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PROVISION OF QUALITY AND CERTIFICATION INTERMEDIARIES[1]

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Abstract

If buyers do not observe the quality of a product and production of qualityis costly,market allocations can be very inefficient. Certi cation intermediaries areinstitutions that provide information about quality to buyers. The amountof informationin the market determines the incentives that producers have to provide highqualitygoods. In this paper, we model information revelation as a strategicvariable o nter-mediaries. The amount of disclosed information is shown to deeplyin- uence both theintermediary s pro ts and the distribution of quality produced inequilibrium. We showthat a monopoly intermediary will provide noisy signals of quality and thatthe qualityproduced in equilibrium is the same as the one that would be chosen by amonopsonisticbuyer who optimally designs a mechanism. Efficiency is increased by theintermediary butless quality is produced in equilibrium than under complete information.

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1 Introduction

Asymmetric information may cause inefficiencies in the allocation of resources. The problems can be particularly severe in cases where consumers have poor information about product quality. In such cases producers may choose to provide low quality and this could even lead to a breakdown of trade. There are ways to get around these problems: warranties and reputation are two possibilitiess. However, in some settings these are only partial solutions and are sometimes completely inadequate.[1] In such cases, the lack of communication between informed and uninformed agents can lead to the emergence of a particular type of market institution: certi cation intermediaries. Such intermediaries play a potentially important role, they search out some of the information of privately informed agents and reveal part of it to uninformed parties.

There are many examples of institutions providing exactly this service: laboratories in markets for industrial products and auditors are the most immediate ones. There are many other examples where certi cation belongs to the range of the activities of an istitution: schools rate the ability of students, investment banks and underwriters evaluate the quality of rms that want to raise capital, bond rating agencies also provide a similar service.

When examining the behavior of these intermediaries, a natural question concerns the extent to which they disclose information to uninformed parties. In order to better understand the role intermediaries play in transmitting information one should consider information revelation as a strategic decision by intermediaries. Indeed, our results show that intermediaries pro ts are deeply affected by the amount of information revelation.

There are many possible roles for certi cation intermediaries. Lizzeri (1995) considered an environment of pure adverse selection and discussed the role of the intermediary under a variety of assumptions on the the market structure and on the nature of the good. In that paper the amount of information revealed by the intermediary could affect the distribution of the surplus and the extent of trade but had no effect on the distribution of quality of the good.

[1] Warranties may be unsatisfactory because of the moral hazard on the part of buyers or because the quality of the good may be very hard to verify ex post. Models of reputation and product quality almost never provide complete solutions to the problems of asymmetric information; moreover in some cases parties to a transaction are short term participants in markets, as in the case of used cars, and this prevents the establishment of reputation.

In the current paper the seller chooses how much quality to produce, the buyer does not observe quality. Thus, market equilibrium without the intermediary would involve no production of quality and therefore no exchange. The intermediary can induce the seller to provide positive quality by revealing information about quality to buyers. Clearly the amount of information revealed and the payment requested by the intermediary for the certification services affect the amount of quality produced by the seller.

In our setting the intermediary represents the unique channel through which information can be trasmitted to consumers. Though extreme, this assumption allows us to highlight how equilibrium quality and intermediary profits vary with the disclosure rules chosen by the intermediary. In particular, we shall compare the distribution of quality in the market when the intermediary can freely choose the disclosure rule and when he is conditioned by external constraints such as minimum legal standards.

We show that the intermediary can maximize profits by choosing a flat fee and an appropriately chosen noisy disclosure rule that only reveals quality with some probability. The amount of quality produced in equilibrium is lower than under full information.

Section 2 presents the structure of the game. Section 3 discusses some results that assume particular disclosure policies by the intermediary. These results are useful for comparison purposes and as an introduction to the analysis of section 4 that describes the equilibrium of the game where the intermediary chooses a disclosure policy optimally.

1.1 Related Literature

A number of authors have suggested that the distortions caused by the presence of asymmetric information may be solved by intermediaries. These suggestions have mostly been informal. The analysis of this paper will touch on issues that are related to those in the literature on the disclosure of private information and the literature on occupational licensing.

The literature on disclosure of private information started with Grossman (1981). The question of disclosure is a big issue in finance and accounting because there are questions about the extent to which laws should require disclosure of proprietary information in financial markets. Answers depend on the incentives that privately informed parties have to voluntarily disclose information. The models in this literature are very different from ours because the information structure is typically imposed exogenously: there is a set of veri able statements that parties can make and the analysis focuses on the question of the equilibrium set of statements; this translates into disclosure results. Our analysis allows intermediary to optimally pick the set of statements that can be made by informed parties. This paper is also a contribution to the disclosure literature because it makes the information structure endogenous.

There are some papers that view occupational licensing as a way to attenuate an adverse selection problem.[2] They view it as a mechanism that sets a minimum standard of quality. In a pure adverse selection setting this cuts the left tail of the quality distribution by making it illegal for the worst quality sellers to trade. In a moral hazard setting (where the quality is an endogenous variable), occupational licensing can raise the minimum investment in human capital thereby affecting the cost of producing quality. These papers mention certi cation as an alternative way of doing the same thing. However these papers provide no formalization of the idea.

