Endogeneous Allocation of the Exit Option between Workers and Shareholders[*]

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Abstract

We deal with efficient allocation of the shut-down decision of a firm in which there is profit sharing. The paper can be considered as a complement to the literature on the endogenous ownership structure of the firm. We examine a variety of cases according to both different schemes of layoff compensation and various degrees of specificity of human and physical capital. It appears that there are circumstances in which granting workers the decision to close can maximize the total payoff accruing to both contenders, with respect to the usual practice of shareholders decision making. Traditional conduct reveals an inefficency that may add to the well known principal-agent concern. Leaving the decision to close to shareholders gives rise to a deadweight loss, since a failure arises in the internal market for highly specific factors. Loss of control over the decision to exit is costly for shareholders. Proper compensation schemes can be devised for efficient transfer and/or sharing of the closing decision.

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1 Introduction

Labor participation in …rms' decisions takes di¤erent forms. It may be con…ned to pro…t-sharing or extended to key decision variables such as investment (Aoki, 1984), entry-exit and layo¤s (Lazear and Freeman, 1996).

Recent literature (Barrett and Pattanaik, 1989; Moretto and Rossini, 1995, 1997) has shown that the shape and extent of pro…t-sharing schemes are in‡uenced by the degree of ‡exibility of both factors of production and by institutional factors. By and large the responsibility and the …nancial burden of the closure is mostly born by the shareholders. In these cases, the opportunity to shut-down, when pro…tability goes below a predetermined threshold level, acts as a credible threat and a¤ects distribution of organizational rent among workers and shareholders.[1]

Casual observation suggests that it may be quite a narrow perspective to assume that the allocation of the decision to close a …rm is exogenously given and controlled only by the shareholders. As a matter of fact, there are other agents, either external or internal to the …rm, who have some in‡uence on it. This is the case of local authorities, trade unions, joint boards consisting of workers and …rm representatives. Even though all this is, by and large, more common for medium-large enterprises, in small size …rms it can often be observed that the executive board does not completely share the views of shareholders because workers may have a voice in it because of closeness between employees and owners. Something similar happens in all cases of codetermination, in some ESOP (Employee Stock Ownership Plan) cases in the United States and in most Japanese-like …rms where managers are mostly insiders who have gone up the whole career ladder within the same …rm and are therefore nearer to workers' views (Drucker, 1976; Aoki, 1984). In Germany codetermination dramatically a¤ects the behavior of many large …rms as far as hiring and …ring workers is concerned (Freeman and Lazear, 1994).

In all these cases workers are no longer passive vis à vis most of the …rm's crucial decisions which should then endogeneized instead of assumed the two polar cases represented respectively by the traditional one where shareholders alone decide on closure and the extreme opposite where workers take the decision and /or have a veto power, there may exist a whole range of allocations of the shut down decision. Then, theoretically, the problem may be seen as one of optimal allocation of the shut down decision.

Traditionally, when shareholders carry out their closure threat they inflict a loss to the workers who are laid off. Efficiency requires that the loss that workers (in game-theoretic terms we name them the victims) suffer be slightly smaller than the benefit (future expected losses) shareholders (the threateners) get from the closure. If the loss is strictly higher, inefficiency, measured by a deadweight loss, arises. If that is the case we face a new form of inefficiency in the theory of firm organization, which may be added to other cases such as those related to the principal-agent relationship.

In which circumstances does this happen? And, if so, is there any escape from it? We shall try to answer these questions, which are at the core of the paper, considering as a status quo the case where shareholders have the possibility of carrying out their closure threat at their earliest convenience.

By and large shutting down implies a private and a social cost and the coincidence of the two costs is ensured only in particular circumstances. The contender who is exogenously supposed to be the decision maker has an advantage since he can affect distribution of profits. The victim may be willing to pay an amount slightly lower than the loss he suffers in order to condition or "buy" the right to decide. This appears quite consistent with Lazear and Freeman (1996) who maintain that "worker ownership can increase worker support for efficient firm policies, even on such a potentially divisive topic as layoffs". Our concern is somewhat similar. If workers are allowed to decide when the firm should stop, are there circumstances in which they may do so efficiently? Or, are there cases in which the firm's policy can be agreed upon by workers as well? In this sense the paper may be considered a complement to the literature on the firm's endogenous ownership structure (Hart and Moore, 1990; Dow, 1988, 1993) since we examine the opportunity by varying the institutional setting and the degree of speci…city of human and physical capital. The context is one of timing decisions of an irreversible investment with stochastic payo¤s. Much of the vast recent research that has considered this issue was recently surveyed by Pindyck (1991), Dixit (1992) and Dixit and Pindyck (1994). We associate it with pro…t sharing during activity and compensation schemes for laid-o¤ workers.

So far, the traditional practice of letting the owners have the right to decide has never been evaluated in terms of maximization of the total payo¤ accruing to both contenders. Nor has been considered the closure threat as a shut down option.

In the …rst section we present the general model and we distinguish the case in which shareholders decide exit from the one in which workers take the hold. In section 3 we design a gain function for each contender. By comparing the two functions we get a net gain function. In section 4 we evaluate the net gain function in di¤erent scenarios and, accordingly, we trace the lines for a compensation scheme that may lead to e¢ciency, i.e. to maximization of total payo¤ for the two contenders (the Appendix proposes a continuous time repeated game representation of one of the many possible compensation schemes). We provide some conclusions in section 5. In

2 The basic model

In this section we lay out the general formulation of the model and analyse the contenders' objective functions. For simplicity we consider an incumbent …rm which exhibits a constant-returns-to-scale technology and is endowed with a capital stock of in…nite life. Each period the …rm produces one unit of output. Cost c; inclusive of labor payment, is known and constant. The labor force is, for the sake of simplicity, normalized to one.

Workers get a share of the pro…t or "organizational rent". The extent of pro…t-sharing may be considered either as the outcome of bargaining between workers and shareholders as envisaged by Aoki (1980, 1984), or, more generas renegotiation processes are highly costly, firms and workers cannot change the sharing rule continuously. Moreover, in the interest of both workers and shareholders, national legislation tends to limit the frequency in changes of sharing criteria (OECD, 1995).[3]

Revenue, coinciding with the market price, is uncertain and driven by a geometric Brownian motion:

$$dp_t = \alpha p_t dt + \sigma p_t dz_t \quad \text{with } p_{t_0} = p_0 \text{ and } \alpha, \sigma > 0; \tag{1}$$

where dz_t is the standard increment of a Wiener process (or Brownian motion), uncorrelated over time and satisfying the conditions that $E(dz_t) = 0$ and $E(dz_t^2) = dt$.

The operating profit (loss), as residual over the unit cost c, at time t is termed organizational rent and is a function of the market price:

$$\pi(p_t) = p_t - c \tag{2}$$

In keeping with the above arguments, this residual is distributed between shareholders and workers according to a constant (over time) sharing rule. Defining with $0 < \theta < 1$ the distributive parameter of profits going to shareholders, the premium earning per employee is simply:

$$\Delta w(p_t; \theta) = (1 - \theta)\pi(p_t) \equiv (1 - \theta)(p_t - c) \tag{3}$$

As the market price may go below c equation (3) becomes negative, and workers and shareholders partake both profits and losses in a symmetric way.[4]

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tional legislation. Profit-sharing is compulsory in Mexico and partially also in France. In Canada it is linked to accumulation of retirement funds (OECD, 1995, Vaughan-Whitehead, 1995, Biagioli, 1995).

[3] Moretto and Rossini (1997) analyse the case of a policy maker who uses profit distribution as a way of regulating the firm, calls the workers and the shareholders to renegotiate their distributive share through new bargaining if profits reach a predetermined level.

[4] A recent study undertaken by the OECD in major industrialized countries has shown that, among the firm characteristics relevant to the choice of a profit sharing scheme, a

When the …rm shuts down, shareholders bear a lump-sum cost $K_s = K_k + K_w$: The capital loss su¤ered by shareholders due to abandonment of the economic activity for which the …rm has idiosyncratic know-how is K_k. This amount is net of scrapping value, and/or shareholders' discounted value of future pro…ts associated with alternative asset investment from exit onwards. The legally required termination allowance the …rm has to pay to laid o¤ workers is represented by K_w. These two capital losses constitute the cost of closure born by shareholders. Also the workers face an exit cost K_{ll} which represents the loss due to their speci…c skill acquired in the …rm. This is the loss of speci…c human capital that cannot be used elsewhere without su¤ering an adjustment cost lower than K_{ll}. Also K_{ll} may be considered net of the workers' discounted value of future earnings associated with an alternative job.[5] Exit costs on both parts imply the existence of rents while the …rm is in operation. We may therefore equally think in terms of rent-sharing as well as pro…t-sharing.

