beta\Delta^2)/2(1 - \beta\Delta^2) > 1$ and ϕ_1 defined in equation (14). The function ϕ_2 is increasing in r, δ, and h. A higher discount or decay rate or a more flexible firm, makes taxes more attractive. In addition, ϕ_2 approaches infinity as h approaches 0, so for sufficiently small (but positive) h, taxes are certainly preferred to quotas. However, for the limiting value of h = 0, taxes and quotas are equivalent. This conclusion follows because ρ_i approach limiting (finite) values as $h \rightarrow 0$, and from

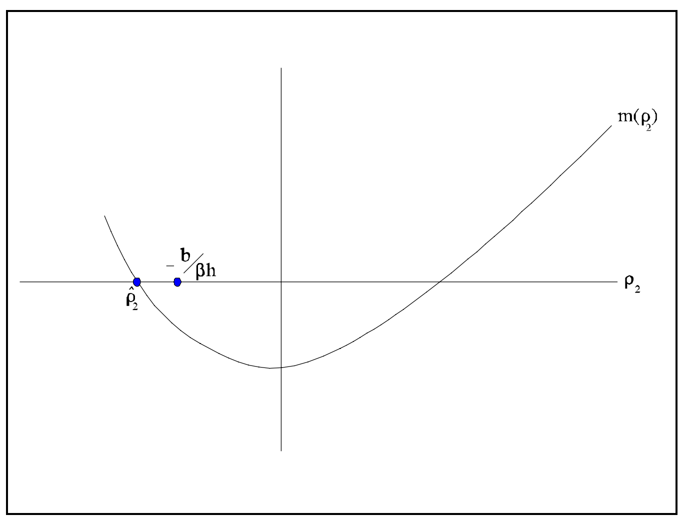

Figure 1 Graph of $m(\rho_2)$ when $_2 < b/\beta h$.

equation (18) we see that in the limit ρ_0 is independent of σ^2.[8] We noted that even for the inflexible regulator, taxes and quotas are equivalent in the limit as $h \rightarrow 0$. (See the discussion of equation (13') in Section 3.) Finally, since $\gamma > 1$, $\phi_2 > \phi_1$, so giving the regulator flexibility increases the set of parameter values for which he prefers to use a tax.

To summarize, we have

Proposition: For the quadratic model with additive uncertainty about abatement costs, increased flexibility, either on the part of the firm or the regulator, makes it more likely that the regulator prefers to use a tax rather than a quota. If the firm is sufficiently flexible, the regulator prefers to use taxes. However, the difference in the regulator's payoff under the two policies approaches 0 in the limit as the firm is able to change its plans arbitrarily quickly. There are two critical values for the relative curvature of the damage and benefit functions, g/b. These values are $\phi_1 \equiv (1 - \beta\Delta^2)\beta h^2$ *and* $\phi_2 \equiv (2 - \beta\Delta^2)/2\beta h^2$. *For* $g/b < \phi_1$ *inflexible taxes are preferred to quotas. For* $\phi_1 < g/b < \phi_2$, *flexible taxes are preferred to quotas, which are preferred to inflexible taxes. For* $\phi_2 < g/b$, *quotas are preferred to both flexible and inflexible taxes.*

As in Section 3, we can approximate the critical value ϕ_2. Expanding the formula in

---

[8] This conclusion requires only that $\sigma^2(h)$ be of a smaller order of magnitude than 1/h, which is obviously a weaker condition than Assumption 1.

equation (21) yields the approximation ϕ_2^a:

$$\phi_2^a \equiv \frac{r + 2\delta + \frac{1}{h}}{2h} \quad . \tag{22}$$

We noted that $\phi_2 > \phi_1$. Since $e^{-(r+2\delta)h} < 1$, a necessary condition for the approximation in equation (22) to be valid is $1 > (r+2\delta)h$. When this inequality holds, we also have $\phi_2^a > \phi_1^a$. Both approximations have the comparative statics properties described in the Proposition.

5. An Empirical Illustration

We apply the results of the previous sections to rank taxes and quotas in controlling CO_2, the major "greenhouse" gas. The control of greenhouse gasses is among the most important, or at least most hotly debated current environmental issues.

Even our parsimonious model stretches the limits of available data. We use two estimates for the slope of marginal damages, *g*: Falk and Mendelsohn's (1993; hereafter, FM) "high" estimate, and Reilly's (1992) estimate which we adapted by converting units from parts per million to tons. We used data from Nordhaus (1991) to estimate the parameter *b*, under a variety of specifications. All of these specifications led to estimates with similar orders of magnitude. We chose the specification which produced the smallest estimate of *b* [= 2.97E(-8)], thus biasing the results in favor of the use of quotas. Table 1 summarizes the estimates of *g* and the ratio *g/b*. The appendix discusses the data more fully.

Table 1: Estimates of g/b*

	FM	Reilly
estimate of g	8.12E(-13)	1.19E(-12)
estimate of g/b	2.73E(-5)	4E(-5)

* Using the estimate of b = 2.97E(-8).