Biglaiser (1993) and Biglaiser and Friedman (1994) also study the problem of certi cation intermediaries (which they call middlemen). The rst paper studies an adverse selection matching model, the second a moral hazard one with experience goods. In one dimension Biglaiser s approach is more ambitious than ours because in his paper, whether the middlemen becomes an expert or not is determined endogenously and he assumes less commitment ability on the part of the intermediary than we do. Our paper on the other hand studies a more general informational structure and investigates the problem of the optimal degree of information revelation and how this information affects the production of quality.

2 Structure of the Game

We assume that there are four agents in the market: one informed seller, two uninformed buyers and one intermediary. The seller produces one unit of an object of quality θ at a cost $c(\theta, t)$, where t is an efficiency parameter: sellers with higher t have lower costs. Buyers have value θ for a product of quality θ and they do not observe quality. Only the seller knows t, buyers and the intermediary have a prior on the value of t which is represented by a cumulative distribution function $F(t)$. $F(t)$ is assumed strictly increasing

[2] Examples are Leland (1979) and Shapiro (1986).

with continuous density on the closed interval $[\underline{t}, \overline{t}]$. This is common knowledge to all participants in the market. The intermediary can nd out quality θ with perfect precision.

We shall adopt the following assumptions on the cost function:

A 1 $\dfrac{\partial c(\theta, t)}{\partial \theta} > 0;$

A 2 $\dfrac{\partial^2 c(\theta, t)}{\partial \theta^2} > 0;$

A 3 $\dfrac{\partial c(\theta, t)}{\partial t} < 0;$

A 4 $\dfrac{\partial^2 c(\theta, t)}{\partial \theta \partial t} < 0;$

A 5 $\exists \theta : \theta - c(\theta, \overline{t}) > 0,$

Thus the cost function is increasing and convex in θ, decreasing in the seller s type and the marginal cost strictly falls with type: higher types are more efficient. Assumption **A5** guarantees that under complete information a positive mass of types would nd worth producing positive levels of quality. An example that we will be using throughout the paper is $c(\theta, t) = \theta^2/2t$

The game is the following:

Stage 1: The intermediary sets a fee P and commits to a disclosure rule D to maximize expected pro ts. The fee P can be any non negative number. The set of disclosure rules from which the intermediary is allowed to choose is assumed to be very large: it can choose to perfectly disclose test results, to only disclose intervals of test results (grades), to disclose Borel measurable sets of scores, to perfectly disclose test results of some values and to only disclose intervals for other values, to disclose nothing or, nally, to disclose a noisy transformation of test results. At the end of stage 1 nature chooses the type of seller according to the distribution F.

Stage 2: Having observed P, D and t, the seller decides the level of quality to produce and whether to avail himself of the intermediary s service, i.e., whether to pay the fee and have the product tested.

Stage 3: If the seller has paid the fee, the product is tested and the quality is observed by the intermediary.

Stage 4: Buyers observe the disclosure rule, the fee, whether the product was tested or not and what the intermediary disclosed.

Stage 5: Buyers bid for the product in a rst price auction.

A strategy for the intermediary is a pair (P, D), i.e., a fee and a disclosure rule. Formally, D is measurable function from Θ into the set $\mathbf{Q}$ of Borel probability distributions on real numbers, where Θ is the range of qualities tested by the intermediary. Denote Ψ the set of such functions. A policy of full disclosure can be represented by a function mapping each level of quality θ to a probability distribution degenerate at θ. An example of a policy of noisy disclosure is the function that maps quality θ to the normal distribution with mean θ and variance σ^2.

The policy of no disclosure of test results should be thought as a policy of releasing a certi cate that says: the seller is certi ed by this intermediary and nothing else. This policy can be represented (for example) by a function that maps all levels of quality θ to some probability distribution degenerate at some number x independent of θ. This speci cation of a disclosure rule also allows the intermediary to disclose partitions. Let $\{\Theta_1, \cdots, \Theta_n\}$ be a partition of Θ; let D be a one to one function that maps each $\theta \in \Theta_i$ to a probability distribution degenerate at some x_i.

A (pure) strategy for the seller is a function $\rho : \Re_+ \times \Psi \times [\underline{t}, \overline{t}] \rightarrow \{0, 1\} \times \Re_+$ that maps the triple (P, D, t) into a decision of going (1) or not going (0) to the intermediary and a choice of quality as a function of type, fee and disclosure rule.

A (pure) strategy for a buyer is a function $\beta : \Re_+ \times \Psi \times \{0, 1\} \times \Re \rightarrow \Re_+$ that maps the observed data of price P, disclosure rule D, whether the seller went to the intermediary (1) or not (0), and the realization of the disclosure rule (a real number that can be thought of as a test score after the resolution of any uncertainty caused by a noisy disclosure rule) into bids for the seller s product. The equilibrium concept we shall use is sequential equilibrium.

2.1 Discussion

The game is highly stylized but it is meant to capture a market environment where pro ts to producers and therefore the amount of quality provided depends on the amount of information that is available to buyers. This information can be manipulated by intermediairies. With no intermediary the market would collapse: the sellers would produce zero quality. This facilitates the analysis of the role of the intermediary.

The reason for assuming that there are two buyers is that this considerably simplifies the analysis of the relationship between buyers and sellers: Given that buyers observe the same information, they simply bid the expected value of quality given any information. Thus buyers' role in the analysis will be extremely limited.

The fact that the intermediary reveals the same information to both buyers is without loss of generality: if the intermediary revealed different information to different buyers, buyers would sometimes make a profit. Since the intermediary is paid by the seller, it has no interest in letting buyers capture some of the surplus.

