

# **A theoretical approach to tourism sustainability**

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## *Non technical summary*

Aim of this paper is to show why it is difficult, if not impossible, to find policies that guarantee that touristic activities can be maintained for a long time without severely impacting on the environment. The analysis is based on very simple and general assumptions on the interactions between the three main compartments of the system: tourists, environment, and services. These assumptions are encapsulated in a so-called minimal model which is used to predict the economic and environmental impact of any given policy.

The paper is organized as follows. Firstly, the minimal model is described in detail, with emphasis on the non linearities concerning the interactions among the three compartments. Then, all possible asymptotic modes of behaviour are detected through bifurcation analysis. The control parameters of the analysis are investment attitude of policy makers, local prices and environmental reclamation. The results are interpreted from an economic point of view by introducing a suitable definition of profitability, and from an environmental point of view by referring to the notion of compatibility. This allows introducing a sharp definition of sustainable policies which is used to lump the results in a few general principles.

The paper has three merits. Firstly, it introduces a new approach, namely that of minimal descriptive models, in a context which has been traditionally dominated by the use of detailed simulation models. Secondly, the specific results are quite interesting. It is shown in fact that tourism sustainability can be obtained provided the agents are not too greedy and/or reluctant to protect the environment and that sustainability is very often at risk, since accidental shocks can easily trigger a switch from a profitable and compatible behaviour to an unprofitable or incompatible one. All these results are in line with common wisdom and observations, but the interesting fact is that they are here theoretically derived from a few very simple and abstract premises. Finally, the third merit is not strictly related with the problem of tourism but with the general topic of sustainability. In fact, this is the first time that notions like profitability, compatibility and sustainability, which are more and more pervasive in the field of resources management, are interpreted in terms of structural properties of the attractors of a dynamical system. This creates an important and promising bridge between the notion of sustainability and bifurcation theory, one of the most important chapters of dynamical systems theory.

## ***Technical summary***

In this paper we analyse tourism sustainability from a modelling point of view. The minimal model we propose is composed of three nonlinear ordinary differential equations describing the dynamics of tourists, environment and services. Many of the parameters involved are concerned with the policy followed by agencies or local decision makers. A bifurcation analysis of the model is performed with respect to these parameters. Then, the notion of profitability and compatibility is introduced and used to interpret the bifurcation diagrams. The results are summarised in a few sustainability diagrams.

**Keywords:** tourism industry, environment, sustainability, nonlinear dynamics.

## ***Introduction***

Tourism industry has remarkably increased in the last decades and has become one of the major sources of income in many countries (Williams and Shaw, 1988; Coccossis and Nijkamp, 1995). In many touristic sites the rewarding phase of development is characterised by a long and intense growth of services that, sooner or later, seriously impact on the environment, thus creating a rather critical situation. In fact some regions, after a long flourishing period, are abandoned by tourists in favour of more attractive sites newly available on the market. In order to compensate this instability, local agents can increase investment and develop special services to attract tourists. Sometimes they are successful, but at the expenses of the environment, which is heavily deteriorated.

Aim of this paper is to show why it is difficult, if not impossible, to find policies that guarantee that touristic activities can be maintained for a long time without severely impacting on the environment. The analysis is very abstract. It is not based on data of one or more special cases, but on very simple and general assumptions on the interactions between the three main compartments of the system: tourists, environment, and services. These assumptions are encapsulated in a so-called minimal model which is used to predict the economic and environmental impact of any given policy. The study is carried out by varying the parameters interpreting the attitude of the agents toward the two main conflicting objectives: economic development and environmental protection (Lindberg, 1991; Smith and Eadington, 1992). The results are in agreement with common wisdom on the argument and, more in general, with the impression that human short-sightedness and greed can transform sustainability into an unrealistic dream (Ludwig *et al.*, 1993; Ludwig, 1993; Arrow *et al.*, 1995; Roe, 1996; Brown *et al.*, 1996).

