

# Does Competition at School Matter? A View Based upon the Italian and the Japanese Experiences

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## **Abstract**

We present a model where the interaction between competition at school, industrial structure and labor market outcomes is characterized by the concept of Nash decentralized equilibrium. We show that the presence of spillovers and strategic complementarities could generate multiple equilibria. In one equilibrium, a competitive schooling system induce individuals to accumulate basic academic skills, large firms hire mainly new school graduates and there is limited labor turnover. In another equilibrium, schooling is less competitive and individuals focus more on idiosyncratic skills, large firms hire mainly experienced workers and labor turnover is important. We argue in the paper that the main features of each equilibrium are consistent with key stylized facts of the Italian and the Japanese labor markets.

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# 1 INTRODUCTION

<sup>1</sup>Education and human capital accumulation are often viewed as key to economic performance and growth. Adequate skills are considered as a critical factor in the development of new ideas and designs. When applied to industry, a higher ability to innovate yields both higher competitiveness and higher growth<sup>2</sup>. School enrollment rates are a key variable in most modern growth accounting exercises<sup>3</sup>. Active policies to improve the stock of on-the-job and off-the-job human capital are often advocated as an effective way to reduce persistently high unemployment rates in Europe<sup>4</sup>. Finally, school quality is considered as important both for productivity and for real earnings<sup>5</sup>.

This paper looks at a closely related but often overlooked question: does selectivity and competition at school matter for economic performance? By forcing individuals to invest in the competition for access to the best schools, a selective schooling system increases the stock of *common basic academic skills*. Examples of these skills include the basics of reading, mathematics and science and problems solving skills. If the accumulation of these skills and training are complements, selective education reduces training costs and increases the relative advantage of adopting complex and highly productive technologies. When competition is excessive, however, it could hamper the development of *individual skills*, because of the strong incentives it places on the development of common basic skills. Examples of individual skills are self-expression, creative thinking and idiosyncratic competences. If individual skills are also important for industrial performance, too much competition in

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<sup>2</sup>See Rebelo (1992) and Bertola and Coen Pirani (1995).

<sup>3</sup>See Barro and Ree (1992) and the references therein.

<sup>4</sup>See OECD (1994).

<sup>5</sup>Card and Krueger (1990) present evidence on the relationship between school quality and earnings. Mankiew, Romer and Weil (1992) use a production function approach and relate real output per head to the rate of human capital accumulation, measured by secondary school enrollment rates.

the schooling system could have negative spillovers on net national output and average per capita productivity.

In this paper, we argue that this potential trade-off between the accumulation of common academic skills and the development of individual skills is exemplified by the economic performance of country as different as (Northern and Central) Italy and Japan<sup>6</sup>. Broadly speaking, Japan has a very competitive schooling system, that is often criticized because it hampers individual development, and an industrial structure that includes clusters of export driven complex technologies<sup>7</sup>. On the other hand, Italy has a less competitive schooling system, with a stronger emphasis on flexible individual development and informal specialization, and an industrial structure characterized by the presence of highly productive industrial districts that often specialize in personalized industrial design<sup>8</sup>. In both countries, the share of small firms is large by international standards.

We use these examples to motivate a simple theoretical model that focuses on the interactions between the degree of competition of the schooling system, the industrial structure and the labor market. Depending on the underlying parameters, these interactions can produce multiple locally stable equilibria. In one equilibrium, schooling is very competitive and individuals invest substantially in the accumulation of common academic skills. The availability of these skills in the market and the complementarity between education and training facilitates the adoption of complex technologies, that require substantial training costs. Large firms, that use these technologies, pay relatively high wages and attract graduates from the best schools. Small firms, that operate in a secondary labor market, use relatively simple technologies and pay the reservation wage. The high expected gain from access

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<sup>6</sup>We exclude Southern Italy because of the structural problems of Italian Mezzogiorno, that makes it different in economic terms from the rest of the country. From now on we shall refer to Northern and Central Italy simply as "Italy".

<sup>7</sup>Examples are electronics (semiconductors) and telecommunications. See Porter (1989). Japanese schools do not "...encourage independence of opinion or support idiosyncratic skill development. Large classes, the routines, the emphasis on group social control, and the strenuous pace required for exam preparation all preclude much individualistic activity.." (Rohlen (1992), p.344).

<sup>8</sup>Industrial districts are often found in the textiles, apparel and personal jewelry sectors. See Porter (1989). "...The unique feature of the Italian economy is that learning takes place mainly outside the schooling system...highly specialized competences are transmitted within households, from a generation to the next..." (Porter (1989), p.511).

into a good school stimulates investment in academic skills.

In another equilibrium, schooling is not as selective and more time is spent for individual development in flexible curricula, both formally at school and informally on the job. Lower accumulation of common academic skills makes the adoption of complex technologies relatively less profitable. At the same time, however, the higher stock of individual skills stimulates the adoption of technologies that are intensive in these skills. Labor turnover is higher and labor market experience matters more than school quality in the reduction of training costs faced by firms. The relative scarcity of large firms, that pay higher wages, and the higher turnover rate further reduces the incentive to accumulate common academic skills, because of the relatively low expected returns.

While the former equilibrium is consistent with a stylized characterization of the Japanese labor market, the latter equilibrium captures the main features of the Italian labor market. Both are decentralized Nash equilibria, and are based on the assumption that each agent (individual or firm) is too small to internalize the effects of her own actions on the optimal strategy of the other agents. In these equilibria, the interactions among decision-makers are characterized both by positive and by negative spillovers. For instance, the individual decision to accumulate common academic skills and to overlook individual skills affects positively the profits of firms adopting complex technologies and negatively the profits of firms using technologies that use intensively individual skills.

Since individual decisions have both negative and positive spillovers on the action of other agents, it is not possible to Pareto rank equilibria as in Cooper and John (1988). Hence, our model suggests that a very competitive schooling system is not necessarily better for net national output than a system where schooling is less competitive. The latter could even be superior when the technologies that use intensively individual skills are highly productive.

The model presented in this paper can be extended in a straightforward way to discuss the relationship between individual creativity and ability to innovate. Both are likely to require either type of skill. While basic academic skills are an important ingredient for creativity and innovation, an excessive accumulation of these skills, triggered by a very competitive schooling system, could hamper both creativity and innovation because it reduces the accumulation of individual skills. The basic idea used in this paper is familiar

in the theory of incentives: concentrating incentives on a single dimension of individual behavior can have undesirable side effects when other dimensions are important<sup>9</sup>.

The material of the paper is organized as follows. In the next section, we provide a stylized characterization of education, industrial structure and labor market performance in Italy and in Japan. Section 3 is the central part of the paper, where the theoretical model is introduced. A discussion of the main results is in Section 4. Conclusions follow.

## 2 THE STYLIZED FACTS

This section briefly illustrates the main features of the Italian and the Japanese labor markets and pin-points the relationship between education systems, the industrial structure and labor market outcomes in the two countries. Needless to say, education systems are the outcome of complex historical, cultural and economic developments. There is no space here for an exhaustive description of these developments and we can only spell out the following key stylized facts.

*1. While the Japanese schooling system emphasizes uniformity, competition and the accumulation of basic skills in mathematics and sciences, the Italian system is both less competitive and less successful, on average, in the provision of these skills to younger cohorts.*

By Western standards, the Japanese schooling system provides limited flexibility in the selection of curricula and little choice to individuals<sup>10</sup>. Historically, "...teaching has been characterized by carefully developed, tightly executed instructions standardized for the entire nation....The system, especially from middle school on, has always been very competitive. Because universities (particularly elite ones) serve as powerful signaling devices in the labor market, competition to enter them is severe.." (Rohlen (1992)). Stern and excessive competition is emphasized also by Porter in his comparative study of competitive advantage. While excessive competition stifles individual creativity, "...the system succeeds in providing the vast majority of students in the whole country with a solid base for further education and

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<sup>9</sup>See Holmstrom and Milgrom (1991).

<sup>10</sup>See Dore and Sako (1989) for details on the education system in Japan..

training. A graduate of a Japanese high school knows as much mathematics as most American college graduates...” (Porter (1989), p.465).

Compared to Japan, the Italian system offers more flexible curricula and substantial freedom of access to all levels of education in exchange for a poorer quality of education. Competition and signalling are less important, as suggested by the virtual absence of an acknowledged system that ranks schools by quality and performance and of cram schools specialized in preparing students for entry examinations, both key features of the Japanese education system. Michael Porter, in his well known analysis of the competitive advantage of nations, emphasizes the relatively poor quality of the Italian schooling system and argues that “..in order to sustain growth and to acquire professional competencies, Italians need to improve their basic knowledge of mathematics, computers and other key disciplines...” (Porter (1989), p. 812.).

Not only is the average level of competence in basic academic skills higher in Japan than in Italy<sup>11</sup>, but also a lower percentage of the population at theoretical age of graduation completes upper secondary education in the latter country. More in detail, less than 60% of individuals at theoretical age of graduation complete upper secondary education in Italy, compared to more than 90% in Japan.

*2. In Japan, large private firms tend to hire new school graduates and schools are very active in the placement of new graduates. In Italy, there is a widely perceived mismatch between the supply of educated labor and the demand by firms operating in the private sector. Italian private industry has historically relied more upon internal and informal training than on formal education.*

A distinctive feature of the Japanese labor market is the willingness, and even the preference, that many firms show for hiring untrained and yet untainted youth just after graduation from school and training them according to their needs. There is also a clear preference for “generalists” over “specialists”. Every year, well over a third of new hires by firms with more than a thousand employees consist of new school graduates, whereas

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<sup>11</sup>It is well known that Asian kids outrank their peers both in the US and in Western Europe in math-and-science comparative tests. See for instance the article “At the Top of the Class” in Newsweek December 2 1996.

only one in eight new hires of firms with less than one hundred employees is a new school graduate<sup>12</sup>. Another key feature in the transition from school to work in Japan is the active role played by schools in the placement of school graduates in the labor market. Schools in Japan often act at least in part as employment agencies. This role is noticeable in upper-secondary schools and above, particularly in junior colleges, science and engineering colleges, and in vocational schools.