Our set-up ignores the possibility that the intermediary may collude with the seller to reveal false information: the announcement of a disclosure rule is assumed to be credible. While it is certainly desirable to understand the issue of collusion, here we ignore the issue to focus on another matter.[3] The results of the current paper can be interpreted as a benchmark against which to evaluate the effects of collusion.

We assume that the intermediary chooses a flat fee. We discuss in section 5 the case where the intermediary can choose a non linear price schedule and then proceed to show that the intermediary can do just as well by choosing a flat fee and a particular type of disclosure rule. Thus restriction to linear prices is without loss of generality.

3 Introductory Results

Let us begin by considering what would happen if the intermediary were to perfectly disclose the information it acquires. We will later show that this is not an optimal policy for the intermediary. However it is a starting point that is interesting for two reasons. First, this analysis permits to have some insights to understand the importance to the intermediary of manipulating the information structure. Second, there may be legal requirements that force the intermediary to disclose fully the information it has acquired.

[3] Tirole (1986) studies a three-layer hierarchy in which the supervisor colludes with the agent in reporting false information to the principal. In an audit model, Kofman and Lawarrée (1993) distinguish internal auditors from external ones. They show that the optimal contract may specify random external audits.

Let $\theta^{FD}(t) = argmax_\theta\{\theta - c(\theta, t)\}, \forall t \in [\underline{t}, \overline{t}]$. This denotes the quality that is optimally chosen by type t under full disclosure by the intermediary. For every P, call $t^*(P)$ the type such that

$$\theta^{FD}(t^*(P)) - c(\theta^{FD}(t^*(P)), t^*(P)) - P = 0. \tag{1}$$

Type $t^*(P)$ is the seller who makes zero pro ts by going to the intermediary to get certi ed. Such a type will exist as long as $0 < P \leq \theta^{FD}(\overline{t}) - c(\theta^{FD}(\overline{t}), \overline{t})$ and will be unique as marginal costs are strictly decreasing in t. Let us simply call $\theta^*(P)$ the quality produced by $t^*(P)$, i.e., $\theta^* = \theta^{FD}(t^*(P))$. Observe that the left-hand side of 1 is negative if $t < t^*(P)$ and strictly positive if $t > t^*(P)$.

Proposition 1 *Suppose the intermediary commits to a policy of full disclosure. IfPsatis es*$0 < P < \theta^{FD}(\overline{t}) - c(\theta^{FD}(\overline{t}), \overline{t})$*then the unique sequential equilibrium of the subgame is the following:if the seller of type* $t < t^*(P)$*, he will produce zero quality and not goto the intermediary; if* $t > t^*(P)$*, the seller will go to theintermediary and produce quality* $\theta^{FD}(t)$*.If* $P > \theta^{FD}(\overline{t}) - c(\theta^{FD}(\overline{t}), \overline{t})$*, theunique sequential equilibrium involves no type of the sellergoing to the intermediary and no quality being produced.*

Proof: Take any positive $P < \theta^{FD}(\overline{t}) - c(\theta^{FD}(\overline{t}), \overline{t})$ and suppose the intermediary fully discloses. Observe rst that in equilibrium any type of the seller that do not go to the intermediary will produce zero quality. To see this, let w be the offer by buyers if the seller does not go to the intermediary. Observe that w cannot depend on θ. Thus, the seller s payoff conditional on not going to the intermediary is $w - c(\theta, t)$. The unique maximizer of this expression is $\theta = 0$ for all t.

If the seller goes to the intermediary and produces quality θ, he gets offers θ. Therefore, the seller s payoff is $\theta - c(\theta, t) - P$. According to equation 1, $t^*(P)$ will be the type that is indifferent between going to the intermediary and producing $\theta^*(P)$ or not going to the intermediary and producing zero quality. Because $c(\theta, t)$ is strictly decreasing in t, all types above $t^*(P)$ will go to the intermediary and produce $\theta^{FD}(t) > \theta^*(P)$, whereas all types less than $t^*(P)$ will not go to the intermadiary and not produce quality. ■

Remarks: (1) Intermediary s pro ts under full disclosure are

$$\Pi(P) = \begin{cases} P[1 - F(t^*(P)] & P < \theta^{FD}(\overline{t}) - c(\theta^{FD}(\overline{t}), \overline{t}) \\ 0 & P \geq \theta^{FD}(\overline{t}) - c(\theta^{FD}(\overline{t}), \overline{t}). \end{cases}$$

(2) If there is no uncertainty about the efficiency of the seller, i.e. $F(\cdot)$ is degenerate, the intermediary can extract the entire surplus. With full disclosure the seller will choose the optimal quality and the seller s pro ts can be extracted through the fee charged by the intermediary. In general, the intermediary cannot extract the whole surplus because higher types can always imitate the behavior of the lower types and get higher pro ts.

Example 1

Let us take a brief look at an example in which F is a uniform distribution on $[0,1]$ and $c(\theta,t)=\theta^2/2t$. Then, $\theta^{FD}(t)=t$ and $t^*(P)=2P$. Lastly, the intermediary s problem is to $max_P[P(1-2P)]$ which yields $P^*=1/4$ and $t^*(P^*)=1/2$. Thus the intermediary makes pro ts $\Pi(FD)=(1/2)1/4=1/8$.