We chose one unit of time equal to one year and set the discount rate r = .03 and the decay rate $\delta = .005$. A review of the literature suggests that $\delta = .005$ is widely accepted as a point estimate for the decay rate for greenhouse gasses (FM and Nordhaus). Reilly uses $\delta = .0083$ in a study which focuses on CO_2. The values we chose for both the discount and decay rate are therefore plausible but conservative (i.e. small), thus tending to bias the results in favor of quotas.

We have no way of estimating the parameter *h*, but for units of time equal to one year, $h = 1$ and $h = 10$ are reasonable bounds. Table 2 presents the critical values ϕ_i and their approximations ϕ_i^a, for *h* ranging from .1 to 100.

Table 2: Critical Values of g/b*

h	ϕ_1	ϕ_1^a	ϕ_2	ϕ_2^a
.1	.4004	.4000	50.350	50.2
1	.0404	.04	.535	.52
10	.0044	.004	.00897	.007
100	.00197	.004	.00199	.00025

* r = .03, $\delta = .005$

Even though we chose parameter values from the plausible range in such a way to make *g/b* large and ϕ_i small (thus making it more likely that quotas dominate taxes) our calculations indicate that taxes lead to higher welfare ($g/b << \phi_i$). This conclusion arises because of the extremely small estimate for the slope of the marginal damages. However, even if the larger (based on Reilly) estimate of g is too small by a factor of 1000, so that the actual value of g/b is approximately .04, taxes would still dominate quotas if the firm and the regulator were "reasonably flexible" ($h = 1$). If the estimate of g is too small by a factor of 100, taxes would still dominate quotas even if the firm and the regulator are inflexible ($h = 10$). Consequently, in spite of the data limitations, our results provide moderately strong support for the use of taxes rather than quantity restrictions to control greenhouse gasses.

6. Conclusion

Previous literature ranked a tax and quota policy when abatement costs and environmental damages both depend on the flow of pollution, and the polluter has better information than the regulator concerning abatement costs. In that case, for linear-quadratic functions with additive uncertainty, the quota dominates the tax if and only if the slope of the marginal damage function is steep, relative to the slope of abatement costs. We studied the situation where environmental damages depend on the pollution stock rather than the flow. In this circumstance, a direct comparison of the two slopes is not meaningful, since the units of the two are not the same.

The intuition provided by the static model continues to hold, insofar as greater convexity of the damage function, or less concavity of the benefits function, make it more likely that a quota is preferred. However, when environmental damages depend on pollution stocks, the ranking of the two policies also depends on the discount and stock decay rates. Higher discount and/or higher decay rates increase the importance of current flows relative to future stock effects. The chief advantage of the quota is that it makes it possible to control exactly the evolution of the stock. Since a higher discount rate and a higher decay rate both decrease the importance of future stock effects, they also decrease the value to the regulator of exact control of the evolution of the stock. Consequently, higher discount and decay rates make it more likely that a tax is preferred.

The ranking of the two policies also depends on the amount of flexibility that the regulator and the firm have. A firm's flexibility increases if it wants to and is able to change its emissions more frequently. An increase in the firms' flexibility always increases the relative attractiveness of the tax policy, regardless of whether the regulator uses an open loop or feedback policy. The regulator has more flexibility with feedback rules, where its policy depends on the current value of the pollution stock. An increase in the regulator's flexibility increases the expected payoff under a tax and has no effect on the expected payoff under a quota. Therefore, an increase in the regulator's flexibility also makes the tax a more attractive policy instrument.

The generality of our conclusions is limited by the restrictive functional forms and the restrictive assumptions about the random term. However, these assumptions play the same role in the dynamic as in the early static models: they enable us to identify the fundamental considerations that determine the ranking of policy. The simplifying assumptions are also extremely useful for empirical work. It is very difficult to estimate the slopes of marginal abatement costs and marginal damages. Even the simple linear-quadratic model strains the existing data base.

We used the theoretical results, together with estimates of marginal benefits and damages, to compare taxes and quotas in the control of greenhouse gasses. The point estimates suggest that taxes strongly dominate quotas. In order to overturn this ranking, we would need to adjust key parameters by a factor of more than 1000.

Appendix

We surveyed the literature on damage and abatement costs associated with greenhouse gasses. The volumes by Bruce et al. (1996), Cline (1992) and OECD (1992) and the papers by Barnes et al. (1993) and Manne (1993) provide background material and summarize previous estimates.

Falk and Mendelsohn (FM, 1993) use data from Nordhaus (1991) to estimate a linear marginal damage function, which provided our first estimate of the parameter *g*. Reilly (1992) estimates damages as a function of the concentration of greenhouse gasses (ppm). In 1990 the concentration of greenhouse gasses was 441 ppm and the stock of CO_2 was 800 billion tons. We used these quantities and the assumption of a linear relation between concentration rate and stock to convert Reilly's estimate, obtaining a second estimate of *g*. The two estimates differ by a factor of approximately two, which we regard as small, given the imprecision of all these numbers.

