Social States of Belief and the Determinants of the Equity Risk Premium in A Rational Belief Equilibrium

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NOTA DI LAVORO 70.97

SEPTEMBER 1997

ETA - Economic Theory and Applications

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This research was supported by Fondazione Eni Enrico Mattei, Milano, Italy. The author thanks Stanley Black, William Brock, Mark Garmaise, Kenneth Judd, Michael Magill, Maurizio Motolese and Martine Quinzii for many helpful suggestions regarding the research reported in this paper. He is grateful to Stanley Black and Maurizio Motolese for dedicated assistance in carrying out the computations. This paper is a drastic revision of Kurz (1997d).

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SUMMARY

In previous models of rational belief equilibria (RBE), individual states of belief were the foundation for the construction of the endogenous state space where individual states of belief were described with the method of assessment variables. This leads to a lack of "anonymity" where the belief of each individual agent has an impact on equilibrium prices but as a competitor he ignores it. Instead we study a replica economy with a finite number of types but with a large number of agents of each type. The state space for this economy is constructed as the set of products of the exogenous states and the social states of belief which are vectors of "type-states" each of which is a distribution of beliefs among members of a type. Such an economy leads to RBE which do indeed solve the problem of anonymity. We then study via simulations the implications of the model for market volatility and for the determinants of the equity risk premium. Under i.i.d. assessments the law of large numbers imply a single social state of belief and we show that the RBE of such economies have the same number of prices as in rational expectations equilibrium (REE). However, the RBE may exhibit large fluctuations if agents are allowed to hold extreme beliefs. Establishing 5% boundary restrictions on beliefs we show that the model with a single social state of belief cannot explain all the moments of the observed distribution of returns. The introduction of correlation among the beliefs of agents leads to the creation of new social states. We then show that under such correlation the model simulations reproduce the values of four key moments of the empirical distribution of returns. The observed equity premium is then explained by two factors. First, investors demand a higher risk premium to compensate them for the endogenous increase in the volatility of returns. Second, at any moment of time there are in the markets both rational optimists as well as rational pessimists and such a distribution leads automatically to a decrease in the riskless rate and to an increase of the risk premium. Correlation among beliefs of agents leads to fluctuations over time in the social distribution of beliefs and such fluctuations cause a higher equilibrium equity risk premium.

Key words: state space, endogenous uncertainty, rational beliefs, rational belief equilibrium, individual state of belief, social states of belief, OLG economy, correlation among beliefs, volatility measures, equity risk premium, riskless rate.

JEL: D58, D84, G12.

NON TECHNICAL SUMMARY

This paper studies two related problems: (i) why is the level of volatility in financial markets much higher than could be explained by the variability of the exogenous fundamentals, and (ii) why do simulation models based on rational expectations equilibria (REE) predict a very high average riskless interest rate and a very low average equity risk premium. This second question is known as the "Equity Premium Puzzle". The paper uses the theory of rational belief equilibria (RBE) to argue that both phenomena are explained by the presence of "Endogenous Uncertainty" in financial markets. This uncertainty is propagated within the economy (hence the term "Endogenous") by the beliefs and expectations of the agents who trade securities.

The theory of RBE was developed in a sequence of papers assembled in a recently published book (See Kurz [1997]); the present paper builds on these results. The earlier papers constructed RBE with finite number of agents in which the state of belief of each agent has an influence on equilibrium prices but as a competitor he ignores it. This is analogous to competitive equilibria with a finite number of firms where each firm has an effect on equilibrium prices but, as a competitor, it ignores it. The natural solution offered in this paper is to use a large replica economy with a small number of "types" but with a large number of agents of each type. This leads to an economy where price volatility is not impacted by the beliefs of any one agent but by the *distribution of beliefs* in the market. The question then becomes: how does the distribution of beliefs affects price volatility and the equity risk premium.

The paper is structured as follows. Section 1 reviews the way in which uncertainty is formulated in standard general equilibrium theory and shows why this formulation is not sufficient for modelling endogenous uncertainty. Section 2 reviews the model of RBE with individual states of belief and explains how in such equilibria the beliefs of individual agents have an effect on equilibrium prices. Section 3 introduces the model of a large replica economy with finite number of types. It shows that in such economies prices are determined by the distribution of beliefs and hence the belief of an individual agent has no effect on prices. Section 5 formulates a simulation model in which the "state of belief" of an agent is described by a conditional probability of prices in the next period. Given the price at date t, the agent can be optimistic, pessimistic or neutral about the probability of high prices at t + 1. All simulation analysis is carried out in Section 5 and is the main basis for the conclusions of the paper. We turn to explain them now.

The theory of RBE shows that individual states of belief must fluctuate when agents disagree. In the models with individual states of belief endogenous price volatility is then caused by the fluctuations of the states of beliefs of the agents.

In the large replica economy agents disagree but if their beliefs are *independent* then the law of large numbers implies that the market distribution of beliefs is constant. Nevertheless, even under this condition an RBE can have much higher volatility than the corresponding REE. For example, compare an REE in which all agents have the same beliefs with an RBE in which 60% of the agents are optimistic and 40% pessimistic about high prices in the next period. The rationality conditions show that if the agents are not split 50% optimists and 50% pessimists then the intensity of pessimism must be stronger than the intensity of the optimists, compensating for the numerical difference. In addition, the intensity of pessimists varies with realised prices and hence prices fluctuate more due to the fluctuations in the *intensity* of pessimism over time. There are strong arguments in support of the view that the beliefs of agents are not independent. In that case fluctuations of the distribution of belief over time will also contribute to price volatility.
Analysis of the equity risk premium shows that the key question is what are the conditions on beliefs which will ensure that the average riskless rate is low and hence the average equity risk premium is high. It turns out that the key condition requires that the impact of the pessimists dominate the market a significant fraction of time. When this occurs they protect their endowment by shorting the stock and increasing their purchases of the safe riskless bill. This tends to bid up the price of the bill and lowers the price of the stock resulting in a lower riskless rate and a higher equity risk premium. The simulation results also show that correlation among the beliefs of the agents can change the frequency at which prices are realised over time and this implies that the correlation can increase the equity premium by increasing the *frequency* of realisation of those prices in which the pessimists have the greater impact on equilibrium prices.

CONTENTS

1. <u>The State Space and the Emergence of Endogenous Uncertainty</u>

The role of the market mechanism in the optimal allocation of risk bearing has been one of the most extensively studied problems in economics. The theory of general equilibrium, as developed by Arrow and Debreu [1954] and by Arrow [1953], provided an extremely fertile framework for the examination of the behavior of markets for uncertain prospects in general and the markets for insurance and risky securities in particular. The rapid development of the field of finance is a noteworthy example of the impact of this framework of analysis. Yet, despite these impressive achievements important foundational questions regarding the nature of social uncertainty remain unresolved. This paper explores alternative equilibrium concepts in which a central role is played by Endogenous Uncertainty, a concept which was defined in Kurz [1974] and explored in a sequence of recent papers (see Kurz [1994a], [1994b], [1997a], [1997b], [1997c] Kurz and Wu [1996], Kurz and Schneider [1996], Kurz and Beltratti [1997] (due to repeated reference we identify this paper as KB [1997]) and Nielsen [1996]). In order to explore the emergence of endogenous uncertainty, it would be useful first to review some of the problematics arising out of the treatment of uncertainty in the Arrow-Debreu [1954] model, the role played by securities in Arrow's [1953] equilibrium and the modifications of the theory by Radner [1968], [1972], [1979].

As is well understood, the full generality of the Arrow-Debreu formulation enables the incorporation of uncertainty merely by a reinterpretation of the symbols employed. In the original Arrow-Debreu [1954] paper terms like "risk" or "uncertainty" *are not even mentioned*. In his explicit treatment of uncertainty Arrow [1953] defines the exogenous "state space" and explicitly introduces markets for state contingent claims on commodity bundles and the utility of such uncertain commodities. He notes that the treatment of the uncertain case is entirely analogous to the case of certainty except for the enlarged dimension of the commodity space (which equals the number of physical commodities *multiplied* by the number of exogenous "states"). Motivated by markets for insurance, Debreu [1959] uses the broader terminology of "events" to identify subsets of states but his formal treatment requires the trading of a complete set of *state* contingent commodities. Apart from the formal interpretation of the concept of a "commodity" the uncertainty interpretation raises only one issue of substance with respect to the assumption of convexity of preferences. Since in the case of uncertainty[1] convexity implies risk aversion, both the existence of competitive equilibrium in the Arrow-Debreu theory as well as the optimality theorem in Arrow [1953] are proved under the assumption of universal risk aversion.

It is clear that the crucial step taken in the Arrow-Debreu formulation of uncertainty within general equilibrium theory is the introduction of the concept of "the state" into the theory. This concept, however, is the cornerstone of the theory of *individual* decision theory and subjective probability. In Savage's [1954] treatment the concept of "the state of the world" is nothing more than a formal description of what a decision maker is uncertain about. Consequently Savage [1954] defines the "world" to be "the object about which the person is concerned" whereas "a state" (of the world) is defined as " ... a description of the world, leaving no relevant aspect undescribed."[2]

Arrow learned mathematical statistics from Hotelling and Wald, and was influenced by Savage's approach to subjective probability. In some early papers he does not even provide a

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[1] And assuming expected utility maximization with preferences which are state independent.

[2] See Savage [1954], page 9.

definition of the concept of the "state" and takes it to be both known as well as naturally applicable to the economic problem at hand (e.g. Arrow [1951], [1953]). In later papers (e.g. Arrow [1971] or Arrow and Hahn [1971]) he provides a precise definition of the "state of the world" as " ... a description of the world so complete that, if true and known, the consequences of every action would be known."[3]

In the context of decision theory the concept of the "state" is no more than a tool for the formulation of the individual decision problem. As such, it is entirely satisfactory and indispensable. In fact, it is hard to visualize how one can formulate a stochastic dynamic decision problem without a concept like a "state." Moreover, the formulation of any decision problem as well as Savage's theory of subjective probability neither require the "state" to be observable nor need its description be communicable to or be understood by other decision makers.

The generality of the decision theoretic framework naturally led Arrow and Debreu to adopt this framework for the formulation of the problem of choice under uncertainty of every economic agent in a competitive economy. The important theoretical step which they took was to endow all the agents *with the same state space* and to provide them with the market opportunity of trading the uncertainty defined by the "state." That means that the concept of "the state" became a major tool of general equilibrium analysis. In contrast with the context of the individual decision problem where the "state of the world" is merely an expression of individual uncertainty, in the general equilibrium framework "the state of the world" becomes a description of commodities, it identifies markets and becomes a basis for specifying contracts and property rights. In such a framework the concept must satisfy the same marketability criteria as "navel

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[3] See Arrow [1971], page 20.

oranges available in Palo Alto, California, on November 29, 1997": it must be precisely defined, commonly observable and unequivocally comprehended by all economic agents. These requirements clearly raise some difficult practical problems of description.[4] However, the theoretical structure of the exogenous state space enabled Arrow and Debreu to achieve a complete integration of the theory of value.

It is noteworthy that the example of insurance motivated the Arrow-Debreu approach to uncertainty. Indeed, for a description of commodities, the concept of "the state of the world" is extremely useful in characterizing markets for insurance. This is so since an insurance policy is a contract in which the owner receives specified compensations if the state of the world belongs to an event such that the insured object meets a long list of described conditions. In this case "the state of the world" description of the commodity has the precise interpretation of the "sample space" in probability models. Insurance markets function well when the contingency conditions are unambiguous and their probability distributions are truly exogenous and cannot be altered by the behavior of the insured.

Notwithstanding the importance of the integrated vision of the Arrow-Debreu theory, it is evident that the construct of markets for claims which are contingent on the exogenous states constitutes an unsatisfactory solution to the problem of allocating risk in a market economy. Arrow [1953] himself observes that outside the insurance framework, markets for commodity claims which are contingent upon the exogenous states do not exist. Moreover, even the insurance markets do not function as visualized in the theory. More specifically, in order to study insurance markets, Malinvaud [1972], [1973] considers a large economy with individual risks for

[4] For a description of the exogenous state see Debreu [1959], page 98.

which a complete set of insurance markets exists in the form of insurance pools that are used for averaging individual risks. In a large economy all risk averse agents clearly purchase fair insurance. It is then shown by Malinvaud that given such pools, in the equilibria of these economies agents trade only in certainty contracts: individual uncertainty disappears from general equilibrium considerations. The implication of the Malinvaud analysis is that in a general equilibrium context the main problems of allocating risk are not associated with the allocation of individual idiosyncratic risks but rather, the allocation of *collective* risk bearing for which the laws of large numbers are not available. We argue in this paper that this conclusion continues to hold when endogenous uncertainty is introduced. Whether exogenous shocks can account for all observed social risks as reflected in the economic fluctuations of quantities and prices is probably the central question at hand. It is evident that the list of *observed* variables which are truly exogenous to the economic universe is very short and the range of their variability and impact are much too small to account for the observed variability of economic variables. Thus, one must conclude that if the exogenous shocks are all that matters then the most relevant components of the "state" are not commonly observable and cannot provide a basis for contingent contracts.

Arrow's [1953] celebrated solution has become the foundation of modern general equilibrium theory of finance. He recognized that without markets for contingent claims one must think of an economy *as a sequence of spot markets linked together by a market for securities* which enable the reallocation of incomes across the different state-date combinations. In Arrow's [1953] formulation and in the extension by Radner [1972], an equilibrium consists of a set of market clearing spot price functions $p_t(s)$ of commodities associated with each of the finite number of the state-date pairs (s, t), and a set of market clearing prices of securities which pay different dividends in different "states." Since the equilibrium is established at the date $t = 0$ which we can think of as "the present," such an equilibrium requires the agents *to know at* $t = 0$ *all prices* $p_t(s)$ *that would prevail at all future dates and all states* s. This assumption of "Rational Expectations" is the foundation of the optimality theorem of Arrow [1953]. It is also the basis for most work in finance which seeks to show that Pareto optimality is obtained whenever the set of securities "spans" the set of exogenous states.

The rational expectations equilibrium concept of Arrow [1953] and its extension by Radner [1972] elevates the exogenously specified "state" substantially above Arrow's own definition (e.g. Arrow [1971], page 20). It is no longer such a complete description that the consequences of all *individual actions* are known; now the requirement is that the knowledge of the exogenously specified state enables every agent to know the consequences of all collective actions as well and, in particular, to know all future *prices* in the economy. These ideas extend further to the treatment of general equilibrium with private information (e.g. see Radner [1979], [1982]). The agent's knowledge of the *price maps* $p_t(s)$ plays a crucial role in the public revelation of private information.

The assumption of rational expectations in the Arrow-Radner equilibrium is viewed, almost universally, as placing excessive and unreasonable demands on the agents: since the map $p_t(s)$ is not observable, how could the agents know it at date 0? The term "rational" in connection to the knowledge of this map appears to mean that agents know the structure of the economy so completely (including technology and resources as well as preferences and endowments of other agents) that for each exogenous state s the agents can carry out all general equilibrium calculations needed to deduce the map $p_t(s)$[5] for all future dates. It is then natural to ask what if the agents do not know the map since they do not have "structural knowledge"[6]. The Arrow-Radner equilibrium theory does not apply since agents cannot carry out, at date 0, the kind of intertemporal planning which the theory calls upon them to do. The needed extension of the theory to the case where agents do not have structural knowledge has been recently proposed (see, for example, Kurz [1994b], [1997b], [1997c], Kurz and Wu [1996], Kurz and Schneider [1996] and Nielsen [1996] all of which are included in the volume of Kurz [1997]) by the theory of Rational Belief Equilibrium (in short RBE). The theory of RBE leads, in a natural way, to the emergence of endogenous uncertainty (see Kurz [1974]) which is that part of social uncertainty (and hence economic fluctuations) which is propagated within the economy rather than being "caused" by exogenous shocks. We now explore this connection in some detail.