We shall now investigate whether the intermediary can choose alternative disclosure rules to increase pro ts. In a pure adverse selection setup, Lizzeri (1995) showed that in the unique equilibrium of the game the intermediary can extract all the surplus in the market by revealing no information at all. We shall now see that in the environment considered here this is not possible.

Proposition 2 *Suppose the intermediary adopts a policy of no disclosure, i.e., he onlyannounces that the seller paid the fee P.Then, for any $P>0$, the unique sequential equilibrium involves none of thesellers going to the intermediary and zero quality being produced. Thus theintermediary makes zero pro ts.*

Proof: The proof is trivial. Because the intermediary does not reveal any information, in equilibrium the payoff of the sellers cannot depend on the quality produced. Thus sellers will produce no quality and buyers will offer zero whether the seller goes to the intermediary or not. Since going to the intermediary is costly no type of the seller will go to the intermediary. ■

We shall now show that the intermediary can do better by choosing a policy of noisy" disclosure. There are many possible noisy disclosure rules; the following is a particularly simple family: The intermediary chooses a cut-off level θ^c, if the seller produces quality $\theta\geq\theta^c$, with probability z the intermediary reveals quality fully, with probability $(1-z)$ it announces only that quality is above the cut-off. If $\theta<\theta^c$, the seller is not certi ed. Let us call $D(z,\theta^c)$ this particular noisy disclosure rule.

Suppose that the intermediary sets $P=P^{FD}$, i.e. the optimal price under full disclosure and commits to a diclosure policy described as follows: if the seller goes to be certi ed, then he is fully revealed with probability z and with probability $(1-z)$ the intermediary only announces that his quality is at least θ^*, where θ^* is the quality produced by the cut-off type under full disclosure, namely $t^*(P)$. The intermediary does not certify any $\theta < \theta^*$.

Proposition 3 *There exists a probability of revelation z and a cut-off such that pro tsunder the disclosure rule $D(z, \theta^c)$ are higher than under fulldisclosure (FD).*

Proof: Let $P = P^{FD}$ (the optimal price under full disclosure) and $\theta^c = \theta^*(P)$ (the minimal quality produced under full disclosure).

If a seller of type t goes to the intermediary and produces quality θ, in expectation he receives offers $z\theta + (1-z)w^e$. In equilibrium w^e is the average quality produced by sellers who go to the intermediary. So a seller maximizes $z\theta + (1-z)w^e - c(\theta, t)$. If this is higher than P the seller goes to the intemediary. Let $\theta^z(t)$ be the optimal choice of quality by type t under disclosure rule $D(z, \theta^*)$. Clearly, if a seller of type t chooses to produce a quality $\theta^z(t) \geq \theta^*(P)$, it must be that he goes to the intermediary and that any type higher than t will also choose to go to the intermediary and produce a quality of at least θ^*. We shall show that there is a $z < 1$ such that $\theta^z(t) \geq \theta^*$ for all $t \in [t_l, \bar{t}]$, where $t_l < t^*(P)$, so that the lowest type under this noisy disclosure rule is lower than under full disclosure. Thus the intermediary makes more pro ts because at the same price more types of the seller get certi ed.

Step 1: There exists a $z \in (0, 1)$ and a $\hat{t} < \bar{t}$ such that $\theta^z(t) > \theta^*$ for $t \in [\hat{t}, \bar{t}]$. This is because, since by the de nition of θ^*,

$$\frac{\partial c(\theta^*, \bar{t})}{\partial \theta} < 1.$$

There is a z sufficiently close to 1 such that

$$\frac{\partial c(\theta^*, \bar{t})}{\partial \theta} < z,$$

i.e. the highest type has a marginal cost strictly less than z by producing θ^*. Now, call $\hat{t} = inf\{t : \partial c(\theta^*, t)/\partial \theta \leq z\}$. This type will produce at least θ^* but all higher types will produce strictly higher levels of quality since their marginal revenue from raising quality is z (they get revealed with probability z) and their marginal cost is less than z.

Step 2: $w^e > \theta^*$. This is an immediate consequence of step 1 and the de nition of w^e. Since the mass of types producing more than θ^* has positive measure, equilibrium offers by the buyers when the intermediary announces only that $\theta \geq \theta^*$ must be higher than θ^*.

Step 3 $t_l < t^*$. By the de nition of $t^*(P)$ and step 2, we can nd z such that

$$z\theta^* + (1-z)w^e - c(\theta^*, t^*) - P > 0$$

Therefore there exist types slightly lower than $t^*(P)$ who can have a positive pay-off by going to the intermediary. ■

The analysis undertaken so far highlights how the intermediary can increase his pro ts by manipulating the amount of information released to consumers. By adopting a noisy disclosure rule as in Proposition 3, the intermediary uses a fraction of the surplus produced by higher types in order to subsidize the ones lying on the left of the cutoff type under full disclosure. If the probability of being fully revealed is sufficiently close to one, there will always exist types close to $\bar{t}$ producing more than the cutoff quality θ^c. When observing a seller going to the intermediary, buyers rationally believe that expected quality must be (strictly) greater than the cutoff quality. Some types lower than the cutoff type under full disclosure will get nonnegative pro ts by producing at least the cutoff quality θ^c and being certi ed. These types will therefore be willing to pay the fee under noisy disclosure when they were not under full disclosure.