As for both actors there is an opportunity cost of abandoning now rather than waiting for new information about market demand conditions, the …rm would rather decide to exit when these conditions become su¢ciently adverse, i.e. only if the price falls below a trigger value $b < c$ which has to be endogenously determined. To this purpose we have to consider future opportunities vis à vis the exit cost. The trigger price is going to change according to who holds the decision to close and may be strongly in‡uenced by possible asymmetry between shareholders and employees as to the exit costs $K_w + K_k$ and $K_w - K_{ll}$. This asymmetry is the major source of con‡icting interests as to the timing of closing and, as a consequence, as to who should hold the option to close. The way sunk costs are allocated is crucial as to both the threatening power of the closing decision and the private and social cost of ending the activity.

As long as shareholders are homogenous in all respects and the relative share μ remains constant over time, the shareholders' expected sum of discounted pro…ts up to the shut down is simply given by:

where $\rho(>\alpha)$ is the cost of capital. On the other hand, incumbent employees are interested in the lifetime amount of earnings they can get by taking part in the firm's production. Under the simplifying assumptions: (a) that workers are risk neutral, (b) that the market wage w is constant over time and (c) that workers are fired only when the firm closes, the level of lifetime income per worker can be represented as:[6]

$$L(p;\theta) = E_0\left\{\int_0^T [w + (1-\theta)(p_t - c)]e^{-\rho t}dt \mid p_0 = p\right\} + \tag{5}$$

$$+E_0\left\{(K_w - K_{ll})e^{-\rho T} \mid p_0 = p\right\}$$

Finally, in both equations $T(b) = \inf(t \geq 0 \mid p_t \leq b)$ indicates the (stochastic) stopping time at which the firm exits.

Some remarks on outside opportunities simplify matters. Since the workers' wage from alternative jobs can be expressed by $\frac{w}{\rho}$; and we can set $K_{ll} = K_l - \frac{w}{\rho}e^{-\rho T}$; the level of lifetime well-being of a worker may be ordered according to the expected discounted sum of the premium earnings up to the shut down. That is:

$$W(p;\theta) = E_0\left\{\int_0^T (1-\theta)(p_t - c)e^{-\rho t}dt \mid p_0 = p\right\} + \tag{6}$$

$$+E_0\left\{[(K_w - K_l)e^{-\rho T} \mid p_0 = p\right\}$$

where $W(p;\theta) \equiv L(p;\theta) - \frac{w}{\rho} > 0$; appears as a participation constraint.[7]

2.1 Exit in the shareholders' hands

Let us consider first the traditional case when the shut down option is owned by shareholders whose objective is maximization of the market value of the firm represented by (4). The stopping time can be specified as $T(b_s) = \inf(t \geq 0 \mid p_t \leq b_s)$ where b_s stands for the shareholders' trigger exit price.

As the opportunity to keep the firm in operation can be considered an asset that is held by shareholders for a series of small intervals of time dt, for a given value of θ; the value $S(p;\theta)$ must satisfy a non-arbitrage condition which requires that the sum of the returns on the investment, given by the dividend flow $\theta\pi(p)dt$ plus the capital gain $E(dS(p;\theta))$; equals the market return on capital investment $\rho S(p;\theta)dt$: Since p_t is driven by (1), applying Ito's Lemma to dS the asset equilibrium condition yields the following differential equation (Dixit and Pindyck, 1994, pag. 147-152):

$$\frac{1}{2}\sigma^2 p^2 S'' + \alpha p S' - \rho S = -\theta(p - c) \quad \text{for } p \in [b_s; \infty); \tag{7}$$

with boundary conditions:

$$\lim_{p \to \infty} [S(p;\theta) - \theta \left(\frac{p}{\rho - \alpha} - \frac{c}{\rho} \right)] = 0 \tag{8}$$

$$S(b_s;\theta) = -K_s \equiv -(K_k + K_w) \tag{9}$$

$$S'(b_s;\theta) = 0 \tag{10}$$

As usual, equation (8) states that, when the market price goes to infinity the value of the firm must be bounded. The second term in (8) represents the discounted present value of shareholders' profit flows over an infinite horizon starting from price level p (Harrison, 1985, p.44). Boundary conditions (9) and (10) stem from consideration of optimal operation. The first one, i.e. (9), says that when the price reaches the trigger level, at which it becomes convenient for the shareholders to exit (b_s), the value of the firm must be equal to its liabilities (matching value condition). The second condition, i.e. (10), rules out arbitrary or suboptimal exercise of the option to exit at a different point (smooth pasting condition).

Since, from (8), the term $A_1 p^{\beta_2}$ indicates the option value to abandon production, the constant A_1 must be positive. Yet, the constant A_1 as well as the trigger value $\hat{p}_s$ are determined by using the boundary conditions (9) and (10):

$$A_1(\hat{p}_s) \equiv \theta A(\hat{p}_s) = -\theta \frac{1}{\beta_2} \frac{1}{\rho - \alpha} \hat{p}_s^{1-\beta_2} > 0; \tag{12}$$

and

$$\hat{p}_s = \frac{\beta_2}{\beta_2 - 1} \frac{\rho - \alpha}{\rho} \left(c - \frac{\rho}{\theta} K_s\right). \tag{13}$$

To make the option to shut down viable for shareholders we add an assumption that guarantees that the exit price $\hat{p}_s \geq 0$:

Assumption 1. $c\theta - \rho K_s \equiv c\theta - \rho(K_w + K_k) \geq 0.$

Trivial comparative statics follow.

Proposition 1 : $\frac{d\hat{p}_s}{dK_s} < 0$; hence $\frac{dA_1}{dK_s} < 0.$

These results derive directly from (12) and (13) by inspection. K_s is the whole sunk cost born by shareholders; as it gets larger the firm stays longer in the market and the value of the option to exit decreases. Finally, substituting (12) and (13) into (11), we can write:

$$S(p;\theta) = \theta V_s(p;\theta) = \theta \left[A p^{\beta_2} + \left(\frac{p}{\rho - \alpha} - \frac{c}{\rho}\right) \right]. \tag{14}$$

where V_s stands for the firm's market value before distribution when the exit policy is in the hands of shareholders.

Let us now consider workers, who do not have any influence on the shut down option. We indicate the discounted sum of the premium earnings (6) with $\hat{W}(p;\theta)$ to mark with the $\hat{}$ that workers do not participate in the choice of the exit policy. The non arbitrage condition now requires that the

Since the exit policy is controlled by shareholders, boundary conditions reduce to (i.e. no smooth pasting condition holds):

$$\lim_{p\to\infty}[\hat{W}(p;\theta) - (1-\theta)\left(\frac{p}{\rho-\alpha} - \frac{c}{\rho}\right)] = 0 \tag{16}$$

$$\hat{W}(b_s;\theta) = K_w - K_l \tag{17}$$

By symmetry with (11), the solution has the form:

$$\hat{W}(p;\theta) = \hat{B}_1 p^{\beta_2} + (1-\theta)\left(\frac{p}{\rho-\alpha} - \frac{c}{\rho}\right) \quad \text{for } p \in [b_s;\infty): \tag{18}$$

The part of the workers' well-being attributable to the exit decision (possessed by shareholders), $\hat{B}_1 p^{\beta_2}$; depends now on the size and magnitude of the net bonus $K_w - K_l$: In fact, applying the value matching condition (17), we get:

$$\hat{B}_1(b_s) = -(1-\theta)\left(\frac{1}{\beta_2}\frac{1}{\rho-\alpha} b_s^{1-\beta_2}\right)\left(\frac{c\theta - \beta_2\rho K_s}{c\theta - \rho K_s}\right) + (K_w - K_l) b_s^{-\beta_2} \tag{19}$$

Considering now the firm's market value before distribution V_s and taking account of (11), (17), (18) and (19), we compute:

$$\hat{W}(p;\theta) = (1-\theta)V_s(p;\theta) + D_1(p) + D_2(p) \tag{20}$$

where:

$$D_1(p) = \frac{1}{\theta}(K_k + K_w)\left(\frac{p}{b_s}\right)^{\beta_2} > 0; \qquad D_2(p) = -(K_k + K_l)\left(\frac{p}{b_s}\right)^{\beta_2} < 0$$

Although $D_1 + D_2 > 0$ is always positive whenever $K_w - K_l > 0$; negative effects cannot be a priori excluded.: Therefore, $D_1 + D_2$ provides, in equation (20), a measure of workers' well-being induced by the asymmetry between the exit costs born respectively by shareholders and employees, i.e. $K_w + K_k$