Recall that it was Arrow's [1953] and Radner's [1972] views that without markets for contingent claims at date $t = 0$ an equilibrium for the economy *is a sequence of market clearing spot prices of the reopened markets at the different dates*. But then at $t = 0$ agents are uncertain about future spot prices at $t = 1, 2, \ldots, T$. If we then follow Savage's [1954] dictate, then future spot prices are part of the "world" about which all agents are uncertain. This means that the state, which is a description of the world, *should include future spot prices*. Agents are therefore uncertain about their future utilities not only because of the effect of exogenous random variables but also because they are uncertain about those future spot prices that would prevail, at any

[5] For this reason the assumption is sometimes called "conditional perfect foresight".

[6] We have introduced this term earlier (see Kurz [1994a]) in order to distinguish knowledge about the *state of the economy* which is considered "information" and knowledge about *the functioning of the economy* which we call "structural knowledge".

configuration of the exogenous variables. However, if prices are part of "the state of the world" then agents cannot view prices as a known equilibrium map like $p_t(s)$. Moreover, from the point of view of each agent the state space does not consist of abstract and unknown objects but rather, in the case of M equilibrium prices, the price state space is simply the set of integers $\{1, \ldots, M\}$ and in the case of a continuum of prices, the space is the unit interval. With this enlargement of the "state space" we lower the concept of "the state of the world" back to where it is merely *a terminology for the description of what agents are uncertain about.* However, this change of the state space has far reaching implications for the way we need to think about uncertainty in a general equilibrium context and for our perspective on what *social uncertainty* is.

Once agents view prices as random variables they must form probability beliefs about future prices in the same way they form beliefs about exogenous variables. Since Savage [1954] Arrow [1953] and Radner [1972] allow agents to have different probability beliefs about what they are uncertain about, it follows that if an equilibrium concept is to permit agents to be uncertain about future prices, then *equilibrium prices at each date must depend upon what agents expect future equilibrium prices to be!!* Formally, suppose that in an economy with K agents we denote by $y_t = (y_t^1, \ldots, y_t^K)$ the date t vector of conditional probabilities of the K agents about all equilibrium events after date t conditional *upon the entire past.* y_t is the "state of belief" in the economy and y_t^k is the state of belief of agent k. The decision functions of the agent at each date take the general form

(1) $$x_t^k = F^k(p_{(t)}, s_{(t)}, y_t^k)$$

where $z_{(t)} = (z_0, z_1, \ldots, z_t)$ denotes the entire history of the variable. Market clearing conditions establish equilibrium prices p_t at each date t as

(2) $$p_t = \varphi_t(y_t, s_{(t)})$$

and in the special and useful case of finite memory equilibria, (2) takes the simpler form

(2') $$p_t = \varphi(y_t, s_t).$$

The map (2) which is unknown to the agents in an RBE corresponds to the Arrow-Radner price map $p_t(s)$ which is assumed to be known to the agents. The crucial difference is the emergence of the state of belief which becomes part of the enlarged state space for the economy. In either case (2) or case (2') the fluctuations of prices over time are in part due to fluctuations of the exogenous shocks s_t and in part to the fluctuations in the state of beliefs y_t.

In a dynamic economy consisting of a sequence of markets, economic risk is an intertemporal phenomenon in the sense that what agents perceive as risk is directly linked to the fluctuations of the economy over time and against such variability they wish to insure themselves. Endogenous uncertainty is then that component of economic fluctuations which is due to the impact of the agent's beliefs on the variability of prices or other endogenous variables. This effect is generated both by the time variability of the states of beliefs of the agents as well as by the structure of the maps (2) or (2'). Since the agents do not know the true equilibrium map between states (y_t, s_t) and prices and since they do not observe states of beliefs, they can learn something from an examination of the data generated by the economy. One of the main conclusions of Kurz [1994a] is that there is no basis to expect that agents will learn the true structure of the maps (2) or (2') and what is the true probability distribution of exogenous shocks.

For this reason the agents form probability beliefs about prices and exogenous states knowing that *the exogenous state space is a partition of the price state space.*

The emergence of endogenous uncertainty in economies where agents do not have structural knowledge points to the observation that in such economies "expectations matter" and have real effects on equilibrium allocations. The theory of Rational Beliefs establishes the limits within which individual conditional probability beliefs may vary if they satisfy the basic rationality principle that such expectations are compatible with the data generated by the economy. An RBE is an equilibrium in which agents do not have structural knowledge and hold rational beliefs.

In what sense should endogenous uncertainty, as defined above, be taken to be "endogenous" and "stochastic"? Observe that endogenous uncertainty is generated by *variations* in the state of beliefs of the agents each of whom selects a rational belief from a set of probability beliefs which satisfy the axioms of rationality. Since the selection of a rational belief is an endogenous phenomenon and their adopted beliefs *cause* aggregate risk and fluctuations, the uncertainty which is induced by these selections is "endogenous" in the sense that it is generated *within* the economy rather than caused by exogenous shocks. In the development below we employ the technique of a Markov model in which each agent uses a *privately generated stochastic assessment variable* (for details on this approach see Kurz and Schneider [1996]). On the basis of this realization the agent determines which of a finite number of transition matrices to use on that date. This leads to a tractable modeling of the aggregate states of belief since the individual state of belief is fully described by the realization of his private assessment variable.

The present paper aims to explore alternative ways of defining the expanded state space of an economy with endogenous uncertainty. One may represent the states of belief in the economy either as vectors of the *states of beliefs of the individual agents* or as *distributions* of individual states of belief. Such two descriptions are obviously closely related but we note that the RBE concept used in all the papers cited above defines the states of belief using the first of these two alternatives. We explain below that with a finite number of agents such an RBE lacks a desired property of "anonymity" in the sense that the belief of an agent has an impact on equilibrium prices but, as a competitor, *he is required to ignore it*. Needless to say, lack of anonymity is a universal problem which is common to all competitive models with a finite number of agents. The interest in the second approach is based on the fact that it has two important implications. On the one hand it leads to a concept of an RBE which possesses the anonymity property and thus demonstrates that in a large economy the belief of any one agent does not matter for aggregate behavior. On the other hand, this view of equilibrium explains how the *distribution* of beliefs affects aggregate behavior and why in applications it is important to focus on the properties of this distribution. Our exploration is carried out both analytically in Section 3 as well as via simulations in Section 4 of this paper. Section 5 concludes.

2. Rational Belief Equilibria (RBE) with Individual States of Belief

2a *A Family of OLG Models with a Finite Number of Equilibrium Prices*

Some of the papers mentioned earlier (i.e. Kurz [1997c], Kurz and Wu [1996], Kurz and Schneider [1996], KB [1997] and Nielsen [1996]) use a standard two period OLG model with a single consumption good but vary in the structure of securities which are available. Nevertheless, the construction of the expanded state space which includes the *vector* of individual states of belief is the same in all of them. Since the aim of the present paper is to show how an endogenous state space can be constructed so as to depend only on *social states of belief* and not on *individual states of belief*, we select one of these models and follow its development. This enables us to explain why RBE with individual states of belief lack anonymity. In Section 3 we show how the use of "social states of beliefs" leads to RBE which have the anonymity property.

We note that in OLG models with a single, homogeneous, consumption good *old* agents do not need to optimize by allocating a budget over alternative consumption vectors. As a result, equilibrium prices do not depend upon the entire history of the economy and under our assumptions such RBE have a finite number of equilibrium prices. Since we aim to study the construction of the state space, we consider the assumption of a single consumption good as a convenient simplification. Our construction continues to hold in an economy with an infinite number of equilibrium prices but is technically more demanding. We now outline the basic model.

The agents in the economy live for two periods. At any date there are K young agents denoted by $k = 1, 2, ..., K$. There are also K old agents in each generation but only the young receive an endowment $\Omega_t^k \in \mathbb{R}_+$ initially assumed constant.[7] The assumption of a constant endowment stream represents, as usual in OLG models, the labor supply of each young agent.

Each young person is a copy of the old person who preceded him where the term "copy" refers to the *utilities, endowments* and *beliefs*. Hence, ours is a model of "dynasties" and we assume that there is a finite number of such dynasties. In addition to a competitive market for the consumption good, two types of financial assets are traded at each date in competitive markets in the economy. The first asset is the common stock of an infinitely lived firm and at date 1 the

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[7] Both Kurz and Wu [1996] as well as Kurz and Schneider [1996] assume the endowment to be constant over time. On the other hand, Nielsen [1996] and KB [1997] assume, for their modeling purposes, that $\{\Omega_t^k, t = 1, 2, ...\}$ is a stochastic process for each k. In the analytical discussion of Section 3 we assume endowment to be constant but adopt the KB [1997] framework in the simulations of Section 4.

supply (equal to 1) of the stock is distributed among the old at that date. The infinitely lived firm is assumed to be simple: it generates *exogenously* a stochastic sequence $\{R_t \in \mathbb{R}_+, t = 1, 2, \dots\}$ of dividends in the form of positive quantities of the homogenous commodity. We assume that the process $\{R_t \in \mathbb{R}_+, t = 1, 2, \dots\}$ is a finite state Markov process which will be specified below. The second asset is a zero net supply real short term bond which is issued at t and pays at $t + 1$ one unit of the consumption good.[8] The notation which we employ in this paper is as follows:

x_t^{1k} - the consumption of k when young at t;

x_{t+1}^{2k} - the consumption of k when old at $t + 1$. This indicates that k was born at date t;

θ_t^k - stock purchase of young agent k at t;

B_t^k - bond purchase of young agent k at t;

Ω^k - the endowment of k when young;

p_t^c - the price of consumption goods at date t;

P_t - the price of the common stock at date t;

q_t - the price of the bond at date t;

$s_t = (R_t, p_t^c, P_t, q_t) \in S$ is the state *from the point of view of the agents*;

$\mathcal{B}(A)$ - the Borel subsets of any measurable set A in a Euclidean space.

We turn now to specify our basic assumptions.

Assumption 2.1: for each k, $u^k(\cdot)$ is a strictly increasing and quasi concave function.

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[8] We note that Nielsen's [1996] economy has a pure fiat money used by young agents as a store of value to transfer income from t to $t + 1$. Kurz and Wu [1996] follow Svensson [1981] and Henrotte [1996] in using Price Contingent Contracts (in short PCC) which enable an agent to contract for the delivery of a unit of the common stock at future dates contingent upon the *prices* which prevail at these future dates.

We restrict attention to a Markovian economy along the lines of Kurz and Schneider [1996]. Thus we assume that $\{R_t, t = 1, 2, ...\}$ is an exogenous dividend process where $R_t \in D \subseteq \mathbb{R}_+$.

Assumption 2.2: D is a finite set with $|D|$ positive quantities; the process $\{R_t, t = 1, 2, ...\}$ is a stable Markov process on D with probability measure Π_D defined on $(D^\infty, \mathcal{B}(D^\infty))$ with a stationary measure m_D.

The price process $\{(p_t^c, P_t, q_t) \in P^*, t = 1, 2, ...\}$ is of interest. The measurable set $P^* \in \mathbb{R}_+^3$ of the appropriate space of feasible prices is of central importance in the analysis below. A belief of agent k is a probability on sets of sequences $\{(R_t, p_t^c, P_t, q_t) \in D \times P^*, t = 1, 2, ...\}$ and, as in Kurz and Schneider [1996], we characterize such beliefs with the technique of *private assessment variables*. An assessment variable y_t^k is a random variable or a parameter that agent k perceives at t. The probability of assessment variables *is part of the identity of the agent in the sense that it is selected by the agent as part of his model of the market.* It is thus clearly allowed to be stochastically interdependent with other economic variables. Putting it differently, the agent has a theory about the market mechanism which is represented by the probability belief Q^k and this belief entails some assessment which will represent the state of belief of the agent. The value of an assessment variable may depend upon market observables and would thus summarize the state of belief of the agent although it will have a random component. The probability belief Q^k is then a joint probability of the observed market data and the assessment variable. We stress that the assessment is a description of the agent's *perception* and may be considered a parameter of his belief. He alone can understand its meaning, it cannot be observed or comprehended by anyone else and should not be confused with "information" or "data" with respect to which our standard rationality of belief conditions apply. As explained by Kurz and Schneider [1996], the method of private assessment variables is introduced to allow us a tractable description of non-stationarity in the dynamics. Given y_t^k the agent selects one from among a *finite* number of Markov matrices to apply at t and hence, for any infinite sequence of y_t^k, his effective belief is the *conditional probability of Q^k given the sequence*. The domain of y_t^k is Y^k and is assumed to be a finite subset in $\mathbb{R}$. Q^k is then a probability on the space $((D \times P^* \times Y^k)^\infty, \mathcal{B}((D \times P^* \times Y^k)^\infty))$. In sum:

Assumption 2.3: For all k, the system $((D \times P^* \times Y^k)^\infty, \mathcal{B}((D \times P^* \times Y^k)^\infty), Q^k, T)$ is stationary and ergodic. Y^k is a finite subset in $\mathbb{R}$ with $|Y^k|$ elements and under Q^k agent k believes that the process $\{(R_t, p_t^c, P_t, q_t, y_t^k),\ t = 1, 2, \ldots;\}$ is a Markov process. The non-stationarity induced by each assessment sequence $y^k \in (Y^k)^\infty$ is a selection, at each date, of a Markov transition function (a matrix if the set of prices is countable) which is determined by the value taken by y_t^k.

Since the *effective* belief of the agent is the conditional probability of Q^k given y_t^k, and this may be time dependent, Assumption 2.3 (of stationarity of the joint system) means that the description of the variables y^k *exhausts all the time dependency which the agent perceives*.