Example 2

A numerical example will point out the increase in pro ts by means of a noisy disclosure rule. Let s recall the results we obtained for Proposition 1, that is $P^* = 1/4, t^*(P^*) = 1/2, \theta(t^*(P^*)) = 1/2$. Now, consider the following noisy disclosure rule: if the seller goes to the intermediary he is fully revealed with probability $z = 9/10$ and with probability $(1-z) = 1/10$ the intermediary announces that the quality is at least $1/2$. The seller s objective function becomes $(9/10)\theta + (1/10)w^e - \theta^2/2t - 1/4$ which gives $\theta^z(t) = (9/10)t$. All types greater than $5/9$ will produce strictly more than $1/2$ because, at $\theta = 1/2$, they have marginal costs strictly less than $9/10$. The new cut-off type t_l will then be de ned by

$$\frac{9}{10}\frac{1}{2} + \frac{1}{10}w^e - \frac{1}{8t_l} - \frac{1}{4} = 0,$$

which gives

$$t_l = \frac{5}{8+4w^e}.$$

At equilibrium, w^e is equal to the expected quality given that a positive mass of types will produce more than 1/2 and an atom of types will provide exactly 1/2, i.e.,

$$w^e = \left[\frac{5}{9} - \frac{5}{8+4w^e}\right]\frac{1}{2} + \left[\frac{4}{9} + \frac{5}{8+4w^e}\right]\frac{7}{10},$$

where $7/10 = E\left[\theta^z(t) \mid \theta^z(t) \geq 1/2\right] = E\left[(9/10)t \mid (9/10)t \geq 1/2\right]$. The equation above yields $w^e \approx 17/24 > 1/2$. Then $t_l = 30/65 < 1/2 = t^*$. The indermediary will obtain pro ts $\Pi(D) = (1-30/65)1/4 > (1/2)1/4 = \Pi(FD)$.

We can exploit the result of Proposition 3 to characterize the situation in which law imposes a minimum standard of quality [4] so that the intermediary only announces whether or not a producer has ful lled the legal requirements.

Corollary 1 *Suppose the intermediary only announces that the quality produced is at least*θ^c *and sets a price* P^**. Then, all sellers who get certi edproduce* θ^c*.*

Proof: Consider the disclosure rule $D(z, \theta^c)$ with $z = 0$. Let w^e be the offers to a seller who gets certi ed. Clearly,

$$w^e \geq \theta^c.$$

The producer will in turn choose θ to maximize

$$w^e - c(\theta, t) - P^*,$$

which gives $\theta(t) = \theta^c, \forall t \in [t^*, \overline{t}]$, where t^* is the cut-off type who will attain zero pro ts by producing θ^c and being tested by the intermediary. Thus, in equilibrium consumers must expect that quality produced by a certi ed seller is θ^c so that $w^e = \theta^c$ ■

It is clear that minimum legal standards normally aim at raising the lowest level of quality provided to consumers. However, the disclosure rule the intermediary is obliged to adopt may frustrate incentives highest types have in providing better quality goods.

[4] A typical case is the minimal quantity of particular ingredients contained in goods as chemical products, drugs etc.

4 Optimal disclosure policy

We now wish to characterize the optimal disclosure policy for the intermediary. This problem is hard to solve directly since the class of disclosure rules to consider is very large and for each of them we have to obtain the distribution of quality produced by the seller.

We shall use an indirect approach. This proceeds in three steps: (1) We rst characterize the optimal choice for the intermediary if it could choose any possible mechanism. (2) We then show that this can be replicated by complete revelation of information and a nonlinear price function. (3) We nally construct a disclosure rule and a at fee such that the intermediary obtains the same pro ts as in the optimal mechanism. The disclosure rule constructed in step 3 must then be optimal.

4.1 The Optimal Mechanism

By the revelation principle, we can restrict attention to direct revelation mechanisms. The intermediary then chooses functions $\theta : [\underline{t}, \overline{t}] \to \Re$ and $\kappa : [\underline{t}, \overline{t}] \to \Re$, where $\theta(t)$ is the quality and $\kappa(t)$ is the payment that the intermediary requests from type t. Thus the intermediary chooses these functions to maximize $\int_{\underline{t}}^{\overline{t}} \kappa(t) f(t) dt$ subject to the incentive compatibility and individual rationality constraints for the seller. By de ning $w(t) = \theta(t) - \kappa(t)$ as the net payment received by the seller after the payment of the fee to the intermediary, we can write the problem as follows:

$$\max_{w(\cdot),\theta(\cdot)} \int_{\underline{t}}^{\overline{t}} [\theta(t) - w(t)] f(t) dt \tag{2}$$

$s.t.$

$$w(t) - c(\theta(t), t) \geq 0, \ \forall t \in [\underline{t}, \overline{t}] \tag{3}$$

$$w(t) - c(\theta(t), t) \geq w(\hat{t}) - c(\theta(\hat{t}), t), \ \forall t, \hat{t} \in [\underline{t}, \overline{t}] \,. \tag{4}$$

This way of writing the problem highlights an interesting feature of the problem: the intermediary acts as a monopsonistic buyer who must design an optimal mechanism for a producer of unobservable cost but observable quality.

This problem can be solved by using standard tools.[5]

[5] See Fudenberg and Tirole (1992), Guesnerie and Laffont (1984).

We shall impose further assumptions on the cost functions and on the distribution of producer s types that simplify the problem considerably. These guarantee that the producer s problem is well de ned and that the optimal solution does not involve any bunching", i.e. regions where different types produce the same quality.[6] Our results would be qualitatively similar without these assumptions.

