Intertemporal and Spatial Depletion of Landfills[1]

Gérard Gaudet
Département de sciences économiques
and Centre de recherche sur les transports
Université de Montréal

Michel Moreaux
ERNA-INRA, GREMAQ and IDEI
Université de Toulouse

Stephen W. Salant
Department of Economics
University of Michigan

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Abstract

This paper generalizes Hotelling's (1931) theory of nonrenewable resources to situations where resource pools and their users are distributed spatially. Extraction and transport costs are assumed to be linear in the rate of extraction, but utilization of each deposit may require a setup cost. While Herfindahl's (1967) analysis of the socially optimal utilization of multiple deposits by a single user can be given a spatial reinterpretation, our contribution is to generalize his results further to the case where there are multiple users who are themselves spatially distributed. While our spatial generalization is important in many resource applications, it is essential to an understanding of solid waste problems. Landfill space may be regarded as a depletable resource, since space extracted today is unavailable tomorrow. But since cities and landfills are dispersed geographically, transshipment of waste commonly occurs within and between countries. Our analysis characterizes socially optimal waste flows over time and space and will facilitate the evaluation of the many government interventions designed to regulate such shipments of solid waste.

1 Introduction

Although geographical dispersion is an important real-world consideration in most resource problems, spatial considerations have largely been ignored in the resource literature which originated with Hotelling's (1931) seminal contribution. Our purpose here is to remedy this situation by generalizing Hotelling's intertemporal theory of exhaustible resources to the case where both resource pools and users are spatially distributed.

One application of our analysis is to the intertemporal and spatial utilization of landfills. There is a strong similarity between landfills and conventional nonrenewable resources[1]. Like mineral deposits or oil pools, landfills are exhaustible resources: the size of a given landfill is finite and "extracting" a unit of landfill space today irreversibly reduces the stock available for extraction tomorrow. In addition to being exhaustible resources, landfills also bring up particularly important spatial considerations. Transshipments of solid waste within different areas of one country as well as between countries occur frequently[2]. Unfortunately, regulations intended to control these flows have been formulated without the analytical framework necessary to evaluate their welfare effects. The theoretical model developed here can be used to evaluate such regulations[3].

Spatial considerations are, of course, also important in the case of more conventional exhaustible resources. Indeed, each of the previous, limited attempts to include spatial considerations in a resource model was motivated by a different application. Laffont and Moreaux (1986) study the case of a continuum of deposits located along a line segment with a single market located at one end of the line segment. Their model was initially formulated to study the optimal extraction of gravel in the vicinity of the city of Bordeaux, where the market for this gravel is located. They take account of the fact that the land being mined for gravel cannot, once mined, continue to be exploited as vineyards. If we abstract from

[1]This similarity has recently been noted and exploited by Chang and Schuler (1990), Dunbar and Berkman (1991) and Ready and Ready (1995).

[2]For a description of the interstate flows of solid waste in the U.S., see McCarthy (1995).

[3]Ley, Macauley and Salant (1997) represents a first attempt to use our model to evaluate regulation of solid waste flows. Their simulations are based on a special case, without set-up costs, of the theoretical model developed here.

this aspect of the problem and assume a discrete number of gravel deposits, then their model can be viewed as a direct reinterpretation of Herfindahl's (1967) analysis of extraction from multiple deposits with constant but differing marginal costs. Herfindahl showed that mines should be exploited in the order of their marginal costs, with no extraction from a higher cost mine while any reserves remain in a mine of lower cost. If we assume that the cost of extraction is the same for all gravel deposits and that the constant cost of transporting a unit of gravel to Bordeaux increases with the distance to the city, then the Herfindahl principle simply states that they should be exploited in strict order of distance from the city starting with the closest. This conclusion would still be valid if the deposits were located anywhere in the plane rather than on a line segment as Laffont and Moreaux assume. But the further generalization of their analysis to *multiple users* who are *also* distributed in the plane is far from obvious.

Kolstad (1994) advances the discussion by assuming multiple users, but he confines attention to one tractable spatial configuration. In particular, he considers the case where a continuum of consumers are distributed uniformly along a line segment with a mineral deposit located at each end. At any point in time, these consumers are easily partitioned into two groups: those patronizing the mine at one end of the line segment and those patronizing the mine at the other end. He studies how the market shares of the two mines evolve over time so as to exhaust them. Unfortunately, it is by no means clear how to generalize Kolstad's approach when customers and mines can *each* be located *anywhere* in the plane. For then it is extremely difficult to identify the set of customers patronizing any given mine at a particular time.

As in Herfindahl, Laffont-Moreaux, and Kolstad, we examine the case where extraction requires no set-up cost. However, set-up costs are an important consideration in the case of landfills and other exhaustible resources. We therefore also extend our analysis to take account of such costs. The Herfindahl framework and principle have been extended in a nonspatial context to take account of set-up costs by Kemp and Long (1984) and Hartwick, Kemp and Long (1986)[4]. They show that one implication of set-up costs is that even if deposits are identical in all respects it is always optimal to deplete one completely before going on to the next deposit. However, if deposits differ in their set-up cost as well as in their marginal cost of extraction, they need not be exploited in the order of their marginal costs. It may be optimal, for example, to postpone depletion of the deposit with the lowest marginal cost of extraction if it has a high set-up cost relative to the other deposits. As they established, net marginal benefit will jump down whenever a new deposit is opened if its set-up cost is positive[5].

Our model provides a number of theoretical insights into the intertemporal and spatial allocation of the scarce resource. We show that three basics principles must hold whether there are set-up costs or not and whether there are many spatially differentiated sources of demand or just one. These principles dictate which landfill site a user should be using at any given time and how his usage should behave when he switches to a different site. In the one-city case with no set-up cost, they reduce to the usual Herfindahl principle. In the one-city case with set-up cost, they collapse to the results of Hartwick, Kemp and Long.

But when there are many spatially distributed cities, phenomena arise which cannot otherwise occur and hence at first seem counterintuitive. For instance, even in the absence of set-up costs, it may be optimal for a city to abandon using a local site in favor of a more distant one even though space remains in the local site and it has the lower transport cost. When there are set-up costs, it may even be optimal for a user to abandon a site at some point and then return to it after an interval of time. We also find a somewhat surprising comparative dynamics result which can occur with or without set-up costs: if the exogenous transport cost per unit of distance goes up, the shadow prices may increase at one landfill

[4]Weitzman (1976) also proposed a method for solving the problem of minimizing the present discounted cost of supplying a fixed flow of a depletable resource from many deposits, when the average cost of extraction from a deposit depends on how much has already been taken out of it. A positive set-up cost with constant marginal cost of extraction can be viewed as a special case of such a cost configuration. His method can in fact provide a generalization of the Herfindahl principle for arbitrary extraction costs and can be used to confirm the results of Hartwick, Kemp and Long.

[5]Hartwick, Kemp and Long (1986) also show that, in general, with set-up costs, the socially optimal path of extraction cannot be decentralized by a competitive price system.

and decrease at another. This means that in a decentralized equilibrium, landfill prices may increase for some users and decrease for other users in response to an increase in the cost of transport.

In the next section, we describe the general problem when many users and many landfills are arbitrarily distributed in the plane. We discuss the optimality conditions which must hold while a landfill site is being used and whenever a new landfill site is set-up. In section 3, we apply these conditions to describe the optimal intertemporal and spatial allocation when there are no set-up costs. In section 4, we extend the discussion to the case with set-up costs. Section 5 explains why an increase in transport costs can cause some landfill prices to fall for some users while it rises for others. In section 6, we explain how the analysis we have presented can be reinterpreted to take account of recycling, waste reduction, incineration, costly expansion of existing sites, and the choice of new sites. Section 7 suggests several extensions and concludes the paper.

2 The general problem: many cities and many landfills at arbitrary locations

There are n landfill sites that can be used to dispose of the solid waste of m cities. These cities and landfills can be located anywhere in the plane. The cities can be considered as m different users of the finite volume of landfill space at each of the n sites.

Assume that the marginal cost of transporting and dumping city k's waste in landfill i is constant and denote it c_{ik}. Further assume, to eliminate even the possibility of indeterminacies over time, that for any city no two sites have the same marginal cost. The given initial volume of landfill space at site i is $S_i^0 > 0$ and its set-up cost is $F_i \geq 0$. We will denote by $S_i(t)$ the remaining volume of space at site i at time t and by $q_{ik}(t) \geq 0$ the space allocated to city k from landfill i at date t. The total amount of landfill space used by city k at date t is therefore $q_{\bullet k}(t) = \sum_{i=1}^{n} q_{ik}(t)$ and the total amount used by all cities of landfill i is $q_{i\bullet}(t) = \sum_{k=1}^{m} q_{ik}(t)$. The total utility derived by city k from its landfill disposal activity at date t is $U_k(q_{\bullet k}(t))$. Total utility is assumed to be an increasing, strictly concave function with $U_k(0) = 0$ and $U_k(0)$ finite. Let r denote the social rate of discount and τ_i denote the date at which the set-up cost is incurred for site i. We focus on the optimization problem of a social planner. A solution to this problem exists even in the presence of set-up costs. Although the competitive equilibrium does not always exist in such cases, when it does exist the allocation it induces solves the planning problem in the cases we examine.