Given that Q^k is jointly stationary on $((D \times P^* \times Y^k)^\infty, \mathcal{B}((D \times P^* \times Y^k)^\infty))$ the standard theorems of dynamic programming apply when each agent knows $(R_t, p_t^c, P_t, q_t, y_t^k)$ in the sense that he *observes* (R_t, p_t^c, P_t, q_t) while he perceives the parameter y_t^k which is *generated privately*. With this in mind we turn to the formulation of the optimization problem of the agents. The problem of agent k when young is then as follows:

(3) $$\max_{(x^{1k},\, \theta^k,\, B^k)} E_{Q^k}[u^k(x_t^{1k},\ x_{t+1}^{2k}(s_{t+1})) \mid s_t, y_t^k]$$

subject to

(4a) $$p_t^c x_t^{1k} + P_t \theta_t^k + q_t B_t^k = p_t^c \Omega^k$$

(4b) $$p_{t+1}^c x_{t+1}^{2k}(s_{t+1}) = \theta_t^k (P_{t+1} + p_{t+1}^c R_{t+1}) + B_t^k p_{t+1}^c.$$

The market clearing conditions for this model are then

(5a) $$\sum_{k=1}^{K} \theta_t^k = 1 \qquad t = 1, 2, \ldots$$

(5b) $$\sum_{k=1}^{K} B_t^k = 0 \qquad t = 1, 2, \ldots$$

If follows from (4a) - (4b) and (5a) - (5b) that when markets clear then

(6a) $$p_t^c x_t^1 + P_t \le p_t^c \Omega \qquad t = 1, 2, \ldots$$

(6b) $$p_t^c x_t^2 \le P_t + p_t^c R_t \qquad t = 1, 2, \ldots$$

where x_t^1, x_t^2 and Ω are the aggregates defined by

(7a) - (7b) $$x_t^i = \sum_{k=1}^{K} x_t^{ik} \qquad i = 1, 2$$

(7c) $$\Omega = \sum_{k=1}^{K} \Omega^k.$$

Under the Markov assumption, the demand functions of all generations take the form

(8a) $$x_t^{1k} = \varphi_c^k(R_t\, , p_t^c\, , P_t\, , q_t\, , y_t^k)$$

(8b) $$\theta_t^k = \varphi_\theta^k(R_t\, , P_t^c\, , p_t\, , q_t\, , y_t^k)$$

(8c) $$B_t^k = \varphi_B^k(R_t\, , p_t^c\, , P_t\, , q_t\, , y_t^k).$$

An equilibrium requires that conditions (5a)-(5b) be satisfied hence

(9a) $$\sum_{k=1}^{K} \varphi_\theta^k(R_t\, , p_t^c\, , P_t\, , q_t\, , y_t^k) = 1$$

(9b) $$\sum_{k=1}^{K} \varphi_B^k(R_t\, , p_t^c\, , P_t\, , q_t\, , y_t^k) = 0.$$

Using the notation $y_t = (y_t^1, y_t^2, \ldots, y_t^K) \in Y \equiv Y^1 \times Y^2 \times \ldots \times Y^K$ we can solve (9a)-(9b) and write the equilibrium map in the form

(10) $$\begin{pmatrix} p_t^c \\ P_t \\ q_t \end{pmatrix} = \Phi^*(R_t\, , y_t) \qquad \text{for} \quad t = 1\, , 2\, , \ldots$$

Solutions of the form (10) are also derived by Nielsen [1996], Kurz and Schneider [1996] and by Kurz and Wu [1996]. In all these models an RBE has the property that the vector of private assessment variables influences prices and consequently the state space for equilibrium analysis is $(D \times Y)$ which is different from the state spaces $(D \times P^* \times Y^k)$ of the individual agents. Note that the number of distinct equilibrium prices cannot exceed $M = |D| \prod_{k=1}^{K} |Y^k|$. Indeed, there exists a finite collection $\{ (p_i^c\, , P_i\, , q_i) \in \mathbb{R}_+^3\, ,\ i = 1, 2, \ldots, M \}$ of equilibrium price vectors such that

(11) $$\begin{pmatrix} p_i^c \\ P_i \\ q_i \end{pmatrix} = \Phi^*(R_i\, , y_i) \qquad \text{for} \quad i = 1\, , 2\, , \ldots, M.$$

To complete the model we specify the true joint distribution of private assessments y_t and dividends as a probability Π_{DY} on the measurable space $((D \times Y)^\infty, \mathcal{B}((D \times Y)^\infty)$. This is an important part of the formulation and we need to explore the restrictions on this measure and, correspondingly, on the beliefs of the agents. First consider the vector y_t of private assessments. The probability of each y_t^k is determined by agent k and hence, each agent *knows* his own distribution. The probability of the signal as perceived by agent k is the *marginal* measure of Q^k on Y^k and we denote it by $Q^k_{Y^k}$ (where Q_Z is the marginal measure of Q on a subspace $(Z^\infty, \mathcal{B}(Z^\infty))$. Given Π_{DY}, the implied probability of y^k is $\Pi_{(DY)_{Y^k}}$. It must then be true that

(12) $$\Pi_{(DY)_{Y^k}} = Q^k_{Y^k} \quad \text{for} \quad k = 1, 2, \ldots, K.$$

The specification of Π_{DY} implies that agents may condition on prices and dividends when forecasting their own future signals. More important is the fact that *the specification permits the private assessments to be correlated with each other and such correlation may be affected by the observed prices and dividends*. Each agent does not know other agent's assessments and does not know the structure of this correlation and cannot take this structure into account in his own optimization. This leads to the emergence of an important market externality.

The fact that assessment signals are *entirely private* yet *correlated* is the result of social communication through which agents interact with each other. In addition agents observe the same data and such common observations act as correlating devices. To put it differently, y_t^j and y_t^i may be positively or negatively correlated and, in general, jointly distributed with observed data in the economy such as prices and dividends, because agents i and j communicate with each other and may influence each other's models. This correlation plays a central role in an

RBE as demonstrated by Kurz and Schneider [1996] and by KB [1997] in the study of the volatility of asset prices. Therefore, it would have been desirable to formulate the structure of social communication as part of the model. We have not done so and the assumption of a fixed structure of communication (implied by Π_{DY}) is a simple representation of the impact of social communication on economic fluctuations. Our assumption is then:

Assumption 2.4: Under Π_{DY} the process $\{(R_t, y_t), t = 1, 2, \ldots\}$ is a Markov process and the dynamical system $((D \times Y)^\infty, \mathcal{B}((D \times Y)^\infty), \Pi_{DY}, T)$ is stable and ergodic with a stationary measure m_{DY}. We denote by m_D and by m_Y the corresponding D and Y marginal measures.

Lemma 1: The price process $\{(p_t^c, P_t, q_t), t = 1, 2, \ldots\}$ is a stable and ergodic process on the finite state space $D \times Y$ with probability Π_P and a stationary measure m_P. The probability Π_P on $((P^*)^\infty, \mathcal{B}((P^*)^\infty))$ is defined by the probability Π_{DY} together with equilibrium price map (11). The measure m_P is also obtained from m_{DY} and the map Φ^* in (11).

To simplify we use the notation $(p^c, P, q) = \Phi^*(R, y)$ to mean $(p_t^c, P_t, q_t) = \Phi^*(R_t, y_t)$ for all t. Now, for any set $A \in \mathcal{B}(D^\infty)$ define

(13) $$\Phi_D^*(A) = \{(p^c, P, q) \in (P^*)^\infty : (p^c, P, q) = \Phi^*(R, y) \text{ for } R \in A, \text{ some } y \in Y^\infty\}$$

and interpret (13) to identify the set of prices associated with any given set of infinite sequences of dividends. It then follows from the equilibrium map (11) that in equilibrium we must have

(14a) $$\Pi_D(A) = \Pi_P(\Phi_D^*(A)) \quad \text{for all } A \in \mathcal{B}(D^\infty)$$

and therefore

(14b) $$m_D(A) = m_P(\Phi_D^*(A)) \quad \text{for all } A \in \mathcal{B}(D^\infty).$$

(11), (12) and (14a)-(14b) provide the tools for stating the rationality conditions of the agents. Note that a belief Q^k is a probability on the space $((D \times P^* \times Y^k)^\infty, \mathcal{B}((D \times P^* \times Y^k)^\infty))$ since the agent is not assumed to know the map Φ^*. However, the data reveals that the empirical distribution of prices and dividends must conform to (14b) and this condition must be satisfied by Q^k. The following is then implied by the Conditional Stability Theorem (see Kurz and Schneider [1996]):

Lemma 2: Under the assumptions of Lemma 1 Q^k is a rational belief relative to Π_P if

i. $\Pi_{(DY)_{Y^k}} = Q^k_{Y^k}$

ii. $Q^k_D(A) = Q^k_P(\Phi^*_D(A)) = m_D(A)$ for all $A \in \mathcal{B}(D^\infty)$

iii. $Q^k_P = m_P$.

Using Lemma 2 we can define a Rational Belief Equilibrium as follows:

Definition 1: $\{\Pi_P, \{ (Q^k, \theta_i^k, B_i^k)$ for $k = 1, 2, \ldots, K$ and $i = 1, 2, \ldots M\}$ and (p_i^c, P_i, q_i) for $i = 1, 2, \ldots M \}$ constitute a *Rational Belief Equilibrium* (RBE) of the heterogenous agent stock market OLG economy if

1. Q^k is a rational belief relative to Π_P for $k = 1, 2, \ldots, K$ and Π_P is defined by Π_{DY} and

by the equilibrium map induced by $(Q^1, Q^2, \ldots, Q^K)$.

2. $(\theta_1^k, \theta_2^k, \ldots, \theta_M^k), (B_1^k, B_2^k, \ldots, B_M^k)$ are optimal agent allocations for $k = 1, 2, \ldots, K$

3. $\sum_{k=1}^{K} \theta_i^k = 1$ for all t and all i

4. $\sum_{k=1}^{K} B_i^k = 0$ for all t and all i.

Theorem 1: Under Assumptions 2.1 - 2.4 there exists an RBE[9].

2b *The Problem of Anonymity of an RBE*

We say that an RBE lacks anonymity *if equilibrium prices depend upon individual states of beliefs.* Consider the special case $D = \{R^H, R^L\}$, $K = 2$ with $Y^k = \{0, 1\}$ for $k = 1, 2$ which we use in the simulations below. Members of D identify the *state of the dividend process* while members of $Y^1 \times Y^2$ identify the *individual states of belief* of the agents. The equilibrium map (11) implies that the state space has 8 members and we think of $V = \{1, 2, \ldots, 8\}$ as *the price state space.* Although the equilibrium map of this RBE takes the form

$$(15) \qquad \begin{bmatrix} p_i \\ q_i \\ q_i \end{bmatrix} = \Phi^*(R_i, y_i^1, y_i^2) \qquad i = 1, 2, \ldots, 8$$

[9] A comment on multiple and sunspot equilibria is warranted at this point. The definition of an RBE does not address directly the issue of multiple equilibria. Keep in mind that we are modeling the economy as a dynamical system in which infinite random draws are associated with *definitive* sequences of realized economic allocations. This means that if at any date the economy can have multiple market clearing outcomes, then as part of the dynamics postulated there is a procedure for selecting a *particular* one of them which, in turn, generates the data observed in the economy. This, indirectly, addresses also the issue of sunspot equilibria. Such equilibria require a device for alternating random selections from among multiple equilibria of some underlying economy over time. If such an equilibrium is to be realized then this selection must be part of the description of the dynamical system. Moreover, a formal coordination among agents is feasible only if one of the observable exogenous variables provides the needed signal for joint action and all agents interpret this public signal in exactly the same way. In that case we must interpret the fluctuations of the economy which are due to the publicly observed sunspot variable as *exogenously caused.*

we can define an equivalent map Φ between the *indices* of the price states $\{1, 2, ..., 8\}$ and the vectors of *dividend states and states of belief* as follows:

$$(16) \qquad \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{bmatrix} = \Phi \begin{bmatrix} R_1 = R^H, y_1^1 = 1, y_1^2 = 1 \\ R_2 = R^H, y_2^1 = 1, y_2^2 = 0 \\ R_3 = R^H, y_3^1 = 0, y_3^2 = 1 \\ R_4 = R^H, y_4^1 = 0, y_4^2 = 0 \\ R_5 = R^L, y_5^1 = 1, y_5^2 = 1 \\ R_6 = R^L, y_6^1 = 1, y_6^2 = 0 \\ R_7 = R^L, y_7^1 = 0, y_7^2 = 1 \\ R_8 = R^L, y_8^1 = 0, y_8^2 = 0 \end{bmatrix}$$

We further assume below that the marginal distributions of the assessments y^1 and y^2 are i.i.d. with $P\{y_t^k = 1\} = \alpha_k$ for $k = 1, 2$. Rationality of beliefs implies that the agents have two pairs of matrices (F_1, F_2) and (G_1, G_2) such that the beliefs Q^1 and Q^2 are characterized as follows:

(17) Q^1 *for agent 1* : adopt F_1 if $y_t^1 = 1$ $\qquad$ Q^2 *for agent 2*: adopt G_1 if $y_t^2 = 1$

adopt F_2 if $y_t^1 = 0$ $\qquad$ adopt G_2 if $y_t^2 = 0$.

The implied rationality conditions are $\alpha_1 F_1 + (1 - \alpha_1)F_2 = \Gamma$ and $\alpha_2 G_1 + (1 - \alpha_2)G_2 = \Gamma$ where Γ is a Markov matrix which defines the stationary measure. It is then clear that RBE defined by (15) or (16) lacks anonymity since a change in the state of belief of an agent causes the equilibrium price to change. It is also clear that the equilibrium concept adopted here requires the agents to ignore their effects on equilibrium prices.

In a finite economy all competitive equilibria fail to be anonymous and hence the problem above is no different from the corresponding problems which arise in equilibria with finite number of agents. The traditional tool to explore anonymity has been the "replica economy" and this is the motivation for our adoption of this tool here. The rest of this paper is an examination of the consequences of this approach.

3. Rational Belief Equilibria with Social States of Belief

3a *Anonymity in a Large Replica Economy*

We start by reconsidering the RBE concept defined in Section 2 to highlight the decreased effect of each individual's belief on the equilibrium outcome as the economy becomes large. The implication is that equilibrium prices are functions of the social *distribution* of beliefs rather than functions of the vectors of individual states of belief. This means that in large economies endogenous uncertainty impacts aggregate economic fluctuations via the *distribution* of beliefs in the economy. Such distributions define the "social states of beliefs" and these are determined by *the structure of correlation among the individual beliefs.*

Consider the model of Section 2 and restrict attention to the financial structure used by KB [1997] which consists of one stock and one bond. Suppose, however, that each one of the K agents in the model is now considered to be a "type" *with N replicas* and that instead of 2K agents (K young and K old) we now have 2KN agents. As is standard, the economy becomes large if N is large. The N replicas have the same utility, endowment and belief but not necessarily the same realized assessment. Hence, for all $n = 1, 2, \dots, N$, $y^{k,n} \in Y^k$ where $y^{k,n}$ is the i.i.d assessment of replica agent n of type k and the central question of interest is the joint distribution of the assessments. Since the beliefs of the agents are determined by the $y^{k,n}$ the N agents of type k may not hold the same conditional belief. In the extreme case these may be perfectly correlated so that all of them take the same value in Y^k. Indeed, one way to interpret the results of a small economy (consisting of, say, two agents) is to observe that they apply to a large economy in which the N assessments are perfectly correlated. This suggests two different types of correlations in society. The first is a correlation among $(y_t^{k,1}, y_t^{k,2}, \ldots, y_t^{k,N})$ which is "within type" correlation that determines the distribution of assessments of type k agents. We define each such possible distribution as a "type-state." The second is the correlation among the type-states themselves which, in turn, determines the aggregate social states. Why are the correlations "within" a type and "across" types not the same? Without a formal model of social communication to explain this assumption we can suggest that one must visualize agents of the same type as associating with each other in a different manner than agents of different types and communicating with each other via different and more complex channels than the public channels used by agents of different types.