A 6 $\partial^3 c(\theta, t)/\partial\theta^2 \partial t \leq 0$;

A 7 $\partial^3 c(\theta, t)/\partial\theta \partial t^2 \geq 0$;

A 8 (*monotone hazard rate*) $\dfrac{f(t)}{1 - F(t)}$ is an increasing function.

Under conditions A1-A8 a necessary and sufficient condition for $\theta(t)$ to be implementable is that it is nondecreasing: for any nondecreasing $\theta(\cdot)$ we can nd a function $w(\cdot)$ that makes it incentive compatible.

By using standard methods, we can reduce the problem to one where only $\theta(\cdot)$ is a choice variable thereby eliminating $w(\cdot)$.[7]

$$\max_{\theta(\cdot)} \int_{\underline{t}} \left[\theta(t) - c(\theta(t), t) + \frac{1 - F(t)}{f(t)} \frac{(\theta(t), t)}{\partial t} \right] f(t) dt \tag{5}$$

s.t.

$$\theta(t) \quad non - decreasing$$

We can now proceed by ignoring the constraint that $\theta(\cdot)$ has to be nondecreasing since, as is well known, assumptions A6-A7-A8 are sufficient conditions for the monotonicity constraint to be satis ed. Moreover, assumption A6 ensures that problem 5 is concave in θ.

We can thus obtain an optimal solution by pointwise optimization of the integrand in 5.

Thus the objective function for the intermediary is the same as that of a buyer who produces the good for himself at a **virtual cost**

$$VC(t) = c(\theta(t), t) - \frac{1 - F(t)}{f(t)} \frac{(\theta(t), t)}{\partial t}. \tag{6}$$

[6] See Fudenberg and Tirole (1992) for more on this.

[7] See Albano and Lizzeri for the details.

Notice that the virtual cost is higher than the actual cost since $\partial c(\theta(t), t)/\partial t < 0$ (Assumption A3). Thus the intermediary acts as if it had perfect information but the cost was actually higher.

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$$t^{\star} = sup\{t : \theta(t) - c(\theta(t), t) + \frac{1 - F(t)}{f(t)} \frac{\partial c(\theta(t), t)}{\partial t} \leq 0\}.$$

We can summarize the above discussion in the following proposition.

Proposition 4 *Assume A1-A8. Then, in the optimal mechanism thequality produced by type t ($\theta^m(t)$) is:*

$$\theta^m(t) = 0, t \in [\underline{t}, t^{\star})$$

And, for $t \in [t^{\star}, \overline{t}], \theta^m(t)$ *solves*

$$1 - \frac{\partial c(\theta^m(t), t)}{\partial \theta} = -\frac{1 - F(t)}{f(t)} \frac{\partial^2 c(\theta^m(t), t)}{\partial \theta \partial t}. \tag{7}$$

Two interesting features of the optimal quality are:

i) All types except for the highest underproduce quality relative to the full information optimum. In order to see this, notice rst that under full information we would have that optimal quality would solve $1 - \frac{\partial c(\theta^{FI}(t),t)}{\partial \theta} = 0$. The right-hand side of 7 is strictly positive for all $t \in [t^{\star}, \overline{t})$, and $c(\theta, t)$ is convex in θ (assumption A2). Thus $\theta^m(t) < \theta^{FI}(t)$ for all t: the optimal quality for the intermediary is below the full information level.

ii) The optimal level of quality is a strictly increasing function of type if it is positive: $\theta^m(t)$ is increasing in t for all $t \in [t^{\star}, \overline{t}]$.

By (ii) we can invert $\theta^m(\cdot)$ on $[t^{\star}, \overline{t}]$

$$u = \theta^m(t), \ \forall t \in [t^{\star}, \overline{t}].$$

By inverting the function above, we can always nd the type who optimally chooses to produce quality u.

$$t = (\theta^m)^{-1}(u).$$

4.2 Optimal Non-linear Price

We can now characterize the optimal non-linear price for an intermediary who chooses full disclosure. Under this regime, a producer will have to solve the following problem:

$$\max_{\theta} \{\theta - c(\theta, t) - P(\theta)\}, \forall t \in [\underline{t}, \overline{t}],$$

which requires the rst order condition

$$1 - \frac{\partial c(\theta, t)}{\partial \theta} = P'(\theta). \tag{8}$$

By substituting $t = (\theta^m)^{-1}$ in the left-hand side of 8, we can obtain the non linear price function that achieves the optimum.

Proposition 5 *The optimal non-linear price is:*

$$P(\theta) = \begin{cases} \int_0^{\theta} [1 - \frac{\partial c(u, \theta^{-1}(u))}{\partial u}] du & \theta \geq \theta^m(t^\star) \\ +\infty & \theta < \theta^m(t^\star). \end{cases}$$

Moreover,

$$0 \leq P'(\theta) < 1, \; P''(\theta) < 0.$$

Proof: Expression 8 clearly shows that $P'(\theta) < 1$, and that $P''(\theta) < 0$ owing to the assumptions on the cost function. ■

Example 3

Let s illustrate these last results by continuing examples 1 and 2. The optimal level of quality will become the solution of the equation

$$1 - \frac{\theta}{t} = (1 - t) \frac{\theta}{t^2},$$

i.e. $\theta^m(t) = t^2$, $\forall t \in [0, 1]$. The non-linear price function will be

$$P(\theta) = \theta \left(1 - \frac{2}{3}\sqrt{\theta}\right).$$

Figure 1: Distribution of quality under different disclosure policies

Example 4

It is worth pointing out that the absence of xed costs in our numerical examples explains the fact that the sets of types producing positive quality coincide under full information and under the optimal mechanism. In order to emphasize the distortionary effect of the intermediary, we now consider a cost function with positive xed costs, i.e.,

$$c(\theta, t) = \frac{\theta^2}{2t} + \frac{1}{16}.$$

Under full information, the optimal level of quality will be $\theta^{FI}(t) = t$. The type realizing zero pro t, t^{FI}, will be the solution of

$$t - \frac{t^2}{2t} - \frac{1}{16} = 0,$$

which gives $t^{FI} = 1/8 \approx 12/100$.