To allocate the landfill spaces over cities and over time in an efficient manner, the planner will:

$$\max_{\{q_{ik}(t)\},\tau_i,T} \int_0^T e^{-rt} \sum_{k=1}^{m} \left[U_k(q_{\bullet k}(t)) - \sum_{i=1}^{n} c_{ik} q_{ik}(t) \right] dt - \sum_{i=1}^{n} e^{-r\tau_i} F_i$$

subject to

$$\dot{S}_i(t) = -\sum_{k=1}^{m} q_{ik}, \quad i = 1, \ldots, n \tag{1}$$

$$S_i(0) = S_i^0, \quad q_{ik}(t) \geq 0, \quad q_{ik}(t) = 0 \ \ \forall t < \tau_i, \quad i = 1, \ldots, n \quad k = 1, \ldots, m. \tag{2}$$

Call this problem **P**. It can be decomposed into two subproblems. Subproblem **P1** consists in solving problem **P** for given $\tau_i \geq 0$'s. Subproblem **P2** then consists in solving

$$\max_{\tau_i \geq 0} J(\tau_1, \tau_2, \ldots, \tau_n)$$

where $J(\tau_1, \tau_2, \ldots, \tau_n)$ is the optimal value of the program generated by **P1**. Given the stationarity of the exogenous functions, it is optimal to incur at least one set-up cost at date 0 as long as it is optimal ever to use some landfill. We will, without further loss of generality, assume $\tau_1 = 0$. This of course does not exclude the possibility that $\tau_i = 0$ for one or more other $i \neq 1$.

The current value Hamiltonian associated with Problem **P1** is:

$$H = \sum_{k=1}^{m} \left[U_k(q_{\bullet k}(t)) - \sum_{i=1}^{n} [c_{ik} + \lambda_i(t)] q_{ik}(t) \right]. \tag{3}$$

It measures the total social value derived by all the cities from landfill usage at date t, net of all the associated costs, including the total imputed value of the landfill space being depleted. The variable $\lambda_i(t)$ denotes the imputed value of a unit of space at site i at time t.

Problem **P1** requires that for $i = 1, \ldots, n$ and $k = 1, \ldots, m$ the following conditions be satisfied for all $t \geq \tau_i$:

$$q_{ik}(t) \geq 0, \quad U_k(q_{\bullet k}(t)) - c_{ik} - \lambda_i(t) \leq 0, \quad [U_k(q_{\bullet k}(t)) - c_{ik} - \lambda_i(t)]q_{ik}(t) = 0 \tag{4}$$

$$\lambda_i(t) = \lambda_i e^{rt}. \tag{5}$$

In addition, at the terminal date T, the following transversality conditions must hold:

$$H(\cdot)|_T = 0 \tag{6}$$

$$\lambda_i(T)S_i(T) = 0, \quad \lambda_i(T) \geq 0, \quad S_i(T) \geq 0, \quad i = 1, \ldots, n. \tag{7}$$

The economic interpretation of these necessary conditions merits discussion. Condition (4) is necessary to maximize at each date the Hamiltonian associated with problem **P1**. If city k is using landfill site i at date t, then condition (4) requires that the marginal utility derived from using site i must equal the *full* marginal cost of doing so. This "full marginal cost" includes not only the costs of transporting and dumping the waste from city k in site i but also the imputed cost ($\lambda_i(t)$) of depleting the extra space in site i. This imputed cost takes account of the net utility foregone in not using that marginal unit of space at another date — perhaps for a *different* city. Condition (4) also says that if the full marginal cost of city k using site i at date t strictly exceeds the marginal benefit, city k should not use that site ($q_{ik}(t) = 0$) at that date.

Condition (5) is nothing but the Hotelling rule of nonrenewable resource exhaustion (Hotelling, 1931) applied to each landfill site i, $i = 1, \ldots, n$. It says that the shadow value assigned to the marginal unit of landfill space remaining at the site must grow at the discount rate r.

The terminal conditions (6) and (7) serve to determine the optimal values of T and of the $\lambda_i(T)$'s. Condition (6) requires that the value of the Hamiltonian (H) — which measures the total surplus net of the opportunity cost of the landfill space being used — be zero at the terminal date T. If instead it were strictly positive, then the landfill depletion program could be improved upon by delaying somewhat the terminal date[6]. By substitution for the $\lambda_i(T)$'s from (4), one verifies that—since marginal cost is independent of the remaining landfill space and the utility functions are strictly concave—condition (6) implies that no landfill site be operating at a positive rate at the terminal date, i.e. $q_{i\bullet}(T) = 0$ for all $i = 1, \ldots, n$. Condition (7) says that the value of the remaining landfill space must be zero at the terminal date T. Again, since marginal cost at any date t is independent of the remaining landfill space, this will require that all landfills be exhausted at the terminal date, with $\lambda_i(T) > 0$ being determined by the condition that $S_i(T) = 0$ be satisfied.

Three basic results follow directly from conditions (4) and (5). First, *it is never optimal for a city to use more than one site over any interval of time.* For suppose city k were using simultaneously both site i and site j over some interval $\Delta t > 0$ ending at time t. Since this means that $q_{ik}(t) > 0$ and $q_{jk}(t) > 0$ over that interval, conditions (4) and (5) imply that $[U_k(q_{\bullet k}(t)) - c_{ik}]e^{-rt} = [U_k(q_{\bullet k}(t-\Delta t)) - c_{ik}]e^{-r(t-\Delta t)}$ and $[U_k(q_{\bullet k}(t)) - c_{jk}]e^{-rt} = [U_k(q_{\bullet k}(t-\Delta t)) - c_{jk}]e^{-r(t-\Delta t)}$. Subtracting the second of these equations from the first, we find that they imply $c_{ik} - c_{jk} = (c_{ik} - c_{jk})e^{r\Delta t}$. But this is impossible since, by assumption, r is positive and $c_{ik} \neq c_{jk}$ for all $i \neq j$.

Second, *amongst all the landfill sites that are already set up, each city will, at any given date, use only the one with lowest full marginal cost.* This result follows since maximization of H at each t with respect to $q_{ik}(t)$ requires that, for each k, we maximize

$$H_k = U_k(q_{\bullet k}(t)) - \sum_{i=1}^{n}[c_{ik} + \lambda_i(t)]q_{ik}(t)$$

with respect to $q_{ik}(t)$, subject to $\sum_{i=1}^{n} q_{ik}(t) = q_{\bullet k}(t) \geq 0$. This maximization can be achieved by first maximizing H_k over $q_{ik}(t) \geq 0$ subject to some specified $q_{\bullet k}(t)$ and then maximizing over all $q_{\bullet k}(t) \geq 0$. Given the linearity of the full total cost and the linearity of the constraint — which implies that the sites are perfect substitutes in use — the first maximization always

[6]Since we assume $U_k(0)$ to be finite for all $k = 1, \ldots, m$, then T will necessarily be finite.

requires that for all t and for all k, $q_{ik}(t)$ be set to zero except for the one for which the full marginal cost, $c_{ik} + \lambda_i(t)$, is smallest amongst all the sites that are set up.

Third, *when a city switches from one landfill site to another, its landfill usage does not jump down.* Indeed, as we have just shown, if city k switches from landfill i to landfill j at some date τ, then it must be the case that $c_{jk} + \lambda_j(\tau^+) \leq c_{ik} + \lambda_i(\tau^-)$. Otherwise, the city would be switching to a site with a strictly larger full marginal cost, in violation of the previous result. Intuitively, the planner would only assign a different landfill to a given city if one became available with a (weakly) cheaper full marginal cost. It follows from condition (4) that $U_k(q_{\bullet k}(\tau^+)) \leq U_k(q_{\bullet k}(\tau^-))$. By the concavity of the utility function, this means that $q_{\bullet k}(\tau^+) \geq q_{\bullet k}(\tau^-)$.