To be concrete suppose that for all k, $Y^k = \{1, 2, \ldots, L\}$ and $y_t^{k,n}$ are i.i.d marginally with probabilities $(\alpha_{k1}, \ldots, \alpha_{kL})$ hence the model of Section 2 has $|D|L^{KN}$ individual states. It follows from the optimization (3)-(4) that the demand functions of agent (k, n) have the form

(18a) $$\theta_t^{k,n} = \varphi_\theta^k(R_t, p_t^c, P_t, q_t, y_t^{k,n})$$

(18b) $$B_t^{k,n} = \varphi_B^k(R_t, p_t^c, P_t, q_t, y_t^{k,n}).$$

(18a)-(18b) point out that all agents of type k with the same realized assessment *have the same demand for securities*. Consider any individual state i of the $|D|L^{KN}$ states. Denote by $s_i(k,\ell)$ the number of agents of type k with assessments taking the value ℓ in state i. It follows from

(18a)-(18b) that the market clearing conditions in the RBE of Section 2 take the exact form

$$\text{(19a)} \qquad \sum_{k=1}^{K}\sum_{\ell=1}^{L} \frac{s_i(k,\ell)}{N}\varphi_\theta^k(R_i, p_i^c, P_i, q_i, \ell) = 1 \qquad i = 1,2,3,\ldots,|D|L^{NK}.$$

$$\text{(19b)} \qquad \sum_{k=1}^{K}\sum_{\ell=1}^{L} \frac{s_i(k,\ell)}{N}\varphi_B^k(R_i, p_i^c, P_i, q_i, \ell) = 0 \qquad i = 1,2,3,\ldots,|D|L^{NK}.$$

Now, each type has only L *different* demand functions hence variability in (19a)-(19b) is caused by the different distributions (type-states) of the assessments of the N agents of type k. To see that the number of distinct distributions $\{\frac{s_i(k,\ell)}{N}, \ell = 1,2,\ldots,L\}$ is dramatically less than L^{KN}, consider the case $L = 2$. For each k there are 2^N permutations of the assessments but the set of distinct values that may be taken by $s_i(k,\ell)$ is $\{0, 1, \ldots, N\}$ with at most N+1 distributions $\{(0,1), (1/N, (N-1)/N), \ldots, (n/N, (N-n)/N), \ldots, (1, 0)\}$. Jointly for all the K types there are only $(N+1)^K$ distinct distributions. For $L = 3$ the number of distinct distributions (i.e. three tuples) for each k is $\sum_{\tau=0}^{N}(\tau+1)$ and for any L this number is $M_{NL} = \sum_{\tau_L=0}^{N}\sum_{\tau_{L-1}=0}^{\tau_L}\ldots\sum_{\tau_4=0}^{\tau_5}\sum_{\tau=0}^{\tau_4}(\tau+1)$. The implication is that the number of *distinct* prices in (19a)-(19b) is the relatively small number of $|D|(M_{NL})^K$ rather than $|D|L^{KN}$ and hence, for large N *most of the equations in (19a)-(19b) are redundant*. We thus arrive at the following:

Observation 1: Even in a finite economy the number of distinct equilibrium prices is much smaller than the number of individual states and hence the equilibrium map in terms of individual states such as (15) or (16) is generically not invertible. The number of distinct prices is determined by the number of distinct vectors of type-states of beliefs and exogenous states.

Under the assumption that the assessments of all agents are independent, Observation 1 implies that a finite replica economy *tends to anonymity as the number of replicas increases* since the effect of the belief of each agent on the type-states becomes small. Consequently, in the case of independence a finite but large economy is approximately anonymous and we may as well assume that the agent neglects the minimal effect his belief has on the type-states and thus on prices. This conclusion is completely analogous to competitive equilibria of a replica economy.

Now suppose that N is large. In the case of independence within and across types the zero-one law implies that with probability 1 the assessments of type k have only *one* limit distribution $(\alpha_{k1}, \ldots, \alpha_{kL})$ and hence at any date the type-state of belief of type k agents is represented by the constant vector $(\alpha_{k1}, \ldots, \alpha_{kL})$. Similarly, there is only one joint distribution for all types. If the state space D of the exogenous process has a dimension |D| then it follows that with probability one the system of equations (19a)-(19b) is reduced to |D| independent equations implying that there are, at most, |D| distinct prices associated with a constant vector of distributions of beliefs. This is the case of <u>a single social state of belief</u> in the limit economy and our first task below is to explore the nature of endogenous uncertainty in such an economy. The case of correlation among the beliefs of the agents leads to very different economies and since such correlation is central to the conclusions of this paper we comment on this issue now.

Extensive work has explored in recent years the implications of alternative patterns of economic interactions[10] and the main conclusion of this literature is that relatively simple local interactions are sufficient to induce a limiting behavior which is a random variable rather than a

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[10] See for example Brock [1993],[1996], Brock and Durlauf [1995], [1997], Durlauf [1993], [1994] and Föllmer [1974]. For a related approach see Benerjee [1992], Bikhchandani, Hirshleifer and Welch [1992] and Scharfstein and Stein [1990].

constant. As an illustration of an explicit analysis of such interactions the reader may consult the procedure used by Brock [1993] in which he utilizes the results of Kac [1968] to derive the limiting behavior of the system. Given the extensive amount of interaction among participants in financial markets, one must therefore conclude that the assumption of independent assessments is an extreme one and the case of correlation among beliefs of agents is the norm.

In the case of correlation we do not have general convergence results but even if the limit distributions exist, one cannot ensure anonymity. This is because the assessment of some agent may become an atom and consequently we can only make the following, self-evident, comment:

Observation 2: Assume the existence of limit distributions of beliefs across all types in the case of correlation. The replica economy tends to anonymity as N becomes large if the limit distributions do not have an atom concentrated on the assessment of any one agent.

When correlation among assessments is present, boththe limit distributions (type-states) of each type as well as the joint distribution over all types are random variables. In the applications below we make the following assumption.

Assumption 2.5: *There is a finite number of type-states of beliefs in the economy.*

Assumption 2.5 holds in any finite economy. It would also be satisfied in an infinite economy in which the limit random variables $\{ \frac{s_i(k, \ell)}{N}, \ \ell = 1, 2, \ldots, L \ , \ k = 1, 2, \ldots, K \}$ are well defined random variables and with probability 1 take only a finite number of values. Anonymity holds if these limit random variables are not correlated with the assessment of any one agent. The thrust of Assumption 2.5 is that only a finite number of market clearing conditions in (19a)-(19b) are applicable to an RBE with social states. This is because almost all market clearing conditions in (19a)-(19b) apply to individual states i that occur with probabilities which tend to zero as N goes to infinity and hence are ignored. We now formulate the concept of "social states of belief".

3b *Social States of Belief as Distributions*

The concept of *social states of belief*, inspired by concepts of *collective risk* developed in Malinvaud [1972], [1973] and in Cass, Chichilnisky and Wu [1996], can now be defined in a natural way. Let M^k_{ML} be the number of distinct distributions $\{ \frac{s_i(k,\ell)}{N}, \ \ell = 1,2,\ldots,L \}$ for each k. In the finite economy one computes these distributions for each of the vectors of individual states i but in the infinite economy one takes the limit as N goes to infinity and ignores individual states with zero probability. We now introduce notation to describe members of this set of distributions. Thus, for each of the M^k_{ML} distributions we use the following notation:

$\zeta^{k,\ell}$:= the proportion of agents of type k with assessment variables taking the value ℓ.

Definition 2: A *type-state* of agents of type k is a distribution of the form

$$\zeta^k = (\zeta^{k,1}, \zeta^{k,2}, \ldots, \zeta^{k,L}) \text{ such that } \zeta^{k,\ell} \text{ are non-negative numbers and } \sum_{\ell=1}^{L} \zeta^{k,\ell} = 1 .$$

Letting $S^k_B = \{ \zeta^k : \zeta^k \text{ is a type state for type k} \}$ then S^k_B has M^k_{ML} members. A *social state of belief in the economy* is a vector of distributions $\zeta = (\zeta^1, \zeta^2, \ldots, \zeta^K)$, $\zeta^k \in S^k_B$.

The set of possible social states of belief is then $S_B = \left\{ \zeta : \zeta = (\zeta^1, \zeta^2, \ldots, \zeta^K) \ , \ \ \zeta^k \in S_B^k \right\}$ and this set has $M_{ML} = \prod_{k=1}^{K} M_{ML}^k$ members. We then define naturally:

Definition 3: A *social state* for the economy is a pair consisting of a dividend state and a state of belief in the economy. It consists of a $|D| + KL$ tuple

(20) $$(d, \zeta) \equiv (d, \zeta^1, \zeta^2, \ldots, \zeta^K) \ , \ \ d \in D \ , \ \ \zeta \in S_B .$$

Denote by M the number of possible social states and we know that $M = |D| M_{NL}$. Now, list the M social states by the index s and this set is then defined by

(21) $$\hat{S} = \{ (d_s, \zeta_s^1, \zeta_s^2, \ldots, \zeta_s^K) \ , \ \ d_s \in D \ , \ \ \zeta_s^k \in S_B^k \ , \ \ s = 1, 2, \ldots, M \} .$$

Since the $y^{k,n}$ are not observable and the agents do not know the equilibrium map, one may think of social states as a *listing of the index* s of the states in (21) and define the price state space to be

(22) $$S = \{1, 2, \ldots, M).$$

The difference between the state spaces in (21) and (22) is analogous to the distinction between the maps (10) and (11). In sum, we have the following observation:

Observation 3: Given the market clearing conditions (19a)-(19b) then with probability 1 there are *at most* M distinct social states which induce at most M different aggregate excess demand functions and hence there are at most M *distinct* equilibrium prices.

It then follows that we may rewrite the system (19a)-(19b) in the form

$$\text{(23a)} \qquad \sum_{k=1}^{K}\sum_{\ell=1}^{L} \zeta_s^{k,\ell}\, \varphi_\theta^k(R_s, p_s^c, P_s, q_s, \ell) = 1 \qquad s = 1, 2, \ldots, M\,.$$

$$\text{(23b)} \qquad \sum_{k=1}^{K}\sum_{\ell=1}^{L} \zeta_s^{k,\ell}\, \varphi_B^k(R_s, p_s^c, P_s, q_s, \ell) = 0 \qquad s = 1, 2, \ldots, M\,.$$

The interpretation of (23a)-(23b) leads to the final clarification of the nature of an equilibrium with social states of belief. Conditional on their assessments agents carry out the optimization in (3)-(4) leading to demand functions which depend upon the private value of their own assessment variable. These private assessments are then aggregated into distributions $(\zeta_s^1, \zeta_s^2, \ldots, \zeta_s^K)$ which constitute social states of belief. Equilibrium prices and dividends are then maps defined on the social state space and are written in the form (R_s, p_s^c, P_s, q_s).

In the redefined economy what matters in equilibrium is the *distribution* of beliefs rather than the belief of any one agent. However, the distribution of beliefs in society may exhibit a complex structure even if the assessment of each agent is i.i.d. As a result, the aggregate implications of our approach depend decisively upon the structure of correlation among agents.

It is generally difficult to study analytically the impact of different correlation structures on the long term volatility of the implied equilibria and the appropriate tool to carry out such an examination is the method of simulation. Accordingly, the simulation work of the next section aims to exhibit how the model of an RBE with social states helps the understanding of the factors which determine asset price volatility in general and the equity premium in particular. More specifically, we focus on the effect of the correlation structure within the model on the volatility characteristics of the equilibrium. The five measures of uncertainty and volatility that we focus on are: (i) the equity premium, (ii) the riskless rate, (iii) the standard deviation of the risky returns,

(iv) the standard deviation of the riskless rate and (v) the variance of the price/dividend ratio.

4. Market Volatility and Correlation Among Social States of Belief: Simulation Analysis

In order to proceed to the simulation results we need first to reformulate the above model to conform to the growth assumptions of KB [1997]. In order to accomplish this we briefly review the assumptions made by KB [1997].

4a *A Brief Review of the KB [1997] Assumptions*

The OLG model used by KB [1997] aims to approximate the model of Mehra and Prescott [1985] (in short MP[1985]). Accordingly, KB [1997] assume an economy with two agents and no replica: $K = 2$, $N = 1$, $L = 2$, $Y^k = \{1, 0\}$ for $k = 1, 2$ with the time additive utility function $\frac{1}{1-\gamma_k} c^{1-\gamma_k}$ and a constant discount factor β_k. Also, the model conforms to the real growth assumptions made by MP[1985]. Under these assumptions $\{R_t, t = 1, 2, \ldots\}$ satisfies

$$R_{t+1} = d_t R_t.$$

The growth rate process $\{d_t, t = 1, 2, \ldots\}$ is then assumed to be a stationary and ergodic Markov process on the state space $\{d^H, d^L\}$ with a transition matrix

$$\begin{bmatrix} \phi, & 1 - \phi \\ 1 - \phi, & \phi \end{bmatrix}. \tag{24}$$

MP [1985] assume that $d^H = 1.054$, $d^L = .982$ and $\phi = .43$. Since this implies that over time agents experience a rise in the level of dividends, it requires us to redefine the budget constraints. We revise the assumption that Ω^k is constant and instead assume that $\omega^k = \frac{\Omega_t^k}{R_t}$ for $k = 1, 2$ are constant over time. This in accord with the assumption often used (see MP [1985]) that the growth rate of the output of the economy *as a whole* is a stationary Markov process with a transition matrix (24). Now denote by $b_t^k = \frac{B_t^k}{R_t}$ the bond/dividend ratio of agent k and by $p_t = \frac{P_t}{R_t}$ the price/dividend ratio at date t. Normalizing by setting $p_t^c = 1$ for all t and using the notation introduced, the budget equations (4a) - (4b) are now written as

(25a) $$x_t^{1k} = [\omega^k - p_t\theta_t^k - q_t b_t^k]R_t$$

(25b) $$x_{t+1}^{2k} = [\theta_t^k(p_{t+1} + 1)d_{t+1} + b_t^k]R_t.$$

The Markov assumptions imply that given assessments (y_t^1, y_t^2), the market clearing conditions are $\theta_t^1(p_t, q_t, d_t, y_t^1) + \theta_t^2(p_t, q_t, d_t, y_t^2) = 1$ and $b_t^1(p_t, q_t, d_t, y_t^1) + b_t^2(p_t, q_t, d_t, y_t^2) = 0$. It is then clear that the implied equilibrium map has the exact form specified in (15), with an index map such as (16) and a price state space $S = \{1, 2, \ldots, 8\}$. KB [1987] also assume that the marginal distributions of the assessment variables of the two agents are i.i.d. with the probability of 1 being α_1 and α_2, respectively.

Denote by $Q^k(j|s, y^k)$ agent k's conditional probability of price state j given price state s and the value of y^k (but ignoring his effect on prices). The first order conditions can then be written in terms of price states for $k = 1, 2$ and $j, s = 1, 2, \ldots, 8$ as follows

(26a) $$-(\omega^k - \theta_s^k p_s - b_s^k q_s)^{-\gamma_k} p_s + \beta_k \sum_{j=1}^{8} (\theta_s^k(p_j + 1)d_j + b_s^k)^{-\gamma_k}(p_j + 1)d_j Q^k(j|s, y^k) = 0$$

(26b) $$-(\omega^k - \theta_s^k p_s - b_s^k q_s)^{-\gamma_k} q_s + \beta_k \sum_{j=1}^{8} (\theta_s^k(p_j + 1)d_j + b_s^k)^{-\gamma_k} Q^k(j|s, y^k) = 0.$$

Once (Q^k, ω^k) are specified for $k = 1, 2$ one computes the demand functions (θ_s^k, b_s^k) as a function of the 8 prices. In equilibrium

(27a) $$\theta_s^1 + \theta_s^2 = 1 \quad \text{for } s = 1, 2, \ldots, 8$$

(27b) $$b_s^1 + b_s^2 = 0 \quad \text{for } s = 1, 2, \ldots, 8.$$

(26a)-(26b) and (27a)-(27b) constitute a system of 48 equations in prices and quantities which are the basis of the simulation results of KB [1997].

4b *Reformulation of the Model to a Replica Economy with Large N*

We now make use of the conclusions of Section 3a. It follows from (23a) - (23b) that for any N the first order conditions (26a)-(26b) remain the same and consequently the implied demand functions are entirely determined by the *type* of an agent and the value of his assessment variable. Since in the simulations below we assume that K = 2 and L = 2, there are two pairs of demand functions $\theta^{k,n} = \varphi_\theta^k(R, p, q, y^{k,n})$ and $b^{k,n} = \varphi_b^k(R, p, q, y^{k,n})$ for n = 1, 2, ... , N and for k = 1, 2.