In Italy, private industry has traditionally been characterized by "low intensity of education" and by reliance on internal training rather than on formal education<sup>13</sup>. As remarked by Michael Porter (1989), the success of Italian *industrial districts*<sup>14</sup> has been based more on informal training, often provided by the extended family, that operates small artisan shops and small and medium firms, than on formal education. The geographical concentration of these districts has also helped in the diffusion of the relevant knowledge and skills. While Japanese private firms are closely involved in networking with schools of different levels and quality, Italian private firms have traditionally exhibited little interest in the national schooling system<sup>15</sup>.

*3. The industrial structure of both countries is characterized by the important presence of small and medium firms.*

Compared both to the US and to Europe, Italy and Japan share the relative importance of small firms, measured by the percentage of employees working in these firms. While in the United States and Germany only 34.8 percent and 45.9 percent of total employment is in firms with less than 100 employees, this percentage rises to 55.6 percent in Japan and reaches 71.4 percent in Italy.

*4. Job turnover is higher in Italy than in Japan, both in small and in large firms.*

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<sup>12</sup>See the annual issues of Japanese Ministry of Labour, *Koyo Doko Chosa Hokoku*.

<sup>13</sup>See Jannaccone Pazzi and Ribolzi (1990) for more details.

<sup>14</sup>By industrial districts we mean networks of closely located small and medium firms that specialize in similar product lines. See Brusco (1992) and Piore and Sabel (1984) for details.

<sup>15</sup>The main employer of individuals with high formal education in Italy is the public sector. Focusing on individuals with a university degree or higher education, about 75 percent of these individuals are employed in the Italian public sector, compared to only 18 percent in Japan.

In Italy, close to 80 percent of workers aged between 15 and 29 are in firms with less than 100 employees, that employ about 70 percent of Italian workers. On the other hand, about 55 percent of workers aged between 40 and 49 are in firms with more than 100 employees, that employ 30 percent of Italian workers. In sharp contrast, in Japan about 34 percent of workers aged 15-29 are in firms with 500 and more employees, that employ 27 percent of Japanese workers. As workers get older, they tend to shift towards small firms<sup>16</sup>.

Small firms in Italy not only hire the large majority of young school graduates but also exhibit a much larger job turnover than large firms. In particular, gross job turnover in firms with less than 100 employees is more than twice as high as gross job turnover in firms with more than 100 employees. According to Contini and Rapiti (1994).. "in the sector of small firms there is a high share of young workers.. who exhibit substantial turnover... young workers who have been through on the job training in a small firm make up the large majority of job-to-job changes occurring in the Italian economy.." (p. 13). In this view, small and medium firms in Italy provide substantial internal training to workers, who often use accumulated skills either to move to larger firms, where wages are higher and labor conditions better, or to set up their own shop<sup>17</sup>.

Although available figures are not exactly comparable between the two countries because of the differences in the coverage of data, they suggest that job turnover is larger in Italy than in Japan. Focusing only on the expansion and contraction of existing firms, job turnover in firms with less than 20 employees is estimated to be about 23 percent in Italy and about 10 percent in Japan. Considering firms with more than 100 employees, the corresponding values are respectively about 9 and 7 percent<sup>18</sup>.

##### *5. Hourly earnings in both countries vary significantly with firm size.*

The relative importance of small and medium firms in Italy and in Japan

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<sup>16</sup>Sources: Bank of Italy, *Indagini sui bilanci delle famiglie italiane*, 1993; Japanese Bureau of Statistics, *Shugyo Kozo Kihon Chosa*, 1993.

<sup>17</sup>See Contini and Revelli (1992). Substantial turnover from small to large firms in Italy has also been promoted by the institutional and legal environment, that has strongly incentivated job-to-job moves rather than hires from the unemployment pool or from out of the labour force. See OECD (1994) and Bertola-Ichino (1995) for a discussion.

<sup>18</sup>Source: Contini et al. (1995).



raises the question whether earnings vary in a significant way with firm size. Evidence based on aggregate data suggest that they do. In particular, earnings in firms with less than 100 employees are respectively 65 and 60 percent of earnings in firms with more than 1000 employees in Italy and in Japan<sup>19</sup>. On the other hand, earnings in firms with 100 to 999 employees in the two countries are respectively 80 and 72 percent of the earnings of firms with more than 1000 employees<sup>20</sup>.

*6. Both earnings and productivity differentials by firm size are wider in Japan than in Italy.*

Earnings differentials are partially matched by productivity differentials. Using data on Italian and Japanese real gross value added per worker for different industries within the manufacturing sector and for different firm sizes in 1989, we find that, while firm size productivity differentials are quite small in Italy (except for small firms in the clothing industry), they are rather large in Japan, especially for foodstuffs and for electrical machinery and motor vehicles, the two key exporting industries<sup>21</sup>. These patterns are likely to reflect, among other things, both the relative importance of highly productive industrial districts in the Italian economy and the relative abundance of low productivity subcontractors in the Japanese economy.

In sum, education systems in Italy and Japan differ rather sharply with respect to flexibility and competitiveness. At the same time, there are similarities in the industrial structure, with either country experiencing both a larger than average share of small and medium firms and significant earnings and productivity differentials by firm size, and important differences in labor

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<sup>19</sup>Sources: For Italy, INPS (Social Security National Institute) administrative archives (1990). For Japan, Ishikawa and Dejima (1995). Differentials are based on hourly earnings for Japan and on annual earnings for Italy. Only regular workers for Japan, all workers with a social security account in Italy.

<sup>20</sup>Similar results hold for industrial data. A comparison based upon data from Eurostat, *Structure and Activity by Industry*, and the Japanese MITI, *Census of Manufactures*, shows that Italian manufacturing small firms pay 72.8 of the labor cost paid by large firms. This compares with 61.4 percent for Japanese firms.

<sup>21</sup>The original values in national currencies are converted into US dollars by using the 1990 based PPP exchange rates computed by the OECD. Focusing on electrical engineering, labor productivity in small Italian firms is 1.64 times labor productivity in small Japanese firms. This ratio falls to 0.53 when large firms are compared. Similar results apply to textiles, foodstuffs and general engineering.

turnover, hiring practices and relative productivity performance by firm size. To simplify, large firms in Japan tend to hire straight from schools; inter-firm labor mobility is relatively low, especially from small to large firms; small and medium firms are often in a subcontracting relationship with large firms. There is a large productivity gap between large and small-medium firms. In Italy, large firms prefer to hire experienced workers, partly trained by small firms, rather than new school leavers; inter-firm labor turnover is higher than in Japan and many small firms belong to innovative and creative industrial districts, with little or no relationship with large firms. Moreover, small and medium firms show no great disadvantage in terms of labor productivity vis-à-vis large firms.

In the next section, we argue that these stylized features can be related to and *partially* explained by differences in the national education systems. More specifically, we present a simple model that replicates some key features of the Italian and the Japanese economies as equilibrium outcomes of the endogenous interaction between education systems and industrial structures that occurs in national labor markets.

### 3 A THEORETICAL MODEL

Consider an economy populated by a given large number of individuals and firms. Each firm employs a single worker<sup>22</sup>. In each period of time, a fraction  $s$  of employed workers quits the labor market forever and is replaced by an equal inflow of new workers. If the total number of firms is normalized to 1, total outflows and total inflows are both equal to  $s$ . For a single firm, new hires can be either new entrants or workers who quit other firms. For the economy as a whole, however, new hires must necessarily come from new entrants.

New entrants differ only in the level of education acquired before entering the labor market. While education is provided free of charge by schools run by the government, schooling systems can vary in the degree of selectivity and competitiveness. A very selective schooling system allocates individuals in upper and lower-layer schools by adopting tough entry standards. Because

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<sup>22</sup>An extension that allows for the number of employees to explicitly vary among firms is briefly discussed in the concluding remarks.

of these standards, only a fraction of individuals investing in education can enter upper-layer schools <sup>23</sup>.

While education is free of charge, competition for entry into upper-layer schools is not costless. Individuals do not differ in their innate ability and share the same allotted time in education. Let allotted time vary between zero and one. A competitive schooling system requires that individuals devote an important part of the allotted time to develop common basic academic skills (such as the ability to solve mathematical problems and language skills), with little time left for individual development based upon more flexible and personalized curricula at school or upon more informal learning on the job. We capture the key aspects of competition in the schooling system by modeling it as a tournament over an entry standard. The stricter the standard, the lower the fraction of individuals who gain access to upper-layer schools.

Let  $\mu \in [0, 1]$  denote the time allocated by an individual to develop basic academic skills, that are used in the competition to enter upper-layer schools. A higher  $\mu$  increases the probability of getting into upper-layer schools. With  $\mu$  spent in the development of basic academic skills, only  $1 - \mu$  can be spent in individual skills. Treating basic academic skills and individual skills as perfect substitutes is clearly a strong assumption, that is useful to sharpen our results. In practice, the relationship between these two types of skill is more likely to be hump-shaped, with a trade-in for relatively low values of  $\mu$  and a trade-off for high values. As briefly discussed in the concluding remarks, we can easily extend our results to this more complex situation.

Notice the inefficiency of the entry tournament. While only a fraction of participants get into upper-layer schools, all participants spend the same amount of time in the development of homogeneous skills.<sup>24</sup>

Firms can choose between three technologies, L, and S and V. The L-technology has the following features: first, monitoring of individual effort spent by employees on the job is imperfect and costly. In particular, the probability of detecting a worker who is shirking on the job is  $\delta$ , with  $\delta$  less than one. Second, output per head is equal to  $\phi y$ , with  $\phi > 1$ . Third, jobs require

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<sup>23</sup>An examples of upper and lower-layer schools is high-ranked versus low-ranked universities.