## ***A minimal model***

The minimal model on which we base our analysis is so crude and abstract that it cannot represent any specific system in detail. Nevertheless, it incorporates the core features of many systems. The model refers to a generic site and has only three variables: the tourists  $T(t)$  present in the area at time  $t$ , the quality of the environment  $E(t)$  and the services  $S(t)$  available to tourists. While the choice of these three compartments is rather natural, their description with a single variable definitely poses some problems. In fact, one might be reluctant to aggregate into a single variable tourists of different ages and cultures, various aspects of the environment and all classes of services, ranging from hotels to parks and from sport facilities to transportation systems. But this aggregation process is mandatory, because we must keep variables and parameters at reasonably low numbers in order to obtain a tractable problem. Seasonal effects are not taken into account since we are interested only in the long term behaviour of the system.

The dynamics of tourists is assumed to be of the form

$$\dot{T}(t) = T(t) \cdot [\hat{a}(T(t), E(t), S(t)) - a]$$

where  $\hat{a}$  is the attractiveness of the site and  $a$  is a reference value indicating the demand of attractiveness of the tourists. Obviously, the parameter  $a$  increases with local prices and decreases with prices of alternative touristic sites. The function  $\hat{a}$  is not an absolute measure of the attractiveness of the site, but a measure of the attractiveness perceived by the tourists. Thus,  $\hat{a}$  depends upon the culture of the tourists and, in particular, upon their sensitivity to environmental quality and their capability of detecting it. The attractiveness is the algebraic sum of three terms since tourists can be sensitive to environmental quality, availability of services, and congestion.

The attractiveness of the environment can be modelled as an increasing and saturating function of  $E(t)$ . In the following, it will be described as a Monod function

$$\mu_E \frac{E}{(E + j_E)}$$

where  $\mu_E$  is the attractiveness associated to high environmental quality ( $E \rightarrow \infty$ ) and  $\varphi_E$  is the half saturation constant, namely the environmental quality at which tourist satisfaction is half maximum. Thus, tourists characterised by low values of  $\varphi_E$  are satisfied by low qualities of the environment because they are unable to perceive environmental quality. For example, a tourist who is unable to perceive if a river is polluted or not, will associate to the river a constant attractiveness  $\mu_E$  independently upon its water quality. The second component of the attractiveness, namely that associated with services, can also be modelled through a Monod function of the estimated available services per capita  $S / (T+I)$ , i.e.

$$\mu_S \frac{S / (T+I)}{S / (T+I) + j_S} = \mu_S \frac{S}{S + j_S T + j_S I}$$

Notice that the attractiveness associated to the environment is a function of  $E$  and not of  $E / (T+I)$ , as prescribed by the theory of public-goods and non-consumptive use (Herfindahl and Kneese, 1974). By contrast, services are consumed by tourists and, therefore, the attractiveness associated with services is a function of  $S / (T+I)$ . Finally, if we assume that congestion is proportional to  $T$  and that attractiveness is linearly decreasing with congestion, we end up with the following formula for  $\hat{a}$

$$\hat{a} = \mu_E \frac{E(t)}{E(t) + j_E} + \mu_S \frac{S(t)}{S(t) + j_S T(t) + j_S I(t)} - a T(t)$$

where the five parameters ( $\mu_E, \varphi_E, \mu_S, \varphi_S, \alpha$ ) identify the culture of the tourist population.

The quality of the environment  $E(t)$ , in the absence of tourists and services, is described by a classical logistic equation

$$\dot{E}(t) = rE(t) \cdot \left(1 - \frac{E(t)}{K}\right)$$

where the net growth rate  $r$  and the carrying capacity  $K$  take into account all local activities. Since tourists and services impact negatively on the environment, the dynamics of  $E(t)$  is

$$\dot{E}(t) = rE(t) \cdot \left(1 - \frac{E(t)}{K}\right) - D(T(t), E(t), S(t))$$

where  $D(T(t), E(t), S(t))$  is the flow of damages induced by tourism. In the following, such a flow will be specified as a bilinear term

$$D = E \cdot (bS + gT)$$

This means that if  $T$  and  $S$  would be kept constant, the environment would still be described by a logistic equation

$$\dot{E}(t) = r^* E(t) \cdot \left(1 - \frac{E(t)}{K^*}\right)$$

with

$$r^* = r - bS - gT$$

$$K^* = \frac{r - bS - gT}{r} K$$

In other words, tourism activities ( $S$  and  $T$ ) reduce the carrying capacity and the net growth rate of the environment in the same proportion.