We also assume that the marginal distributions of $y^{k,n}$ are i.i.d. with parameters α_1 and α_2. We have noted that independence among the assessments of each type implies that for large N the type-state is almost surely constant at $(\alpha_1, 1-\alpha_1)$. Hence we can express the correlation among the assessments of a type by specifying the type-states to be a random variables with distributions to be specified. Size limitations in the simulations below lead us to make simplifying assumptions on the joint distribution of the two type-states in the model below:

1. The support of each of the distributions of the type-states contains three points.

2. The marginal distribution of the type-states is i.i.d.

3. In most calculations we set $\alpha_1 = \alpha_2 = .57$ as will be motivated later. We use the α_k notation for general discussion.

More specifically, in all simulations we assume that the support of the distribution of the two type-states is

$$\{(.85, .15), (.57, .43), (.25, .75)\} \quad \text{for} \quad k = 1, 2,$$

This reflects the idea that correlation within each type results in probability being placed not only on the type-state (.57, .43) which is sure to occur without correlation, but also on two other states. We shall also assume that marginal distributions of the two type-states are i.i.d. with probabilities

$$(.5(1-\chi_1)\,,\, \chi_1\,,\, .5(1-\chi_1))$$
$$(.5(1-\chi_2)\,,\, \chi_2\,,\, .5(1-\chi_2)).$$

These assumptions are compatible with Assumptions 2.4-2.5 and with the standing assumption that the marginal distribution of $y^{k,n}$ for each k and n is i.i.d. The special assumption of i.i.d. type-state marginals is justified by the technical fact that the representation of correlation among social states is simplified by i.i.d. marginals of the type-states.

In all cases considered below we have 2 dividend states, 2 agent types and 3 type-states for each agent type. This implies that there are 18 possible equilibrium prices and 9 social states of belief. <u>The equilibrium map</u> is defined by the following: prices 1 - 9 are

$$\begin{array}{llll} & 1.\ (d^H,(.85, .15), (.85, .15)) & 4.\ (d^H,(.57, .43),(.85, .15)) & 7.\ (d^H,(.25, .75),(.85, .15)) \\ (28) & 2.\ (d^H,(.85, .15), (.57, .43)) & 5.\ (d^H,(.57, .43),(.57, .43)) & 8.\ (d^H,(.25, .75),(.57, .43)) \\ & 3.\ (d^H,(.85, .15), (.25, .75)) & 6.\ (d^H,(.57, .43),(.25, .75)) & 9.\ (d^H,(.25, .75),(.25, .75)). \end{array}$$

Prices 10-18 are defined similarly but with d^L replacing d^H. Turning to the stochastic structure of the joint process of dividend growth and social states of belief we assume that it is a stable Markov process. As in KB [1997], we specify the *stationary measure* by selecting the following 9×9 matrix to be the transition among the 9 social states of belief. The marginals of this matrix conform to the specified marginal i.i.d. of the type-states

$$A = \left[\begin{array}{l}
a_1^1,\ a_1^2,\ \frac{1}{2}(1-\chi_1)-a_1^1-a_1^2,\ a_1^3,\ a_1^4,\ \chi_1-a_1^3-a_1^4,\ \frac{1}{2}(1-\chi_2)-a_1^1-a_1^3,\ \chi_2-a_1^2-a_1^4,\ a_1^1+a_1^2+a_1^3+a_1^4-\frac{1}{2}(\chi_1+\chi_2) \\
a_2^1,\ a_2^2,\ \frac{1}{2}(1-\chi_1)-a_2^1-a_2^2,\ a_2^3,\ a_2^4,\ \chi_1-a_2^3-a_2^4,\ \frac{1}{2}(1-\chi_2)-a_2^1-a_2^3,\ \chi_2-a_2^2-a_2^4,\ a_2^1+a_2^2+a_2^3+a_2^4-\frac{1}{2}(\chi_1+\chi_2) \\
a_3^1,\ a_3^2,\ \frac{1}{2}(1-\chi_1)-a_3^1-a_3^2,\ a_3^3,\ a_3^4,\ \chi_1-a_3^3-a_3^4,\ \frac{1}{2}(1-\chi_2)-a_3^1-a_3^3,\ \chi_2-a_3^2-a_3^4,\ a_3^1+a_3^2+a_3^3+a_3^4-\frac{1}{2}(\chi_1+\chi_2) \\
a_4^1,\ a_4^2,\ \frac{1}{2}(1-\chi_1)-a_4^1-a_4^2,\ a_4^3,\ a_4^4,\ \chi_1-a_4^3-a_4^4,\ \frac{1}{2}(1-\chi_2)-a_4^1-a_4^3,\ \chi_2-a_4^2-a_4^4,\ a_4^1+a_4^2+a_4^3+a_4^4-\frac{1}{2}(\chi_1+\chi_2) \\
a_5^1,\ a_5^2,\ \frac{1}{2}(1-\chi_1)-a_5^1-a_5^2,\ a_5^3,\ a_5^4,\ \chi_1-a_5^3-a_5^4,\ \frac{1}{2}(1-\chi_2)-a_5^1-a_5^3,\ \chi_2-a_5^2-a_5^4,\ a_5^1+a_5^2+a_5^3+a_5^4-\frac{1}{2}(\chi_1+\chi_2) \\
a_6^1,\ a_6^2,\ \frac{1}{2}(1-\chi_1)-a_6^1-a_6^2,\ a_6^3,\ a_6^4,\ \chi_1-a_6^3-a_6^4,\ \frac{1}{2}(1-\chi_2)-a_6^1-a_6^3,\ \chi_2-a_6^2-a_6^4,\ a_6^1+a_6^2+a_6^3+a_6^4-\frac{1}{2}(\chi_1+\chi_2) \\
a_7^1,\ a_7^2,\ \frac{1}{2}(1-\chi_1)-a_7^1-a_7^2,\ a_7^3,\ a_7^4,\ \chi_1-a_7^3-a_7^4,\ \frac{1}{2}(1-\chi_2)-a_7^1-a_7^3,\ \chi_2-a_7^2-a_7^4,\ a_7^1+a_7^2+a_7^3+a_7^4-\frac{1}{2}(\chi_1+\chi_2) \\
a_8^1,\ a_8^2,\ \frac{1}{2}(1-\chi_1)-a_8^1-a_8^2,\ a_8^3,\ a_8^4,\ \chi_1-a_8^3-a_8^4,\ \frac{1}{2}(1-\chi_2)-a_8^1-a_8^3,\ \chi_2-a_8^2-a_8^4,\ a_8^1+a_8^2+a_8^3+a_8^4-\frac{1}{2}(\chi_1+\chi_2) \\
a_9^1,\ a_9^2,\ \frac{1}{2}(1-\chi_1)-a_9^1-a_9^2,\ a_9^3,\ a_9^4,\ \chi_1-a_9^3-a_9^4,\ \frac{1}{2}(1-\chi_2)-a_9^1-a_9^3,\ \chi_2-a_9^2-a_9^4,\ a_9^1+a_9^2+a_9^3+a_9^4-\frac{1}{2}(\chi_1+\chi_2)
\end{array}\right].$$

Apart from the parameters χ_1 and χ_2 which are determined by the agents, the matrix A has 36 parameters which specify the joint distribution and hence the correlation among the social states of belief. These are not free parameters and we specify below the restrictions on them. To allow for the possibility of a dividend effect on the distribution of assessments we employ a second matrix B which has the same structure as A except that it is defined by parameters b_i^j. As in KB [1997] the stationary measure is identified by the 18×18 Markov transition matrix of the form:

(29) $$\Gamma = \begin{bmatrix} \phi A, & (1-\phi)A \\ (1-\phi)B, & \phi B \end{bmatrix}$$

where A and B are 9×9 matrices as defined above. Each is characterized by the 36 parameters $a=(a^1,a^2,a^3,a^4)$ where $a^j=(a_1^j,a_2^j,\ldots,a_9^j)$, $j=1,2,3,4$ and $b=(b^1,b^2,b^3,b^4)$ where $b^j=(b_1^j,b_2^j,\ldots,b_9^j)$, $j=1,2,3,4$. The first 9 rows of the matrix Γ are identified with d^H and the 9 specified states of belief while the second 9 rows of Γ are identified with d^L and the 9 states of belief. With this identification Γ satisfies the required properties: the marginal of Γ on the dividends is the matrix (24) and the marginals on the type-states are as specified. The simulation model would be completed once we specify the beliefs of the two types of agents. The rationality conditions are similar to KB [1997].

An inspection of the matrices A, B and Γ reveal that there are feasibility conditions which must be satisfied by the parameters a and b. More specifically there are 90 inequality constraints which the parameters must satisfy and these are as follows: for i = 1, 2, ... , 9

$$
(30)\qquad
\begin{array}{ll}
a_i^1 + a_i^2 \le \frac{1}{2}(1-\chi_1) & b_i^1 + b_i^2 \le \frac{1}{2}(1-\chi_1) \\
a_i^3 + a_i^4 \le \chi_1 & b_i^3 + b_i^4 \le \chi_1 \\
a_i^1 + a_i^3 \le \frac{1}{2}(1-\chi_2) & b_i^1 + b_i^3 \le \frac{1}{2}(1-\chi_2) \\
a_i^2 + a_i^4 \le \chi_2 & b_i^2 + b_i^4 \le \chi_2 \\
a_i^1 + a_i^2 + a_i^3 + a_i^4 \ge \frac{1}{2}(\chi_1+\chi_2) & b_i^1 + b_i^2 + b_i^3 + b_i^4 \ge \frac{1}{2}(\chi_1+\chi_2)
\end{array}
$$

In addition we have the 8 conditions which specify that the rows of A and B sum to 1.

The marginal distributions of $y^{1,n}$ and $y^{2,n}$ are i.i.d. with $P\{y_t^{k,n}=1\}=\alpha_k$ for k = 1, 2. This means that as in (17) the agents have two pairs of matrices (F_1, F_2) and (G_1, G_2) such that the conditional beliefs $Q^{1,n}$ and $Q^{2,n}$ given the assessments are as follows:

$$
Q_t^{1,n}(j\,|\,s,y_t^{1,n}) = \begin{cases} F_1^{sj} & \text{if} \quad y_t^{1,n}=1 \\ F_2^{sj} & \text{if} \quad y_t^{1,n}=0 \end{cases}
\qquad
Q_t^{2,n}(j\,|\,s,y_t^{2,n}) = \begin{cases} G_1^{sj} & \text{if} \quad y_t^{2,n}=1 \\ G_2^{sj} & \text{if} \quad y_t^{2,n}=0 \end{cases}
$$

for $n = 1, 2, \ldots, N$ where C_1^{sj} is the (s, j) element of matrix C_1. Rationality of beliefs requires

(31) $\alpha_1 F_1 + (1-\alpha_1)F_2 = \Gamma$ and $\alpha_2 G_1 + (1-\alpha_2)G_2 = \Gamma$.

The matrices (F_1, F_2, G_1, G_2) are defined by two sets of 18 parameters $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_{18})$ and $\mu = (\mu_1, \mu_2, \ldots, \mu_{18})$ which will be motivated later. To describe how they are constructed we introduce the notation for the row vectors of A and B:

$$A^j = (a_j^1,\ a_j^2, \ldots,\ a_j^1 + a_j^2 + a_j^3 + a_j^4 - \frac{1}{2}(\chi_1 + \chi_2))$$
$$B^j = (b_j^1,\ b_j^2, \ldots,\ b_j^1 + b_j^2 + b_j^3 + b_j^4 - \frac{1}{2}(\chi_1 + \chi_2)).$$

With this notation define the 4 matrix functions of a vector $z = (z_1, z_2, \ldots, z_{18})$ of real numbers:

(32a)
$$A_1(z) = \begin{bmatrix} z_1 A^1 \\ z_2 A^2 \\ \cdots \\ z_9 A^9 \end{bmatrix}, \qquad A_2^{\phi}(z) = \begin{bmatrix} \dfrac{1-\phi z_1}{1-\phi} A^1 \\ \dfrac{1-\phi z_2}{1-\phi} A^2 \\ \cdots \\ \dfrac{1-\phi z_9}{1-\phi} A^9 \end{bmatrix}$$

(32b)
$$B_1(z) = \begin{bmatrix} z_{10} B^1 \\ z_{11} B^2 \\ \cdots \\ z_{18} B^9 \end{bmatrix}, \qquad B_2^{\phi}(z) = \begin{bmatrix} \dfrac{1-(1-\phi) z_{10}}{\phi} B^1 \\ \dfrac{1-(1-\phi) z_{11}}{\phi} B^2 \\ \cdots \\ \dfrac{1-(1-\phi) z_{18}}{\phi} B^9 \end{bmatrix}.$$

Using (32a)-(32b) define the matrices

(33)
$$F_1(\lambda) = \begin{bmatrix} \phi A_1(\lambda),\ (1-\phi)A_2^{\phi}(\lambda) \\ (1-\phi)B_1(\lambda),\ \phi B_2^{\phi}(\lambda) \end{bmatrix} \qquad G_1(\mu) = \begin{bmatrix} \phi A_1(\mu),\ (1-\phi)A_2^{\phi}(\mu) \\ (1-\phi)B_1(\mu),\ \phi B_2^{\phi}(\mu) \end{bmatrix}$$

and (F_2, G_2) determined by (31). The selection of the vectors (λ, μ) is restricted by 108 inequality constraints which define the feasible region. These constraints are as follows:

$$\begin{array}{llll}
 & \lambda_s \le \dfrac{1}{\phi} & \mu_s \le \dfrac{1}{\phi} & \text{for } s = 1, 2, \dots, 9 \\[2ex]
 & \lambda_s \le \dfrac{1}{1-\phi} & \mu_s \le \dfrac{1}{1-\phi} & \text{for } s = 10, 11, \dots, 18 \\[2ex]
(34) & \lambda_s \le \dfrac{1}{\alpha_1} & \mu_s \le \dfrac{1}{\alpha_2} & \text{for } s = 1, 2, \dots, 18 \\[2ex]
 & \lambda_s \ge \dfrac{\alpha_1 + \phi - 1}{\phi \alpha_1} & \mu_s \ge \dfrac{\alpha_2 + \phi - 1}{\phi \alpha_2} & \text{for } s = 1, 2, \dots, 9 \\[2ex]
 & \lambda_s \ge \dfrac{\alpha_1 - \phi}{(1-\phi)\alpha_1} & \mu_s \ge \dfrac{\alpha_2 - \phi}{(1-\phi)\alpha_2} & \text{for } s = 10, 11, \dots, 18
\end{array}$$

To motivate this construction note that the intensity parameters λ_s and μ_s are multiplied by the rows of A and B and hence are proportional changes of the conditional probabilities of the two sets of nine states (1, 2, ..., 9) and (10, 11, ..., 18) *relative to the stationary measure* represented by Γ. It should be clear that up until now the assessment variables of the agents had no economic meaning. They attain meaning only when the agents specify how they interpret these variables in generating conditional probability beliefs. For example, $\lambda_s > 1$ implies *increased* probabilities of states (1, 2, ..., 9) in F_1 relative to Γ of an agent of type 1 given that he is in state s. This means that the assessment variables induce more "optimism" or "pessimism" about the prospects of prices (1, 2, ..., 9) at $t + 1$ relative to Γ. To see why, suppose that $\lambda_1 > 1$ and that at some date t state $s = 1$ occurs so that (p_1, q_1) is realized. In that case type 1 agents with assessments $y_t^{1,n} = 1$ use matrix F_1 to forecast prices at t +1 and by (32a)-(32b) they are more optimistic (relative to Γ) about the probabilities of $((p_1, q_1), (p_2, q_2), \ldots, (p_9, q_9))$ at $t + 1$. The equilibrium map (28) shows that conditionally on (p_1, q_1), 85% of type 1 agents are then optimistic about the prospects of the first 9 prices.