<sup>24</sup>The well known "exams-hell" type of competition for upper-layer schools in Japan certainly implies that more than the required number of individuals invest substantial time in the development of the skills required to pass exams.

that new hires be trained at the cost. Skills are firm-specific and training is fully paid by firms. Let  $\tau_l$  be the cost borne by the standard firm<sup>25</sup>. If the hired employee is a new labor market entrant who has just graduated from an upper-layer school, her formal education and investment in the development of basic academic skills interact well with the requirements of this technology, thereby enhancing her trainability. Hence, her training cost is reduced at the rate  $1 - \beta_g\mu$  with  $\beta_g > 0$ .

On the other hand, if the hired employee is from a lower-layer school, the training cost is simply  $\tau_l$  (for the standard firm). An alternative to hiring new school graduates is to hire experienced workers, who have already undertaken some training with the S- or V-technology. While skills are firm-specific and cannot be fully transferred from firm to firm, labor market experience is useful in that it reduces training costs in the L-technology at the rate  $\lambda$ , with  $\lambda \in [0, 1]$ .<sup>26</sup> The obvious implication of these assumptions is that a firm choosing the L-technology will never hire new graduates from lower-layer schools, who are more costly to train. Thus both education in upper-layer schools and labor market experience are productive in this model because they reduce the training costs borne by firms choosing the L-technology.<sup>27</sup>

The S-technology is characterized by perfect and costless monitoring of workers. Moreover, output per head and training costs for new hires are both lower than in the L-technology and equal respectively to  $y$  and  $\tau_s$  (again, in the standard firm). Compared to the L-technology, schooling and investment  $\mu$  are assumed to be ineffective in the reduction of training costs in this type of technology. Hence, firms choosing the S-technology are simply indifferent among graduates coming from different types of school.

Intuitively, it is useful to think of the L-technology as a complex environment that yields higher output but is more difficult to manage. Higher

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<sup>25</sup>The meaning of 'standard' is explained later.

<sup>26</sup>In a broad sense, skills developed in firms using the S- or V-technology are partially transferrable to the L-technology. See Stevens(1995) for a discussion of transferrable skills. In this paper, we assume that labour market experience reduces training costs equally for all movers, independently of their education. This is a simplification and is equivalent to assuming that the relative advantage of being a new graduate from an upper-layer school rapidly decays with labour market experience.

<sup>27</sup>See Rosen (1976) and the references therein for a similar interpretation of the economic role of education. In this paper we assume that the cost of on-the-job training is fully borne by firms. Following Becker, costs and returns are, in general, shared by individuals and firms.

complexity requires adequate skills. These skills are provided to employees by firms and are more easily acquired by individuals with better (and more) education, prior to that being dissipated in work life. Hence, better education reduces training costs burdened by firms adopting this technology. On the other hand, firms adopting the S technology have a simpler and less productive environment that require simpler skills. The cost of acquiring these skills, also provided by the hiring firm, depends neither on the quality nor on the quantity of education.

Finally, the V-technology shares with the S-technology both the perfect and costless monitoring of workers and the training costs  $\tau_s$ . To help intuition, it is useful to think of the V-technology as venture business or as a trial-and-error self-employment sector that relies on the flexibility, the creativity and the specialization provided by individual skills. Output per head in this technology,  $vy$ , is higher than in the S-technology, but lower than in the L-technology. Hence,  $\phi > v > 1$ .<sup>28</sup> Compared to the L-technology, training costs in the V-technology are lower the higher the level of individual skills held by the hired employee. Since individual skills reduce training costs by  $\tau_s [1 - \beta_v (1 - \mu)]$ , where  $(1 - \mu)$  is the level of individual skills, training costs burdened by firms adopting the V-technology are an increasing function of  $\mu$ . We capture this relationship by a convenient reparametrization of training costs in the normal firm as  $\tau_s(1 + \beta_c\mu)$ .

We assume that the training costs associated to the different technologies are related as follows

$$\tau_s < \tau_s(1 + \beta_c) < \tau_l(1 - \beta_g) \quad (1)$$

and

$$\tau_s < \tau_s(1 + \beta_c) < \tau_l\lambda. \quad (2)$$

Hence, either basic academic skills accumulated in the education process or experience in the labor market can reduce training costs faced by L-technology firms but cannot make training in that technology cheaper than training in the V- or S-technologies.<sup>29</sup>

While individuals do not differ in their innate ability, firms differ in their underlying *managerial ability*. Abler managers are more likely to attract

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<sup>28</sup>We assume hereafter that  $(\phi - v)y > \frac{r+s}{\delta}$ , where  $r$  is the real rate of interest.

<sup>29</sup>Notice that these inequalities hold for any value of  $\mu \in [0, 1]$ .

hires from upper-layer schools and to manage and properly organize the complexity of the L-technology, that requires higher training costs. On the other hand, the simpler S-technology is more likely to be chosen by less able managers. Managerial ability,  $\alpha$ , is assumed to be distributed uniformly in the population of firms as

$$\int_{\alpha_{\min}}^{\alpha_{\max}} \frac{1}{\alpha_{\max} - \alpha_{\min}} d\alpha \equiv \int_{\alpha_{\min}}^{\alpha_{\max}} f(\alpha) d\alpha = 1 \quad (3)$$

and affects on-the-job training costs as follows

$$\frac{\tau_z}{\alpha_i} < \frac{\tau_z}{\alpha_j} \text{ iff } \alpha_i > \alpha_j$$

where  $z = l, s$ . Hence, firms endowed with higher managerial ability face lower training costs (and the "standard" firm referred to earlier on is the firm with managerial ability equal to unity).

In this model, individuals invest in education by allocating their time between common academic and individual skills and firms select the appropriate technology. Each agent is assumed to play a Nash non-cooperative game and select the optimal action by taking the action of all other agents as given. Thus each individual chooses  $\mu$  by taking both the action of other individuals and the allocation of firms to technologies as given. This is equivalent to assuming that individuals are too small compared to the size of the market to internalize the effect of their individual choice of  $\mu$  on the allocation of firms. On the other hand, each firm chooses the most suitable technology by taking both the choice of other firms and the decision of individuals as given. Hence, each firm is also too small relative to the size of the market to explicitly take into account the impact of its choice of the appropriate technology on the educational choice of individuals. These features of the game played by each agent imply the presence both of spillovers and of strategic complementarities. As a consequence, there could be multiple equilibria.<sup>30</sup>

To solve for the steady state decentralized Nash equilibrium, we start from the optimal allocation of firms to technologies when the individual choice of  $\mu$  is given.<sup>31</sup> Firms in this frictionless economy live forever and fill immediately

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<sup>30</sup>See the discussion in Cooper and John (1988).

<sup>31</sup>By optimal allocation we mean that each firm in the market selects the technology that yields the highest profit for a given value of  $\mu$ .

vacant jobs created by separations with new hires. Define with  $J$  the asset value of lifetime profits from a filled job and with  $R$  the asset value from a vacant job. In the steady state, the following relation must hold

$$J_i = \frac{\xi_i + sR_i + (1-s)J_i}{1+r} \quad (4)$$

where  $i$  indicates the firm and  $\xi$  are current operating profits gross of training costs. Since vacant jobs are immediately filled and training costs are fully borne by firms, it must be that

$$R_i = J_i - \tau_i$$

Substituting this relationship into equation (4) we get that  $rJ_i = \xi_i - s\tau_i \equiv \pi_i$  and we can simply focus hereafter on current operating profits net of expected training costs,  $\pi_i$ .

In the steady state, profits depend on the selection of the technology. In particular, for a firm choosing the L-technology the current operating profit net of training costs (hereafter, simply 'profit') is defined as

$$\pi_{pi} = \phi y - w_p - \frac{s\tau_l(1 - \beta_g\mu)}{\alpha_i} \quad (5)$$

if it chooses to hire a new school graduate in case of a vacancy, and

$$\pi_{mi} = \phi y - w_m - \frac{s\lambda\tau_l}{\alpha_i} \quad (6)$$

if it chooses to hire an experienced worker from the labor market. Notice that  $w_p$  and  $w_m$  are (real) wages when firms hire either from schools or from the market of experienced workers and that the profit  $\pi_{pi}$  is a positive function of  $\mu$ , the investment in basic academic skills. This is the source of positive spillovers on the L-technology.

Let  $F$  be the number of firms that choose the L-technology, with  $P$  firms hiring new graduates and  $M = F - P$  firms hiring experienced workers from other firms. In what follows, we shall adopt the convention of identifying a firm adopting the L-technology and hiring new school graduates as a  $P$ -firm, and a firm hiring an experienced worker and using the same technology as a  $M$ -firm.

Let  $S$  be the number of firms using the S-technology and  $V$  be the number of firms using the V-technology. Clearly,  $S + V = 1 - F$ . Let  $Q$  be the total number of quits. Since only  $M$ -firms hire experienced workers from the market to replace  $sM$  separations, in the steady state equilibrium it must be that  $Q = sM$ . It is shown below that  $P$ - and  $M$ -firms pay the same wage to their workers and that  $S$ - and  $V$ -firms pay a wage  $w_s$  strictly lower than the wage paid by the L-technology firms. Consequently, quits can only occur profitably from  $S$ - and  $V$ -firms to  $M$ -firms, and the endogenous quit rate  $q$  is given by

$$q = \frac{sM}{1 - F} \quad (7)$$

With endogenous quits, the profit for a firm  $i$  choosing either the V- or the S-technology is respectively

$$\pi_{vi} = vy - w_s - \frac{(s + q)\tau_s(1 + \beta_c\mu)}{\alpha_i} \quad (8)$$

and

$$\pi_{si} = y - w_s - \frac{(s + q)\tau_s}{\alpha_i}. \quad (9)$$

Importantly, profits in the V-technology are a negative function of  $\mu$ . Hence, the degree of competition of the schooling system has negative spillovers on the selection of this technology.