Finally, the rate of change of touristic services is the balance of an investment flow  $I$  and a depreciation flow  $\delta S$ , i.e.

$$\dot{S}(t) = I(T(t), E(t), S(t)) - dS(t)$$

The function  $I$  could be specified in many different ways for interpreting different investment policies. Indeed, one might imagine to impose special constraints on the function in order to avoid degenerate dynamics. Alternatively, one could derive the structure of the function  $I(T, E, S)$  using optimality arguments, like in Shah (1995). In the present paper we will assume that investments are a fixed proportion of total revenues due to tourism activities and that such revenues are proportional to the stock of tourists, i.e.

$$I(T(t), E(t), S(t)) = \epsilon T(t)$$

Thus, the parameter  $\epsilon$ , called for shortness investment rate, is increasing with local prices.

In conclusion, our minimal model turns out to be

$$\dot{T}(t) = T(t) \cdot \left[ m_E \frac{E(t)}{E(t) + j_E} + m_S \frac{S(t)}{S(t) + j_S T(t) + j_S} - aT(t) - a \right] \quad (1)$$

$$\dot{E}(t) = E(t) \cdot \left[ r \left(1 - \frac{E(t)}{K}\right) - bS(t) - gT(t) \right] \quad (2)$$

$$\dot{S}(t) = -dS(t) + \epsilon T(t) \quad (3)$$

It is a positive model since  $T(0), E(0), S(0) \geq 0$  implies  $T(t), E(t), S(t) \geq 0$  for all  $t \geq 0$ . Being nonlinear, model (1)-(3) can have multiple attractors in the positive orthant. Some of them are not strictly positive since they lie on the face  $(T, S)$  (see eq. (2) with  $E(t) = 0$ ) or on the  $E$ -axis (see eqs. (1) and (3) with  $T(t) = S(t) = 0$ ). The model has twelve parameters, which are all positive, except  $\alpha$  which is negative if tourists like crowding. The parameter that local agents and decision makers can more easily control is the rate of investment  $\epsilon$ , while price control influences also  $a$ . Reclamation of the environment gives rise to lower values of  $\beta$  and/or  $\gamma$ , while increased competition of alternative touristic sites

can be viewed as an increase of  $a$ . These are the parameters we will vary with the aim of detecting all possible modes of behaviour of the system.

### ***Bifurcation analysis***

Aim of this section is to identify all stable modes of behaviour (attractors) of model (1)-(3) in a suitable range of two parameters, namely investment rate  $\varepsilon$  and competition  $a$ . Since the model is a “third order” model, its attractors can be equilibria and limit cycles, but also chaotic attractors (Guckenheimer and Holmes, 1983; Strogatz, 1994). Nevertheless, our analysis shows that model (1)-(3) does not behave chaotically, at least in the range of parameter values we have considered. Each point of the two dimensional parameter space  $(\varepsilon, a)$  corresponds to one model of our family (1)-(3) and therefore to one specific set of attractors, saddles and repellers. If point  $(\varepsilon, a)$  is slightly changed, i.e. if at least one of the two parameters is slightly perturbed, by continuity the position and the form of the attractors, saddles and repellers in state space will vary smoothly (e.g. a cycle might become slightly bigger and faster) but all trajectories will remain topologically the same (e.g., an attracting cycle will remain an attracting cycle). Only at some particular points in parameter space the above continuity argument will fail. At these points, called *bifurcation points*, small variations of the parameters entail significant changes in the model behaviour. For example, a strictly positive equilibrium  $(\bar{T}, \bar{E}, \bar{S})$  can be stable (i.e., attract all nearby trajectories) for a given parameter setting, but lose its stability if competition  $a$  is increased even of an infinitesimal amount. If this is the case, after the parameter has been varied, the state of the system will not tend toward the equilibrium  $(\bar{T}, \bar{E}, \bar{S})$  but toward another attractor. If this new attractor is infinitely close to the old one, the bifurcation is called *non catastrophic*. By contrast, it is called *catastrophic* if a microscopic variation of the parameter gives rise to a macroscopic transition from one attractor to another. For example, this would be the case if a small increase of competition would force the system to switch from a strictly positive equilibrium  $(\bar{T}, \bar{E}, \bar{S})$  to an equilibrium  $(0, \bar{E}, 0)$  characterised by the absence of touristic activities.