The solution to problem **P2** will determine the optimal set-up dates for each landfill site. Let $M_i(\tau_i) \subseteq \{1, 2, \ldots, m\}$ denote the subset of cities that switch to landfill i when it is set-up at τ_i and let $h(k) \in \{1, 2, \ldots, n\}$, $h(k) \neq i$, denote the landfill site used by city k just before switching to site i. Then the following conditions must be satisfied at τ_i:

$$\sum_{M_i} G_k(q_{h(k)k}(\tau_i^-)) + rF_i = \sum_{M_i} G_k(q_{ik}(\tau_i^+)) \tag{8}$$

and

$$\sum_{M_i} G_k(q_{h(k)k}(\tau_i^-))\dot{q}_{h(k)k}(\tau_i^-) \leq \sum_{M_i} G_k(q_{ik}(\tau_i^+))\dot{q}_{ik}(\tau_i^+), \tag{9}$$

where $G_k(q_{ik}) = U_k(q_{ik}) - U_k(q_{ik})q_{ik}$. Since when landfill i is used by city k, $U_k(q_{ik}) = c_{ik} + \lambda_i e^{rt}$, we can rewrite this as $G_k(q_{ik}) = U_k(q_{ik}) - (c_{ik} + \lambda_i e^{rt})q_{ik}$. Hence, it is the gross utility which city k obtains from using q_{ik} units of space in site i net of the *full* cost (including the imputed cost) of shipping that much waste to site i.[7].

Condition (8) is a first-order condition. In the one-city case, it insures that usage will not

[7]Condition (9) must hold in order for the Hessian of $J(\tau_1, \tau_2, \ldots, \tau_n)$ to be locally negative semi-definite. Condition (8) must hold in order for the derivatives of $J(\tau_1, \tau_2, \ldots, \tau_n)$ with respect to τ_i, $i = 1, \ldots, n$, to be zero. Notice that the first-order condition for the corner case at $\tau_i = 0$ can be obtained by replacing the first term on the left-hand side of condition (8) by zero and its equality sign by a weak inequality sign indicating that the right-hand side is at least as large as the left-hand side. That is, if it is optimal to open site i at $t = 0$, then the net benefit from opening it must be at least as large as the interest that would be saved by delaying its opening.

jump in the absence of set-up costs as shown by Herfindahl (1967) and must jump up in the presence of set-up costs as shown by Hartwick, Kemp, and Long (1986). In our multi-city generalization, the condition implies that it is optimal to delay setting up site i if the interest saved by postponing the set-up cost is strictly greater than the loss in net consumer surplus from having to use the old sites a little longer. In making this calculation, the planner must take into account the loss in consumer surplus at *every city* which would have switched to site i if it were opened at τ_i.

Condition (9) is a second-order condition. Since $G_k(q_{ik}) = -U_k(q_{ik})q_{ik}$ and, by the necessary conditions to problem **P1**, $U_k(q_{ik})\dot{q}_{ik} = \dot{\lambda}_i = r\lambda_i$, this condition is best rewritten as

$$\sum_{M_i} \lambda_{h(k)}(\tau_i^-) q_{h(k)k}(\tau_i^-) \geq \lambda_i(\tau_i^+) \sum_{M_i} q_{ik}(\tau_i^+). \tag{10}$$

It says that the imputed value of the total landfill space used just after the switch to site i must be no greater than the total imputed value of that used just before the switch.

3 The case of no set-up costs

Assume the set-up costs are negligible at all the landfill sites ($F_i = 0$)[8]. Before turning to the case of many cities and many sites, we first consider briefly the simpler case of one city.

3.1 One city and many landfill sites with no set-up costs

The case where one city may ship to n landfills which have already been set up corresponds to the well known basic problem solved by Herfindahl (1967): the city should completely draw down the landfill space from low cost sites before moving on to higher cost ones. For no matter what path of usage is designated for the lone city, it is cheaper in present value terms to provide it by using all of a lower cost resource before using any of a higher cost resource. Thus when a switch occurs in the course of the optimal program, it must be to a site with a higher marginal cost. This can also be seen directly from the second-order

[8]This is equivalent to assuming that all the sites have already been set-up.

condition (10). Suppose at date τ_j the city were to switch from site i, with costs c_{ik}, to site j, with costs $c_{jk} < c_{ik}$. Then from condition (4) and the fact that by (8) there is no discontinuity in usage ($q_{\bullet k}(\tau_j^+) = q_{\bullet k}(\tau_j^-)$), we must have $\lambda_j(\tau_j) > \lambda_i(\tau_j)$. But since only one site is used at any given date, this would violate the second-order condition (10). Hence, in the optimal program, the city must always switch to a landfill with a higher cost.

When there is only one city, it is immediate from condition (8) that if it switches from site i to site j at date τ_j, then we must have $q_{\bullet k}(\tau_j^+) = q_{\bullet k}(\tau_j^-)$. In other words, if there are no set-up costs, there must be no discontinuity in the usage of landfill space when a change of site occurs. Hence there must be no discontinuity in the marginal utility (or price, in a competitive equilibrium which decentralizes the optimal solution)[9]. From conditions (4) and (5), usage will decrease continuously at a rate which will result in the shadow value of the marginal unit of landfill space growing at the rate of discount. Marginal utility (or price) hence follows a continuously increasing path, with kinks at the dates where switches in sites occur (see Figure 1). These kinks reflect the fact that, when switching from one landfill to another with the same full marginal cost but a higher marginal cost, the shadow value of the landfill adopted is lower than that of the one abandoned and the full marginal cost of the adopted site must therefore be growing more slowly.

3.2 Many cities and many landfill sites with no set-up costs

As mentioned in the introduction, Herfindhahl's results facilitate the incorporation of spatial considerations into the Hotelling model when there is a single user. Since at any given time, however, a landfill may be used by more than one city, a first obvious direction of generalization of the Herfindahl framework consists in introducing many cities, arbitrarily distributed over space. Kolstad (1994) constitutes the only attempt at studying nonrenewable resource extraction when consumers of the resource are distributed over space. He considers a situation where two mines of potentially different size and different extraction costs are located at each end of a line segment with a continuum of consumers uniformly distributed over

[9] In the absence of set-up costs, such a competitive equilibrium always exists.

the line segment. The two mine operators are price takers[10] and the unit transport cost is an increasing function of distance. He derives the relationship that holds between the equilibrium rents at each mine and studies how the consumers will be partitioned between the two mines and how this partition will evolve over time, assuring that the two mines are eventually exhausted.

Put in our framework, two landfill sites would serve many cities located on the line segment between them. Kolstad's framework is restrictive, because it does not allow for the cities to be distributed arbitrarily over space. Furthermore, the approach makes it very difficult to deal with the case of more than two landfill sites, since it requires that at every point in time the set of cities be partitioned between the sites. As we will see, redirecting the focus from the landfill sites to the cities using them permits any spatial configuration to be studied and facilitates the analysis of the case of many cities and many sites. It also allows an immediate comparison with the intuitive Herfindahl results.

Consider then the case of many cities and many landfill sites, both arbitrarily located in the plane. Assume every landfill has been set-up at the outset. We know that each city will at any given time use only the lowest full marginal cost site. Since a site which is of lowest full marginal cost for one city will not necessarily be of lowest full marginal cost for other cities, it will generally be observed, in contrast to the case of one city, that *many sites are being used at the same time*. Clearly, Herfindahl's result does not generalize in this particular way. However, it does generalize if one focuses on individual cities. It remains true, as shown previously, that *no city will use more than one site at the same time*. Moreover, it remains true that each city will use landfills in the order of their marginal costs: if city k switches from site j to site i, it must be that $c_{ik} > c_{jk}$[11]. In the absence of set-up costs, one can in fact devise a straightforward algorithm to solve the entire set of first-order conditions associated

[10] He also considers a situation where one is a monopolist and the other is a price taker, but the presence of imperfect competition raises issues which are beyond the scope of the present paper.

[11] The proof is exactly the same as that made above in the one-city case. Again, since this implies that $\lambda_j(t) > \lambda_i(t)$ at the switch date and since, with $F_i = 0$, condition (8) implies that $q_{ik}(\tau_i^+) = q_{h(k)k}(\tau_i^-) = q_{\bullet k}(\tau_i)$, the second-order condition (10) is satisfied.

with the planning problem[12]. Since this problem is concave, solution of these conditions suffices to identify the optimum when — in the absence of set-up costs — multiple landfills serve multiple cities at arbitrary locations. Note that although in the optimal program no city uses more than one site at the same time, it is quite possible that many cities will simultaneously use the same site.

A fundamental difference with the one-city case and the Herfindahl principle based on that case may however arise in the multiple city case. This is that *a city may now very well be using some landfill site while a lower cost one is still available and being used by one or more other cities.* The reason is that the imputed value of a unit of space at site i, λ_i, will now depend not only on that city's cost and demand parameters, but on those of other cities as well. Hence, although city k will always use only the landfill site for which $c_{ik} + \lambda_i(t)$ is smallest, this does not necessarily correspond to the one with the lowest c_{ik} available to it at time t.

