

# **PRODUCTION SMOOTHING AND THE SHAPE OF THE COST FUNCTION**

by

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Abstract. We present estimates of inventory models based on firm level panel data and investigate whether over-simplified specification of the production technology may account for the frequent failure to find technological incentives to smooth production in the context of the standard linear-quadratic model of inventory behavior. In particular, we argue that if the role of quasi-fixed factors is not modeled properly, this may lead to inconsistent estimates of marginal costs and, therefore, to erroneous conclusions about the convexity/concavity of the cost function. The model is accordingly extended to allow for a general restricted quadratic cost function, on the assumption that capital is costly to adjust. The evidence obtained by estimating the standard inventory model on a panel of Italian manufacturing firms suggests that marginal costs are decreasing. However, this result is overturned when one allows for the general quadratic cost function with capital as a quasi-fixed input, implying that the firm's technology provides incentives to smooth production.

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## 1. Introduction

The linear-quadratic model introduced by Holt, Modigliani, Muth, and Simon (1960) has been a widely used tool in inventory research, but its estimation has yielded mixed results. The model is often rejected on the basis of standard misspecification tests, while the parameter estimates sometimes fail to support the proposition that firms use inventories to smooth production. The basic idea of the model is that a profit-maximizing firm facing a convex cost curve will have an incentive to smooth production in response to fluctuating sales, building up stocks when demand is relatively low and decumulating them when it is high. The variance of production should therefore be smaller than that of sales. Instead, it turns out that output is often more variable than sales at the aggregate as well as at industry and firm level. To explain this and to improve the empirical performance of the model, stockout-avoidance motives for holding inventory have been allowed for (Blanchard, 1983; West, 1986) and cost shocks have been introduced (Blinder, 1986; Miron and Zeldes 1988; Eichenbaum, 1989). Stockout motives or cost shocks would explain why inventories move procyclically and why they are used to bunch rather than smooth production. Although these extensions often make the model more data-coherent, at times its performance is less than fully satisfactory.<sup>1</sup> Moreover, there is also evidence that a more radical departure from the production smoothing model may be required, in that the cost structure may not be convex. This is the conclusion reached by Ramey (1991), although other researchers have obtained different results (Eichenbaum, 1989; Kashyap and Wilcox, 1993; Durlauf and Maccini, 1995; Fuhrer, Moore, and Schuh, 1995; Kollintzas, 1995).

The present paper addresses the issue of whether the shape of the firm's cost function provides an incentive to smooth production. Unlike most previous literature, which uses aggregate industry data, we base our empirical work on a rich panel of

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<sup>1</sup> See West (1995) for a review of the literature.

several hundred Italian firms over the period 1982 to 1992.<sup>2</sup> We start by estimating standard augmented linear-quadratic models. However, these models may lead to inconsistent estimates of marginal costs and hence of the incentives to use inventory adjustments in order to smooth production. In fact, changes in the stock of quasi-fixed factors will shift the firm's short-run cost function, so that ignoring these inputs may lead to incorrect conclusions concerning its concavity or convexity. In addition, the quadratic term in the change in output, which is typically included to capture the adjustment costs associated with changing capital or labor stock (Blanchard, 1983), misrepresents the fact that the firm can adjust on two margins, by using more of its capacity or by expanding its capacity, at a cost. For this reason we extend the basic model to allow for the existence of quasi-fixed factors using a quadratic cost function that can be considered a second order approximation to any cost function. This extension has a profound effect on our empirical conclusions concerning the shape of marginal costs.

The structure of the paper is as follows. In section 2 we briefly review the standard augmented linear-quadratic model. In section 3 we introduce a more general specification of the cost function. Data panel and variable construction are described in section 4. Section 5 presents the econometric estimates, after which brief conclusions close the paper.

## **2. The Standard Linear-Quadratic Model of Inventory Behavior**

The standard linear-quadratic model of inventory behavior posits a convex short-run cost function and assumes that adjusting output entails costs. Because of increasing marginal costs and the cost of changing production levels due to the cost of adjusting inputs, a firm will choose to smooth production when faced with time-varying

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<sup>2</sup> See Schuh (1993) for an investigation of the consequences of aggregating micro units using panel data, and Guariglia and Schiantarelli (1995) for the role of financing constraints in linear-quadratic inventory equations using U.K. panel data.

sales, by accumulating inventories during periods of low sales and depleting them when sales are high. However, if production costs are subject to stochastic movements, firms may bunch production in periods of low costs. Moreover, since sales are stochastic, firms may also desire to hold inventories as a buffer against stockouts, should sales unexpectedly exceed the sum of inventories and production, resulting in delay of delivery, a possible loss of sales and damage to customer relations. This will lead inventory accumulation to move together with sales.

In all the models considered here, firms produce a single homogeneous good and take input prices as given. A necessary condition for profit maximization is the minimization of the sum of discounted costs for any given stream of sales. Costs are broadly defined to include production costs, adjustment costs, the cost of holding inventories, and the cost of stockouts. Differences between the models lie in the form of the cost function.