We observe that conditionally upon (p_1, q_1) (i.e. in state 1), 15% of type 1 agents have an assessment $y_t^{1,n} = 0$ and consequently use matrix F_2 to forecast prices at $t + 1$. If $\lambda_1 > 1$ it follows from (32a)-(32b) that they are more pessimistic (relative to Γ) about the probabilities of the nine prices $((p_1, q_1), (p_2, q_2), \ldots, (p_9, q_9))$ at $t + 1$. The converse applies when $\lambda_s < 1$. We also note that the possible dependence of the deviations (λ_s, μ_s) from Γ on the state s is very important since this is a way for the agents *to condition beliefs on prices*. Formally, if λ_s or μ_s vary with s then we say that *the impact of the assessment variables on the forecasts of the agents is price dependent*. This fact is central to the interpretation of our results below.

We note in summary that a simulation model requires the specification of 108 parameters: 36 for matrix A, 36 for matrix B and 36 intensity parameters (λ, μ). However, these belief parameters are restricted by the following 242 rationality conditions:

(i) 98 equality and inequality restrictions (30) on the matrices A and B.

(ii) 36 direct rationality conditions (31) on the structure of the matrices (F_1, F_2, G_1, G_2)

(iii) 108 inequality restrictions (34) on the choices of (λ, μ)

The simulations focus on the factors which generate endogenous uncertainty in the replica RBE with types and the determinants of the equity risk premium There are four such factors:

(i) *Deviations over time of the intensity parameters (λ, μ) from 1* reflecting the non-stationarity of beliefs of the agents. Hence, aggregate volatility may be caused by the fact that the conditional probability beliefs of the agents may vary over time.

(ii) *Correlation of assessments within types* represented by the existence of type-states other than the type-state $(\alpha_k, 1 - \alpha_k)$.

(iii) *Correlation among type-states* (i.e. across types) represented by the vectors (a, b) of parameters inducing a joint distribution of the assessments which is Markov and not i.i.d.

(vi) *Price dependency of the intensity variables* λ_s *and* μ_s.

The objective of the parameter specification below is to study the configurations which generate equilibrium volatility and equity premia. These specifications do not represent *illustrations* of parameters which generate volatility and premia but rather, they are the only configuration which we found to generate volatility and premia in the range observed in the U.S. economy and hence their interpretation provides an explanation of the volatility and premia which arise in the real economy. Some discussion of the results is provided below.

4c *Simulation Results*

As in KB [1997] the focus of the simulation results is the equity premium and related statistics. We thus report in each table the following key variables:

ρ - the long term mean equity risk premium; historically around 6%,

σ_r - the long term standard deviation of the risky returns on equity; historically about 18%[11],

r^F - the long term riskless rate on one period loans; historically .5%-1.0%,

σ_F - the long term standard deviation of the short term riskless rate; historically about 6%,

σ_p^2 - the long term variance of the price/dividend ratio; observations on σ_p^2 do not

[11] This corrects the confusing practice in Kurz and Beltratti [1997] and Kurz [1997c] of reporting the variance of the risky returns as $\frac{1}{100}\sigma_r^2$. Thus, they report the variance as 3.42% instead of 342%.

correspond to the economic concept due to tax and accounting distortions in reported earnings.

The historical estimates vary depending upon definitions, data sources and periods of estimation. We disregard these fine details and focus on the order of magnitudes involved.

4c.1 *Rational Expectations Equilibria*

To enable comparison with results obtained under rational beliefs we report in Table 1 the results for rational expectations equilibria. These are in accord with the standard results which gave rise to the equity premium puzzle debate: a very high riskless rate over 5%; a very low

Table 1: Rational Expectations Equilibria with Selected Variables

		$\gamma_2 = 2.75$ $\beta_2 = .92$	$\gamma_2 = 2.75$ $\beta_2 = .96$	$\gamma_2 = 3.25$ $\beta_2 = .92$	$\gamma_2 = 3.25$ $\beta_2 = .96$
$\gamma_1 = 2.75$ $\beta_1 = .92$	ρ	.41 %	.41 %	.44 %	.44 %
	r^F	5.16 %	5.15 %	5.12 %	5.11 %
	σ_r	4.04 %	4.04 %	4.07 %	4.06 %
	σ_F	.83 %	.82 %	.85 %	.84 %
	σ_p^2	.0049	.0049	.0056	.0056
$\gamma_1 = 2.75$ $\beta_1 = .96$	ρ	.41 %	.41 %	.44 %	.44 %
	r^F	5.14 %	5.13 %	5.11 %	5.09 %
	σ_r	4.04 %	4.04 %	4.06 %	4.06 %
	σ_F	.82 %	.82 %	.84 %	.84 %
	σ_p^2	.0049	.0049	.0056	.0056
$\gamma_1 = 3.25$ $\beta_1 = .92$	ρ	.44 %	.44 %	.49 %	.49 %
	r^F	5.12 %	5.11 %	5.08 %	5.06 %
	σ_r	4.07 %	4.06 %	4.09 %	4.09 %
	σ_F	.85 %	.84 %	.87 %	.87 %
	σ_p^2	.0056	.0056	.0065	.0064
$\gamma_1 = 3.25$ $\beta_1 = .96$	ρ	.44 %	.44 %	.49 %	.49 %
	r^F	5.11 %	5.09 %	5.06 %	5.05 %
	σ_r	4.06 %	4.06 %	4.09 %	4.08 %
	σ_F	.84 %	.84 %	.87 %	.86 %
	σ_p^2	.0056	.0056	.0064	.0064

equity premium of less than .5% and a very low standard deviation of the risky returns on equity around 4.1%. We also report here the extremely low variance of the price dividend/ratio which we consider to be an important indicator. Price volatility is the primary phenomenon associated with endogenous uncertainty and hence we are inclined to pay particular attention to it. Table 1 also shows that the results are not sensitive to parameter values in the realistic range. This conclusion does not hold for RBE where the results are sensitive to parameter values (see KB [1997]). Since we focus in this paper on social states and on the effects of correlation, we restrict ourselves to the fixed set of parameters $\gamma_1 = \gamma_2 = 3.25$, $\beta_1 = \beta_2 = .92$.

4c.2 *Rational Belief Equilibria I: a Constant, Single, Social State of Belief and No Correlation with* $\chi_1 = \chi_2 = 1$

We start the study of the equity risk premium by assuming a constant social state of belief hence $\chi_1 = \chi_2 = 1$. This economy should be considered to be the limit of a replica economy under the assumption of no correlation among the assessments of the agents and no price dependency in the intensities (λ, μ) of deviation from the Markov matrix Γ. Under the assumption of independence, the single social state of belief is $((\alpha_1, 1 - \alpha_1), (\alpha_2, 1 - \alpha_2))$ and the two social states are

$$\{(d^H, (\alpha_1, 1 - \alpha_1), (\alpha_2, 1 - \alpha_2)), (d^L, (\alpha_1, 1 - \alpha_1), (\alpha_2, 1 - \alpha_2))\}.$$

It follows from the equilibrium map (28) that in such RBE there are only two prices which are associated with these two social states and this is the same number of prices as in the rational expectations equilibria reported in Table 1. This means that in such RBE endogenous uncertainty does not lead to the emergence of additional prices but rather, it changes the two rational expectations equilibrium prices. Indeed, we shall shortly see that it can induce dramatic increases in the volatility of equilibrium prices. We call such an effect a *volatility amplification* effect.

Under the assumption of no price dependency we must have $\lambda_s = \lambda^0$, $\mu_s = \mu^0$ for all s and given this assumption let us adopt the convention of selecting $\lambda^0 > 1$ and $\mu^0 > 1$. We can then interpret the model to be one in which a proportion α_k of agents of type k are *always* relatively (to Γ) optimistic about the states of high prices in the next period and a proportion $(1 - \alpha_k)$ of agents of type k are *always* relatively pessimistic. The beliefs of individual agents fluctuate over time between optimism and pessimism but over the long run every agent is relatively optimistic a fraction α_k of the time and relatively pessimistic a fraction $(1 - \alpha_k)$ of the time. The parametrization of the model is then reduced to the four parameters $(\alpha_1, \lambda^0), (\alpha_2, \mu^0)$ and we need to consider the effect of the feasibility restrictions (30), (31) and (34).

Note that as we vary the four parameters $(\alpha_1, \lambda^0), (\alpha_2, \mu^0)$ over the feasible region we reach boundary points at which some of the inequalities in (30) or (34) are satisfied with equality. It can be checked that at these boundary points some probabilities in the matrices F_1, F_2, G_1 or G_2 become zero. More specifically, we adopt in this section the following rules:

(35)
1. For each α_1 select the *largest* feasible λ^0.
2. For each α_2 select the *largest* feasible μ^0.

To illustrate, suppose that we select $\alpha_1 = .5$ and $\alpha_2 = .4$. A single social state of belief implies that we must select $\chi_1 = \chi_2 = 1$, $a^1 = a^2 = a^3 = b^1 = b^2 = b^3 = 0$, $a^4 = b^4 = 1$. It follows from (34) that we must also have the following four restrictions:

$$\lambda^0 < \frac{1}{\phi} = 2.3256 \quad , \quad \mu^0 < \frac{1}{\phi} = 2.3256$$
$$\lambda^0 < \frac{1}{1-\phi} = 1.7544 \quad , \quad \mu^0 < \frac{1}{1-\phi} = 1.7544$$
$$\lambda^0 < \frac{1}{\alpha_1} = 2.000$$
$$\mu^0 < \frac{1}{\alpha_2} = 2.500$$

In this case the binding constraint is 1.7544. Other constraints will be binding if we wanted to select the *smallest* feasible λ^0 or μ^0.

To see the meaning of the criteria specified in (35) keep in mind that under the above specifications the matrices F_1, F_2, G_1 and G_2 are, in effect all 2×2 matrices. Hence, a zero probability in, say, the matrix F_1 means that given that some state of low or high prices is obtained at date t, the agent who uses the matrix F_1 is *certain* that at date t + 1 high or low prices will be realized. This is a rather extreme belief. Note also that given the rationality condition $\alpha_1 F_1 + (1 - \alpha_1)F_2 = \Gamma$, an extreme optimism about high prices when using F_1 must be associated with extreme pessimism when using F_2. Note also that some boundary restrictions apply only to the first 9 states and others only to states 10 - 18 (see (34)). Hence, under the criteria (35) we know that a *positive fraction* of the agent will hold conditional probabilities with zero entries *some of the time*.

Table 2 reports the volatility results for RBE simulated under several configurations of the parameters (α_1, λ^0) and (α_2, μ^0) derived under the criteria (35). There are two important conclusions that can be drawn from the table. First, it shows that although the RBE with a single

**Table 2: RBE with a Single, Constant Social State of Belief ($\chi_1 = \chi_2 = 1$)
Derived Under (35) and No Correlation**

		$\lambda^0 = 1.754$ $\alpha_1 = .5$	$\lambda^0 = 1.754$ $\alpha_1 = .57$	$\lambda^0 = 1.666$ $\alpha_1 = .6$	$\lambda^0 = 1.428$ $\alpha_1 = .7$
$\mu^0 = 1.754$ $\alpha_2 = .5$	ρ r^F σ_r σ_F σ_p^2	.98 % 6.05 % 16.34 % 14.01 % 4.5417	4.94 % 3.55 % 23.51 % 19.65 % 9.8228	3.92 % 3.17 % 16.41 % 12.37 % 4.4648	2.88 % 3.71 % 13.32 % 9.59 % 2.7414
$\mu^0 = 1.754$ $\alpha_2 = .57$	ρ r^F σ_r σ_F σ_p^2	4.94 % 3.55 % 23.51 % 19.65 % 9.8228	10.00 % .43 % 31.00 % 24.30 % 16.7917	7.69 % .45 % 21.70 % 15.88 % 8.1316	6.14 % 1.25 % 18.00 % 12.87 % 5.4623
$\mu^0 = 1.666$ $\alpha_2 = .6$	ρ r^F σ_r σ_F σ_p^2	3.92 % 3.17 % 16.41 % 12.37 % 4.4648	7.69 % .45 % 21.70 % 15.88 % 8.1316	5.23 % 1.46 % 13.43 % 8.61 % 2.6852	3.96 % 2.32 % 10.43 % 6.15 % 1.3887
$\mu^0 = 1.428$ $\alpha_2 = .7$	ρ r^F σ_r σ_F σ_p^2	2.88 % 3.71 % 13.32 % 9.59 % 2.7414	6.14 % 1.25 % 18.00 % 12.87 % 5.4623	3.96 % 2.32 % 10.43 % 6.15 % 1.3887	2.88 % 3.10 % 7.75 % 3.91 % .5543

social state has only two prices, which is the same number as in the REE, the two equilibria are dramatically different. The crucial difference between them is found in the fact that in the RBE, half of the agents have optimistic probability beliefs relative to Γ about the prospects of $((p_1, q_1), (p_2, q_2), \ldots, (p_9, q_9))$ while half of the agents have pessimistic beliefs (relative to Γ) about these prices. This in contrast with the REE in which all agents hold Γ as their belief at all dates. Table 2 then demonstrates a new property of the model of the replica economy with types: volatility does not necessarily emerge as a result of an increase in the number of social states of beliefs but may arise as a result of *the nature of the distribution of beliefs* in each social state. Compare this conclusion with the observations made in the papers in the volume by Kurz [1997] that endogenous uncertainty is induced by the variability, over time, in the states of belief. This idea is explained in detail by Kurz [1997a] (page 32) and is based on RBE of models with individual states of beliefs. One of the important results of the model with types and social states is that volatility may be propagated simply by the social distribution itself and not by any variations over time in the social states of belief.

The second conclusion that we draw from Table 2 is that the amplification of volatility in RBE with a constant social state of belief can be very dramatic if agents are allowed to adopt boundary beliefs. Indeed, these are the maximal volatility measures and equity premia that this specification of the model can generate. It is interesting, however, that both at low as well as high α_k the equity premium is low and the riskless rate is high. The largest equity premium is realized in the middle of the table where α_1 and α_2 are close to .57 but in those cells the standard deviations of both the riskless rates as well as those of the risky returns are much too large. As α_1 and α_2 move away from .57 the volatility of both the riskless rate as well as the risky returns falls dramatically. As a result of these facts there is no cell which fits the historical record of all four moments $(\rho = 6\%,\ r^F = .5\%,\ \sigma_r = 18\%,\ \sigma_F = 6\%)$.

Under the axioms of the theory of rational beliefs agents may hold extreme beliefs but this does not mean that such beliefs must be observed in the market. Indeed, we shall shortly argue that one may choose between two alternative hypotheses by imposing restrictions on beliefs based on known facts about the distribution of beliefs in the market. The question then becomes *which of the two alternative hypotheses performs better under the stipulated restrictions*. To motivate these restrictions we note that although high degrees of optimism or pessimism are observed in the beliefs of investors in security markets, it is evident that *certainty beliefs are rarely* *encountered.* We then propose to restrict the beliefs of the agents so as not to permit them to hold boundary beliefs. Formally we require

$$(36) \qquad f_{ij}^{k} \geq .05\Gamma_{ij} \qquad g_{ij}^{k} \geq .05\Gamma_{ij} \qquad k = 1, 2$$

where f_{ij}^{k} and g_{ij}^{k} are the (ij) elements of the matrices F_k and G_k. (36) specifies that any deviations from the stationary measure should not result in probabilities which are less than 5% of the corresponding probabilities in Γ. Observe that lower bound restrictions *imply* upper bound restrictions due to the rationality conditions $\alpha_1 F_1 + (1 - \alpha_1)F_2 = \Gamma$. We call the collection of all such restrictions <u>the 5% boundary restrictions on beliefs</u>. It is clear that under these restrictions the beliefs used in Table 2 are not allowed.