To see this, consider the following example. Suppose there are two cities (city A and city B) and two landfill sites. Assume that, for $k = A, B$, $c_{jk} > c_{ik}$. Assume further that $c_{jB} - c_{iB} > c_{jA} - c_{iA}$ so that, although $\lambda_i > \lambda_j$, we have $c_{jB} + \lambda_j > c_{jA} + \lambda_j > c_{iB} + \lambda_i > c_{iA} + \lambda_i$. The situation is depicted in Figure 2. For both cities, the cost of transporting and dumping a unit of waste to landfill site i is lower than to landfill site j. However it costs less for city A to switch from site i to site j. Initially, the full marginal cost of using site i is lower than that of using site j for both cities. So both cities are using site i at first. At t_A,

$$c_{iA} + \lambda_i e^{rt_A} = c_{jA} + \lambda_j e^{rt_A} \tag{11}$$

[12]The simulation model in Ley, Macauley, and Salant (1997) uses such an algorithm. The algorithm proceeds as follows: (1) assign to each landfill an initial multiplier and let each multiplier subsequently grow at the exogenous rate of interest; (2) for each of the m cities, assemble the set of the n full marginal costs available at each date and assign to that city at that date the landfill with the smallest full marginal cost; (3) assume that each city uses the designated landfill to the point where the marginal benefit from additional usage equals the full marginal cost; (4) determine the cumulative usage of each landfill over time and across cities and compare the cumulative usage to the initial size of each landfill; (5) if, for each landfill, cumulative usage exactly matches the initial stock, then the optimum has been identified; (6) otherwise, the multipliers must be revised and the process repeated.

and city A begins shipping its waste to site j instead of site i. At $t_B > t_A$,

$$c_{iB} + \lambda_i e^{rt_B} = c_{jB} + \lambda_j e^{rt_B}, \tag{12}$$

site i is completely filled up, and city B begins using site j. Therefore, for $t \in [t_A, t_B]$, city A is using landfill site j while space remains in site i and $c_{iA} < c_{jA}$.

Given the results in the one-city case, one might expect that costs could be lowered if city A were to transfer some of its waste from site j to site i. Indeed, transferring a unit of waste from site j to site i at $t \in [t_A, t_B]$ results in a cost saving of $e^{-rt}(c_{jA} - c_{iA}) > 0$. But since site j will be filled up by city B at t_B, this transfer must be offset by a similar transfer from site i to site j by city B at some date $(t + \Delta t) \in [t, t_B]$. This last transfer will result in a cost increase of $e^{-r(t+\Delta t)}(c_{jB} - c_{iB})$. Hence, the total change in cost, in present value terms, is

$$\begin{aligned} & e^{-rt}[(c_{jA} - c_{iA}) - e^{-r\Delta t}(c_{jB} - c_{iB})] \\ = \; & e^{-rt}(e^{r(t_B - \Delta t)} - e^{rt_A})(\lambda_i - \lambda_j) \leq 0, \end{aligned}$$

where the cost differences in the first line have been replaced in the second line by differences in shadow values (appropriately discounted) using conditions (11) and (12). Therefore the program cannot be improved upon by such a transfer and it is indeed optimal for city A to switch to the higher cost landfill site j before the lower cost site i is filled up.

Nonetheless, it will never be the case, in this situation, that a city would come back at a later date to the site it had abandoned. For this would mean switching to a low marginal cost site from a high marginal cost one, which has been shown not to be optimal in the absence of set-up costs.

4 The case of set-up costs

The costs which must be incurred in setting up landfill sites before they can be used are at times substantial and, despite the analytical complexity involved, it is important that they be taken into consideration. We therefore turn now to the analysis of the intertemporal and spatial allocation of landfill sites with set-up costs. We briefly review some known results for the one-city case before turning to the more general case of many cities.

4.1 One city and many landfill sites with set-up costs

The conclusions of Hartwick, Kemp and Long (1986) apply immediately to the landfill problem with one city and many landfill sites. As in the case without set-up costs, the city must exploit the landfill sites in strict sequence: each site must be exhausted before going on to the next. However, unlike the case without set-up costs, sites need not be exploited in order of their marginal costs; it may be optimal, for example, to postpone usage of a site with the lowest marginal cost if opening it involves relatively high set-up costs.

It is easy to see that opening fills in order of their marginal costs is no longer necessary for an optimum in the presence of set-up costs. For suppose the lone city is city k and that it switches at τ_j from site i to site j, which requires that a set-up cost of $F_j > 0$ be incurred at that date. Then, remembering that it never uses more than one site, we deduce from the first-order condition (8), which must hold at τ_j, that $q_{jk}(\tau_j^+) > q_{ik}(\tau_j^-)$: the city's landfill usage jumps up. The second-order condition (9), which must also hold at τ_j, can be written

$$(\lambda_i(\tau_j^-) - \lambda_j(\tau_j^+))q_{ik}(\tau_j^-) \geq \lambda_i(\tau_j^+)(q_{jk}(\tau_j^+) - q_{ik}(\tau_j^-)).$$

Since the right-hand side is strictly positive, so must be the left-hand side, and hence $\lambda_i(\tau_j^-) > \lambda_j(\tau_j^+)$. Now the city's landfill usage path must also satisfy, at all t, the first-order condition (4), and, as a result,

$$c_{jk} + \lambda_j(\tau_j^+) < c_{ik} + \lambda_i(\tau_j^-).$$

This does not require $c_{jk} > c_{ik}$[13].

4.2 Many cities and many landfill sites with set-up costs

In the case of many cities and many landfills sites with set-up costs, the three basic results established earlier still hold: each city never uses more than one site at any given time, always uses the one with the lowest full marginal cost amongst all those that are set-up and never reduces its usage when it switches sites. But they admit a very remarkable possibility as well. Not only can it be optimal for a city to exploit a high marginal cost site before a low marginal cost one — because of the presence of fixed costs — or to abandon a low marginal cost site in favor of a higher cost one while the low cost one is still available — because there are other cities using the sites whose behavior has an impact on their full marginal cost — but *a city may well abandon a site and come back to it at a later date.* In optimizing, the planner may thus appear to vacillate.

Clearly, this requires that the city switch from a higher cost site to a lower one when it abandons the site originally and then from a lower cost site to a higher cost site when it resumes shipping to the abandoned site. This can never happen in the case of a single city with or without set-up costs since in all such cases a landfill must be fully depleted before it is abandoned and there can be no point in returning to it. Nor can this ever occur with multiple cities in the absence of set-up costs. For, while it is possible for a city to switch to a higher cost site while a lower cost site is still available, it is never optimal to *return* to the low cost site since transitions for any given city must always be to sites with higher marginal costs. The possibility requires *both* multiple cities *and* set-up costs. As we have discussed, when there are multiple cities it is sometimes optimal for a city to abandon a site even though that site is not fully depleted. When there are also set-up costs, the site which that city abandons may be a high cost site which has been exploited before a low cost one has been set-up. Once the lower cost site is set up, however, the city must switch to it

[13] Recall that in the one-city case with no set-up cost, the first-order condition (4) implies $c_{jk} + \lambda_j(\tau_j) = c_{ik} + \lambda_i(\tau_j)$, which requires $c_{jk} > c_{ik}$, since $\lambda_i(\tau_j) > \lambda_j(\tau_j)$ by condition (10).

since it would be, among the sites set up, the one with the lowest full marginal cost. Once that site is exhausted, however, a return by that city to the abandoned landfill would be a standard transition to a site of higher marginal cost.

This counterintuitive result is best understood with the help of an example. Assume there are two cities (cities A and B) and three landfill sites (sites 1,2,3). For the sake of the discussion, we may assume that the cost of transporting a unit of waste to a particular landfill is strictly proportional to the distance from the city to the site and that, other than the set-up cost, this is the only cost. Assume that sites 1 and 2 have no set-up cost ($F_2 = F_1 = 0$) but that site 3 has a very large set-up cost ($F_3 > 0$). Site 2 is small, whereas sites 1 and 3 are larger. Site 1 is closer to city A than it is to city B ($c_{1A} < c_{1B}$) and it is closer to city A than is site 2 ($c_{1A} < c_{2A}$). Site 3 is located at city A ($c_{3A} = 0$), while city B is equidistant from sites 2 and 3 ($c_{2B} = c_{3B}$). The location of the cities and the landfill sites might be as depicted in Figure 3.