2.2 Exit in workers' hands

Let us now consider the case where the shut down option is owned by workers, whose objective is maximization of their lifetime well-being (6). The exit time becomes now $T(b_w) = \inf(t \geq 0 \mid p_t \leq b_w)$; where b_w is the workers' exit price. By using a procedure similar to the one adopted above, the lifetime well-being W is the solution of the following differential equation:

$$\frac{1}{2}\sigma^2 p^2 W'' + \alpha p W' - \rho W = -(1-\theta)(p-c) \quad \text{for } p \in [b_w; \infty); \qquad (21)$$

with limit conditions:

$$\lim_{p \to \infty}\left[W(p;\theta) - (1-\theta)\left(\frac{p}{\rho-\alpha} - \frac{c}{\rho}\right)\right] = 0 \qquad (22)$$

$$W(b_w;\theta) = K_w - K_l \qquad (23)$$

$$W'(b_w;\theta) = 0 \qquad (24)$$

Letting B_1 be a constant to be determined and β_2 the negative root of the characteristic equation $\Phi(\beta)$; the solution of (21) is given by:

$$W(p;\theta) = B_1 p^{\beta_2} + (1-\theta)\left(\frac{p}{\rho-\alpha} - \frac{c}{\rho}\right) \quad \text{with } p \in [b_w; \infty) \qquad (25)$$

Applying the value matching condition (23) and the smooth pasting condition (24), we obtain:

$$B_1(b_w) \equiv (1-\theta)B(b_w) = -(1-\theta)\frac{1}{\beta_2}\frac{1}{\rho-\alpha}b_w^{1-\beta_2} > 0 \qquad (26)$$

and

$$b_w = \frac{\beta_2}{\beta_2-1}\frac{\rho-\alpha}{\rho}\left(c + \frac{\rho}{1-\theta}(K_w - K_l)\right) \qquad (27)$$

Proposition 2 If $K_w - K_l > 0$, then $\frac{db_w}{d(K_w - K_l)} > 0$ and hence $\frac{dB_1}{d(K_w - K_l)} > 0$.

If the (positive) net bonus increases, workers would like to exit earlier. The increase in the net transfer boosts the value of the exit option to workers. The opposite happens if $K_w - K_l < 0$. Substituting (26) and (27) into (25), we write:

$$W(p;\theta) = (1-\theta)V_w(p;\theta) = (1-\theta)\left[Bp^{\beta_2} + \left(\frac{p}{\rho-\alpha} - \frac{c}{\rho}\right)\right]; \qquad (28)$$

where V_w indicates the firm's market value before profit distribution and exit policy owned by workers.

Let us now turn back to shareholders. Referring to (4) which we indicate with $\hat{S}(p;\theta)$ since shareholders do not decide the exit policy, the non arbitrage condition leads to:

$$\frac{1}{2}\sigma^2 p^2 \hat{S}'' + \alpha p \hat{S}' - \rho \hat{S} = -\theta(p-c) \quad \text{for } p \in [b_w, \infty); \qquad (29)$$

Shareholders' optimization problem is similar to the one seen in the previous section. The boundaries are

$$\lim_{p \to \infty}\left[\hat{S}(p;\theta) - \theta\left(\frac{p}{\rho-\alpha} - \frac{c}{\rho}\right)\right] = 0; \qquad (30)$$

$$\hat{S}(b_w;\theta) = -K_s \equiv -(K_k + K_w). \qquad (31)$$

The solution is:

$$\hat{S}(p;\theta) = \hat{A}_1 p^{\beta_2} + \theta\left(\frac{p}{\rho-\alpha} - \frac{c}{\rho}\right) \quad \text{for } p \in [b_w, \infty); \qquad (32)$$

while applying the value matching condition (31) we get:

where:

$$D_3(p) = -\frac{\theta}{1-\theta}(K_w - K_l)(\frac{p}{b_w})^{\beta_2} ?; \quad D_4(p) = -(K_k + K_w)(\frac{p}{b_w})^{\beta_2} < 0$$

Again, although the asymmetry due to the respective exit costs and a reversed allocation of the shut down option, may induce a reduction in the flow of profits accruing to shareholders, i.e. $K_w - K_l > 0$ assures that $D_3 + D_4$ is always negative, positive effects cannot be a priori excluded.. Then we may indicate $D_3 + D_4$ as the shareholders' loss (gain) induced by the exit cost asymmetry.

From (14), (20), (28) and (34), it is apparent that the threatening power has a distributive effect. Carrying out the threat (i.e. closing) by one of the contenders is privately optimal, but we do not know whether and in which circumstances the individually optimal program leads to maximization of the two contenders aggregate payoff. The next section will be devoted to analysis of the efficiency of workers' and shareholders' choices by using a gain function.

3 The (net) gain function

Whenever we attribute the decision to close to one contender we actually give him the power to carry out a threat (Moretto and Rossini, 1995). Carrying out the threat is privately optimal. For instance, shareholders receive a net benefit with respect to the alternative case in which workers decide to close. Workers lose when shareholders decide. They are laid off without having the possibility to halt the shareholders's decision.[9] As Klein and O'Flaherty (1993) and Shavell and Spier (1996) have pointed out, this situation may give rise to inefficiency if the benefit shareholders get when they exercise their threat (close) is strictly smaller, in absolute value, than the loss workers bear. Then, it is in the interest of both contenders to ".....negotiate around the inefficiency and avoid the dead weight loss associated with carrying out conditions will allow us to figure out some institutional scenarios providing suggestions to solve real questions of the firm's internal organization. For this purpose we resort to individual gain functions and then to a net (aggregate) gain function. The individual gain function indicates how much a contender holding (not holding) the decision to close benefits (losses) with respect to the case in which he does not (he does) control the decision. The net gain function aggregates algebraically the two individual gain functions, providing information as to the desirability of attributing the decision to one of the two contenders. If the benefit from closing is smaller than the loss inflicted to the victim we face a deadweight loss and total payoff is not maximized.

Taking for granted that the decision to close should be attributed to the shareholders (our benchmark position) is equivalent to assuming that this deadweight loss never shows up. In this section we shall see that this is not always the case and that, because of the nonlinerarity of the abandonment option coupled with the asymmetry of the exit costs, the usual conduct rule of a firm owned and directed by shareholders may lead to inefficiency.

Let us first consider the gain function of shareholders defined as the difference $S - \hat{S}$. Substituting (11) and (32) we obtain:

$$S(p;\theta) - \hat{S}(p;\theta) = (A_1 - \hat{A}_1)p^{\beta_2} > 0; \qquad (35)$$

By the same arguments, defining the gain function of workers as $W - \hat{W}$, substituting (18) and (25) we get:

$$W(p;\theta) - \hat{W}(p;\theta) = (B_1 - \hat{B}_1)p^{\beta_2} > 0 \qquad (36)$$

As long as the discounted value of expected profit, when the firm is active forever, is independent of who has the right to decide the exit the difference concerns only the part of the value of the firm coming from the shut down option, that is: $A_1 - \hat{A}_1$ and $B_1 - \hat{B}_1$. Moreover, as the optimal policy is to exit at b_s for shareholders and at b_w for workers, by Bellman's principle we realize that $A_1 - \hat{A}_1 > 0$ and $B_1 - \hat{B}_1 > 0$. Therefore, for any given price exit. Traditional organization is efficient and no deadweight arises. If $G < 0$ the reverse applies and the workers' loss is larger in absolute terms than the shareholders' gain. Maximization of the total payoff implies attributing the closing decision to workers, for instance, by giving them veto power. Otherwise the deadweight loss appears equal to G: If $G = 0$ it is irrelevant who decides to exit. Contenders agree on the exit decision.

Recalling from (12) and (33) (or equivalently (14) and (34)) that constants A_1 and $\hat{A}_1$ are functions of the trigger prices b_s and b_w, their difference can be simplified as:

$$A_1 - \hat{A}_1 = A_1(b_s) - A_1(b_w) + \left[\frac{\theta}{1-\theta}(K_w - K_l) + (K_k + K_w)\right] b_w^{-\beta_2} \quad (38)$$

The gain from holding the exit option for the shareholders can be split into two parts. The first captures the difference (positive or negative) in the value of the shareholders' shut down option evaluated at two distinct trigger prices, one chosen optimally by shareholders and the other chosen by workers $A_1(b_s) - A_1(b_w)$: The second part gives the gain or loss accruing to shareholders due to the asymmetry in sunk exit costs.

Symmetrically, for the workers from (19) and (26) (or equivalently (20) and (28)) we can write:

$$B_1 - \hat{B}_1 = B_1(b_w) - B_1(b_s) + \left[-\frac{1-\theta}{\theta}(K_k + K_w) - (K_w - K_l)\right] b_s^{-\beta_2} \quad (39)$$

$B_1(b_w) - B_1(b_s)$ is the gain or loss in exit option evaluated in correspondence of the two trigger prices when the decision to exit is granted to workers. The second part accounts for the exit costs asymmetry between the two contenders.