The choice of the most profitable technology depends both on managerial ability and on the level of wages paid to workers. Notice first that both the V- and the S-technology imply perfect and costless monitoring of workers. With no unemployment, competition among workers joining these firms drives the real wage down to  $b$ , the reservation level. Hence

$$w_s = b. \quad (10)$$

On the other hand, costly monitoring and limited information on worker effort force firms choosing the L-technology to pay efficiency wages in order to motivate workers and to achieve efficient production levels. Consider an  $M$ -firm. If a worker employed in such a firm is detected shirking, she is fired and moves to an  $S$ - or  $V$ -firm, where wages are lower.<sup>32</sup> If she is not detected,

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<sup>32</sup>See Jones (1987) for an efficiency wage model in a dual labour market set-up.



she stays on. With risk neutrality, individual utility is given by

$$U = w - e \quad (11)$$

where  $e$  is individual effort, that can either be equal to zero or to one. In the steady state, the returns from not shirking are

$$rE_m = w_m - 1 - sE_m \quad (12)$$

where  $E_m$  is the asset value from employment in an  $M$ -firm in the absence of shirking. Alternatively, the returns from shirking are

$$rE_m^S = w_m + \delta [E_c - E_m^S] - sE_m^S \quad (13)$$

where  $E_m^S$  is the asset value from employment in the presence of shirking and  $E_c$  is the asset value from employment in the competitive sector of the labor market, composed of the firms adopting either the S- or the V-technology. Hence, a worker who shirks on the job gains utility because her effort is equal to zero but faces the nonzero probability of being dismissed and forced to take a job in the competitive sector where wages are lower<sup>33</sup>. Finally, define the returns from employment in the competitive sector as

$$rE_c = b - 1 + q[E_m - E_c] - sE_c. \quad (14)$$

With endogenous quits, workers dismissed by  $M$ -firms can find their way back into the high wage sector by transiting in the sector of S- or V-firms and by moving into another  $M$ -firm with probability  $q$ . Using the no shirking condition  $rE_m = rE_m^S$  and equations (12)-(14), we obtain

$$w_m = b + \frac{(r+s)}{\delta} + \frac{q}{\delta} > w_s = b \quad (15)$$

As expected,  $M$ -firms pay a premium over the reservation wage in order to motivate workers to spend the efficient level of effort. Next, consider  $P$ -firms. These firms share with  $M$ -firms the L-technology and imperfect monitoring.

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<sup>33</sup>We are ruling out the possibility that M-firms hire straight from P-firms or other M-firms. With a positive wage differential, M-firms recognize that workers coming from P- or M-firms must be shirkers and avoid hiring them. Identification of workers as previous shirkers is difficult, however, if they are hired from S- or V-firms.

For a given quit rate  $q$ , it is easy to check that the efficiency wage paid by  $P$ -firms is also given by equation (15), so that <sup>34</sup>

$$w_m = w_p \quad (16)$$

Turning to the allocation of firms to the alternative technologies, we can use equations (7), (15) and (16) into the definitions of current profits for firms choosing the L-technology to get

$$\pi_{pi} = \phi y - b - \frac{(r+s)}{\delta} - \frac{sM}{\delta(1-F)} - \frac{s\tau_l(1-\beta_g\mu)}{\alpha_i} \quad (17)$$

and

$$\pi_{mi} = \phi y - b - \frac{(r+s)}{\delta} - \frac{sM}{\delta(1-F)} - \frac{s\tau_l\lambda}{\alpha_i}. \quad (18)$$

Recalling that each firm chooses the most adequate technology by taking the decision of other firms as given, equations (17) and (18) clearly suggest that, independently of the value of  $\alpha$ , firms using the L-technology will prefer to hire new graduates rather than experienced workers if

$$\mu \geq \frac{1-\lambda}{\beta_g} \equiv \underline{\mu}, \quad (19)$$

that is, if, for a given  $\mu$ , the degree of positive interaction between basic academic skills and the L-technology is high ( $\beta_g$  is high) and/or if previous labor market experience is of little use for training in the L-technology ( $\lambda$  is high). To put it differently, firms will adopt the P-hiring policy when investment  $\mu$  exceeds a certain critical level,  $\underline{\mu}$ .<sup>35</sup>

If condition (19) holds, then *all* firms choosing the L-technology hire exclusively from schools. With no hirings of experienced workers, there can be neither endogenous quits nor  $M$ -firms in the optimal allocation. We call this situation the *P-regime*, where  $P = F$ . On the other hand, if condition

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<sup>34</sup>This is only true *ex-ante*, under the assumption that each firm, independently of the type, chooses the (real) wage by taking the quit rate  $q$  as given. We shall show below that  $w_m \neq w_p$  in an *ex-post* sense, because the endogenous quit rate varies between the two possible regimes.

<sup>35</sup>Notice that each firm takes  $M$  as given when choosing between hiring new graduates and hiring experienced workers in the L technology. If every firm chooses  $P$  then  $M = 0$  in the ex-post equilibrium and profits  $\pi_p$  are higher *ex-post* than *ex-ante*.

(19) does not hold, then *all* firms using the L-technology hire experienced workers from the market. Endogenous quits are nonzero and there are only  $M$  firms. This is called the *M-regime*, where  $M = F$ . We shall consider both regimes in turn.

### 3.1 The P-regime.

In this regime, firms using the L-technology hire only from school and the endogenous quit rate  $q$  is equal to zero. Hence, the separation rate in this regime is given by  $s$  for each type of firm. The allocation of firms to technologies L, S and V can be characterized as follows. First, consider the choice between the S- and the V-technology. Using equations (8), (9) and (10), the former is preferred iff

$$\alpha_i \geq \frac{s\mu\tau_s\beta_c}{y(v-1)} \equiv \alpha_V, \quad (20)$$

so that firms with the level of managerial ability higher than  $\alpha_V$  will find the V-technology more profitable than the S-technology. Notice that both profit functions are concave in  $\alpha$ . Moreover, it is easy to show that

$$\frac{\partial}{\partial \alpha} [\pi_v - \pi_s] > 0 \quad \text{iff} \quad \mu > 0$$

This implies that, with  $\alpha_V \geq \alpha_{\min}$ ,  $[\pi_v - \pi_s] < 0$  when  $\alpha$  is at its minimum value  $\alpha_{\min}$ .

Next, consider the choice of the L-technology in the P-regime. Assumptions (1) and (2) ensure that

$$\frac{\partial}{\partial \alpha} \pi_p > \frac{\partial}{\partial \alpha} \pi_v > \frac{\partial}{\partial \alpha} \pi_s. \quad (21)$$

Thus the profit function with the L-technology is steeper than the other two functions. It is also reasonable to assume that

$$[\pi_p - \pi_s] < 0 \quad \text{when} \quad \alpha = \alpha_{\min} \quad (22)$$

so that the marginal firm, that is endowed with the lowest level of managerial ability, prefers the S- to the L-technology.

One can easily check that the profit function for firms choosing the L-technology can intersect the profit function for the V-technology either to the left or to the right of  $\alpha_V$ . Since the allocation of firms to technologies is given by the envelope of the profit functions, in the former case there are no firms choosing the V-technology and in the latter case all the three technologies attract a positive number of firms. Hence, there are two possible allocations.

In the case when the  $\pi_p$  curve intersects the  $\pi_v$  curve to the left of  $\alpha_V$ , only the S- and the L-technology attract firms and the allocation depends on the condition

$$[\pi_p - \pi_s] \gtrless 0$$

that is, the L-technology is chosen if

$$\alpha_i \geq \frac{s [\tau_l(1 - \beta_g\mu) - \tau_s]}{y(\phi - 1) - \frac{(r+s)}{\delta}} = \alpha_{P'} \quad (23)$$

In this case, the equilibrium share of firms that choose the S-technology is

$$\int_{\alpha_{\min}}^{\alpha_{P'}} f(\alpha) d\alpha = S \quad (24)$$

and the share of firms choosing the L-technology is  $F = 1 - S$ .

Alternatively, when the  $\pi_p$  curve intersects the  $\pi_v$  curve to the right of  $\alpha_V$ , there is another critical point,  $\alpha_P$ , defined by

$$\alpha_i \geq \frac{s [\tau_l(1 - \beta_g\mu) - \tau_s(1 + \beta_c\mu)]}{y(\phi - v) - \frac{(r+s)}{\delta}} \equiv \alpha_P \quad (25)$$

such that all firms with managerial ability higher than  $\alpha_P$  will choose the L-technology. In this case, the number of firms selecting the V-, L- and S-technologies are respectively

$$\int_{\alpha_V}^{\alpha_P} f(\alpha) d\alpha = V, \quad (26)$$

$$\int_{\alpha_P}^{\alpha_{\max}} f(\alpha) d\alpha = P = F, \quad (27)$$

and

$$S = 1 - F - V. \quad (28)$$

Using definition (3),  $F$  is seen to be related to  $\alpha_P$  by

$$\alpha_P = \alpha_{\max} - F(\alpha_{\max} - \alpha_{\min}). \quad (29)$$

The following lemma characterizes the allocation of firms to technologies in the P regime.

**Lemma 1** *There is a critical value of  $\mu$ , such that allocations with no firms adopting the V-technology exist if  $\mu \geq \bar{\mu}$ . This critical value does not necessarily lie in the domain  $\mu \in [0, 1]$ .*

**Proof:** First notice that  $\alpha_V(\mu)$  is increasing and  $\alpha_P(\mu)$  is decreasing in  $\mu$ . Moreover,  $\alpha_V$  goes to zero and  $\alpha_P$  goes to a positive number as  $\mu$  tends to zero. Hence, the two curves must have a unique intersection point, say, at  $\mu = \bar{\mu}$ . This intersection is given by

$$\frac{\tau_l - \tau_s}{\tau_s \beta_c} = \frac{\mu}{(v-1)y} \left[ (\phi - v)y - \frac{(r+s)}{\delta} \right] + \frac{(\tau_l \beta_g + \tau_s \beta_c) \mu}{\tau_s \beta_c} \equiv k(\mu). \quad (30)$$

It is easy to see that  $k(\mu)$  is an increasing and linear function of  $\mu$ , with  $k(0) = 0$  and  $\lim_{\mu \rightarrow 1} k(\mu) \rightarrow k_1$ , a positive constant. A sufficient condition for  $\bar{\mu}$  to be less than 1 is that  $k_1$  be larger than the left hand side of (30).