Now pick the shadow values of the landfill space such that $\lambda_2 > \lambda_3 > \lambda_1$, $c_{1A} < \lambda_3 - \lambda_1$ and $c_{1B} - c_{2B} > \lambda_1$. We then have

$$c_{2A} + \lambda_2 > c_{1A} + \lambda_1 > c_{3A} + \lambda_3$$

and

$$c_{1B} + \lambda_1 > c_{2B} + \lambda_2 > c_{3B} + \lambda_3.$$

The resulting time paths of marginal utility at each city are depicted in Figure 4.

Given the fact that sites 1 and 2 are already set-up and that site 3 has a very large set-up cost, at first city A uses site 1 and city B uses site 2. Site 2 is relatively small and is exhausted by city B at some date τ_3, while site 1 still has some space left. However, because the cost of transporting the waste from city B to site 1 is very high, it becomes optimal to set-up site 3, even though this means incurring a set-up cost of F_3. Once setup, it becomes advantageous to have city A switch to site 3 as well, since the full marginal cost of site 3 to city A is smaller than that of site 1 ($c_{1A} + \lambda_1(\tau_3) > c_{3A} + \lambda_3(\tau_3)$). Because $\lambda_3(t)$ and $\lambda_1(t)$

both grow at the same constant rate and $\lambda_3 > \lambda_1$, $c_{3A} + \lambda_3(t)$ will eventually cross $c_{1A} + \lambda_1(t)$ from below at, say, date t . It therefore becomes optimal for city A to return to site 1 at t , since beyond that date $c_{1A} + \lambda_1(t) < c_{3A} + \lambda_3(t)$.

Along each path just described, the landfill usage is given by equalizing marginal utility at each city to the full marginal cost of the site being used, calculated using the shadow values picked initially. The necessary condition (4) is therefore satisfied. So is the necessary condition (5), by having the shadow values grow at the same constant rate of discount. The switching date τ_3 is chosen to satisfy the necessary condition (8). As for the second-order condition (10), which requires that the imputed value of the total landfill space used not jump up when a switch occurs, it may be written

$$\begin{aligned}&(\lambda_3(\tau_3^+) - \lambda_1(\tau_3^-))q_{3A}(\tau_3^+) + (\lambda_3(\tau_3^+) - \lambda_2(\tau_3^-))q_{3B}(\tau_3^+)\\&\quad \le \lambda_1(\tau_3^-))(q_{3A}(\tau_3^+) - q_{1A}(\tau_3^-)) + \lambda_2(\tau_3^-))(q_{3B}(\tau_3^+) - q_{2B}(\tau_3^-)).\end{aligned}$$

Notice that although it constrains the λ_i's, the fact that $\lambda_3(\tau_3^+) - \lambda_1(\tau_3^-) > 0$ and $\lambda_3(\tau_3^+) - \lambda_2(\tau_3^-) < 0$ does not prevent the condition from being satisfied. Hence, it is possible to set the multipliers and F_3 so that the necessary conditions are all satisfied and then to choose initial stocks so that the usages required in this example are exactly available.

5 The comparative dynamics of transport costs

In a context where there are many landfill sites serving many cities, a change in transport cost can have effects which cannot be observed in the simple one-city case. When only one city is being served by one or more landfill sites, it is clear that an increase in the cost of transporting the waste to the sites must always have the effect of decreasing the shadow value of landfill space. But when there are many cities being served by the same sites, an increase in the transport cost may in fact reduce the shadow value of landfill space at one site and raise it at another site. As a consequence, we may observe the counterintuitive result that the full marginal cost to some cities goes up while that to some other cities goes down. To illustrate this, consider the following example.

Suppose there are two landfill sites (sites 1 and 2) from which to supply landfill space to three cities (cities A, B and C). Site 1 is of initial size S_1^0 and site 2 of initial size S_2^0. Assume the only costs are transport costs and that these are proportional to the distance between the city and the site. Thus if c is the transport cost per unit of distance and d_{ik} is the distance between city k and site i, we can write $c_{ik} = cd_{ik}$. We will assume $d_{1A} < d_{2A}$, city A being located close to site 1 and far from site 2, while $d_{2B} < d_{1B}$, city B being located close to site 2 but far from site 1. City C on the other hand is located at site 2 ($d_{2C} = 0$) and a long distance from site 1 ($d_{1C} > 0$).

Assume that the utility function at city k is given by

$$U(q_{\bullet k}(t)) = v_k \min\{q_{\bullet k}(t), \bar{q}_k\}.$$

This means that when $cd_{ik} + \lambda_i e^{rt} < cd_{jk} + \lambda_j e^{rt}$, $i \neq j$, $i, j = 1, 2$, the Hamiltonian will be maximized if, for $k = A, B, C$, we have $q_{jk}(t) = 0$ and[14]

$$q_{ik}(t) = \begin{cases} \bar{q}_k & \text{if } cd_{ik} + \lambda_i e^{rt} \leq v_k \\ 0 & \text{if } cd_{ik} + \lambda_i e^{rt} > v_k. \end{cases}$$

We will assume that city A values waste disposal less than it costs to transport the waste to site 2 ($cd_{2A} > v_A$) and city C values it less than it costs to transport it to site 1 ($cd_{1C} > v_h$). Therefore it will never be optimal for city A to use site 2, nor for city C to use site 1.

This means that, provided $cd_{1A} + \lambda_1 < v_A$, city A will be using site 1 at date $t = 0$ and will continue to do so without switching until some date T_A, given by

$$cd_{1A} + \lambda_1 e^{rT_A} = v_A. \tag{13}$$

[14] In a decentralized pricing system, this utility function would generate a "rectangular" demand, the quantity demanded in market k being $\bar{q}_k$ for any price smaller than v_k, zero for any price higher than v_k and some indeterminate quantity between zero and $\bar{q}_k$ for a price exactly equal to v_k.

At date T_A, its landfill usage drops to zero.

Similarly, provided $\lambda_2 < v_C$, city C will be using site 2 at date $t = 0$ and will stick to it until some date T_C, given by

$$\lambda_2 e^{rT_C} = v_C. \tag{14}$$

At T_C, city C's landfill usage also falls to zero .

As for city B, if in equilibrium we had $\lambda_2 \leq \lambda_1$, then it would be using only site 2, since by assumption $d_{2B} < d_{1B}$ and both shadow prices are growing at the same rate. On the other hand, if $\lambda_2 > \lambda_1$ and $cd_{1B} + \lambda_1 \leq cd_{2B} + \lambda_2$, we would necessarily have $cd_{2B} + \lambda_2 e^{rt} > cd_{1B} + \lambda_1 e^{rt}$ for all $t > 0$ and it would be using only site 1 for all $t > 0$. We will hereafter restrict attention to the more interesting case where $\lambda_2 > \lambda_1$, but $cd_{1B} + \lambda_1 > cd_{2B} + \lambda_2$. In that case, city B will be using site 2 initially and will switch to site 1 at some date $t_B < S_2^0/\bar{q}_B$, which must satisfy[15]

$$cd_{2B} + \lambda_2 e^{rt_B} = cd_{1B} + \lambda_1 e^{rt_B}. \tag{15}$$

Having switched to site 1 at t_B, city B will use it until it reaches the terminal date T_B, which must satisfy

$$cd_{1B} + \lambda_1 e^{rT_B} = v_B. \tag{16}$$

In order for these landfill consumption paths to be feasible, the total required landfill space must not exceed that which is available at the two sites. To guarantee this, we must impose

$$\bar{q}_A T_A + \bar{q}_B (T_B - t_B) = S_1^0 \tag{17}$$

and

$$\bar{q}_C T_C + \bar{q}_B t_B = S_2^0. \tag{18}$$

The six equations (13) to (18) together determine the six variables λ_1, λ_2, t_B, T_A, T_B

[15]The case where $t_B < S_2^0/\bar{q}_B$ is the only one of interest, for if $t_B = S_2^0/\bar{q}_B$ is to occur in equilibrium, we must also have $\lambda_2 \geq v_C$ and hence $T_C = 0$. This means that city C would be using neither of the landfills.

and T_C as a function of c and all the other parameters. Since there is no direct cost other than the transport cost, the variable λ_i can be thought of as the tipping fee at site i, that is the "on-site price" of dumping waste at landfill i in a decentralized market. As we will show, there exist admissible values of the parameters such that $\frac{d\lambda_1}{dc} < 0$ and $\frac{d\lambda_2}{dc} > 0$. The underlying intuition for this result can be summarized as follows. For the values of the parameters generating the result, an increase in the unit cost of transporting waste would cause city B to delay switching to the more distant site: $\frac{dt_B}{dc} > 0$. If that site's tipping fee (λ_1) did not fall to induce additional usage by city A then site 1 would be underutilized. As for site 2, which now has more usage from city B, it would be overutilized if its tipping fee (λ_2) did not rise. Therefore, the tipping fee paid by city A (λ_1) will fall while that paid by city C (λ_2) will rise.