Since $A_1(b_s) + B_1(b_s) = -\frac{1}{2}\frac{1}{\rho - \alpha} b_s^{1-\beta_2} \equiv A(b_s)$ and $A_1(b_w) + B_1(b_w) =$

4 Optimal allocation of the exit decision

We now see how the net gain function (40) varies as we change both the institutional setting and the degree of factor speci…city, captured respectively by the parameter μ and the exit costs K_w, K_k and K_l: The question we face may be considered a complement to the literature on the …rm's optimal ownership structure (Hart and Moore, 1990; Dow, 1986, 1993) in environments with di¤erent degrees of factor speci…city and di¤erent compensation schemes for layo¤s.

To reduce the complexity of the net gain function (40) we consider 3 "main" cases which may be taken as representative of the totality of factor speci…city within the …rm. In the …rst capital and labor have the same degree of speci…city. In the second capital is speci…c while labor can be deployed elsewhere without costs. The third is the opposite of the second. Numerical simulations have been undertaken in these cases.

4.1 Equal speci…city between capital and labor

Capital and labor can be deployed elsewhere by bearing a sunk cost equal for both of them. Laid-o¤ labor receives a positive compensation. That is:

Assumption 3. $K_k = K_l > 0$ and $K_w > 0$:

Before analyzing the net gain function G, we show that under assumptions 1, 2 and 3 the feasible set where the exit option is viable for both actors can be split into two subsets according to whether the shareholders' trigger value is higher or lower than the workers' trigger value. Considering the ratio of the two trigger prices, from (13) and (27) we get:

$$\frac{b_w}{b_s} = \frac{c + \frac{\rho}{1-\mu}(K_w - K_l)}{c - \frac{\rho}{\mu}(K_w + K_k)}: \qquad (41)$$

If $K_w > K_l$ workers receive a positive net exit bonus and wish to exit

FIGURE 1 ABOUT HERE

Parallelogram ABCE indicates the area where $b_w > b_s$; triangle CDE where $b_w < b_s$, while along the line CE we have $b_s = b_w$.

The net gain function (40), under assumption 3 and normalization $K_l = K_k = 1$; becomes:

$$G(\theta; 1; K_w) = A(b_s) - B(b_w) + \left[\frac{1-2\theta}{1-\theta} + \frac{1}{1-\theta}K_w\right] b_w^{-\beta_2} + \left[\frac{1-2\theta}{\theta} + \frac{1}{\theta}K_w\right] b_s^{-\beta_2} \qquad (42)$$

In Figure 1 we have also traced the net gain function $G(\theta; 1; K_w) = 0$ using dashed line GG. The values of the relevant parameters follow Dixit (1989): the operating cost of the productive activity amounts to 30% of the capital exit cost (i.e. $c = 0.3K_k(K_l)$); the discount rate is $\rho = 0.1$. To avoid paradoxical results due to presence of inflation in the price process but not on the side of costs, we set $\alpha = 0$. The price's instantaneous volatility is $\sigma = 0.25$ (for instance 25% per year) so that the price elasticity of the shut down option becomes $\beta_2 \simeq -3$.

Above and on the right of GG the gain shareholders obtain by deciding to exit is larger than what workers would get in the same role, i.e. $G > 0$. Below and on the left of GG it is just the opposite, i.e. $G < 0$. Noting that $G(0.5; 1; 0) = 0$; from Figure 1 we can distinguish three areas within the feasible set.

The first one, indicated by the parallelogram ABCE; where $G > 0$. If shareholders own the exit decision (benchmark case) they have a greater advantage than what workers would have in the same role. The time horizon of shareholders is longer than that of workers (and consequently also the life of the firm), i.e. $b_w > b_s$.

payoff maximization. However, as the workers' net bonus is negative the firm's expected life is shorter than what it would be under workers' control.

These results and in particular the one highlighted by the second area that constitute the core of the paper can be summarized in the following proposition:

Proposition 3 Under assumption that both physical and human capital have the same degree of specificity ($K_k = K_l$) we find that: There are combinations of profit distributions and exit costs where the traditional conduct of shareholders deciding to exit does not maximize the total payoff. In this case laid-off workers receive an exit compensation lower than their exit sunk cost ($K_w < K_l$); which induce them to desire a longer life for the firm to recover losses. It would be better for the players to negotiate and avoid the deadweight loss associated with carrying out the threat of closing too soon.

4.2 Capital non specific, labor specific

In this case there is a strong asymmetry in favor of shareholders. Capital is reversible while labor can be deployed elsewhere by bearing a sunk cost. Labor receives a positive compensation when the firm stops production.

Assumption 4. $K_k = 0$; $K_l > 0$ and $K_w > 0$:

Under assumptions 1, 2 and 4, the feasible set where the exit option is viable for both actors can be split into two subsets according to whether the shareholders' trigger value is higher or lower than the workers' trigger value. From (41), when $K_w > K_l$ workers wish to exit earlier than shareholders for the entire set of parameters. If $K_w < K_l$ workers would like to exit earlier only if $\frac{K_w}{K_l} > \mu$:

Normalizing $K_l = 1$; the feasible set for the trigger prices can be represented by the area ABCD in Figure 2 wherein both trigger prices b_s and b_w

$$+[\frac{1}{\mu}K_w + 1]b_s^{i\,-\,2}$$

Noting that $G(0; 1; 0) = 0$ and that $G > 0$ to the right of the line GG in Figure 2, we can distinguish two areas. The first one is on the left of GG and below AB; where $G < 0$: If workers decide to exit they gain more than what shareholders lose. Unlike the previous case, here $b_w > b_s$: Workers receive a negative net bonus ($K_w < K_l$), but the low level of the share parameter μ speaks in favor of shareholders, who have a larger (per unit of time) exit cost $\frac{\rho}{\mu}K_w$ (even if $K_k = 0$) than their contenders. Then they would let the firm live longer than workers (see equation (41)). The second area is indicated by the remainder of the parameter set on the right of GG; where shareholders show a higher gain: $G > 0$, inside the triangle ACD: The owners do not let the firm live longer since $b_w < b_s$: Aggregate payoff maximization does not go hand in hand with the longest possible life expectancy of the firm.

FIGURE 2 ABOUT HERE

We then summarize:

Proposition 4 Under the assumptions that labor is specific while capital is not, we find that: There is a small area at low levels of μ and K_w where efficiency would require letting workers take over the exit policy. Laid-off workers receive a compensation lower than their exit cost ($K_w < K_l$), while, if they stay active they receive a high profit share $(1 - \mu)$ which reduces substantially their per unit of time exit cost $\frac{\rho}{1-\mu}(K_w - K_l)$: They, then, exit earlier than shareholders. A similar short sightedness is shown by shareholders in a near area where $G > 0$. In both cases maximization of the total payoff does not coincide with the firm's longest life expectancy.

4.3 Capital specific labor non-specific

Assumption 5. $K_k > 0; K_l = 0$ and $K_w > 0$:

From (41) we see that the level of the trigger set by workers is larger than the one set by shareholders, i.e. $\frac{b_w}{b_s} > 1$: Then, under assumptions 1, 2 and 5, the feasible set where the exit option is viable for both actors is given by the triangle ABC in Figure 3. Setting $K_k = 1$, the net gain function (40) becomes:

$$G(\mu; 1; K_w) = A(b_s) - B(b_w) + [1 + \frac{1}{1 - \mu} K_w] b_w^{-\beta_2} + \qquad (44)$$
$$+ [\frac{1 - \mu}{\mu} + \frac{1}{\mu} K_w] b_s^{-\beta_2}$$

In this case we just get one area, the triangle ABC where $G > 0$: If shareholders decide to exit they do so efficiently and this is consistent with a longer firm's expected life, since $b_w > b_s$. This is the traditional case analyzed by Moretto and Rossini (1995).

FIGURE 3 ABOUT HERE

Then we write:

Proposition 5 When labor is not firm specific and the cost of exit is born entirely by shareholders, efficiency requires that shareholders control exit. Yet, life expectancy of the firm is longer since $b_w > b_s$:

5 Towards an agreed upon efficient exit policy

the joint organizational rent. We have seen in the previous section that the …rm may stay on the e¢cient frontier if ownership by shareholders is in some circumstances delinked from control over the …rm's life expectancy.