Using equations (25) and (29) it is straightforward to show that

$$\frac{\partial F}{\partial \mu} > 0, \quad \frac{\partial^2 F}{\partial \mu^2} = 0 \quad (31)$$

so that the optimal number of firms selecting the L-technology in the P-regime is a linear and increasing function of the investment in basic academic skills  $\mu$ .

The foregoing discussion can be geometrically summarized in the  $(\mu, \alpha)$  plane as in Figure 1. Since  $\alpha$  is bounded by  $\alpha_{\max}$  from above and by  $\alpha_{\min}$  from below, and also  $\mu$  is bounded by 1 from above and by 0 from below, our consideration is restricted to the rectangular domain defined by these boundaries. Furthermore, this rectangular domain is separated into two segments by a vertical line at  $\mu = \bar{\mu}$  as defined by equation (23). The P-regime occurs on the right hand side and the M-regime on the left hand side of this vertical line. We have drawn a downward sloping  $\alpha_P = \alpha_P(\mu)$  curve and an

upward sloping  $\alpha_V = \alpha_V(\mu)$  curve that intersect at  $\mu = \bar{\mu}$ <sup>36</sup>. To the right of this point, the  $\alpha_V = \alpha_V(\mu)$  curve disappears and the  $\alpha_{P'} = \alpha_{P'}(\mu)$  curve continues to slope downward. For later reference we shall call the union of  $\alpha_P(\mu)$  and  $\alpha_{P'}(\mu)$  curves as the PP curve and write it as  $\alpha_P = \tilde{\alpha}_P(\mu)$ ., i.e.,

$$\tilde{\alpha}_P(\mu) \equiv \begin{cases} \alpha_P(\mu) & \text{if } \mu \leq \bar{\mu} \\ \alpha_{P'}(\mu) & \text{if } \mu > \bar{\mu} \end{cases} \quad (32)$$

Similarly, we shall call the  $\alpha_V = \alpha_V(\mu)$  curve the  $V_P V_P$  curve. For any given level of  $\mu$ , say  $\mu = \mu'$ , the selection of technology by a firm is represented by the location of its managerial ability  $\alpha$  along the vertical line  $\mu = \mu'$ . The relative size of S, V, and L can then be measured by the relative distance of the vertical line segments as dissected by the above curves.

Now consider the educational investment of individuals. The optimal allocation of firms to technologies implies that new school graduates can be hired either by firms with the L-technology at the wage  $w_p$  or by firms with the S- or the V-technology at the wage  $b$ . In a steady state equilibrium, individuals going through the schooling system take these possible outcomes into account in their selection of investment in basic academic skills,  $\mu$ .

With a constant population, the steady state equilibrium requires that in each period of time there are  $s$  individuals graduating from school and entering the labor market. As discussed above, schooling systems can vary in their degree of selectivity. We measure selectivity by the exogenous parameter  $\Phi$ , the number of slots available in upper-layer schools, with  $\Phi < s$ . The lower  $\Phi$ , the more selective is the schooling system. Among individuals graduating from school and entering the labor market,  $\Phi = s\Theta$  are graduates from upper-layer schools and  $s - \Phi = s(1 - \Theta)$  are graduates from lower-layer schools, where  $\Theta$  is the proportion of new graduates coming from upper-layer schools. In the P regime, only graduates from upper-layer schools can be hired with probability  $\frac{F}{\Theta}$  by firms adopting the L-technology and paying the higher wage  $w_p$ . In what follows, we assume that the government chooses exogenously a selectivity parameter  $\Theta$  to ensure that

$$\Theta \geq F \quad (33)$$

Hence, the net return that individual  $j$  can expect from access to an upper-

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<sup>36</sup>In the diagram, we assume that  $\bar{\mu} < 1$ .

layer school is

$$EU_{Gj} = \frac{F}{\Theta} E_{pj} + \left(1 - \frac{F}{\Theta}\right) E_{cj} - (\mu - \mu_0) \quad (34)$$

where  $(\mu - \mu_0)$  is the cost of investment in education with the following properties

$$- (0) = 0, \quad - ' \leq 0 \text{ as } \mu \leq \mu_0, \quad - '' > 0, \quad (35)$$

$(1 - \frac{F}{\Theta})$  is the probability of finding a job in an  $S$ - or in a  $V$ -firm and  $\mu_0$  is the minimum level of investment in basic academic skills, that yields zero investment costs.  $E_{pj}$  is the same as  $E_m$  defined by equation (12) except for a change in subscripts from  $m$  to  $p$ , and  $E_{cj}$  is defined by equation (14) with  $q$  set to zero under the P-regime.

On the other hand, the expected net return from entry into a lower-layer school is

$$EU_{Bj} = E_{cj} - (\mu - \mu_0) \quad (36)$$

With homogeneous individuals, entry into upper-layer schools is restricted by the selectivity parameter  $\Theta$  and is modeled in this paper as a tournament against an entry standard. Given a standard of performance  $\sigma^*$ , individuals who perform at least up to the standard get into upper-layer schools, while those who perform less than the standard remain in lower-layer schools. Since participants are homogeneous, they end up investing the same amount  $\mu$ <sup>37</sup>.

More in detail, define the probability of passing the standard as  $\text{Prob}\{\sigma \geq \sigma^*\}$  and let individual performance in the schooling race be measured as

$$\sigma = \mu + \varepsilon \quad (37)$$

where  $\varepsilon$  is luck, that varies according to a standard normal distribution  $G$ . Individuals choose  $\mu$  to maximize

$$\text{Prob}\{\sigma \geq \sigma^*\} EU_{Gj} + \text{Prob}\{\sigma < \sigma^*\} EU_{Bj} \quad (38)$$

By substituting equations (34), (36), and (37) into (38), the first order condition of this maximization problem is

$$g(\sigma^* - \mu) \left[ \frac{F}{\Theta \delta} \right] = - '(\mu - \mu_0) \quad (39)$$

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<sup>37</sup>See Malcomson (1984) for a detailed discussion of similar tournaments.

where  $g$  is the normal density function and each individual sets  $\mu$  by taking both  $F$  and  $\Theta$  as given. *Ex-post*, the probability of entering an upper-layer school must equal the proportion of available seats. Hence,

$$1 - G(\sigma^* - \mu) = \Theta \quad (40)$$

where  $G$  is the distribution function and we have used the fact that homogeneous individuals set the same value of  $\mu$ <sup>38</sup>.

The comparative static properties of the choice of  $\mu$  by each individual is summarized by the following lemma.

**Lemma 2** *In the P-regime, individual investment in basic academic skills increases either (i) as the number of L-technology firms increases, or (ii) as the selectivity of upper-layer schools tightens.*

**Proof:** Noting that  $\mu$  and  $\sigma^*$  are simultaneously determined by equations (39) and (40), we totally differentiate the system to obtain:

$$\begin{pmatrix} -\frac{F}{\delta\Theta}g' & -\frac{F}{\delta\Theta}g' \\ g & g \end{pmatrix} \begin{pmatrix} d\mu \\ d\sigma^* \end{pmatrix} = \begin{pmatrix} \frac{gF}{\delta\Theta^2} \\ 1 \end{pmatrix} d\Theta + \begin{pmatrix} -\frac{g}{\delta\Theta} \\ 0 \end{pmatrix} dF \quad (41)$$

where  $g$ ,  $g'$ , and  $\delta\Theta$  are respectively evaluated at the optimum.

(i) When  $d\Theta = 0$ ,  $d\mu = d\sigma^*$  from the second part of (41). Substituting this into the first part gives

$$\frac{d\mu}{dF} = \frac{g}{\delta\Theta} > 0. \quad (42)$$

(ii) When  $dF = 0$ ,  $d\sigma^* = d\mu - (1/g)d\Theta$ , and substituting this into the first part of (41) gives

$$\frac{d\mu}{d\Theta} = -\frac{F}{\delta\Theta} \left( \frac{g}{\Theta} + \frac{g'}{g} \right). \quad (43)$$

The expression within parentheses in the RHS of this expression can be re-written as

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<sup>38</sup>We are supposing here that the government sets the standard  $\sigma^*$  passively given the decision of individuals. Alternatively we could assume that the government sets  $\sigma^*$  by acting as a Stackelberg leader. The qualitative features of the comparative static properties discussed below, however, are not affected.



$$\left(\frac{g}{\Theta} + \frac{g'}{g}\right) = \left(\frac{g}{1-G} + \frac{g'}{g}\right) \equiv \chi(\varepsilon)$$

Because  $g$  is normal,  $g'/g = -\varepsilon$ . On the other hand, the first term is known as the inverse Mills' ratio, and we denote it as  $\zeta(\varepsilon)$ . Hence

$$\chi(\varepsilon) = \zeta(\varepsilon) - \varepsilon$$

Since we know that

$$\zeta(\varepsilon)(\zeta(\varepsilon) - \varepsilon) \in (0, 1)$$

and that  $\zeta(\varepsilon)$  is positive,  $\chi(\varepsilon)$  must also be positive<sup>39</sup> and we can unambiguously sign the expression (43) to be negative. (QED.)

We shall hereafter write the optimal level of investment  $\mu$  in the P-regime as

$$\mu = \mu_P(F, \Theta). \quad (44)$$

The above proof shows that, for a given  $\Theta$ , the difference  $\sigma^* - \mu$  stays constant and so does  $g$ , at the optimum. Hence we have:

**Corollary 3** *If the cost of investing in homogeneous skills is quadratic, i.e.,*

$$- (\mu - \mu_0) = \frac{\psi}{2} (\mu - \mu_0)^2 \quad (45)$$

*then the optimal choice of education is a linear function of  $F$ , given  $\Theta$ .*

Geometrically, the optimal choice of  $\mu$  for a given  $F$  can be expressed by a downward sloping relationship between  $\alpha_P$  and  $\mu$  (recall the relationship (29)) in the  $(\mu, \alpha)$  plane of Figure 1. Using the quadratic investment cost function (45), this relationship is drawn as a straight line segment emanating from the point  $(\mu_0, \alpha_{\max})$  with slope

$$-\frac{\psi\delta\Theta(a_{\max} - a_{\min})}{g(\sigma^* - \mu)}. \quad (46)$$

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<sup>39</sup>See Greene (1990), p. 718.