The non linearity of the six-equation system[16] makes it impossible to get a closed form solution which would make it possible to directly verify this result. However, if we totally differentiate the system of equations and let

$$K = \frac{cd_{1B}\bar{q}_B}{v_B - cd_{1B}} + \frac{cd_{1A}\bar{q}_A}{v_A - cd_{1A}} > 0 \quad \text{and} \quad X = \left(\frac{\lambda_2 - \lambda_1}{\lambda_1} + \frac{\lambda_2}{\lambda_1}\frac{\bar{q}_B}{\bar{q}_C}\right)\frac{\bar{q}_A + \bar{q}_B}{\bar{q}_B} > 0,$$

we find that

$$\frac{d\lambda_1}{dc} = -\frac{\lambda_2}{c}\frac{\frac{\lambda_2 - \lambda_1}{\lambda_1} + KX}{1 + X} < 0. \tag{19}$$

$$\frac{d\lambda_2}{dc} = \frac{r\lambda_2\bar{q}_B}{\bar{q}_C}\frac{dt_B}{dc}. \tag{20}$$

and

$$\frac{dt_B}{dc} = \frac{\left(\frac{\lambda_2 - \lambda_1}{\lambda_1} - K\right)\frac{\bar{q}_A + \bar{q}_B}{\bar{q}_B}}{rcX}. \tag{21}$$

We therefore see immediately that an increase in the transport cost reduces the equilibrium shadow value of a unit of landfill space at site 1. As for the effect of the transport cost

[16]Taking logarithms, we find that $\ln(\lambda_2 - \lambda_1)$ appears in equation (15).

on λ_2, we see from (20) that it is of the same sign as the effect on t_B, given by (21)[17]. Hence it is positive if and only if

$$\frac{\lambda_2 - \lambda_1}{\lambda_1} > K. \tag{22}$$

There remains to show that admissible examples can be constructed which have this property in equilibrium.

To do this, treat λ_2 and λ_1 as parameters by setting $\lambda_2 = 1$ and $\lambda_1 = 1/\alpha$, where α is any number greater than one, and hence $\lambda_2 - \lambda_1 = (\alpha - 1)/\alpha > 0$. If we now treat the system of equations (13) to (18) as one in v_C, S_1^0, t_B, T_A, T_B and T_C, with λ_2 and λ_1 taking the parameter values just assigned, it is easily verified that the system is linear in logarithms and yields as solution for v_C and S_1^0

$$\begin{aligned}
\ln \tilde{v}_C &= r\frac{S_2^0}{\bar{q}_C} - \frac{1}{\bar{q}_C} \ln \left[\frac{\alpha c(d_{1B} - d_{2B})}{\alpha - 1}\right] \\
\tilde{S}_1^0 &= \frac{1}{r}\left(\bar{q}_A \ln[\alpha(v_A - cd_{1A})] + \bar{q}_B \ln[\alpha(v_B - cd_{1B})] - \ln\left[\frac{\alpha c(d_{1B} - d_{2B})}{\alpha - 1}\right]\right).
\end{aligned}$$

Clearly there exist admissible values of the parameters for which both $\tilde{v}_C$ and $\tilde{S}_1^0$ are positive.

If we now set $v_C = \tilde{v}_C$ and $S_1^0 = \tilde{S}_1^0$ in equations (14) and (18) and revert to treating λ_2 and λ_1 as variables, the solution to the original system of equations (13) to (18) for λ_2 and λ_1, with those specific values of the parameters, must be $\lambda_2 = 1$ and $\lambda_1 = 1/\alpha$. Therefore, for any value of $\alpha > K + 1$, we have $dt_B/dc > 0$ and hence $d\lambda_2/dc > 0$. Note that the equilibrium values of the remaining variables are easily calculated as well, but are of no direct interest for our purposes.

We have therefore established that in some cases an increase in the unit transport cost will raise the shadow value of landfill space at site 2 while it lowers it at site 1. This is because, in those cases, the increase in the transport cost creates an incentive to have city B switch from site 2 to site 1 at a later date than it otherwise would. This in turn requires that city B be allocated more of the limited space available at site 2 and less of that available at

[17]Equation (20) follows immediately from equations (14) and (16).

site 1.

As a direct result, the full marginal cost of landfill space necessarily goes up at city C, since the opportunity cost λ_2 is the only cost to city C using site 2. As for the full marginal cost of landfill space to city A, it falls if d_{1A} is not too large (for instance, if $d_{1A} = 0$). This means that in a decentralized competitive equilibrium, the increase in the per unit cost of transporting waste would have the paradoxical effect of increasing the "delivered price" of landfill space at city C and decreasing it at city A, the delivered price being equal to the marginal cost of transporting the waste to the site plus the tipping fee at that site[18].

6 Reinterpretations

To this point, we have assumed that user k's utility at t is a strictly increasing function of the exhaustible resources used then. This formulation seems entirely appropriate for applications to conventional exhaustible resources. However, it may at first seem inapplicable when the exhaustible resource is instead unused landfill space. After all, one tends to think of a city as generating waste at a fixed rate and having no choice but to dispose of it. In fact, cities need not landfill all their waste. At a cost, they can engage in waste reduction, recycling, or incineration. As shown below, no changes need be made in our model to incorporate any of these features of the real-world problem.

Recycling can be accommodated within our model by reinterpreting the utility function. Assume the waste stream of city k (denoted w_k) cannot be reduced. Instead it must be either (1) recycled or (2) landfilled somewhere. Denote the total cost of recycling at rate z as $C_R(z)$, where this function is strictly increasing and strictly convex with $C_R(0) = 0$. Then we can interpret $U_k(q_{\bullet k}(t))$ as the recycling costs saved by landfilling at rate $q_{\bullet k}(t)$ so that

[18]In the example, the full marginal cost to city B — hence the delivered price in a market solution — goes up at both sites. This is obvious for site 2, since both λ_2 and c have gone up. It must also be the case at site 1 even though λ_1 has gone down, for otherwise city B would have switched to site 1 earlier, not later (i.e., t_B would have gone down, not up).

recycling occurs at the reduced rate $w_k - q_{\bullet k}(t)$:

$$U_k(q_{\bullet k}(t)) = C_R(w_k) - C_R(w_k - q_{\bullet k}(t)).$$

Note that the foregoing utility function satisfies the properties assumed throughout: it is strictly increasing, strictly concave, passes through the origin and has a finite first derivative at zero. When city k uses landfill i, the necessary condition $U_k(q_{\bullet k}(t)) = c_{ik} + \lambda_i(t)$ implies $C_R(w_k - q_{\bullet k}(t)) = c_{ik} + \lambda_i(t)$. Hence, the planner recycles city k's waste to the point where the marginal cost of recycling additional waste equals the full marginal cost of landfilling it at the least costly of the n sites.

A similar reinterpretation of the utility function permits incorporation of waste reduction as well. Assume that the potential waste stream of city k (denoted w_k) can at a cost be reduced by $x_k(t)$ so that the amount which must be recycled or landfilled at t is only $w_k - x_k(t)$. Denote the total cost of waste reduction at rate $x_k(t)$ as $C_D(x_k(t))$, where this function is strictly increasing and strictly convex with $C_D(0) = 0$. Then we can interpret $U_k(q_{\bullet k}(t))$ as the costs saved by landfilling city k's waste at rate $q_{\bullet k}(t))$ so that the amount of waste reduction and recycling is reduced to $w_k - q_{\bullet k}(t)$:

$$U_k(q_{\bullet k}(t)) = \max_{x_k(t)\geq 0}\{C_D(\bar{x}_k(t)) + C_R(w_k - \bar{x}_k(t)) - C_R(w_k - x_k(t) - q_{\bullet k}(t)) - C_D(x_k(t))\},$$

where $\bar{x}_k(t)$ is the optimal amount of waste reduction if nothing is landfilled. Let $x_k(q_{\bullet k}(t), t)$ denote the unique maximizer at date t; therefore, $\bar{x}_k(t) = x_k(0, t)$. Note that this utility function also satisfies the properties we have assumed throughout[19]. If the planner disposes of city k's waste at landfill i at time t, $U_k(q_{\bullet k}(t)) = c_{ik} + \lambda_i(t)$. Hence, $C_R(w_k - x_k(q_{\bullet k}(t), t) - q_{\bullet k}(t)) = c_{ik} + \lambda_i(t)$. At the margin, the cost of further recycling is again equal to the cost of additional landfilling. But since $x_k(q_{\bullet k}(t), t)$ is set optimally, the marginal cost of recycling

[19] $U_k(0) = 0, U_k(q_{\bullet k}(t)) = C_R(w_k - x_k(q_{\bullet k}(t), t) - q_{\bullet k}(t)) > 0, U_k(q_{\bullet k}(t)) = -C_R(w_k - x_k(q_{\bullet k}(t), t) - q_{\bullet k}(t)) \cdot (1 + \frac{dx_k}{dq_{\bullet k}}) < 0$. Hence, as before, this new utility function is also strictly increasing, strictly concave, and passes through the origin. In addition, $U_k(0) = C_R(w_k - \bar{x}_k(t))$ is finite.

is also equal to the marginal cost of waste reduction: $C_R(w_k - x_k(q_{\bullet k}(t), t) - q_{\bullet k}(t)) = C_D(x_k(q_{\bullet k}(t), t))$. Hence the marginal cost of landfilling, recycling, and waste reduction are equated whenever it is optimal at time t for city k to use all three of these activities (at strictly positive rates) to dispose of its waste.