Bifurcation points are located on *bifurcations curves* in the parameter space  $(\varepsilon, a)$  and these curves partition the parameter space into subregions. All the models corresponding to the same subregion have qualitatively the same behaviour. Thus, by determining all bifurcation curves we can produce a diagram showing where the different modes of behaviour of a system occur in parameter space. This requires an understanding of bifurcation theory (in particular normal forms) (Guckenheimer and Holmes, 1983) and can be done numerically using specialised software implementing suitable continuation techniques. Once a single bifurcation point in the parameter space is found, this software produces automatically the entire bifurcation curve passing through that point. The extreme points of this curve are, in general, the extreme points of other bifurcation curves. They are called codimension-2 bifurcation points or *organizing centers*, and their discovery is strategically important for constructing the whole bifurcation diagram (Kuznetsov, 1995).

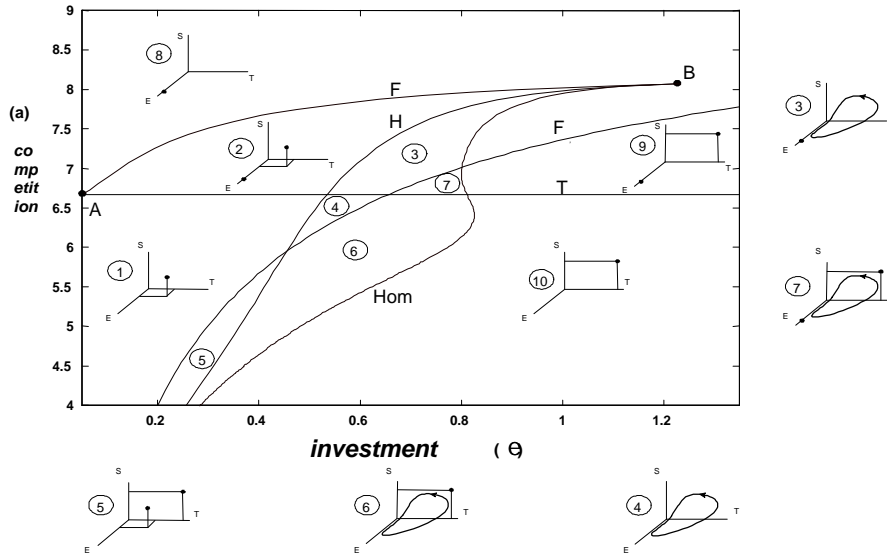


FIG 1

Without going into more details, we show the results of this analysis in Fig. 1, for the parameter setting indicated in the caption. The organizing centers are points *A* and *B* and a third point where the three lower curves merge. Point *B* is the well know Bogdanov-Takens codimension-2 bifurcation point (Guckenheimer and Holmes, 1983). The nature of each bifurcation curve is identified by one of the following symbols

- T*      Transcritical bifurcation of equilibria
- F*      Fold bifurcation of equilibria
- F<sub>p</sub>*    Planar fold bifurcation of equilibria
- H*      Hopf bifurcation
- Hom*    Homoclinic bifurcation

All these bifurcations, except the homoclinic one, are *local bifurcations* characterised by some kind of degeneracy of the eigenvalues of a suitable Jacobian matrix, and are, therefore, evaluated with high precision. They have been obtained by means of LOCBIF



(Khibnik *et al.*, 1993), a software for local bifurcation and normal form analysis. By contrast, homoclinic bifurcations are global bifurcations and are much more difficult to compute (Champneys and Kuznetsov, 1994). The bifurcation curve *Hom* of Fig. 1 has been obtained by means of AUTO (Doedel and Kernévez, 1986) through the continuation of a limit cycle of very high period.