We shall call this the  $E_P E_P$  curve. The geometry given in Figure 1 makes it clear that the interaction between optimal individual investment and the optimal allocation of firms to technologies determines a steady state equilibrium as an intersection of the  $E_P E_P$  curve and the PP curve. It is easy to see that there may be two intersections, one intersection, or no intersection at all, depending on the parameters of the model. Because of the constraint (33), the resulting value of  $F$  must not exceed  $\Theta$  even if an intersection exists. Moreover, in order for an equilibrium to be locally stable the slope of the  $E_P E_P$  curve given by the expression (46) must be less than that of the PP curve which is shown to be given by:

$$\frac{d\widetilde{\alpha}_P(\mu)}{d\mu} = \begin{cases} -\frac{s(\tau_l\beta_g + \tau_s\beta_c)}{(\phi-v)y - \frac{r+s}{\delta}} & \text{if } \mu \leq \bar{\mu} \\ -\frac{s\tau_l\beta_g}{(\phi-1)y - \frac{r+s}{\delta}} & \text{if } \mu > \bar{\mu} \end{cases} . \quad (47)$$

i.e., the  $E_P E_P$  curve must be steeper than the PP curve. This stability condition rules out one of the intersection points as unstable when there are two intersection points.

We can summarize the foregoing discussion as:

**Proposition 4** *An interior locally stable steady state equilibrium in the P-regime exists under a quadratic investment cost function for a given selectivity parameter  $\Theta$ , if there exists a pair  $(\mu^*, F^*)$  such that*

$$(i) \quad \underline{\mu} < \mu^* \leq 1, \quad 0 < F^* \leq \Theta$$

(ii)

$$F^* = \frac{\alpha_{\max} - \widetilde{\alpha}_P(\mu^*)}{\alpha_{\max} - \alpha_{\min}}$$

and

$$\mu^* = \mu_P(F^*, \Theta).$$

(iii)

$$\frac{\psi\delta\Theta(a_{\max} - a_{\min})}{g(\sigma^* - \mu^*)} > -\frac{d\widetilde{\alpha}_P(\mu^*)}{d\mu}.$$

*There is at most one locally stable steady state equilibrium in the P-Regime.*

In the P-regime, separations are exogenous and firms fill their vacant positions by hiring new school graduates. While firms adopting the L-technology

strictly prefer to hire graduates from upper-layer schools, S- and V-firms hire indifferently graduates from either type of school. There is no unemployment and P-firms pay higher wages to their employees in order to motivate them to produce the desired level of effort. S- and V-firms pay the reservation wage. There is no inter-firm mobility from the S- and V-sector to the L-sector. If we interpret high managerial ability as the ability to attract and organize not only trainable workers from upper-layer schools but also large capital stock, we can characterize the equilibrium in terms of firm size, measured by the capital-labor ratio. While large firms typically choose the L-technology, small and medium firms choose either the S- or the V-technology. We find this characterization of the equilibrium fairly suggestive of the Japanese model, where labor turnover is limited and large firms hire mainly from upper-layer schools. In what follows we shall call the equilibrium in the P regime *J-equilibrium*.

### 3.2 The M-regime.

Compared to the P-regime, nonzero quits are possible in the steady state equilibrium of this regime. These quits occur because firms using the L-technology pay efficiency wages that are strictly higher than the reservation wage and prefer to fill their vacancies with experienced workers. These workers must come from S- or V- firms, where wages are lower<sup>40</sup>. Since workers are homogeneous, the endogenous quit rate is defined, in equilibrium, by equation (7), with  $M = F$ . Notice that endogenous quits are costly to firms choosing the S- or the V-technology because they increase training expenses by raising separations<sup>41</sup>.

In this regime firms will prefer the V- to the S-technology if

$$\alpha_i \geq \frac{\left(\frac{s}{1-F}\right) \mu \tau_s \beta_c}{y(v-1)} = \alpha_V. \quad (48)$$

Notice the change caused to equation (20) by the presence of non-zero quits. Similarly, comparison of profits in the L-technology and the V-technology

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<sup>40</sup>Exogenous separations are flows out of the labour force. Moreover, the no shirking condition ensures that no worker is fired from firms using the L-technology.

<sup>41</sup>With an exogenous reservation wage and a given number of firms, the marginal firm must be profitable even with higher separations. This we assume hereafter. Alternatively, we can endogenize the level of the reservation wage by introducing a zero profit condition for the firm run by the least able manager.

is also conditioned by the presence of quits. In particular, under the assumption that the optimal allocation of firms to technologies has a positive number of firms for each technology<sup>42</sup>, the choice between the L- and the V-technology depends on the following condition

$$\alpha_i \geq \frac{s \left[ \tau_l \lambda - \frac{\tau_s}{1-F} (1 + \beta_c \mu) \right]}{y (\phi - v) - \frac{(r+s)}{\delta} - \frac{sF}{\delta(1-F)}} = \alpha_M \quad (49)$$

so that all firms with managerial ability higher than  $\alpha_M$  will choose the L-technology. The shares of firms selecting the V-, L- and S-technologies are again determined by equations (26), (27), and (28), respectively, except that  $\alpha_M$  replaces  $\alpha_P$ . We shall express the dependence of  $\alpha_M$  on  $\mu$  by writing  $\alpha_M = \alpha_M(\mu)$ .

Total differentiation of (49) shows that the relationship between the share of L-technology firms,  $F$ , and the level of individual investment in basic academic skills,  $\mu$ , are increasing in this regime, as it was in the previous regime. The following Lemma characterizes the main properties of the function  $\alpha_M(\mu)$  (the proof is relegated to an Appendix).

**Lemma 5** *Given the boundary level of educational investment  $\underline{\mu} = (1-\lambda)/\beta_g$ , if the condition*

$$\alpha_P(\underline{\mu}) > \delta \tau_s (1 + \beta_c \underline{\mu}) \quad (50)$$

*holds, then the function  $\alpha_M = \alpha_M(\mu)$  is well-defined for all  $\mu \in [0, \underline{\mu}]$ . Moreover, it has the following properties: (i)  $\alpha_M(\underline{\mu}) > \alpha_P(\underline{\mu})$ , (ii)  $\alpha_M(\mu)$  is monotonically decreasing in the domain  $\mu \in [0, \underline{\mu}]$ .*<sup>43</sup>

We shall hereafter assume that condition (50) is satisfied. In Figure 1,  $\alpha_M(\mu)$  is a downward sloping curve in the domain of the M-regime, ending at  $(\underline{\mu}, \alpha_M(\underline{\mu}))$ . We shall call it the MM curve. On the whole, the curve

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<sup>42</sup>We can easily establish in this regime a lemma similar to Lemma 1.

<sup>43</sup>We can show that if  $\alpha_P(\underline{\mu}) < \delta \tau_s (1 + \beta_c \underline{\mu})$ ,  $\alpha_M(\mu)$  is not well-defined in the neighborhood of  $\mu = \underline{\mu}$ . However, we can still show that under the following regularity condition

$$\frac{s(\lambda \tau_l - \tau_s)}{(\phi - v)y - \frac{r+s}{\delta}} > \delta \tau_s$$

there exists a certain  $\mu' < \underline{\mu}$  such that  $\alpha_M(\mu)$  is well-defined for  $\mu \in [0, \mu']$  with the property that  $\alpha'_M(\mu) < 0$ .

representing the critical value of  $\alpha$  that determines the share of L-technology firms not only has a kink but also jumps at  $\mu = \underline{\mu}$ . Similarly, the curve representing the critical value of  $\alpha$  that divides the choice between the V- and the S-technology,  $\alpha_V$ , to be called the  $V_M V_M$  curve, has also a kink as well as a jump at  $\mu = \underline{\mu}$ .

Turning to the choice of education by individuals, notice that firms choosing the L-technology in this regime prefer to hire experienced workers rather than new school graduates. On the other hand, firms choosing either the S- or the V-technology are indifferent to the type of schools each new hire has graduated from. Therefore, the individual incentive to participate in the competition to enter upper-layer schools, that is to choose  $\mu$  such that  $\mu > \mu_0$ , is simply zero.

Notice also that there is no incentive for individuals to set  $\mu < \mu_0$ . This is because the model in this paper assumes that workers are paid only their reservation wage in spite of the improved productivity effect of their investment in individual skills (that is, their small investment in  $\mu$ ). This feature is justified by the underlying assumption that increases in productivity can only be triggered by firm-specific training fully paid by firms. If these training costs were shared between V-firms and hired workers, individuals would have an incentive to decrease  $\mu$  even at a cost. While this possibility is ruled out in the current paper, we do not expect it to change our results in a qualitative way.

The privately optimal level of educational effort in the M-regime is simply  $\mu_0$ , the minimum level of  $\mu$  in terms of cost, quite independently of the distribution of firms among available technologies. In Figure 1, this choice is represented by the vertical line  $\mu = \mu_0$ . We call it the  $E_M E_M$  curve. It is immediate to see that a steady state equilibrium is established by the intersection of  $E_M E_M$  and  $MM$  curves,  $(\mu_0, \alpha_M(\mu_0))$ . More formally, we have

**Proposition 6** *Under the conditions established in Lemma 5, a steady state equilibrium exists in the M-regime if*

$$\alpha_M(\mu_0) < \alpha_{\max}.$$

*There is at most one such an equilibrium, and, if it exists, it is locally stable.*

Figure 1 shows that the equilibrium share of L-technology firms is smaller in the M-regime than that in the P-regime. On the other hand, the share of

V and S firms is respectively higher and lower in the M-regime than in the P-regime. If we characterize the equilibrium in terms of firm size, with large firms typically choosing the L-technology and small and medium firms choosing either the S- or the V-technology, the M-regime exhibits both significant labor market flows from small to large firms, with the small firms taking care of initial training in the labor market, and the limited role played by the schooling system in the matching of new entrants with private industry, that prefers experienced workers to new graduates of relatively low average quality. Compared to the P-regime, this regime has also a more important presence of V-firms, that have higher productivity than S-firms and use the advantage of a larger pool of individual skills developed either at school or informally on the job. We find this characterization of the equilibrium fairly suggestive of the Italian economy. For this reason we call the equilibrium in the M regime *I-equilibrium*.