Cities can, of course, also incinerate their waste. No change in the model is necessary to accommodate this feature of the solid waste problem. Incinerators can simply be regarded as landfills with extremely large (or infinite) initial stocks. That is, incineration can be regarded as a spatially located "backstop."

Since the initial reserves of a mine are fixed while landfills can in fact be expanded at a cost, our formulation may again seem more applicable to conventional resources than to solid waste disposal. To at least some extent, capacity expansion can also be accommodated without changing our model. Capacity expansion is typically "lumpy" in the sense that the cost of opening a space of standard size is the same as the cost of opening anything smaller. In that case, one can simply regard the expansion of the existing landfill as the opening of a second landfill (with initial capacity reflecting the "lumpiness") adjacent to the first.

Since mines can only be located near mineral deposits, while landfills and incinerators can be located "virtually anywhere," our formulation may again seem at first more applicable to extraction of conventional resources than to solid waste disposal. However, with suitable interpretations, our model captures this aspect of the solid waste problem as well. Let n be the number of *potential* sites of a landfill or incinerator. As long as n is finite, our model applies. The planner can then be viewed as choosing which subset of a possibly large but finite set of potential sites to develop.

7 Conclusion

We have generalized the standard model of nonrenewable resource depletion to the common situation where reserves and users are located arbitrarily over space. We have shown how the received results in the literature following Herfindahl (1967) can be given a spatial reinterpretation where a single user is regarded as exploiting many deposits at arbitrarily specified locations. From this perspective, the contribution of our paper is to generalize these spatial results further to the common case where there are also multiple, spatially distributed users. As we have shown, a variety of new phenomena arise. It may be socially optimal, for example, for some landfill users to abandon a nearby landfill in preference for a more distant one even though space remains in the nearby fill and other users continue to exploit it. Moreover, in the presence of set-up costs, it may be socially optimal for every city patronizing a particular landfill to abandon it even though space remains — and then for some users to return to it after a delay. Finally, in a decentralized competitive equilibrium, increases in the transport cost may cause some landfill prices to rise while others fall.

In addition to providing new insights into the spatial and intertemporal allocation of scarce resources, our model also constitutes a useful framework for analyzing policies aimed at regulating the spatial allocation of a scarce resource such as landfill space. To that end, a number of natural extensions of the model can easily be envisioned. The introduction of investment in new capacity would allow for the possibility of scarce landfill space to be increased either by the expansion of existing sites or the creation of new ones. This is the counterpart of investing in discovery and development in the case of minerals and oil. Another useful extension would be to take into account the value of alternative uses of each site in order to determine the initial allocation of land between landfills and other uses. This would, in effect, endogenize the initial space allocated to each landfill (S_i^0) and perhaps the setup cost itself (F_i). These extensions, which would permit a still more accurate characterization of the real-world solid-waste situation, would rely heavily on the analysis presented here; but we leave them for future work.

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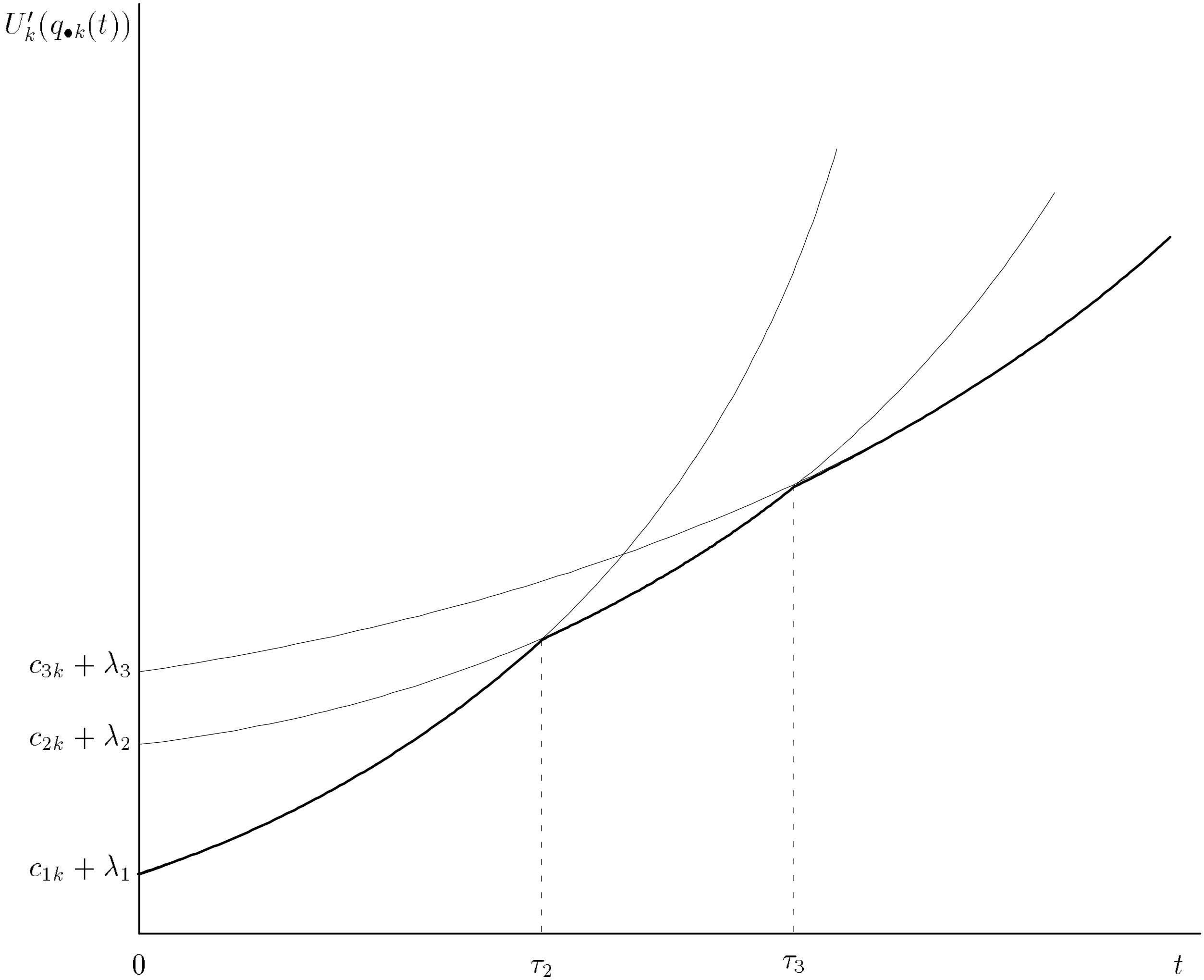
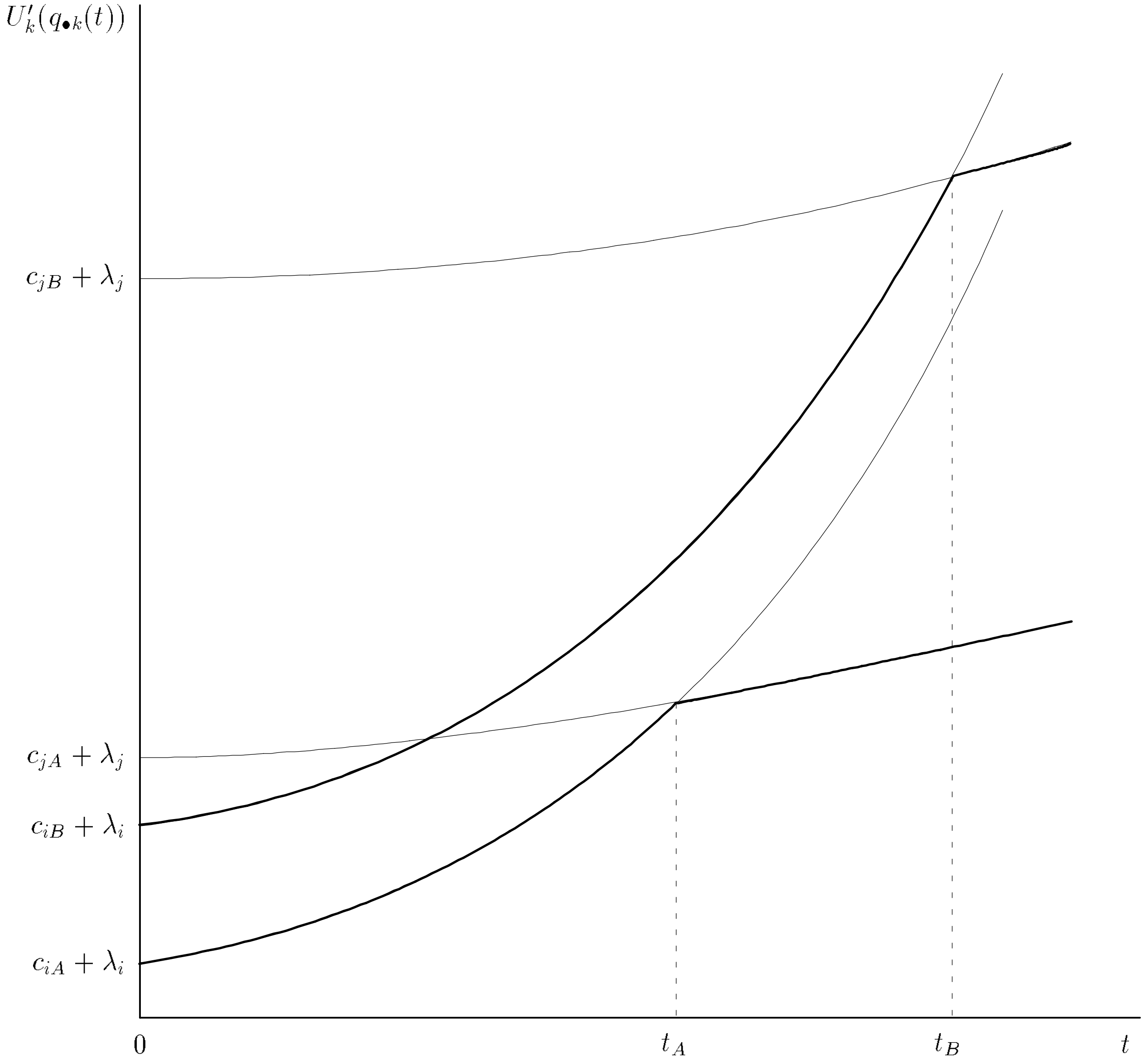