Turning our attention to the optimal mechanism described by proposition 4, type $t^\star$ will be the solution in the interval $[0, 1]$ of the following

$$t^2 - \frac{t^4}{2t} - \frac{1}{16} + (1-t)\frac{3}{2}t^2 = 0,$$

which yields $t^\star \approx 17/100 > 12/100 = t^{FI}$.

Figure 1 illustrates the distribution of quality under different disclosure policies. Figure (1.a) compares the policies of fully discosure and non linear price when there are no xed costs to produce quality; gure (1.b) illustrates that positive xed costs cause the set of types producing quality to shrink under the optimal mechanism with respect to the case of full information.

4.3 Disclosure Rule

We shall now use the result of the previous section to obtain the optimal disclosure rule.

Denote $P^*(\theta)$ the optimal non-linear price and $\Theta \equiv [\underline{\theta}, \overline{\theta}] = [\theta^m(t^\star), \theta^m(\overline{t})]$ the range of quality produced under a policy of non-linear price. Call $G(\theta)$ the distribution function of quality induced by $F(t)$ *via* the direct revelation mechanism described in Proposition 4. We de ne

$$\hat{P} \equiv \int_\Theta (P^*(\theta))\, dG(\theta)$$

and $\hat{\theta}$ such that $P^*(\hat{\theta}) = \hat{P}$. The integral above is always well de ned since $P^*(\theta)$ is a bounded function on a compact interval.

The construction of the disclosure rule is as follows: we rst identify a type $\hat{\theta}$ such that the price this type would pay under the non-linear price is equal to the average price paid by all sellers who go to the intermediary. Then, we construct a noisy disclosure such that, by charging $\hat{P}$, the full range of quality, Θ, is reproduced and the intermediary attains the highest amount of expected pro ts.

Proposition 6 *Let the price charged by the intermediary be $\hat{P}$ and the disclosurerule be asfollows: For any quality $\theta \in \Theta$ the quality will be fully revealedwith probability$q(\theta)$, and with probability $(1 - q(\theta))$ the intermediary releases anumber N which is independent of the actual level of quality.Where $q(\hat{\theta}) = 1$ and $q(\theta) = 1 - (\hat{P} - P^*(\theta))/(\hat{\theta} - \theta)$ for all $\theta \neq \hat{\theta}$. If $\theta \notin \Theta$, the intermediary doesnot certify the seller.This price and disclosure rule are optimal for the intermediary, and inequilibrium thedistribution of quality coincides with the one under full disclosure withoptimalnon-linear price $P^*(\theta)$. Types in $[t^\star, \overline{t}]$ go to theintermediary, types in $[\underline{t}, t^*)$ do not.*

Proof: We rst need to make sure that $q(\theta)$ is a well-behaved probability function, that is, $0 \leq q(\theta) \leq 1$, for all $\theta \in \Theta$.

The expression is not de ned at $\hat{\theta}$ but that s not a problem because $q(\hat{\theta}) = 1$ by de nition.

i)

$$\frac{\hat{P} - P(\theta)}{\theta - \hat{\theta}} \leq 0,$$

since $\hat{P} > P(\theta)$ for $\theta \in [\underline{\theta}, \hat{\theta})$ and $\hat{P} < P(\theta)$ for $\theta \in (\hat{\theta}, \overline{\theta}]$; thus $q(\theta) \leq 1$.

ii)

$$\frac{P(\theta) - \hat{P}}{\theta - \hat{\theta}} \leq 1.$$

De ne $d(\theta) = P(\theta) - \hat{P}$ and $e(\theta) = \theta - \hat{\theta}$.

By proposition 5, $d'(\theta) \leq 1$ for all $\theta \in \Theta$ and $d(\theta)$ is strictly concave. But $e'(\theta) = 1$ and $e(\hat{\theta}) = d(\hat{\theta})$. Hence, $d(\theta) > e(\theta), \theta \in [\underline{\theta}, \hat{\theta}), d(\theta) > e(\theta), \theta \in (\hat{\theta}, \overline{\theta}]$ thus the absolute value of $d(\theta)$ is always less than the absolute value of $e(\theta)$. Thus $q(\theta) \leq 1$.

The rest of the proof is built on two steps.

First step: Under a policy of non linear price and full disclosure, a seller of type t who produces quality θ gets pro ts of

$$\theta \; - \; P^*(\theta) \; - \; c(\theta, t), \tag{9}$$

The disclosure rule that is described in the statement of the proposition is constructed precisely so that the pro ts to a seller of producing quality θ are the same as under the optimal nonlinear price. Let $m(\theta)$ be the offers expected by a seller who produces quality θ. We want

$$m(\theta) \; - \; \hat{P} \; = \; \theta \; - \; P^*(\theta), \tag{10}$$

so that under the optimal noisy disclosure rule the distribution of quality produced in equilibrium will be the same as under the optimal non-linear price. Clearly, no type of the seller will ever produce a quality ouside Θ since then they would not get certi ed and they would get zero offers in equilibrium.