Table 3 presents the results for RBE with the same values of (α_1, α_2) as in Table 2 but under the 5% boundary restrictions on beliefs. The results reported in Table 3 represent the largest possible volatility measures and equity premia that can be generated by the RBE under the restriction of no correlation and a constant social state of belief. One can see that once the 5% restriction is imposed, the model cannot generate statistics which are even close to the historical record: the equity premia are too low, the riskless rates are too high and the volatility of the riskless rate is too low. We remark that a comparison of the results of Table 2 and 3 is complicated by the fact that the impact of the 5% restrictions varies across the cells of the table and each of those restrictions may affect different segments of the agents and only part of the time. However, the results in Table 3 show that in order for the RBE with a constant social state of belief to generate high volatility and large equity premia it is necessary that some of the agents

Table 3: RBE with a Single, Constant, Social State of Belief ($\chi_1 = \chi_2 = 1$) and with the 5% Boundary Restrictions on Agents' Beliefs

		$\lambda^0 = 1.72$ $\alpha_1 = .5$	$\lambda^0 = 1.72$ $\alpha_1 = .57$	$\lambda^0 = 1.63$ $\alpha_1 = .6$	$\lambda^0 = 1.41$ $\alpha_1 = .7$
$\mu^0 = 1.72$ $\alpha_2 = .5$	ρ	1.12 %	2.10 %	1.85 %	1.52 %
	r^F	5.05 %	4.32 %	4.33 %	4.47 %
	σ_r	10.16 %	11.97 %	10.09 %	8.45 %
	σ_F	7.02 %	8.51 %	6.63 %	5.07 %
	σ_p^2	1.3487	2.0779	1.3026	.7676
$\mu^0 = 1.72$ $\alpha_2 = .57$	ρ	2.10 %	3.23 %	2.85 %	2.38 %
	r^F	4.32 %	3.47 %	3.56 %	3.79 %
	σ_r	11.97 %	13.87 %	11.74 %	9.92 %
	σ_F	8.51 %	10.00 %	7.93 %	6.25 %
	σ_p^2	2.0779	2.9743	1.9493	1.2334
$\mu^0 = 1.63$ $\alpha_2 = .6$	ρ	1.85 %	2.85 %	2.45 %	2.01 %
	r^F	4.33 %	3.56 %	3.71 %	3.96 %
	σ_r	10.09 %	11.74 %	9.70 %	7.99 %
	σ_F	6.63 %	7.93 %	5.99 %	4.42 %
	σ_p^2	1.3026	1.9493	1.1457	.2668
$\mu^0 = 1.41$ $\alpha_2 = .7$	ρ	1.52 %	2.38 %	2.01 %	1.62 %
	r^F	4.47 %	3.79 %	3.96 %	4.20 %
	σ_r	8.45 %	9.92 %	7.99 %	6.41 %
	σ_F	5.07 %	6.25 %	4.42 %	2.97 %
	σ_p^2	.7676	1.2334	.2668	.2720

hold, some of the time, conditional beliefs which are rather extreme.

One of the conclusions of this paper is that an equilibrium with a single social state cannot generate data which match all four moments under examination. However, an RBE with a constant state of belief is a relatively simple model that can provide an intuitive explanation of the mechanism which generates equity premium in the model with types. This fact is compatible with one of the aims of this paper which is to give an intuitive explanation of the mechanism which generates an equity risk premium in an RBE. Thus, before we proceed to study the model with correlation among the beliefs of agents, let us pause to explain the results reported in Tables 2 and 3 and the particular role played by the value of .57 taken by α_k.

Note at the outset two facts about the equilibrium model which generate the results in Tables 2 and 3. On the one hand, a change in α_k results in a change of the proportion of type k agents who are optimistic at any moment of time about future capital gains. Since the social state of belief is constant this proportion is constant. On the other hand, the rationality conditions $\alpha_1 F_1 + (1 - \alpha_1)F_2 = \Gamma$ imply that as α_k changes the *intensity* of optimism and pessimism must change so as to compensate for the number of agents who are optimistic or pessimistic. "Intensity" is measured in terms of the probability with which the agents forecast higher or lower prices. The volatility characteristics of the economy are then determined by the interplay between the proportion of agents who are optimistic or pessimistic and the intensity of their optimism/pessimism. The crucial variable that needs to be understood in this connection is the behavior of the riskless rate.

To explore the behavior of the riskless rate observe at the outset that the mean risky rate of return on equity remains in the 6% - 8% range for almost all cells of Tables 2 and 3; the main determinant of the premium is therefore the equilibrium value of the riskless rate. Now consider the number and intensity of belief of those agents who expect at date t a recession and hence lower prices to be realized at date t + 1. It is clear that as α_k increases, the number of such agents decreases. However the rationality conditions induce a non-linear relationship between the *number* of such agents and the level of their *intensity*. The structure of this non-linear relation has three parts:

- (i) For small α_k the rationality conditions limit the intensity of pessimists and even if their number is larger than the optimists, the intensity of the optimists is at a very high level. Since the intensity with which the optimists want to borrow is relatively high in relation to the intensity with which the pessimists want to lend, the results are high riskless rates, low premia and low volatility.

- (ii) As α_k increases the intensity of the pessimists rises and is maximized at (.57, .57); it cannot increase beyond that point. Around .57 the intensity of the pessimists dominates the rising number of optimists and the result is a decline in the riskless rate and a rise in the premium. The rise in the volatility of prices and risky returns in this region is a result of the fact that the intensity of both sides is at the high level and this results in more drastic changes of excess demand in response to fluctuations in the realized dividend growth.
- (iii) As α_k increases beyond .57 the intensity of the pessimists remains constant but their number declines. As the relative number of optimists rises, their intensity declines, the level of volatility falls dramatically and the riskless rate rises again.

In sum, the equity risk premium is the result of the interplay between the number and intensity of beliefs of the optimists vs. the pessimists and hence it is determined by the distribution of beliefs in the economy. For low α_k the *intensity* of the optimists has the stronger impact and for large α_k their *number* has the dominant impact. The non-linearity induced by the rationality conditions results in the middle region in which the *intensity and number* of the pessimists just outweighs the optimists, causing increased volatility and a lowered riskless rate. This structure is made much more complicated in a world of correlation in which there are more social states with more configurations of belief and intensities.

The alternative model with which we propose to explain the data is a model where correlation among the beliefs of agents turns the social state of belief into a random variable. Although the mechanism which generates an equity premium is more complicated, the insight provided by the model with a single state of belief remains correct. We turn now to this subject.

4c.3 *Rational Belief Equilibria II: The Effect of Correlation Among the Beliefs of Agents*

Correlation among the beliefs of agents is a complicated phenomenon due to the fact that it may take several forms. Hence, in order to study the effect of correlation we need to clarify the terms used to characterize it. Here are the basic terms which we use:

I. **Correlation within types** is characterized by the assumption that $\chi_1 < 1$ and $\chi_2 < 1$. Under the specifications above we have 3 type-states and hence 9 social states of belief.

II. **Correlation across types** is characterized by the fact that the matrices A and B are *not* transition matrices of a joint process of i.i.d random variables. For each value of χ_1 and χ_2 the type-states are jointly i.i.d if the following are the values of the parameters in A and B (which are then parameters of the matrix Γ):

$\chi_1 = \chi_2 = .5$

(37a) For all s: $a_s^1 = b_s^1 = .0625$, $a_s^2 = b_s^2 = .125$, $a_s^3 = b_s^3 = .125$, $a_s^4 = b_s^4 = .25$.

$\chi_1 = \chi_2 = .2$

(37b) For all s: $a_s^1 = b_s^1 = .16$, $a_s^2 = b_s^2 = .08$, $a_s^3 = b_s^3 = .08$, $a_s^4 = b_s^4 = .04$.

$\chi_1 = \chi_2 = .1$

(37c) For all s: $a_s^1 = b_s^1 = .2025$, $a_s^2 = b_s^2 = .045$, $a_s^3 = b_s^3 = .045$, $a_s^4 = b_s^4 = .01$.

III. **Price dependency** is characterized by the fact that the parameters λ_s and μ_s are *dependent on s*.

We comment on these by noting that the conditions $\chi_1 < 1$ and $\chi_2 < 1$ could be associated with two situations. First, we may have a large but finite economy which is approximately anonymous in which the existence of multiple type-states is a natural fact. The assumption of three type-states is then an assumption about the nature of correlation (in addition to being a computational simplification). Second, we may have an infinite replica economy and the assessments are not i.i.d. Our assumption that the type-states are marginally i.i.d makes sense only if there is correlation among the assessments within a type.

The distinction between correlation among the type-states and price dependency is important. The correlation among the type-states is a technical condition stipulating that the assessments are random variables which are statistically correlated. Price dependency is not a condition of statistical correlation; rather, it stipulates the *commonality in the interpretation of the assessments by the agents.*

The terms defined above show that in order to specify a model with correlation, we need to specify feasible values of (χ_1, χ_2), (a, b) and (λ, μ). It follows from (30) that the parameters (a, b) depend upon (χ_1, χ_2) so that as we vary (χ_1, χ_2) we must also vary (a, b) in accord with the feasibility conditions (30). It is therefore impossible to isolate the net effect of varying (χ_1, χ_2). In the analysis below we assume $\chi_1 = \chi_2 = \chi$, taking the three values .5, .2 and .1. Correspondingly, we vary (a, b) in a reasonably similar manner but exact comparability is impossible. We, therefore, focus only on simulations in which (χ_1, χ_2) are fixed.

Parameter Specifications. The basic specification takes the case $\alpha_1 = \alpha_2 = .57$. The corresponding RBE under the 5% boundary restrictions on beliefs is the "reference RBE." This is motivated by our aim to examine what would be the contribution of models of correlation. Hence, the reader should keep in mind the results for this reference case as reported in Table 3 (i.e. the case with $\alpha_1 = \alpha_2 = .57$, $\lambda_\bullet = \mu_\bullet = 1.72$). We thus compare the reference RBE with RBE under the following specifications:

(I) χ takes the values .5, .2 and .1.

(II) The intensity variables are specified as follows:

(i) For RBE with i.i.d. assessments and without price dependency we specify $\lambda_\bullet = \mu_\bullet = 1.72$.

(ii) For RBE with *price dependency* we specify

$$\lambda_1 = \lambda_2 = \lambda_3 = .46 \quad , \quad \lambda_4 = \lambda_5 = \lambda_6 = 1 \quad , \quad \lambda_7 = \lambda_8 = \lambda_9 = 1.72$$

$$\lambda_{10} = \lambda_{11} = .46 \quad , \quad \lambda_{12} = \lambda_{13} = \lambda_{14} = \lambda_{15} = \lambda_{16} = \lambda_{17} = \lambda_{18} = 1.72$$

$$\mu_1 = .46 \ , \ \mu_2 = \mu_3 = 1.72 \ , \ \mu_4 = \mu_5 = \mu_6 = 1 \ , \ \mu_7 = .46 \ , \ \mu_8 = \mu_9 = 1.72$$

$$\mu_{10} = .46 \ , \ \mu_{11} = \mu_{12} = 1.72 \ , \ \mu_{13} = \mu_{14} = \mu_{15} = 1 \ , \ \mu_{16} = \mu_{17} = \mu_{18} = 1.72.$$

(III) The (a, b) parameters which are dependent upon χ are specified in the Appendix.

Table 4 presents the results for $\chi = .5$. The reference RBE under $\chi = 1$ is reproduced in

Table 4: RBE with Correlation among Beliefs, with $\chi = .5$ and with the 5% Boundary Restrictions on Beliefs

	RBE (Reference) with a constant social tate of belief $\chi_1 = \chi_2 = 1$	RBE i.i.d type-states no price dependence	RBE i.i.d type-states with price dependence	RBE correlation across types with price dependence
ρ	3.23 %	2.87 %	3.92 %	4.18 %
r^F	3.47 %	3.63 %	2.25 %	2.10 %
σ_r	13.87 %	12.60 %	10.92 %	11.97 %
σ_F	10.00 %	9.68 %	6.51 %	7.94 %
σ_p^2	2.9743	2.4287	1.8498	2.0854

Column 1. A comparison of columns 1 and 2 of the table shows that the reference RBE with a

single social state of belief exhibits about the same volatility characteristics as the RBE with correlation within types but with i.i.d. type-states. In column 3 we see, however, that price dependency increases the premium, reduces the riskless rate but also leads to a reduction in volatility. The addition of correlation *across* type-states raises the premium to 4.18% and restores some volatility. Altogether, the results reported in Table 4 do not match the data very well and leads to the conclusion that if correlation is to generate more volatility, we must explore parameter configurations which place less probability on the social states of belief ((.57, .43), (.57, .43)). We thus explore the two other cases $\chi = .2$ and $\chi = .1$. Since $\alpha_1 = \alpha_2 = .57$, these specifications imply that the correlation among the assessments leads the probabilities to be "spread" away from ((.57, .43), (.57, .43)) which is the constant social state of belief that would be realized under i.i.d. assessments. For $\chi = .2$ and $\chi = .1$ most of the probability is placed on the type-states (.85, .15) and (.25, .75). Table 5 reports the results which are our main results regarding the effects of correlation:

<u>In column 1</u> we repeat the "reference RBE" with a constant social state of belief as in Table 3.

<u>In column 2</u> we report the results for RBE with three type-states which are i.i.d.(hence with correlation within types) and without price dependence. It is evident that these specifications contribute little *by themselves*.

<u>In column 3</u> we report the results for the effect of price dependence. It is clear that in conjunction with the correlation within types and the specification $\chi \leq .2$, price dependence has a strong effect. <u>In column 4</u> we report the added effect of full correlation across types. It contributes about 1% to the premium and substantially contributes to the volatility of returns.

Table 5: RBE II under Correlation among Beliefs with $\chi = .2$ and $\chi = .1$ and with the 5% Boundary Restrictions on Beliefs

RBE (Reference) with a constant social tate of belief $\chi_1 = \chi_2 = 1$			RBE i.i.d type-states no price dependence	RBE i.i.d type-states with price dependence	RBE correlation across types with price dependence
3.23 %	$\chi = .2$	ρ	2.77 %	5.02 %	5.83 %
3.47 %		r^F	3.61 %	1.23 %	.66 %
13.87 %		σ_r	11.85 %	11.73 %	13.75 %
10.00 %		σ_F	9.29 %	7.72 %	10.52 %
2.9743		σ_p^2	2.1450	2.3004	2.9487
3.23 %	$\chi = .1$	ρ	2.76 %	5.42 %	6.54 %
3.47 %		r^F	3.58 %	.87 %	.25 %
13.87 %		σ_r	11.59 %	12.08 %	15.84 %
10.00 %		σ_F	9.13 %	8.20 %	12.81 %
2.9743		σ_p^2	2.0566	2.5036	3.9960

It is instructive to note that the introduction of correlation within types (i.e. $\chi < 1$) *by itself* contributes little to explaining volatility. However, as we add price dependency and correlation across types, the results reported in the last two columns of Table 5 emerge as a result of a *combined* effect of all three forms of correlation. This indicates *a strong interaction effect among the three factors of correlation involved*. We now offer some intuitive explanation of the specification of the matrices A and B which regulate the long term correlation across type-states.