The bifurcation curves define ten different regions and the attractors (equilibria and limit cycles) of each region are sketched in Fig. 1. In regions 1, 4, 8, 10 the attractor is unique, while in the other regions there are two or even three (region 7) alternative attractors. In these cases the system is “fragile” because an accidental shock can suddenly perturb the state of the system and bring it from an attractor  $A_i$  into the basin of attraction of another attractor  $A_j$ . Thus, after the perturbation has ceased, the state of the system will tend toward the new attractor  $A_j$  and remain there until a new shock comes. Another important remark is that in regions 3, 4, 6, 7 the attractor, or one of the alternative attractors, is a strictly positive limit cycle. This means that in these regions the asymptotic behaviour of the system can be characterised by periodic ups and downs of the three state variables. Close to curve *H* in regions 3, 4, 6 oscillations of  $T$ ,  $E$ ,  $S$  are relatively small because approaching the Hopf bifurcation curve *H* from the right, the strictly positive limit cycle shrinks and finally is substituted by a strictly positive equilibrium in regions 2, 1, 5. By contrast, approaching the homoclinic curve *Hom* from the left, the limit cycle present in regions 3, 7, 6 becomes longer and longer and finally disappears when crossing curve *Hom*. This means that the limit cycles are degenerate on the boundaries of their region of existence: on curve *H* they have zero amplitude, while on curve *Hom* they have zero frequency (i.e., infinite period).

### ***Profitable, compatible and sustainable policies***

The parameters of the model can be subdivided into *policy parameters* and *system parameters*. The first ones identify the behavioural characteristics of agents and decision makers, while system parameters describe the other actors involved in the game, namely environment, tourists and alternative touristic sites. A particular parameter setting can therefore be viewed as a particular policy applied to a particular system. In order to judge the economic impact of a policy in a given system, we should therefore be able to associate a value judgement concerning the tourism industry to any parameter setting. Of course, the most crude choice is to associate a zero-one value judgement (i.e. “bad” or “good”) to each parameter setting. Moreover, if the judgement is based on some *structural characteristic* of the attractors, namely on a characteristic that can change only at bifurcation points, the same value judgement will be given to all parameter settings belonging to the same subregion in parameter space. In other words, the zero-one value judgement will be obtained as a side product of bifurcation analysis.

Let us therefore qualify the economic impact of a policy applied to a given system by saying that the policy is *profitable* if at least one of the associated attractors is characterised by  $T(t) > 0$  for all  $t$  (notice that this implies  $S(t) > 0$  for all  $t$ ). This means that the application of that policy to that particular system will have the chance to sustain the tourism industry forever. The property “ $T(t) > 0$ ” is a structural property of the attractors, as it can be verified from Fig. 1. From the same figure it follows that only region 8 corresponds to non-profitable policies. Indeed, in all other regions, at least one