Even if we take for granted the traditional institutional environment with closure in the hands of owners, there seems to be a possibility for the shareholders themselves to be better o¤ if they switch to a di¤erent allocation of the decision to exit. Take for instance the case seen in Proposition 3 where $b_w < b_s$. If shareholders controlled exit they would stop production earlier than workers. In this case the loss of workers is higher (in absolute value) than the shareholders's gain. It may be in the interest of workers to make an o¤er to shareholders to let the …rm live longer by increasing the well being of workers and shareholders at the …rm. But, in the absence of a binding commitment any lump sum transfer will be ine¢cient (Klein and O'Flaherty, 1993 and Shavell and Spier, 1996). The victims, workers, know that the threateners, shareholders, have an incentive to carry out the threat, stopping the activity, the …rst time the price hits the lower barrier b_s. This is intuitively apparent since closing the …rm is equivalent to putting an end to the game played by shareholders and workers. Shareholders have the power to set the time horizon of the game, while workers do not share this power. If the payment by workers is made the …rst time p_t hits b_s immediately after shareholders would strictly prefer to carry out the threat regardless of the (previous) payment made by workers, unless $p_t > b_s$. On the contrary, if the parties agree that the payment is due just when p_t hits b_w then workers won't make any payment. By backward induction the same outcome appears for any scheme with a …nite number of compensations. Workers do not have any incentive to pay in order to delay closure. Threateners do not expect to receive any compensation, and then it is optimal for them to carry out the threat the …rst time the price hits b_s: In game-theoretic terms this means that the unique subgame-perfect equilibrium is ine¢cient since the threat is carried out despite the fact that the victim's loss may or may not be greater than the threatener's payo¤.[11]

for workers to "subsidize continuously the shareholders" till the price hits the workers' optimal lower barrier b_w, and then exit. In terms of operating profits (losses), as long as the market price p_t stays above b_s nothing is done. If ever p_t goes below b_s workers should give up part of their organizational rent in favor of shareholders: workers pay a compensation $s_t \geq 0$ to keep shareholders indifferent with respect to exit. This compensation should go on, until the (exogenous) price process p_t hits, for the first time, the workers' lower barrier b_w. When b_w is touched the compensation stops and both contenders should be at their indifference position, i.e. $S(b_s, \theta) = -K_s$ and $W(b_w, \theta) = K_w - K_l$. Compensation is triggered at T_s and stops at T_w. closure maximizes the total payoff of the two contenders.[12]

This is the scheme proposed, in a discrete-time and constant-payoffs game, by Shavell and Spier (1996). It is quite easy to show the efficiency of their scheme if the threateners use a simple trigger strategy with maximal punishments: Constant payment streams over time a are made by the victim and threats are never carried out in equilibrium (proposition 2, p.12).[13]

Although referring to the Shavell and Spier's scheme can be appealing, the framework where here shareholders and workers make their decisions is quite different. First, decisions must be taken in continuous time. This requires a refinement of the notion of trigger strategy to deal with a continuous time repeated game.[14] Continuous time can be seen as discrete-time with a length

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[12] It may be easy to design a compensation when $b_w > b_s$ (see proposition 4). Workers would exit at b_w. However, the decision is in the hands of shareholders and they must wait till the price falls to b_s. In this case the compensation is simply a lump sum transfer to keep the shareholders indifferent at b_w, i.e. $S(b_w, \theta) + s = S(b_s, \theta)$.

[13] The simple trigger strategy with maximal punishment used by Shavell and Spier (1996) requires that if the victim deviates by paying less than the equilibrium subsidy, the threathner belives that the victim will pay nothing in the future, and the victim belives that the threathner will carry out the treat at his first opportunity. After a single deviation, the continuation equilibrium becomes the inefficient stationary equilibrium.

In Shavell and Spier, the efficient subgame-perfect equilibrium is characterized by a constant per-period payment within a range which depends on the magnitude of the discount factor. When the per-period payment is at its lower bound, the threatener is as well off as of reaction (or information lag) that becomes infinitely negligible to allow the threateners to respond immediately to the workers' actions.[15]

Second, the compensation takes place within a finite (yet stochastic) time-span, $(T_w - T_s)$, but in continuous time. Therefore a participation constraint by workers must be determined: Their payoff must be larger with the compensation scheme than if they exit at b_s:

Third, by the stochastic nature of the game, neither player is able to perfectly predict the value of the state variable p_t at each date: Therefore the payment made by workers is contingent upon realizations of p_t and may depend on the price history.[16] Yet, since the market price follows a random walk there is, for each time interval of small length dt; a constant probability that the price will move up or down, i.e. that the game will continue one more period.[17] That is, the game ends in finite (stochastic) time with prob-

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[15] We may refer either to Simon and Stinchcombe (1989) or to Bergin and MacLeod (1993) contributions. In the former a class of continuous strategies is defined so that any increasingly narrow sequence of discrete-time grids generates a convergent sequence of game's outcomes whose limit is independent of the sequence of the grid. In the latter a class of inertia strategies represent a delay in response: that is an action an agent chooses at time t must also be chosen by the agent for some small period of time after t; with this small period of time tending to zero.

[16] The idea behind a stochastic game is that the history of the game at each period can be summarized by a state variabale (price) that follows a Markov process, and current payoff depends on current action (subsidy). A perfect equilibrium of a stochastic game allows the strategies of each player to be functions of the (entire) history of the game while a Markov perfect equilibrium concerns a smaller class of strategies where the past influences current actions only through the current realization of the state variable.(Fudenberg and Tirole, 1991, p. 503-504).

[17] Applying Itô's lemma to $P_t = \ln p_t$; where dp_t is defined by (1) we get that, for initial condition $P_{t_0} = \ln p_{t_0}$ and for any time $t > 0$, P_t is normally distributed with mean $(P_{t_0} + \mu t)$; where $\mu = \alpha - \frac{1}{2}\sigma^2$; and variance $\sigma^2 t$: Yet, dP_t is derived as the continuous limit of a discrete-time random walk, where in each small time interval of length Δt the variable P either moves up or down by Δh with probabilties (Cox and Miller, 1965, pp. 205-206):

A p__ ! A p__ !

ability one, but everything is as if the horizon were infinite.[18] Workers find convenient at each instant t to continue the game paying a compensation to shareholders until the participation constraint bites. This may happen before T_w is touched. Workers will then stop the payment and let the shareholders close the firm.

Several compensation mechanisms can be devised, once the participation constraint is satisfied, where the initial holder of the decision is able to obtain a rent as a result of a lack of a market allocation of the shut-down decision. In the Appendix a compensation scheme is devised, applying a one-sided regulator $s_t \geq 0$ to the state variable p_t: Referring to the formalization of regulated stochastic processes (Brownian Motion) by Harrison and Taksar (1983), and Harrison (1985), the subsidized price $\tilde{p}_t \equiv p_t + s_t$ is obtained from p_t by imposition of a lower control barrier at b_s; with the compensation defined as:

$$1 + s_t \equiv \max_{T_s \leq v \leq t} \left(\frac{b_s}{p_v} \right)$$

The subsidy s_s increases faster enough to keep $\tilde{p}_t$ always greater than b_s; and indicates the cumulative amount of control (subsidy) exerted on the sample path of p_t up to time t: That is, the subsidized price $\tilde{p}_t$ equals the amount by which p_t exceeds the minimum value of p_t over the interval $[T_s; t]$ (see fig.4): The Appendix also shows the efficiency and the subgame-perfectness

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$$\Pr[\Delta p = (e^{-\Delta h} - 1)p] = \frac{1}{2}\left(1 - \frac{\mu \Delta h}{\sigma^2}\right)$$

That is, for small Δt; Δh is of order of magnitude $a(\sqrt{\Delta t})$ and both probabilities become $\frac{1}{2} + a(\sqrt{\Delta t})$; that is not too different from $\frac{1}{2}$: The probability distribution of future values depends only on where the process is now, and the probability that it will move up or down in each period is independent of what happened in previous periods (Markov property).

[18] It is well known that infinitely repeated games are equivalent to repeated games that terminate in finite time. At each period there is a constant probability that the game continues (Fudenberg and Tirole, 1991, p.148). Consider again the price process $P_t = \ln p_t$;

of the equilibrium where the magnitude of the subsidy is related to the history of the game and to past price realizations.

All these schemes di¤er by the extent of the rent shareholders are able to reap. The rent is due to the commitment of employees to pay a subsidy to shareholders. The closing decision has a cost for the contender not controlling it who may be willing to "buy" the decision. The possessor of the decision to close receives a rent.

These compensation devices may become important since, as we pointed out in the introduction, there are many cases in which workers have directly or indirectly a voice in the closing decision. The extreme situation may be that in which workers have some veto power on the shareholders' closing decision. This leads just to the opposite of what was seen above. If workers would like to exit earlier they may do so by quitting. In that case it could be in the interest of shareholders to make an o¤er to workers to let the …rm live longer by avoiding massive quitting. There results an increase in the well being of workers and shareholders (for example in the …rst area with equal speci…city between the two factors or proposition 5). In a world in which workers share pro…ts and losses the …rm may be faced with massive quitting. Workers may be halted by increasing K_w (since this is received if the …rm closes but not if they voluntarily quit). Lowering K_l through retraining that makes employment elsewhere less costly is not in the interest of the …rm. In other words, as Lazear and Freeman (1996) have observed, it is possible to design ‡exible organizations with the consent of workers even when they have some conditioning power.