We have already noted that variations of the parameter χ induce changes in the feasibility conditions (30) so that it is impossible to vary this parameter while keeping constant the parameters (a , b) of correlation across type-states. The main facts behind the selection of (a, b) is that the 9 prices associated with the states of expanding dividends are higher than the 9 prices associated with the states of declining dividends. In addition, within these two categories of states the prices $((p_1, q_1), (p_5, q_5), (p_9, q_9), (p_{10}, q_{10}), (p_{14}, q_{14}), (p_{18}, q_{18}))$ are the high prices while the "crash" prices are $((p_{12}, q_{12}), (p_{16}, q_{16}))$. Other prices are "medium" prices. The parameters $a = (a^1, a^2, a^3, a^4)$ are selected subject to feasibility so that there is high probability of transition from the very high prices to crash and medium prices. In addition, these parameters aim to maximize transition probabilities from crash prices to very high prices and from all other prices to medium and high prices. The parameters $b = (b^1, b^2, b^3, b^4)$ are selected to maximize transition probabilities to the very high prices, subject to feasibility. This parametrization of the transition probabilities contributes to price volatility. However, keep in mind that the feasibility conditions leave limited room for such selections so that the nature of these transition probabilities and the implied correlations across type-states may be very different for different values of χ (see the specifications of the (a, b) vectors for the different values of χ in the Appendix).

Recall that all simulations in Table 5 have been conducted under the 5% boundary restrictions on beliefs. Comparing the results in columns 2 - 4 with the results in column 1 or in Table 3, we conclude that the model with correlation among beliefs of agents performs much better than the model with a single state of belief. We have seen in Table 3 that the model with a single state of belief could not generate a riskless rate which is smaller than 3% - 4%. These simulations were conducted under the assumption that $\lambda_s = \lambda^0$, $\mu_s = \mu^0$ for all s which means that price dependency was not allowed whereas price dependency is compatible with a single state of belief. We have sampled extensively in the parameter space and can report that allowing price dependency has not changed the essential results of Table 3: the riskless rate in all our simulations was never below 3%. We conclude that under the 5% boundary restrictions on beliefs the model with a single social state of belief cannot generate data which will match the observed values of the four moments which we have been examining.

In contrast to the above conclusion, under the same 5% boundary restrictions on beliefs, the model specification with correlation among the beliefs of agents generates statistics which match all four empirical moments rather well. The standard deviation of the risky returns is somewhat smaller than the historical record and the standard deviation of the riskless rate is somewhat larger than the record. To gain more insight into these results let us examine some variants of the case $\chi = .1$, $\alpha_1 = \alpha_2 = .57$ by perturbing α_1 and α_2 over the values .54, .57, .60. The results of these calculations are presented in Table 6. The table shows that the results are

Table 6: RBE II under Correlation among Beliefs with $\chi = .1$ and with the 5% Boundary Restrictions on Beliefs

		$\alpha_1 = .54$	$\alpha_1 = .57$	$\alpha_1 = .6$
$\alpha_2 = .54$	ρ	4.87 %	5.80 %	5.05 %
	r^F	1.58 %	.82 %	1.32 %
	σ_r	14.51 %	14.62 %	12.88 %
	σ_F	11.31 %	11.94 %	10.28 %
	σ_p^2	2.7746	3.3381	2.4664
$\alpha_2 = .57$	ρ	5.83%	6.54 %	5.65 %
	r^F	.78 %	.25 %	.86 %
	σ_r	14.65 %	15.84 %	13.92 %
	σ_F	11.97 %	12.81 %	11.10 %
	σ_p^2	3.3459	3.9960	2.9654
$\alpha_2 = .6$	ρ	5.20 %	5.76 %	4.82 %
	r^F	1.19 %	.76 %	1.45 %
	σ_r	13.02 %	14.00 %	12.06 %
	σ_F	10.35 %	11.07 %	9.29 %
	σ_p^2	2.5227	2.9908	2.0821

rather sensitive to parameter values but there is a significant region in the parameter space that can give rise to statistics which are compatible with the empirical moments. Key variables that would change the results in the table are the values of the probabilities (χ_1, χ_2) and the social distribution of beliefs defined in our models by the type-states (.85 , 15) and (.25 , .75).

4d *Understanding How An Equity Risk Premium is Generated Under Rational Belief*

Ever since the publication of the paper by Mehra and Prescott [1985] on the equity premium, numerous theories were offered to explain the empirically observed premium. For example, Mankiw [1986] proposed to explain the premium by the presence of nondiversifiable risks; Reitz [1988] proposed to explain it by the introduction of big crash states; Weil [1989] and Epstein and Zin [1990] suggest that a non-expected utility model may be used to explain the data and Constantinides [1990] initiated a large literature on the use of habit forming utility functions to explain the data. This paper complements the earlier paper by KB [1997] and proposes the theory of rational belief as an explanation of the data. The model of an RBE with types offers an intuitive explanation to which we now turn.

The basic assumption of the theory of rational belief is that agents do not observe the social states and do not know the equilibrium map. The consequence of the rationality axioms is that agents form beliefs about prices, not about social states, and may have diverse beliefs about the probabilities of future prices. The important conclusion of the theory is that *if agents disagree then their state of belief must fluctuate over time.* To understand why, observe that if agents disagree then they must deviate from the stationary measure. However, deviations from the stationary measure at one date must be compensated by other deviations at other dates so that the time average of the deviations tends to zero in order to satisfy the rationality axioms. These fluctuations over time in the states of belief of the agents is the mechanism which generates endogenous uncertainty in an RBE and is reflected in the volatility of equilibrium prices and quantities. It then follows that the first component of explaining the risk premium in an RBE is the presence of endogenous uncertainty. All risk averse agents who perceive the extra endogenous volatility of returns will require the compensation of an added risk premium in order to be willing to hold the more risky equity. This argument is, however, insufficient since agents who disagree may be more or less optimistic with respect to future events and thus require a higher or lower premium depending upon their probability assessment. The first basic argument must be then supplemented by an explanation of how the diversity of beliefs by itself can add to equilibrium equity premium.

When some agents are optimistic and some are pessimistic, trading opportunities naturally become available but this need not have anything to do with the equity risk premium. However, when such optimism or pessimism is defined with respect to the future risky rates of return on equity then it will have an effect on the premium. For example, if at price vector 1 the level of pessimism about future equity returns of an agent increases he will select a portfolio with lower weight on equity and higher weight on riskless debt and this will tend to reduce the price of equity and increase the price of riskless debt resulting in increased premium in state 1. The situation is substantially complicated by the rationality conditions which hold that an agent who is relatively optimistic at some date must be relatively pessimistic at some other date. In a large economy with a single social state the proportions of optimists and pessimists are fixed and in the simulations above we allowed these proportions to vary across models. When the proportion of optimists changes, the rationality conditions imply that the *intensity* of optimism and pessimism must change. This shows that at any time both the proportion of pessimists as well as their intensity matter to market equilibrium. We have observed in Tables 2 and 3 that a simultaneous change in the proportions and intensities of the optimists and the pessimists (via changes in α_1 and α_2) has a non-linear effect on market excess demand and hence on the premium. The implication of this observation is that the distribution of beliefs in the market at any date is the crucial factor which determines the equity risk premium at that date. This observation extends to the model with correlation.

In the general model with correlation we cannot think of the equilibrium premium as being determined by a fixed proportion of optimists and pessimists. Since the social state of belief is a random variable these proportions vary but the observation made in the model with a constant state of belief remains valid: *at any date the risk premium is determined by the distribution of beliefs at that date.* But then, any parameter that has an impact on the distribution of beliefs and on the frequencies at which the states of belief are realized over time will have an effect on the average premium of the economy. It is appropriate to think of time dependency and correlation among the assessments of agents as <u>belief externalities</u> which affect the distribution of beliefs in the following two ways:

(i) Price dependence has the effect of changing the number of optimists and pessimists given any price. For example consider price vector (p_1, q_1) defined in the models above by the social state $(d^H, (.85, .15), (.85, .15))$. If λ_1 is price dependent, it will have the following effect: if $\lambda_1 > 1$ then in this first state 85% of type 1 agents are *optimistic* about high prices the next period and if $\lambda_1 < 1$ then in this first state 85% of type 1 agents are *pessimistic* about high prices the next period.

(ii) Correlation among type-states is an externality which can increase the frequency over time of states of beliefs which generate higher premium. The externality also creates new distributions of belief which an agent cannot deduce from his own belief. For example, although the simulations in Tables 4 - 5 postulate RBE in which $\alpha_1 = \alpha_2 = .57$, the correlation among beliefs leads to the emergence of social states of belief which are different from ((.57, .43), (.57, .43)) but the agents do not know the structure of this externality.

It should then be clear that the exact interpretation of the parametrizations of (A, B, λ, μ) in the various models in Tables 4 - 6 is less important than the function of these parametrizations in *regulating the distribution of the states of belief and the frequencies of their realization.* Correspondingly, all four moments of the distribution of the risky and riskless returns are determined by the frequencies of the realized states of belief. From this perspective the reason why models of RBE can generate theoretical moments with high volatility, low riskless rate and high equity premium can be summarized as follows:

(1) In the typical RBE there are relative pessimists at all dates and there is always a range of parameter values where either the number or the intensity of the pessimists dominate and have the impact of pushing the riskless rate down and hence the premium up. The volatility in prices and returns is then a consequence of the fact that due to the rationality conditions the relative impact of the pessimists and optimists vary in such equilibria across states and market prices naturally reflect these changes. Although the simulated RBE with a single social state have the property that the pessimists are in the minority and their intensity dominates the bond market, we cannot be certain of the generality of this conclusion since there are other forms of pessimism and optimism which we have not studied. The general principle proposed by the theory of RBE is, however, clear. At all dates there are, in the economy, optimists and pessimists and either the number or the intensity of the pessimists is dominant: it pushes the riskless rate

down and the equity risk premium up.

(2) The correlation among the beliefs of agents has a dual impact on an RBE. First, it can change the relative number of optimists and pessimists at each state by making the intensity parameters price dependent and this allows the attainment of a low riskless rate and higher premium even when the intensity of the pessimists is not extreme. Second, it can change the stationary distribution and hence the long run frequency at which the different price states are realized. This changes the relative probabilities of states with high premium and consequently the average premium over time.

Let us close with a methodological note. The 5% boundary restrictions on beliefs were not derived from axioms of the theory of rational belief but rather from empirical observations. Using this restriction we argued that the model with correlation among the beliefs of agents is superior to a model with i.i.d. assessments in which there is a single, constant social state of belief. Since not all rational beliefs need to be observed in our economy, in future research we may generalize this approach as follows. One needs to start by obtaining more empirical information about the social distribution of beliefs. Given such data one may then ask what could be the type configurations and the sets of parameters characterizing the beliefs of the agents that would "rationalize" the data. Given that the distribution of beliefs is approximately rationalized, one can then proceed to test if the model with the specified family of beliefs can explain the observed volatility characteristics of the market.

Appendix

Specification of the Parameter (a , b) in Tables 2 - 5

For $\chi = 1$:

$a_s^1 = a_s^2 = a_s^3 = b_s^1 = b_s^2 = b_s^3 = 0$, $a_s^4 = b_s^4 = 1$ for $s = 1, 2, \ldots, 9$.

For $\chi = .5$:

$a^1 = (.0001, .0001, .2498, .0001, .0001, .0001, .2498, .0001, .0001)$

$a^2 = a^3 = (.2498 , .2498 , .0001 , .2498 , .2498 , .2498 , .0001 , .2498 , .2498)$

$a^4 = (.0003, .0003, .4998, .0003, .0003, .0003, .4998, .0003, .0003)$

$b_s^1 = .2498$, $b_s^2 = b_s^3 = .0001$, $b_s^4 = .4998$ for $s = 1 , 2 , \ldots , 9$.

For $\chi = .2$:

$a^1 = (.0001, .0001, .25, .0001, .0001, .0001, .25, .0001, .0001)$

$a^2 = a^3 = (.1998 , .1998 , .1480 , .1998 , .1998 , .1998 , .1480 , .1998 , .1998)$

$a^4 = (.0001, .0001,.0001,.0001,.0001,.0001,.0001,.0001,.0001)$

$b_s^1 = .3998$, $b_s^2 = b_s^3 = .0001$, $b_s^4 = .1998$ for $s = 1 , 2 , \ldots , 9$.

For $\chi = .01$:

$a^1 = (.0001, .0001, .35, .0001, .0001, .0001, .35, .0001, .0001)$

$a^2 = a^3 = (.0998 , .0998 , .0998 , .0998 , .0998 , .0998 , .0998 , .0998 , .0998)$

$a^4 = (.0001, .0001,.0001,.0001,.0001,.0001,.0001,.0001,.0001)$

$b_s^1 = .4498$, $b_s^2 = b_s^3 = .0001$, $b_s^4 = .0998$ for $s = 1 , 2 , \ldots , 9$.

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(xvi) Paper presented at the International Workshop on "The Political Economy of Economic Policy - The Organization of Government" European Science Foundation and Fondazione Eni Enrico Mattei, Castelgandolfo (Rome), September 5-10, 1995

(xvii) This paper was presented at the Workshop on "Corporate Governance and Property Rights" organized by the Corporate Governance Network and by Fondazione Eni Enrico Mattei, Milan, 16-17 June 1995

(xviii) This paper was presented at the International Workshop on "Creation and Transfer of Knowledge: Institutions and Incentives" organized by the Fondazione Eni Enrico Mattei and the Beijer International Institute of Ecological Economics, Castelgandolfo (Rome), September 21-23, 1995

(xix) This paper was presented at the International Workshop on "Environment and Transport in Economic Modelling" organized by the Department of Economics - Ca' Foscari University, Venice for the "Progetto Finalizzato Trasporti 2" CNR and in cooperation with Fondazione Eni Enrico Mattei, Venice, November 9-10, 1995

(xx) This paper was presented at the Conference on "Technology, Employment and Labour Markets" organized by the Athens University of Economics and Business and Fondazione Eni Enrico Mattei, Athens, May 16-18, 1996

(xxi) This paper was presented at the Conference on "Applications of Environmental Accounting", sponsored by the Fondazione Eni Enrico Mattei and the State Science and Technology Commission of the People's Republic of China, Beijing China, March 11-13, 1996

(xxii) This paper was presented at the Conference on "Economics of Tourism", Fondazione Eni Enrico Mattei and University of Crete, Crete, October 13-14, 1995

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(xxiv) This paper was presented at the Conference on "Pressure Groups, Self-Regulation and Enforcement Mechanisms", Fondazione Eni Enrico Mattei, Milan, January 10-11, 1997

(xxv) This paper was presented at the Conference on "Plotting our Future. Technology, Environment, Economy and Society: a World Outlook", Fondazione Eni Enrico Mattei, Milan, October 24-26, 1996

(xxvi) This paper was presented at the Conference on "International Environmental Agreements on Climate Change", Fondazione Eni Enrico Mattei, Venice, May 6-7, 1997

(xxvii) This paper was presented at the "First European Corporate Governance Network Conference", Fondazione Eni Enrico Mattei, Directorate General for Industry of the Commission of the European Communities and the Politecnico of Milan, Milan, March 7-8, 1997

(xxviii) This paper was presented at the Conference on "Trade and Technology Diffusion: The Evidence with Implications for Developing Countries", Fondazione Eni Enrico Mattei and The World Bank - International Trade Division, Milan, April 18-19, 1997

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