attractor is characterised by  $T(t) > 0$ , i.e. by a permanent touristic activity. There is, nevertheless, an important difference among these regions. In fact, in some of them there are also attractors characterised by  $T = 0$ , i.e. by the absence of tourism industry. When this happens, we say that the policy is profitable but *risky*, because an unexpected accidental shock, like a war, an epidemics or an episode of xenophobia, can perturb the state of the system and cause a transient ending in an attractor characterised by no tourism industry. Regions 2, 3, 7, 9 of Fig. 1 correspond to profitable but risky policies, while all regions below the transcritical bifurcation curve  $T$  are profitable and *safe*. Thus, if competition is sufficiently low, all policies are profitable and safe, while if competition is very high (i.e., above the value corresponding to point  $B$ ) all policies are non-profitable, a quite reasonable result. Following a similar line of reasoning for judging the environmental impact of a policy in a given system, we can say that a policy is *compatible* when at least one of its associated attractors has  $E(t) > 0$  for all  $t$ . From this definition it follows that only region 10 in Fig. 1 is not compatible. Again, we can distinguish between safe and risky policies and find out that all regions above the planar bifurcation curve  $F_p$  correspond to safe compatible policies. In line with the theory of conflict resolution in multiobjective analysis (Keeney and Raiffa, 1976) and in accordance with some of the most recent ideas on sustainability (Forum on “Science and sustainability” (1993) *Ecological Applications* **3**, 545-589), we can say that a policy is *sustainable* if one of its associated attractors is characterised by  $E(t) > 0$  and  $T(t) > 0$  (and hence  $S(t) > 0$ ) for all  $t$ , i.e. when one of its attractors is strictly positive. This implies that a sustainable policy has the chance to maintain the tourism industry forever without jeopardising the environment. Obviously, a sustainable policy is profitable and compatible, while the converse is not true, as one can easily check by looking at the attractors in region 9 of Fig. 1. Of course, a sustainable policy can be safe (and is certainly such if the attractor is unique). But it can also be risky for the environment and/or for the economy. Following these definitions, one can easily derive from Fig. 1 the region of sustainable policies and subdivide it, as shown in Fig. 2, into a region of safe policies and various regions of risky policies. Fig. 2, from now on called *sustainability diagram*, shows that the region of sustainable policies is delimited by two catastrophic bifurcation curves (the fold  $F$  and the homoclinic  $Hom$ ). This means that any parameter variation implying the loss of sustainability will be accompanied by a catastrophic collapse of the environment and/or of the tourism industry. By contrast, the subregion of sustainable and safe policies is delimited by two bifurcation curves (the transcritical  $T$  and the planar fold  $F_p$ ) which are non-catastrophic for the strictly positive attractor. Such a region is rather limited and characterised by low competition and investment rate. As soon as competition becomes a bit too heavy the policy becomes risky for the economy, while if local agents are a bit too greedy the policy is risky for the environment.

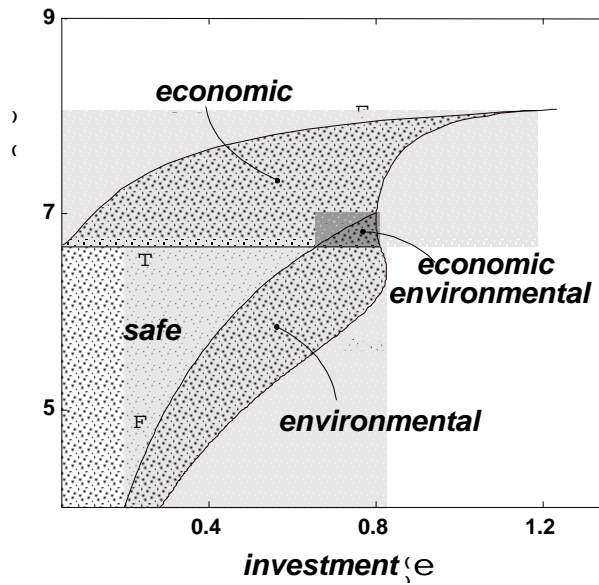
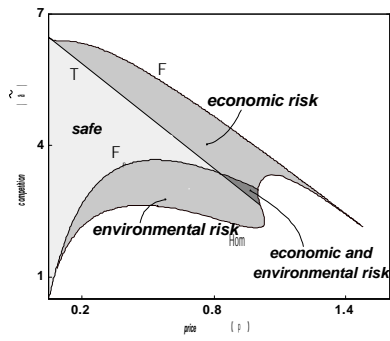