Let $w(N)$ be buyers offers when N is announced. Buyers beliefs upon hearing N are such that $E(\theta|N) = \hat{\theta}$: their expectation of quality conditional on N being announced by the intermediary, is exactly $\hat{\theta}$. Given these beliefs, $w(N) = \hat{\theta}$. The second step of the proof shows that these beliefs are consistent.

Given the disclosure rule we thus have that in equilibrium: $m(\theta) = \theta q(\theta) + \hat{\theta}(1 - q(\theta))$. Substituting for $q(\theta)$ we get $m(\theta) = \theta + \hat{P} - P(\theta)$. Therefore condition 10 is satis ed.

Second step

At a sequential equilibrium, it must be true that

$$w(N) \;=\; \hat{\theta} \;=\; E(\theta \mid N). \tag{11}$$

That is, buyers beliefs must be consistent.

By Bayes rule,

$$Pr(\theta \mid N) = \frac{Pr(N \mid \theta)\, dG(\theta)/d\theta}{Pr(N)} = \frac{(1 - q(\theta))\, dG(\theta)/d\theta}{Pr(N)},$$

and

$$Pr(N) = \int_{\Theta} (1 - q(\theta))\, dG(\theta).$$

Thus, condition 11 becomes

$$\hat{\theta} \;=\; \int_{\Theta} \frac{(1 - q(\theta))}{Pr(\hat{\theta})}\, \theta\, dG(\theta),$$

which gives

$$\int_{\Theta} (1 - q(\theta))\, (\theta \; - \hat{\theta})\; dG(\theta) \;=\; 0.$$

By plugging the expression for $q(\theta)$, we obtain

$$\int_{\Theta} \frac{P^{*}(\theta) - \hat{P}}{\theta - \hat{\theta}}\, (\theta - \hat{\theta})\, dG(\theta)$$

which is always zero owing to the de nition of $\hat{P}$. ■

Corollary 2 *In the optimal disclosure rule, the probability that quality is revealed isincreasing inquality:* $q'(\theta) > 0$.[8]
endCor

[8] this is false at one point: $\hat{\theta}$. However$q(\cdot)$ can be rede ned so that it also holds at $\hat{\theta}$.

Proof*:*

$$q'(\theta) = \frac{-P'(\theta)(\theta - \hat{\theta}) + P(\theta) - \hat{P}}{(\theta - \hat{\theta})^2}.$$

Then,

$$q'(\theta) \geq 0 \Leftrightarrow -P'(\theta)(\theta - \hat{\theta}) + P(\theta) - \hat{P} \geq 0.$$

The latter inequality holds strictly for all θ since $P(\theta)$ is strictly concave.

■

The following is an example of the optimal disclosure rule in the case where xed costs are zero and the other parameters are the same as throughout our other examples.

Example 5

Recall that the optimal mechanism implies that $\theta^m(t) = t^2$ for all $t \in [0, 1]$, and $P(\theta) = \theta(1 - \frac{2}{3}\sqrt{\theta})$.

$G(x)$ is de ned as

$$Prob(\theta^m(t) \leq x) = Prob(t^2 \leq x) = Prob(t \leq \sqrt{x}) = \sqrt{x}$$

since t is uniformely distributed on $[0, 1]$. We can calculate $\hat{P}$ and $\hat{\theta}$ as follows

$$\int_0^1 \theta\,(1 - \frac{2}{3}\sqrt{\theta})\,\frac{1}{2\sqrt{\theta}}\,d\theta = \frac{1}{6};$$

$$P(\hat{\theta}) = \frac{1}{6} \Rightarrow \hat{\theta} = \frac{1}{4}$$

The expression of $q(\theta)$ will turn out to be

$$1 + \frac{2}{3}\,\frac{1 - 6\theta + 4\theta\sqrt{\theta}}{4\theta - 1}.$$

5 Discussion and Concluding Remarks

The results of section 4.3 conclude our formal analysis of the equilibrium choices of participants in this market. We have shown that an intermediary who can verify the quality of a privately informed seller can increase its profits by manipulating the amount of information it reveals. We showed that the intermediary will choose to reveal a garbled signal of the information it *obtains: it chooses a noisy disclosure rule. This can be also be interpreted as the intermediary choosing to perform a test of quality that is stochastic. Thus the quality is not revealed for sure but there is information about quality in the results of the test.*

The remarkable feature of the analysis is that by appropriately choosing the noisy disclosure rule the intermediary can do as well as in any other possible mechanism. This was shown by rst solving for the optimal mechanism and then constructing a disclosure rule that obtained the same distribution of quality and payoffs for all participants. In the equilibrium the intermediary acts exactly like a monopsonistic buyer.

In the equilibrium, efficiency is higher than when there is no intermediary (since then no quality is produced) but quality is lower than under full information. The reason for this is that the intermediary only reveals quality with some probability and therefore the marginal returns to the seller for producing quality are lower. This can also be interpreted in the standard mechanism design language: the intermediary acts as if the seller produced with a virtual cost with higher marginal cost than the true cost function. This is because of the informational incentive constraints.

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