### 3.3 The Possibility of Multiple Regime Equilibria

We have seen that there is at most one locally stable interior equilibrium in the P-regime (with  $\mu > \mu_0$ ) and that there is also at most one locally stable equilibrium in the M-regime. While an equilibrium could also occur at the boundary of the two regimes, that is where the  $E_P E_P$  curve intersects the vertical line at  $\mu = \underline{\mu}$ , such a point cannot be a (locally) stable equilibrium. Hence, we have

**Proposition 7** *Depending on the parameters of the model, the economy can be characterized by (i) a single P-regime equilibrium, (ii) a single M-regime equilibrium, (iii) multiple regime equilibria, or (iv) no equilibrium.*

The case of multiple regime equilibria, that is, an equilibrium in the P-regime (*J-equilibrium*) and an equilibrium in the M-regime (*I-equilibrium*) for the same values of the underlying parameters, is illustrated in Figure 1. In such a case, historical accident decides which equilibrium the economy actually falls in. In the *J equilibrium*, investment  $\mu$  tends to be high because expected returns are high and large firms hire graduates from upper-layer schools. On the other hand, in the *I equilibrium* investment  $\mu$  is low, individual skills are more abundant and labor market turnover is relatively high. As we shall see below, exogenous shocks such as a reduction in the selectivity of schooling can shift the economy from one equilibrium to the other.

## 4 DISCUSSION

Interesting features of the model presented above are both the presence of multiple regime equilibria and the possibility that an economy shifts from an equilibrium to the other as a result of exogenous shifts in the parameters. Consider again Figure 1, where the *I-equilibrium* in the M-regime and the *J-equilibrium* in the P-regime coexist.

Now suppose that initially the economy is in the *J-equilibrium* and that this steady state equilibrium is perturbed by an exogenous change in the selectivity of the schooling system in the direction of less keen competition, i.e., greater  $\Theta$ . Because of Lemma 2, this implies a leftward shift of the  $E_P E_P$  curve. There is no other change in the system so that the  $E_M E_M$ , PP, and MM curves are unchanged. Thus the new steady state equilibrium moves in the upper-left direction along the PP curve. If the change in  $\Theta$  is sufficiently large, individual investment in basic academic skills falls to the point that large firms find it convenient to hire experienced workers from the market. The *J-equilibrium* disappears and the only equilibrium left in the economy is the *I-equilibrium*, that is unaffected by the change in  $\Theta$ , no matter how large. In the new equilibrium, workers are willing to quit since large firms pay higher wages because of internal efficiency reasons. Less time spent in accumulating basic academic skills leaves more space to the development of individual skills, encouraging firms to choose the V-technology. As the decentralized Nash equilibrium switches from the P-regime to the M-regime, the share of V-firms increases and the share of S-firms decreases.

Importantly, a reversal of the original shock, that increases the selectivity of the schooling system back to its original level, is unlikely to produce a shift of the system back to the original *J-equilibrium* in the P regime. With no firms hiring from upper-layer schools, individuals gain nothing from increased competition and have no incentive to increase their effort over its minimum value,  $\mu_0$ . The economy is stuck in the M regime. In this particular sense, the *I-equilibrium* in the M regime is characterized by *hysteresis*.

Other comparative static properties can be described using Figure 1. Suppose that the initial equilibrium is either in a single regime or in a multiple regime *J-equilibrium*. Consider either a reduction in  $v$ , the ratio of output per head in the V-technology to output per head in the S-technology, or an increase in  $\phi$ , the ratio of output per head in the L-technology to output per head in the S-technology. It is easily seen from equations (20), (23), (25),

and (30) that in the case of a reduction in  $v$ , the  $V_P V_P$  curve shifts upward while the  $\alpha_P$  segment of the PP curve shifts downward (the  $\alpha_{P'}$  segment remains unaffected). Similarly, an increase in  $\phi$  leads to a downward shift in the PP curve (the whole segment), while the  $V_P V_P$  curve remains unaffected. Both cases result in a smaller value of  $\bar{\mu}$ . Notice that the  $E_P E_P$  curve is not affected at all (by condition (39)). Thus, the new equilibrium moves to the lower right along the EE curve (except when the initial equilibrium is located in the  $\alpha_{P'}$  segment and a reduction in  $v$  occurs). The result is an increase in the share of L- and S-technology firms and a reduction in the share of V-technology firms. Investment in basic academic skills,  $\mu$ , increases.

On the other hand, when the initial equilibrium is an *I-equilibrium*, a reduction in  $v$  results in a downward shift in the MM curve and an upward shift in the  $V_M V_M$  curve. Since the  $E_M E_M$  curve is unchanged, the new *I-equilibrium* moves vertically downward, again expanding the sector of L- and S-technology firms and reducing the sector of V-technology firms. Similarly, an increase in  $\phi$  results in a downward shift in the MM curve, other curves remaining unchanged. The result is an unchanged  $\mu_0$ , and an expansion in the share of L-technology firms at the cost of a lower share of V-technology firms. The share of S-firms remains unchanged. In the circumstances considered here, the shifting of the equilibrium from one regime to another (under the multiple regime equilibria case) is unlikely to occur<sup>44</sup>.

A key feature of the decentralized Nash equilibrium described in our model is the assumption that each agent (individual or firm) is too small to take into explicit account the effect of her own action on the optimal decision of other agents. This has implications both on the equilibrium level of educational investment  $\mu$  and on the optimal number of firms using the L-technology,  $F$ .

To see why, consider first the P-regime and notice that  $F$  is an increasing function of  $\mu$ . While an increase in effort by a single individual has little

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<sup>44</sup>The discussion here is valid only for a small change in  $v$  or  $\phi$ . As the share of L-technology firms expand, it eventually reaches  $\Theta$ . From that point onwards, our formal characterization of the *J-equilibrium* becomes inadequate because firms cannot hire enough graduates from upper-layer schools. This fact is likely to induce L-technology firms to compete for scarce upper-layer school graduates by pushing up their wages over the efficiency wage level. At the same time, such a situation would certainly generate political pressure on the government to raise  $\Theta$  and increase the supply of slots in upper-layer schools. In fact, this appears to have been the case in Japan during the rapid growth period.



effect on  $F$ , a coordinated and symmetric increase in  $\mu$  increases the number of P-firms hiring from upper-layer schools. This in turn makes individual investment more attractive. In a symmetric cooperative equilibrium, positive spillovers and strategic complementarities are internalized and equilibrium investment  $\mu$  is higher.

Turning to the M regime, individual investment is at its minimum level,  $\mu_0$ . Since the asset value from employment in the S- or V-sectors,  $E_c$ , is a function of the endogenous quit rate,  $q$ , that depends on  $F$ , the positive relationship between  $F$  and  $\mu$  implies that a coordinated increase of  $\mu$  over  $\mu_0$  increases individual (expected) utility. The reason is that, by so doing, agents can increase both the endogenous quit rate and their chances of landing a job in the high wage sector composed of M-firms. Since the margin  $\alpha_M$  is decreasing in  $\mu$ , the equilibrium level of  $F$  increases too.

The argument above suggests that a decentralized Nash equilibrium always lead to under-investment in basic academic skills  $\mu$ . If individuals could coordinate their action, they would achieve an equilibrium with higher average output and wages per head. Notice, however, that the presence of negative spillovers for firms that choose the V-technology makes it difficult to rank equilibria in a Pareto sense. More in detail, an equilibrium with higher values of  $F$  and  $\mu$  is not Pareto superior to an equilibrium with lower values of both variables in the following sense: while individuals investing in education are always better off in the former equilibrium, a sub-set of firms is worse off. If the equilibrium is in the P-regime, only firms choosing the V-technology are worse off. On the other hand, if the equilibrium is in the M-regime, the increase in the endogenous quit rate reduces profits both in M and in V firms.

If the presence of positive spillovers and strategic complementarities means under-investment in  $\mu$  in the Nash equilibria, market failure in the market for education due to imperfect information on individual investment points to the opposite direction. Suppose that the economy is in the P-regime. If the social planner could impose the socially desirable level of investment, she would select only  $sF = s\Theta$  individuals for investment and leave the remaining individuals with no investment at all. The reason is that investment in  $\mu$  is useful only in L-technology firms. In this sense, there is too much investment in the decentralized Nash equilibrium.

The bottom line of this discussion is that the *I equilibrium*, that has both a lower  $\mu$  and a lower  $F$ , cannot be generally ranked as Pareto inferior to the

*J equilibrium* and *vice versa*. A simple way to compare the relative efficiency of the two equilibria is to compute total net output in each equilibrium. If lump-sum transfers are possible, the equilibrium with higher net output should be preferred on efficiency grounds. Total net output  $Y$  is equal to

$$\begin{aligned} Y = & V(\mu) [vy - 1] - \int_{V(\mu)} \frac{s\tau_s(1 + \beta_c\mu)}{\alpha} f(\alpha) d\alpha \\ & + S(\mu) [y - 1] - \int_{S(\mu)} \frac{s\tau_s}{\alpha} f(\alpha) d\alpha + F(\mu) [\phi y - 1] \\ & - \int_{F(\mu)} \frac{s\tau_l(1 - \beta_g\mu)}{\alpha} f(\alpha) d\alpha - s_-(\mu - \mu_0) \end{aligned}$$

in the *J-equilibrium* and to

$$\begin{aligned} Y = & V(\mu) [vy - 1] - \int_{V(\mu)} \frac{(s + q)\tau_s(1 + \beta_c\mu)}{\alpha} f(\alpha) d\alpha \\ & + S(\mu) [y - 1] - \int_{S(\mu)} \frac{(s + q)\tau_s}{\alpha} f(\alpha) d\alpha + F(\mu) [\phi y - 1] \\ & - \int_{F(\mu)} \frac{s\tau_l\lambda}{\alpha} f(\alpha) d\alpha \end{aligned}$$

in the *I-equilibrium*. The *J-equilibrium* yields a higher investment in  $\mu$  and a higher value of  $F$ . This investment reduces the training costs in the highly productive sector of large firms but makes the venture business sector less efficient. While net output is higher in the sector of large firms, it is lower in the sector of small venture business firms. On the other hand, the *I-equilibrium* yields minimum investment in  $\mu$ ,  $\mu_0$ , and a lower value of  $F$ . Contrary to the previous case, this leads to higher net output in the sector of small venture business and to lower net output in the sector of large firms. Whether total net output is higher in the former or in the latter case cannot be established a priori and depends on the parameters of the model.