However these compensation schemes are far from being easily implemented since, whenever we change μ and/or K_w also trigger prices change and the optimal plans are revised by both contenders.

We must actually say that once the allocation of the exit decision is in the hands of one of the two contenders it may be quite di¢cult to avoid falling into ine¢cient practices unless we are able to …nd the proper compensation mechanism to shift responsibility to the other contender who is contingently on endogenous …rm’s ownership structure, in this case employees are better o¤ by self-management (Dow, 1993). If self-management seems too a radical solution, letting closure be decided jointly may be a clear improvement.

6 Conclusions

We have tried to answer a question concerning the optimal organization of the …rm as far as closing is concerned. The issue is complementary to the literature on the optimal …rm’s ownership structure. We have con…ned our attention to the cases in which employees get a …xed share of the organizational rent or pro…t. Closing down a¤ects the welfare of workers and shareholders in di¤erent ways. The control over the closing decision has different opportunity costs according to who holds it. Respective sunk costs of shareholders and employees, i.e. the relative degree of factor speci…city, and the institutional environment represented by severance rules are crucial. Outside options make each agent prefer to stay rather than leave before or after the other contender. The decision to close plays the role of a threat that may in‡uence the pro…t share distribution within the …rm, as shown by Moretto and Rossini (1995).

Taking for granted that owners are the sole subject entitled with the closing decision implies assuming away an ine¢ciency in …rm organization that may be quite large and comparable to that of principal-agent kind. To show that we set up a taxonomy in terms of the maximization of the total payo¤ of both contenders as to the decision to close. We …nd that if we were to design an optimal distribution of the decision to close, in some cases we would have to take it away from shareholders, as, for instance, when the degree of speci…city of human capital is fairly high vis à vis physical capital. Otherwise an ine¢ciency arises.

We take for granted that the most common institutional setting associates property with the right to close. Several compensation schemes can be designed as a way out of the ine¢ciency that this may give rise in par-

There are no handy escape routes from ine¢cient allocation of the exit decision once it has been taken for granted. In some cases the given allocation of the decision to close gives rise to a deadweight loss. Avoiding this makes the holder of the decision able to reap a rent out of most compensation schemes. In this sense history matters. Present institutional settings are justi…ed because most …rms start their activity with highly speci…c physical capital and non speci…c human capital. Human capital becomes speci…c afterwards while physical capital is speci…c from the beginning.

An open strand of research comes from the fact that the credibility of the shut down threat should not be taken for granted because it implies that the …ght for ownership of the decision to close has already taken place while it may, in some context, be an open struggle that may take place simultaneously with determination of the share parameter.

A Appendix

This appendix proposes a compensation mechanism by which workers subsidize shareholders to avoid them closing the firm, when closure implies a loss to workers larger (in absolute value) than the gain shareholders get. This is the case expressed in proposition 3, when $\underline{b}_w < \underline{b}_s$:

The compensation scheme corresponds a perfect equilibrium belonging to the class of efficient perfect equilibria (which may be very large) for the continuous time threat-game described in the text. The compensation mechanism fills workers' desire of capturing part of the deadweight loss by keeping shareholders at their indifferent position $S(\underline{b}_s;\theta) = -(K_k + K_w)$; until the price hits the workers' optimal lower barrier $\underline{b}_w$:

To avoid complications we see the continuous-time representation as discrete-time with a reaction (or information) lag that becomes infinitely negligible. That is, the treatheners are able to respond immediately to the workers' actions.[19] Finally, we begin assuming that the participation constraint is never binding.

Let us start describing the compensation mechanism.

As long as the market price p_t stays above $\underline{b}_s$ nothing is done. If ever p_t goes below $\underline{b}_s$ workers give up part of their organizational rent in favor of shareholders. The transfer is the negative increment dp_t that allows the price process p_t to stay at $\underline{b}_s$: Formally, we define[20] a regulated process $\bar{p}_t$;as:

$$\bar{p}_t \equiv p_t Z_t; \quad \text{for } \bar{p}_t \in [\underline{b}_s; \infty) \tag{45}$$

or in terms of compensation:

$$s_t(p_t; Z_t) = \bar{p}_t - p_t \equiv (Z_t - 1)p_t$$

where:

- i) Z_t is an increasing and continuous process, with $Z_0 = 1$;

Moreover, workers pay the compensation $s_t \geq 0$; defined by (45), to keep shareholders indifferent with respect to exit, every time b_s is hit. In other words, as long as the process p_t stays above the barrier b_s shareholders do not want to exit, i.e. $\tilde{p}_t \equiv p_t$ with initial condition $\tilde{p}_0 \equiv p_0 = p$; and $Z_t = 1$: The first time the barrier is touched, i.e. at T_s; workers start compensating shareholders to keep $\tilde{p}_t = b_s$: The regulation goes on, until the (exogenous) price p_t hits for the first time the workers' lower barrier b_w, i.e. at $T_w \equiv T(b_w) = \inf(t \geq 0 \mid p_t < b_w)$. When this happens $\tilde{p}_t = b_s$; compensation is equals to b_s/b_w and both contenders must be at their indifference position, i.e. $S(b_s; \theta) = -K_s$ and $W(b_w; \theta) = K_w - K_l$: The trigger price b_s plays the role of a reflecting barrier for $\tilde{p}_t$, while b_w is an absorbing barrier for the primitive process p_t:.[21]

A crucial feature of this setup is that as the regulator Z_t depends only on the primitive exogenous process p_t also for the endogenous regulated process $\tilde{p}_t$ the strong Markov property hold (Harrison, 1985, proposition 7, pp.80-81).

The shareholders' expected sum of discounted profits with compensation paid by workers, up to the shut down, becomes (Harrison, 1985, p. 84; Moretto, 1995, p. 100):

$$S^c(p; \tilde{p}; \theta) = E_0 \left\{ \int_0^{T_w} \theta(\tilde{p}_t - c) e^{-\rho t} dt \right\} - E_0 \left[K_s e^{-\rho T_w} \right] \qquad (46)$$

[21] By Itô's Lemma applied to (45) we get:

$$d\tilde{p}_t = \alpha \tilde{p}_t dt + \sigma \tilde{p}_t dz_t + \frac{\tilde{p}_t}{Z_t} dZ_t \quad \text{with } \tilde{p}_0 = p_0 = p$$

The process Z_t describes the price compensation. Then $\frac{\tilde{p}_t}{Z_t} dZ_t$ is the infinitesimally small level of organizational rent given up by workers to let the current profit distributed to shareholders to stay above the level $\theta(b_s - c)$: In fact, by (45), if $\tilde{p}_t = b_s$; we get $d\tilde{p}_t = 0$ and the rate of variation of Z_t is equal to that of p_t to keep $\tilde{p}_t$ constant.

or:[22]

$$S^c(p,\bar{p};\theta) = g(\bar{p};\theta) - g(\bar{p}_{T_w}, Z_{T_w};\theta) - E_0\left\{K_s e^{-\rho T_w}\right\} =$$

$$= E_0\left\{\int_0^{\infty}\theta(\bar{p}_t - c)e^{-\rho t}dt\right\} -$$

$$- E_0\left\{\int_{T_w}^{\infty}\theta(\bar{p}_t - c)e^{-\rho t}dt\right\} - E_0\left\{K_s e^{-\rho T_w}\right\} \qquad (48)$$

where, the stopping time can now be specified as $T_w = \inf(t \geq 0 \mid \bar{p}_t \leq b_s, Z_t \equiv \bar{p}_t/p_t \leq b_s/b_w)$, in order to account for the compensation Both trigger levels $\bar{p}_{T_w} = b_s$ and $p_{T_w} = b_w$ determine the stopping time from which we can immediately see the dependence of S^c on the two processes p_t and $\bar{p}_t$.