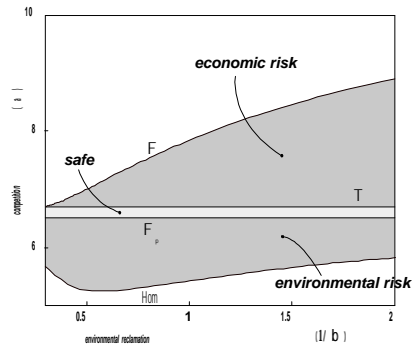
FIG 2

The analysis described in Figs. 1 and 2 concerning the effects of competition and investment has been repeated for other pairs of parameters in order to obtain a better understanding of the problem. The results are shown in Fig. 3, where four different sustainability diagrams are reported. In all diagrams on the horizontal axis we have a policy parameter, like price, environmental reclamation, and investment, while on the vertical axis there is a system parameter describing the attractiveness of alternative sites (competition) or some behavioural characteristics of the tourists, like their appreciation of services ( $\mu_s$  in eq. (1)) and their capability of perceiving the quality of the environment ( $\varphi_E$

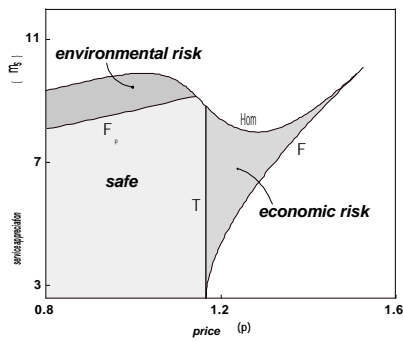
in eq. (1)). In all cases, the boundary of the sustainable region is composed of two catastrophic bifurcation curves (a fold and a homoclinic).



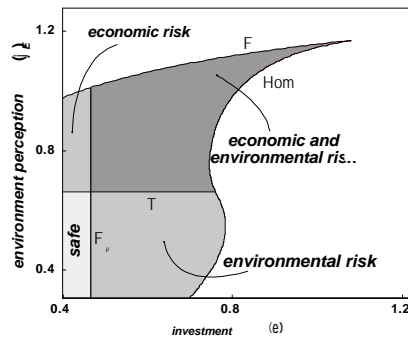
(a)



(b)



(c)



(d)

In general, safe policies are “surrounded” by risky policies, and a continuous increase of the system parameter transforms a safe sustainable policy first into a risky sustainable policy and then into a non sustainable policy. But there are also cases in which a safe sustainable policy can suddenly become non sustainable, like in Fig. 3c where a small increase of service appreciation can cause a catastrophic transition from safe sustainability to non sustainability. In general, sustainability requires low prices, low investments and high environmental reclamation, i.e. policies which are often in antagonism with the aggressiveness of tourism agents.

Another property which is often mentioned in the context of sustainability is *adaptivity* (Holling, 1986; Walters, 1986; Holling, 1993). It corresponds to the possibility of changing policy parameters on the basis of perceived variations of the system parameters in such a way that sustainability can still be guaranteed. A typical question that agents must be able to answer is, for example, the following: if competition increases slowly but continuously, should the policy also be varied in order to avoid or, at least, delay negative consequences? A question like this can be qualitatively answered by looking once more at our sustainability diagrams. In the specific case one should look at Figs. 2, 3a and 3b which have on their vertical axis the system parameter “competition” and on the horizontal axis a policy parameter. Thus, if competition increases and the policy remains unchanged the consequences can be detected by looking at what happens when moving up vertically in each diagram. The three figures show that a safe sustainable policy will first become economically risky and finally non sustainable, so that the problem of avoiding these consequences through adaptation is a well posed problem. Fig. 2 indicates that nothing can be done to avoid or delay the time at which the policy becomes risky by varying the investment rate, because the upper boundary of the safe region (i.e., the transcritical bifurcation curve  $T$ ) is horizontal. The same happens in Fig. 3b, while Fig. 3a points out that the policy can adapt to increasing competition and remain safe, at least for a longer time, if prices are slowly decreased. Thus, in conclusion, if the aim is to avoid economically risky situations, price control seems to be the proper action to cope with increasing competition. By contrast, if risk is accepted and the target is to avoid or delay the time at which the policy becomes non sustainable, all three policy parameters are good control candidates for achieving the task, and the diagrams suggest to increase investment rate and environmental reclamation and decrease prices. In practice it might be rather hard to find the right mix of control actions and one can see from the diagrams that exaggerated reactions could actually generate an environmental crash instead than simply avoiding an economic crash. Moreover, when many system parameters vary at the same time it becomes more difficult to adapt the policy, because some of the sustainability diagrams may suggest conflicting actions. For example, if service appreciation also increases (as it does nowadays) Fig. 3c suggests to increase prices to maintain safe conditions, while Fig. 3a suggests just the opposite to cope with increasing competition.