## 5 CONCLUDING REMARKS.

Rather than summarizing our results, it is useful to conclude by pointing out directions of further research. First, we have treated basic academic skills and individual skills as perfect substitutes. In practice, however, the relationship is more likely to be hump-shaped, and individual skills to increase with basic academic skills when investment  $\mu$  is under a threshold and to decrease when  $\mu$  is above. To put it differently, the development of idiosyncratic skills requires that individuals acquire a degree of proficiency in basic academic skills (such as the ability to solve mathematical problems). An excessive focus on the latter, however, is likely to hamper the development of the former. Our model can easily be extended to include the case of a positive relationship between individual and academic skills. The two key differences are that the V-technology is always superior to the S-technology and that there are no negative spillovers. Hence, equilibria with higher values of  $\mu$  and  $F$  can be ranked as Pareto superior and, provided that competition is not excessive, more competition at school always yields higher net output.

Second, our model does not explicitly distinguish firms by firm size. We speculate in the paper about the association of different technologies to capital stock per head but never address the observed differences in the number of employees. Given the stylized facts discussed in the beginning of this paper, firm size is important. One way to introduce this variable is the following. Suppose that firms using the L-technology are composed of  $k > 1$  plants or units or tasks or jobs, each with one employee. The parameter  $k$  is given by technology. Firms using the S- or V-technology have a single unit and one employee. Introducing the parameter  $k$  implies novel features in the model, but the main qualitative results do remain unchanged.

Third, we have not explicitly evaluated the relative magnitude of wage differentials by firm size in the two countries. These differentials could be driven not only by endogenous turnover, as discussed in the paper, but also by differences in the reservation wage (and in the unemployment rate). Alternatively, the V-sector could be modelled as a much more fluid sector than that in the paper, with substantial gross job creation and destruction. Greater worker turnover and more frequent unemployment spells in the V-sector as compared with the S- and L-sector could generate compensating wage differentials in the former sector. Finally, workers in the V-sector could share in the productivity gain associated with more independent education.

Clearly, these are the important issues that have high priority in our research agenda.

## 6 Appendix: Proof of Lemma 5.

Define a function

$$H(F, \mu) \equiv \frac{s \left[ \lambda\tau_l - \frac{\tau_s}{1-F} (1 + \beta_c\mu) \right]}{(\phi - v)y - \frac{r+s}{\delta} - \frac{sF}{\delta(1-F)}}$$

in the domain  $F \in [0, 1]$  and  $\mu \in [0, \underline{\mu}]$ , and denote its numerator and the denominator by  $N(F, \mu)$  and  $D(F, \mu)$ , respectively. Define also the value of  $F$  for each value of  $\mu$  for which the numerator and the denominator becomes zero as  $F_N(\mu)$  and  $F_D(\mu)$ , respectively. Note that  $F_N(\mu)$  and  $F_D(\mu)$  are well-defined for all  $\mu \in [0, \underline{\mu}]$ . We observe that

$$\begin{aligned} F_N(\mu) &= 1 - \frac{\tau_s(1 + \beta_c\mu)}{\lambda\tau_l} < 1 \\ F'_N(\mu) &= -\frac{\tau_s\beta_c}{\lambda\tau_l} < 0 \\ F_D(\mu) &= \frac{\delta \left\{ (\phi - v)y - \frac{r+s}{\delta} \right\}}{s + \delta \left\{ (\phi - v)y - \frac{r+s}{\delta} \right\}} < 1 \\ F'_D(\mu) &= 0. \end{aligned}$$

First we show the following subsidiary lemma.

*Subsidiary Lemma:* For all  $\mu \in [0, \underline{\mu}]$ ,  $F_N(\mu) \geq F_D(\mu)$  if and only if  $\alpha_P(\underline{\mu}) \geq \delta\tau_s(1 + \beta_c\underline{\mu})$ . The equality  $F_N(\underline{\mu}) = F_D(\underline{\mu})$  occurs only when  $\mu = \underline{\mu}$  and  $\alpha_P(\underline{\mu}) = \delta\tau_s(1 + \beta_c\underline{\mu})$ .

(Proof) Because  $F'_D(\mu) > 0$  and  $F'_N(\mu) < 0$ , it is enough to show that  $F_N(\underline{\mu}) \geq F_D(\underline{\mu})$  if and only if  $\alpha_P(\underline{\mu}) \geq \delta\tau_s(1 + \beta_c\underline{\mu})$ . By substituting the expressions above for  $F_N(\mu)$  and  $F_D(\mu)$  and setting  $\mu = \underline{\mu}$ , the inequality  $F_N(\underline{\mu}) \geq F_D(\underline{\mu})$  can be rewritten as

$$\frac{\lambda\tau_l - \tau_s(1 + \beta_c\underline{\mu})}{\lambda\tau_l} \geq \frac{\delta \left\{ (\phi - v)y - \frac{r+s}{\delta} \right\}}{s + \delta \left\{ (\phi - v)y - \frac{r+s}{\delta} \right\}},$$

which, after rearrangement of terms, becomes

$$\frac{s \left\{ \lambda\tau_l - \tau_s(1 + \beta_c\underline{\mu}) \right\}}{(\phi - v)y - \frac{r+s}{\delta}} \geq \delta\tau_s(1 + \beta_c\underline{\mu}).$$

But since  $\lambda\tau_l = (1 - \underline{\mu}\beta_g)\tau_l$ , the left hand side of this inequality is nothing but  $\alpha_P(\underline{\mu})$ . (q.e.d.)

We also notice that

$$\alpha_P(\underline{\mu}) = H(0, \underline{\mu}).$$

Second, we examine the gradient of H with respect to F for each  $\mu \in [0, \underline{\mu}]$ . By multiplying both the numerator and the denominator by  $(1 - F)$  and partially differentiating with respect to  $F$ , and after rearrangement, we obtain

$$H_F(F, \mu) = \frac{\frac{s}{\delta} \left\{ (\phi - v)y - \frac{r+s}{\delta} \right\} \left\{ \frac{s\lambda\tau_l - \tau_s(1 + \beta_c\mu)}{(\phi - v)y - \frac{r+s}{\delta}} - \delta\tau_s(1 + \beta_c\mu) \right\}}{\left[ (1 - F) \left\{ (\phi - v)y - \frac{r+s}{\delta} \right\} - \frac{s}{\delta}F \right]^2}.$$

Since the denominator and the first term on the numerator are positive,

$$\text{sign } H_F(F, \mu) = \text{sign} \left\{ \frac{s \left\{ \lambda\tau_l - \tau_s(1 + \beta_c\mu) \right\}}{(\phi - v)y - \frac{r+s}{\delta}} - \delta\tau_s(1 + \beta_c\mu) \right\}.$$

Let the inside of the bracket and its first term be denoted as  $\Psi(\mu)$  and  $\gamma(\mu)$ , respectively. Clearly,  $\gamma'(\mu) < 0$  and  $\Psi'(\mu) < 0$  for all  $\mu \in [0, \underline{\mu}]$ . Moreover,  $\gamma(\underline{\mu}) = \alpha_P(\underline{\mu})$ . Thus, by our assumption,  $\Psi(\underline{\mu}) > 0$  and since  $\Psi(\mu)$  is decreasing,  $\Psi(\mu) > 0$  for all  $\mu \in [0, \underline{\mu}]$ . We have thus shown that

$$H_F(F, \mu) > 0 \text{ for all } \mu \in [0, \underline{\mu}].$$

Third, we examine the existence of the schedule  $\alpha_M(\mu)$  and its properties. Note that  $\alpha_M(\mu)$  is defined by the value of  $H(F, \mu)$  with the value of  $F$  solved as a solution to the equation

$$H(F, \mu) = \alpha_{\max} - F(\alpha_{\max} - \alpha_{\min}).$$

We shall employ a geometrical apparatus on the  $(F, \alpha_M)$  plane. The RHS of the above equation is a downward sloping straight line connecting  $(0, \alpha_{\max})$  and  $(1, \alpha_{\min})$ , while the graph of the LHS for a given level of  $\mu$  has been shown to be upward sloping. Because of *Subsidiary Lemma*  $F_D(\mu) < F_N(\mu)$ , so that we only need to consider the domain  $F \in [0, F_D(\mu)]$ . In fact, as  $F$  tends towards  $F_D(\mu)$ , the graph of  $H(F, \mu)$  becomes asymptotic to the vertical line  $F = F_D(\mu)$ . Drawn in Figure 2 below is the  $H(F, \underline{\mu})$  curve, with the vertical intercept equal to  $\alpha_P(\underline{\mu})$ . Clearly it has a unique intersection point with the

downward sloping line, which facilitates the value of  $\alpha_M$  for  $\mu = \underline{\mu}$ . Thus  $\alpha_M(\underline{\mu})$  is well-defined, and

$$\alpha_M(\underline{\mu}) > \alpha_P(\underline{\mu}).$$

This shows part (i) of the lemma. Furthermore, since it is easily seen that

$$H_\mu(F, \underline{\mu}) < 0,$$

$\alpha_M$  is also well-defined for any  $\mu < \underline{\mu}$ , with the property that

$$\alpha'_M(\underline{\mu}) < 0 \text{ for all } \mu \leq \underline{\mu}.$$

This proves the part (ii). QED.

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