The first integral, $g(\bar{p};\theta)$, on the r.h.s. of (48) indicates the shareholders' expected sum of discounted profits, if the price does not go below the reflecting barrier b_s when there is no absorbing barrier. The second integral, $g(\bar{p}_{T_w}, Z_{T_w};\theta)$, accounts for the loss due to absorbing at time T_w. By the strong Markov property of $\bar{p}_t$ it can be proved that $g(\bar{p}_{T_w}, Z_{T_w};\theta) = E_0\left\{g(b_s;\theta)e^{-\rho T_w}\right\}$ and therefore (48) reduces to[23]:

$$S^c(p,\bar{p};\theta) = g(\bar{p};\theta) - E_0\left\{[g(b_s;\theta) + K_s]e^{-\rho T_w}\right\} =$$

$$= E_0\left\{\int_0^{\infty}\theta(\bar{p}_t - c)e^{-\rho t}dt\right\} - E_0\left\{[g(b_s;\theta) + K_s]e^{-\rho T_w}\right\} \qquad (49)$$

Equation (49) shows how the expected discounted value of profits accruing to shareholders, when a reflecting and an absorbing barrier are considered, is represented by the difference between two terms depending upon the joint evolution of the two processes $\bar{p}_t$ and p_t. The first, $g(\bar{p};\theta)$, indicates the

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[22] If we divide the life time of the firm into two parts, before b_s is touched for the first time, T_s, and the period between T_s and T_w, we can write:

$$\left\{\int_{T} \right\}$$

expected value of profits with only the lower barrier b_s beyond which profits do not go. The second, $E_0\{[g(b_s;\theta)+K_s]e^{-\rho T_w}\}$ is the cost of exit at T_w with the inclusion of profits lost because of the decision.

Considering only the reflecting barrier and going through the usual asset equilibrium condition, the shareholders' well-being is the solution of the following differential equation:

$$\frac{1}{2}\sigma^2\bar{p}^2 g'' + \alpha\bar{p}g' - \rho g = \theta(\bar{p}-c) \quad \text{for } \bar{p} \in [b_s;\infty); \tag{50}$$

yet boundary conditions change:

$$\lim_{\bar{p}\to\infty}\left[g(\bar{p};\theta) - \theta\left(\frac{\bar{p}}{\rho-\alpha} - \frac{c}{\rho}\right)\right] = 0 \tag{51}$$

$$g'(b_s;\theta) = 0 \tag{52}$$

The smooth pasting condition (52) is the first order derivative of the expected present value of a function of a Brownian motion. It does not involve any optimizing role of the barrier and requires only the continuity of the first derivative of s in b_s (Dixit, 1993, p. 27). Finally we need an overall value matching condition saying that the shareholders at the exit are indifferent with and without compensation.

$$S^c(b_w;b_s;\theta) = S(b_s;\theta) = -K_s \tag{53}$$

By the linearity of the differential equation (50) and making use of (51), (52) and (53), the general solution of (49) takes the form:

$$S^c(p;\bar{p};\theta) = g(\bar{p};\theta) - [g(b_s;\theta)+K_s]\left(\frac{p}{b_w}\right)^{\beta_2} \tag{54}$$

where:

That is, as long as the compensation period is stochastic, the expected loss during the subsidization is zero. At each time $t \geq T_s$ the compensation keeps shareholders better off (at least indifferent) between exiting now or putting off the decision. Compensation starts at T_s and the value of the firm to shareholders will be greater, i.e. $S^c(p_t;\bar{p}_t;\theta) - S(b_s;\theta) \equiv g(\bar{p}_t;\theta) - S(b_s;\theta) \equiv g(\bar{p}_t;\theta) + K_s \geq 0$ for all $t \in [T_s;T_w)$.

While the compensation keeps shareholders at least indifferent between exiting now or keeping on, workers are not in the same condition. Indicating with $W^c(p;\bar{p};\theta)$ the workers' well-being less the compensation they pay, we get:

$$W^c(p;\bar{p};\theta) = E_0\left\{\int_0^{T_w}(1-\theta)(p_t - s_t - c)e^{-\rho t}dt\right\} + E_0\left[[K_w - K_l]e^{-\rho T_w}\right] \qquad (56)$$

or, using $s_t = (Z_t - 1)p_t$:

$$W^c(p;\bar{p};\theta) = E_0\left\{\int_0^{T_w}(1-\theta)(p_t - c)e^{-\rho t}dt\right\} -$$

$$- E_0\left\{\int_0^{T_w}(1-\theta)(\bar{p}_t - p_t)e^{-\rho t}dt\right\} + E_0\left[[K_w - K_l]e^{-\rho T_w}\right]$$

$$= W(p;\theta) - E_0\left\{\int_0^{T_w}(1-\theta)(\bar{p}_t - p_t)e^{-\rho t}dt\right\}: \qquad (57)$$

Whilst the first integral represents the workers' well-being without compensation, the second accounts for the transfer. Using again the strong Markov property of $\bar{p}_t$; it can be proved that:

$$W(p;\theta) - W^c(p;\bar{p};\theta) = m_1(\bar{p};\theta) - E_0\left[m_1(b_s;\theta)e^{-\rho T_w}\right] - m_2(p;\theta)$$

$$= E_0\left\{\int_0^{\infty}(1-\theta)\bar{p}_t e^{-\rho t}dt\right\} - E_0\left\{\int_0^{\infty}(1-\theta)\bar{p}_t e^{-\rho t}dt\right\}$$

with boundary conditions:

$$\lim_{\bar{p}\to\infty}\left[m_1(\bar{p};\theta) - (1-\theta)\frac{\bar{p}}{\rho-\alpha}\right] = 0; \tag{60}$$

$$m_1'(b_s;\theta) = 0; \tag{61}$$

$m_2(p;\theta)$ is the solution of:

$$\frac{1}{2}\sigma^2 p^2 m_2'' + \alpha p m_2' - \rho m_2 = (1-\theta)p \text{ for } p \in [b_w;\infty); \tag{62}$$

with boundary condition:

$$\lim_{p\to\infty}\left[m_2(p;\theta) - (1-\theta)\frac{p}{\rho-\alpha}\right] = 0. \tag{63}$$

Finally, as for shareholders, we need an overall value matching condition:

$$W(b_w;\theta) - W^c(b_w;b_s;\theta) = 0 \tag{64}$$

By the linearity of the two differential equations (59) and (62), and making use of the boundary conditions, the general solution of (58) becomes:

$$W(p;\theta) - W^c(p;\bar{p};\theta) = M(p;\bar{p};\theta) \equiv m_1(\bar{p};\theta) - m_1(b_s;\theta)\left(\frac{p}{b_w}\right)^{\beta_2} - m_2(p;\theta) \tag{65}$$

where:

$$m_1(\bar{p};\theta) = B_1^c \bar{p}^{\beta_2} + (1-\theta)\frac{\bar{p}}{\rho-\alpha};$$

$$m_2(p;\theta) = B_2^c p^{\beta_2} + (1-\theta)\frac{p}{\rho-\alpha};$$

future subsidies. Therefore, in game-theoretic terms it may be represented, in the strategy space of shareholders, as:

$$\chi(p_t; Z_t) = \begin{cases} \text{Not to exercise the threat at time } t = T_s \text{ or if workers} \\ \text{have payed the subsidy } s_t = (Z_t - 1)p_t \text{ for } t' < t \\ \\ \text{Exit immediately if workers have deviated from} \\ s_t = (Z_t - 1)p_t \text{ at any } t' < t \end{cases}$$

Where $\chi(p_t; Z_t)$ = [Exit, Non Exit] is the action chosen by shareholders at time t with history $(p_t; Z_t)$. In our formulation of the shareholders's "trigger" strategy the threat is exercised if the victims deviate paying less than s_t or by abandoning $s_t = (Z_t - 1)p_t$ as decision rule to evaluate future subsidies. Carrying out the threat depends on the expected sum of future discounted profits (i.e. up to the shut down). Then, shareholders must believe that the compensation rule viewed from the initial date and state $(T_s; b_s)$; will be kept in use for all the (stochastic) planning horizon. If workers deviate the threateners believe that the victims will use a different rule in the future, and then the workers believe that the threateners will carry out the threat at their first opportunity, that is immediately. In this way, workers are punished for deviating of using the announced subsidy rule.

In particular, the threatener does not carry out the threat in period t if $s_{t'} \geq \bar{p}_{t'} - p_{t'}$ for all $t' \leq t$: Shareholders do not exit because they expect payments to continue in the future with the same rule and $S^c(p_t; \bar{p}_t; \theta) \geq S(b_s; \theta) = -K_s$ for all $t \geq T_s$. However, if $s_{t'} < \bar{p}_{t'} - p_{t'}$ for some $t' < t$ shareholders think that the victims are going to follow a different rule and will carry out the threat by immediately closing the firm. That is, they switch from $S^c(p_t; \bar{p}_t; \theta)$ to $S(b_s; \theta) = -K_s$ putting an end to the game. Workers will then get $\hat{W}(b_s; \theta) = K_w - K_l$:

On the other hand, workers do not have any incentive to pay more than s_t

shareholders do not exit, i.e. $\phi(p_t; Z_t)$ = "Non Exit" for all $t \geq T_s$. This situation is equivalent to one in which at the initial date T_s workers choose a decision rule that specifies which subsidy is to be taken at each realization of p_t and for any date $t \in [T_s; T_w)$ (Fudenberg and Tirole, 1991, ch.13).[24]

Moreover, as the strategy ϕ is efficient for any subgame starting at an intermediate date and state $(t; p_t)$ pair at which the option to exit has not been exercised yet (as any subgame beginning at a point at which exit has not taken place is equivalent to the whole game), then we may conclude that it is also subgame perfect.