## ***Discussion and conclusion***

The problem of tourism sustainability has been considered in this paper using a minimal model with three state variables: tourists, environment and services. In our opinion the

paper has three merits.

Firstly, it introduces a new approach, namely that of minimal descriptive models, in a context which has been traditionally dominated by the use of detailed simulation models. In other fields of sciences, like epidemiology, plant and animal ecology, renewable resources management, industrial economics and macroeconomics, this happened long ago and the approach is now appreciated and well established.

Secondly, the specific results are quite interesting. We have shown in fact that tourism sustainability can be obtained provided the agents are not too greedy and/or reluctant to protect the environment. We have also seen that sustainability is very often at risk, since accidental shocks can easily trigger a switch from a profitable and compatible behaviour to an unprofitable or incompatible one. Moreover, adaptivity of sustainable policies is also possible, but is very difficult to be realised in practice, and can at most delay the occurrence of a catastrophe but not avoid it if competition among touristic sites continues to grow. All these results are in line with common wisdom and observations, but the interesting fact is that they are here theoretically derived from a few very simple and abstract premises.

Finally, the third merit is not strictly related with the problem of tourism but with the general topic of sustainability. In fact, this is the first time that notions like profitability, compatibility and sustainability, which are more and more pervasive in the field of resources management, are interpreted in terms of structural properties of the attractors of a dynamical system. This creates an important and promising bridge between the notion of sustainability and bifurcation theory, one of the most important chapters of dynamical systems theory.

The weaknesses of the paper are the typical weaknesses of minimal models. The three compartments are too aggregated. For example, one feels the need for a more detailed description of the tourist population. Indeed, different tourists have different cultures and can presumably be described by the same submodel, namely eq. (1), but with different parameter values. Our analysis shows (see for example Figs. 3c and 3d) that the behaviour of the system can be radically different if tourists are different, at the point that in some cases tourism industry cannot survive. On the other hand, we know that in the real world a touristic site is rarely abandoned by tourists but is more likely visited by lower and lower classes of tourists. This fact could perhaps be studied through a slightly extended minimal model with two or three different classes of tourists acting as competing exploiters of the same resource, thus obeying the principle of competitive exclusion (Hardin, 1960). But even without modifying the present aggregation level, some of the assumptions encapsulated in the minimal model could be relaxed in order to study other cases of interest. For example, one could try to see if the introduction of suitable constraints on investment based on environmental quality and/or services has the power of amplifying the class of sustainable policies. Finally, one could also try to look at the consequences of competition from a different angle, by inserting a fourth compartment for a competitor. This should allow one to prove that tourists fluctuations in time are sometimes related to fluctuations between alternative sites. All these problems, as well as many others, deserve further attention.

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Figure 1: Bifurcation diagram of model (1)-(3) in the parameter space  $(\varepsilon, a)$ . Other parameter values are:  $r = K = \alpha = \beta = \gamma = \varphi_s = 1$ ,  $\delta = 0.1$ ,  $\varphi_E = 0.5$ ,  $\mu_E = \mu_S = 10$

Figure 2: Sustainability diagram of model (1)-(3) with respect to investment and competition. Parameter values are as in Fig. 1

Figure 3: Sustainability diagrams of model (1)-(3) with competition  $a$  and investment  $\varepsilon$  depending upon price  $p$  through the formulas  $a = \tilde{a} + 4p$ ,  $\varepsilon = 0.8p$ . In (a) the component  $\tilde{a}$  is on the vertical axis. In (b)  $\varepsilon = 0.6$ . In (c)  $\tilde{a} = 2$ . In (d)  $a = 6$ . All other parameters are as in Fig. 1