To get an idea of the range of plausible estimates of the slope of marginal damages, Table 3 reports estimates of the cost to the world (in billions of 1990 dollars) resulting from a doubling of the atmospheric stock of CO_2. Where the original study estimates the cost of damages for the US economy only, we assumed for the rest of the world the same ratio between damages and GDP as in the US. Using this ratio and data on world GDP we can then estimate the economic cost of damages for the world Thus, we can compare estimates across the studies.

Table 3 Damage Estimates In Billions of 1990 Dollars
Resulting from a Doubling of Tons of Carbon in Atmosphere

Cline (1992)	Fankhauser (1995)	FM (1993)	Maddison (1995)	Nordhaus (1991)	Nordhaus (1993)
220	260	50 (low) 400 (high)	300	50	266

These estimates vary by a factor of 8. Thus, it seems unlikely that estimates of g based on these studies (if such estimates were possible to construct) would vary by a factor of more than 1000. We noted in the text that the ranking of taxes over quotas would survive a thousand-fold increase in *g*.

Nordhaus (1991) reports estimates of total and marginal costs associated with different percentage reductions in greenhouse gasses. We converted these percentages to tons of greenhouse gas at 1990 levels, thus obtaining 15 "observations" of abatement and associated marginal and total costs. We used these data to estimate marginal abatement costs under a variety of specification (e.g., regressing total costs against abatement and $(\text{abatement})^2$ with and without an intercept; regressing marginal costs against abatement with and without an intercept). Our estimates of *b* ranged from 2.97E(-8) to 4.2E(-8). We used the smallest value of *b* in our calculations, in order to make it more likely that quotas would be preferred.

Maddison (1995) estimates a cubic abatement cost function, using percentages rather than absolute level. We converted his estimates to levels and fit a quadratic function through the resulting curve. The resulting estimate of *b* was of the same order of magnitude as the estimates we obtained using Nordhaus's data.

References

Barns, D.W., J.A. Edmonds, and J.M. Reilly (1993) "The Edmonds-Reilly-Barns Model" in The Cost Cutting Carbon Emission: Results from Global Models, OECD, Paris.

Bruce, J.P., H. Lee, and E.F. Haites, eds. (1996), Climate Change 1995: Economic and Social Dimensions of Climate Change IPCC, Contribution of Working Group III to the Second Assessment Report of the Intergovernmental Panel on Climate Change, Cambridge University Press.

Cline, W.R. (1992), The Economics of Global Warming, Institute for International Economics, Washington DC.

Dasgupta, P., Hammond, P. and Maskin, E. (1980), "On imperfect information and optimal pollution control", *Review of Economic Studies* 47, 857-60.

Fankhauser, S. (1995), Valuing Climate Change. The Economics of the Greenhouse, Earthscan, London.

Falk, I., and R. Mendelsohn (1993), "The Economics of Controlling Stock Pollutants: An Efficient Strategy for Greenhouse Gases," *Journal of Environmental and Economic Management* 25, 76-88.

Kwerel, E. (1977), To tell the truth: imperfect information and optimal pollution control", *Review of economic studies* 44, 595-601.

Maddison, D. (1995), "A Cost-Benefit Analysis of Slowing Climate Change," *Energy Policy* 23, 337-346.

Manne, A.S. (1993) "Global 2100: Alternative Scenarios for Reducing Carbon Emissions" in The Cost Cutting Carbon Emission: Results from Global Models, OECD, Paris.

McKitrick, R. (1997), "Optimal pollution taxes under asymmetric information", mimeo, University of Guelph.

Nordhaus, W.D. (1991), "To Slow or Not to Slow: The Economics of the Greenhouse Effect," *The Economic Journal* 101, 920-937.

Nordhaus, W.D. (1993), "Rolling the ’DICE’: An optimal transition path for controlling greenhouse gases," *Resource and Energy Economics* 15, 27-50.

OECD (1992), Global Warming: The Benefits of Emission Abatement, OECD, Paris.

Reilly, J.M. (1992), "Climate-Change Damage and the Trace-Gas-Index Issue," in Reilly, J.M. and M. Anderson, eds. Economic Issues in Global Climate Change: Agriculture, Forestry, and Natural Resources.

Roberts, M. and Spence, A.M. (1976), "Effluent charges and licenses under uncertainty", *Journal of Public Economics* 5, 193-208.

Staring, K. (1995), "Regulering av forurensing under usikkerhet - en dynamisk modell" (Regulation of pollution under uncertainty - a dynamic model). Master degree thesis, Department of Economics, University of Oslo.

Stavins, R.N. (1996), "Correlated uncertainty and policy instrument choice", *Journal of Environmental Economics and Management* 30, 218-32.

Weitzman, M. (1974), "Prices versus quantities", *Review of Economic Studies* 41, 477-91.

Watson, W.D. and Ridker, R.G. (1984), "Losses from effluent taxes and quotas under uncertainty", *Journal of Environmental Economics and Management* 11, 310-26.

Yohe, G.W. (1977), "Comparisons of price and quantity controls: A survey", *Journal of Comparative Economics* 1, 213-33.