We are now able to introduce the workers' participation constraint. Although the value matching condition (64) assures that, at time T_w, $W^c(b_w; b_s; \theta) = K_w - K_l$; there is no condition that a priori warrants that the loss from having the threat carried out at T_s; i.e. $W(b_s; \theta) - \hat{W}(b_s; \theta)$; is smaller than the (expected) present discounted value of the payment $E_{T_s}\left\{\int_{T_s}^{T_w} s_t e^{-\rho t} dt\right\}$: Then the compensation scheme may be interrupted before reaching the terminal time T_w: However, by the random evolution of the subsidized price $\bar{p}_t$; at each date $t \geq T_s$ there is no reasons for both players to commit for a stochastic period of length lower than $(T_w - t)$: Then workers still use (56) to evaluate participating and will stop paying s_t if, at some time t; they discover that $W^c(p_t; \bar{p}_t; \theta) < \hat{W}(b_s; \theta) = K_w - K_l$: Since the compensation starts when b_s is reached for the first time (T_s); by (65) they would be willing to continue paying if $W^c(p_t; \bar{p}_t; \theta) = W(p_t; \theta) - M(p_t; \bar{p}_t; \theta) > \hat{W}(b_s; \theta) = K_w - K_l$: Up to the interruption time, workers are strictly better off, since $W^c(p_t; \bar{p}_t; \theta) > \hat{W}(b_s; \theta)$.

So far we have implicitly assumed that, once started at time T_s; the compensation flow goes on until the state variable p_t touches the lower barrier b_w at time T_w: Earlier interruption can only be due to the participation constraint. However, by the characteristics of optimal Brownian paths, there always exists a probability, starting at an interior point of the range $(b_s; b_w)$ between the two barrier, that the state p_t touches again b_s before reaching b_w: In this case, workers may be willing to stop compensating shareholders.

option to exit since they are at their indifferent position.

Not only the participation constraint but also the rise of the price above b_s seem to imply a sudden end of the compensation flow and of the game before touching T_w: However, as shareholders anticipate this and the decision rule strategy ϕ depends on the history of the game (is not a Markov strategy), s_t is not subgame perfect any longer.

If we consider the possibility that workers' compensation terminates before reaching T_w, the shareholders' expected sum of discounted profits starting at any $t \in [T_s; T_w)$; becomes:

$$S^c(p_t; \bar{p}_t; \theta) = E_t \left\{ \int_t^{T_s' \wedge T_w} \theta(\bar{p}_v - c) e^{-\rho(v-t)} dv \right\} - E_t \left\{ K_s e^{-\rho(T_s' \wedge T_w - t)} \right\}; \tag{66}$$

where $T_s' \wedge T_w \equiv \min[T'(b_s); T(b_w)]$; and $T_s' = \inf(t \geq T_s \mid p_t \leq b_s)$ indicates the first hitting time of b_s when the compensation scheme is in operation (the prime indicates that b_s has been already touched). E_t represents the expected value operator conditional to all the random variables with the inclusion of the stopping time $T_s' \wedge T_w$: Triggers b_s and b_w play the role of absorbing barriers for the primitive process p_t.

The probability of reaching b_s before b_w is equal to (Cox and Miller, 1965, pp.232-234; Dixit, 1993, p.54.):

$$\Pr(T_s' < T_w \mid p_t) \equiv P(p_t) = \frac{(p_t)^{-2\mu/\sigma^2} - (b_w)^{-2\mu/\sigma^2}}{(b_s)^{-2\mu/\sigma^2} - (b_w)^{-2\mu/\sigma^2}};$$

with $\mu = (\alpha - \frac{1}{2}\sigma^2)$[25]. Then the shareholders' well-being can be rewritten as:

$$S^c(p_t; \bar{p}_t; \theta) = P(p_t) E_t \left\{ \int_t^{T_s'} \theta(\bar{p}_v - c) e^{-\rho(v-t)} dv - K_s e^{-\rho(T_s' - t)} \right\} +$$

we get:

$$S^c(p_t;\tilde{p}_t;\theta) = g(\tilde{p}_t;\theta) - P(p_t)\left[g(\tilde{p}_{T_s'};\theta) + K_s\right]\left(\frac{p_t}{b_s}\right)^{\beta_1}; \qquad (67)$$

where $\beta_1 > 1$ is the positive root of $\Psi(\beta)$. Although (67) satisfies the overall value matching condition $S^c(b_s;\tilde{p}_{T_s'};\theta) = S(b_s;\theta) = -K_s$, this is not true for all $t \in [T_s;T_s')$. To see this consider the case in fig. 4.

FIGURE 4 ABOUT HERE

By (45), at t the subsidy is such that $\tilde{p}_t = b_s$, and the shareholders will not exercise the option to exit if their well-being is at least equal to $-K_s$. In addition, as $\tilde{p}_{T_s'} = (b_s/p_{t'})b_s > b_s$ we may write $g(\tilde{p}_{T_s'};\theta) = -K_s + \Delta\varepsilon$, with $\Delta\varepsilon > 0$. Then substituting into (67) the well-being becomes:

$$S^c(p_t;b_s;\theta) = -K_s - P(p_t)\Delta\varepsilon\left(\frac{p_t}{b_s}\right)^{\beta_1} < -K_s$$

References

[1] Aoki M., (1980), "A Model of the Firm as a Stockholder-Employee Co-operative Game", American Economic Review, 77, 600-610.

[2] Aoki M. (1984), The Cooperative Game Theory of the Firm, Clarendon Press: Oxford.

[3] Barrett C.R. and P. Pattanaik, (1989), "Bargaining over Employment and Wages with Threats of Strikes and Lockouts", European Journal of political Economy, 5, 333-345.

[4] Biagioli M., (1995), "Italy: Decentralization of Wage Bargaining and Financial Participation", in D. Vaughan-Whitehead (ed.), Workers Financial Participation: East-West Experiences, ILO, Geneve, 85-105.

[5] Cox D.R. and H.D. Miller, (1965), The Theory of Stochastic Process, Chapman and Hall: London.

[6] Dixit A., (1989), "Entry and Exit Decisions under Uncertainty", Journal of Political Economy, 97, 620-638.

[7] Dixit A., (1993), The Art of Smooth Pasting, Harwood Academic Publishers: Chur CH.

[8] Dixit A., and R. Pindyck, (1994), Investment under Uncertainty, Princeton University Press: Princeton.

[9] Dow G., (1986), "Control Rights, Competitive Markets and Labor Management Debate", Journal of Comparative Economics, 10, 48-61.

[10] Dow G., (1993), "Why Capital Hires Labor: a Bargaining Perspective", American Economic Review, 83, 118-134.

[11] Drucker P., (1976), The Unseen Revolution: how Pension Funds Social

[13] Fudenberg D., and J.Tirole, (1992), Game Theory, The MIT Press: Cambridge MA.

[14] Harrison J.M., and M.T. Taksar, (1983), "Instantaneous Control of Brownian Motion", Mathematics and Operations Research, 8, 439-453.

[15] Harrison J.M., (1985), Brownian Motions and Stochastic Flow Systems, Wiley: New York.

[16] Hart O., and J. Moore, (1990), "Property Rights and the Nature of the Firm", Journal of Political Economy, 98, 1119-158.

[17] Klein D., and B.O'Flaherty, (1993), "A game Theoretic Rendering of Promises and Threats", Journal of Economic Behavior and Organization, 21, 295-314.

[18] Lazear E.P., and R.B. Freeman, (1996), "Relational Investing: the Worker's Perspective", NBER Working Paper 5436.

[19] Moretto M., (1995), "Controllo ottimo stocastico, processi browniani regolati e optimal stopping", Rivista Internazionale di Scienze Economiche e Commerciali, 41, 93-124.

[20] Moretto M., and S. Pastorello, (1995), "Entry-Exit Timing and Pro…t Sharing", Working Paper 228, Department of Economics, University of Bologna.

[21] Moretto M., and G. Rossini, (1995), "The Shut-Down Option and Pro…t Sharing", Journal of Comparative Economics, 21, 154-186.

[22] Moretto M., and G. Rossini, (1997), "Pro…t Sharing Regulation and Repeated Bargaining with a Shut-Down Option", Economic Design, 43, 1-31.

[23] OECD, (1995), "Pro…t-sharing in OECD Countries", Employment Out-