$U'_k(q_{\bullet k}(t))$
$c_{3k} + \lambda_3$
$c_{2k} + \lambda_2$
$c_{1k} + \lambda_1$
0
τ_2
τ_3
t

Figure 1



$U'_k(q_{\bullet k}(t))$
$c_{jB} + \lambda_j$
$c_{jA} + \lambda_j$
$c_{iB} + \lambda_i$
$c_{iA} + \lambda_i$
0
t_A
t_B
t

Figure 2

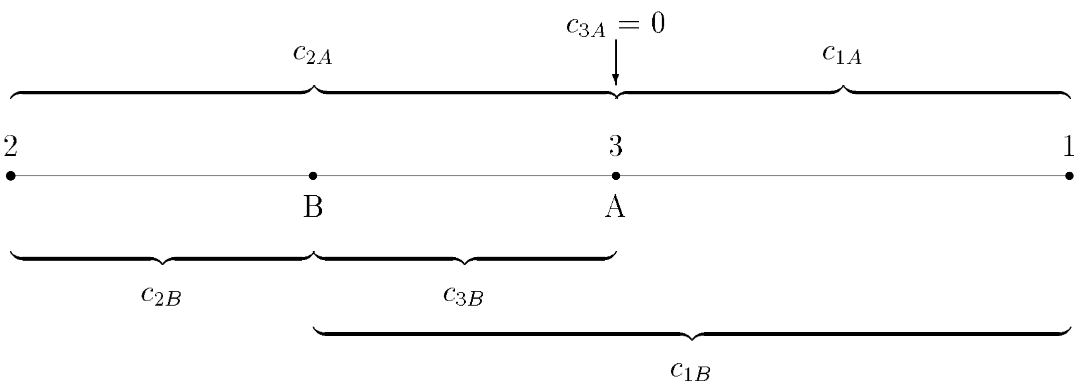


Figure 3

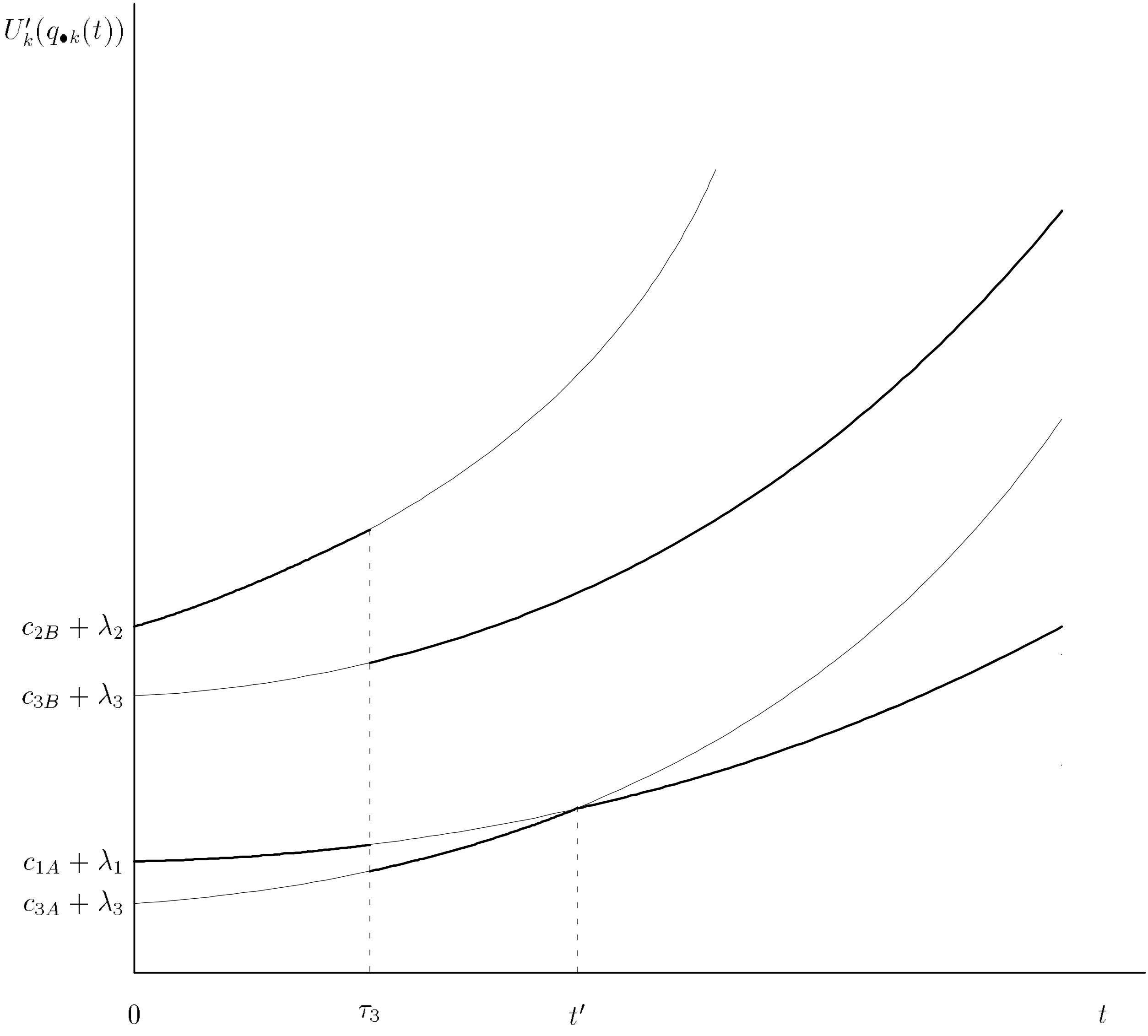

$U'_k(q_{\bullet k}(t))$
$c_{2B} + \lambda_2$
$c_{3B} + \lambda_3$
$c_{1A} + \lambda_1$
$c_{3A} + \lambda_3$
0
τ_3
t'
t

Figure 4