In the simplest version of the linear-quadratic model the firm solves the following problem (see, for instance, West, 1986):

$$\max_{H_j} E_t \sum_{j=t}^{\infty} b^j \{P_j S_j - C(Y_j, \Delta Y_j) - \Phi(H_{j-1}, S_j)\} \quad (1)$$

subject to  $Y_j = H_j - H_{j-1} + S_j$ , where  $Y$  is real production,  $S$  real sales,  $H$  real end-of-period inventories,  $P$  the market price of the product, and  $b$  a discount term satisfying  $0 < b < 1$ . The functional forms are as follows:

$$C(Y_t, \Delta Y_t) = 0.5a_0(\Delta Y_t)^2 + 0.5a_1 Y_t^2 + u_{1,t} Y_t \quad (2)$$

$$\Phi(H_{t-1}, S_t) = 0.5a_2(H_{t-1} - a_3 S_t)^2 + u_{2,t} H_{t-1} \quad (3)$$

The cost of production,  $C(Y_t, \Delta Y_t)$ , is composed of adjustment costs, the short-run production cost function, and the stochastic term,  $u_{1,t}$ , a shock to the marginal production cost. Note that the absence of linear terms (except for the shock terms) is

inconsequential, as it merely introduces a constant in the Euler equations resulting from the first order condition of problem (1). The  $\Phi(H_{t-1}, S_t)$  function has become a standard method of capturing inventory holding costs and stockout costs.<sup>3</sup> Desired inventories are assumed proportional to next-period sales, and deviations are costly, either from having to hold too large an inventory (positive deviations) or because of the increased danger of completely depleting inventories (negative deviations). The specification of these costs includes the possibility of random shocks to marginal holding costs through the term  $u_{2,t}$ . This specification of stockout costs is convenient but not fully satisfactory (Kahn, 1992; Naish, 1994); however, in this paper we do not pursue this issue and concentrate on the specification of production costs.

Optimal behavior requires that the following first-order condition must be satisfied:

$$E_t \{ a_0 (\Delta Y_t - 2b\Delta Y_{t+1} + b^2\Delta Y_{t+2}) + a_1 (Y_t - bY_{t+1}) + a_2 b (H_t - a_3 S_{t+1}) + u_{1,t} - b u_{1,t+1} + u_{2,t} \} = 0 \quad (4)$$

This ensures that the firm cannot profit from changing its production schedule over any two periods: the marginal cost of increasing production today and holding that additional unit in inventory equals the expected reduction in cost of marginally reducing production tomorrow.<sup>4</sup>

Ramey (1991) extends this model by including a cubic term in output (to permit non-linear marginal costs) and a term that allows for observable cost shocks to input prices. The current period cost function has the form:

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<sup>3</sup> The precise timing of the variables in the stockout cost function varies across studies. Our specification is the same as in West (1995). However, this does not affect the nature of the results.

<sup>4</sup> Equation (4) is just one of the first-order conditions for profit maximization. The other, which we do not use here, endogenizes sales by equating marginal cost to marginal revenue.

$$C(Y_t, \Delta Y_t, w_t) = 0.5a_0(\Delta Y_t)^2 + 0.5a_1Y_t^2 + (1/3)a_4Y_t^3 + a_5w_tY_t + u_{1,t}Y_t \quad (5)$$

where  $w_t$  is a (column) vector of observable costs in units of output and  $a_5$  a (row) vector of coefficients. The first order condition becomes:

$$E_t \{ a_0(\Delta Y_t - 2b\Delta Y_{t+1} + b^2\Delta Y_{t+2}) + a_1(Y_t - bY_{t+1}) + a_2b(H_t - a_3S_{t+1}) + a_4(Y_t^2 - bY_{t+1}^2) + a_5(w_t - bw_{t+1}) + u_{1,t} - bu_{1,t+1} + u_{2,t} \} = 0 \quad (6)$$

In these models a firm's incentive to smooth production can be inferred from the second derivative of the present value of the cost function with respect to output, which is equal to  $\frac{\partial^2 \Sigma b^t C_t}{\partial Y_t^2} = a_1 + (1+b)a_0$  when the cubic term in output is excluded and to  $\frac{\partial^2 \Sigma b^t C_t}{\partial Y_t^2} = a_1 + (1+b)a_0 + 2a_4Y_t$  when it is included. This is very convenient, because a single figure captures the effect of output decisions on both marginal production costs and adjustment costs.

Ramey (1991) finds that the estimated cost structure is significantly concave in all production-to-stock industries and in the automobile industry, with four of the second derivatives significantly negative at the 0.05 level. She concludes that firms behave as if they faced declining marginal costs and therefore bunch production. However, other authors have obtained results consistent with increasing marginal costs of production (West, 1986; Eichenbaum, 1989; Kashyap and Wilcox, 1993; Durlauf and Maccini, 1995; Fuhrer, Moore, and Schuh, 1995; Kollintzas, 1995). In general the results appear to be sensitive to the parameter combination one decides to estimate, to the choice of normalization rule, and to the estimation method. Even when marginal costs have been found to be increasing, the coefficient of the quadratic term in output,  $a_1$ , is often not significant (Blanchard, 1983; West, 1990; Kashyap and Wilcox, 1993) and the coefficient of the adjustment costs term,  $a_0$ , is at times negative or insignificant (Kollintzas, 1995). In any case all the results are obtained on the basis of aggregate data and using a

specification for the firm's cost function that captures the cost of changing factors of production by a quadratic term in the first difference of output.<sup>5</sup>

### **3. Generalizing the Firm's Cost Function**

This paper uses panel data to test whether there are technological incentives to production smoothing; we seek to make sure that, whatever the conclusion reached, it is not based on an oversimplified description of the firm's technology. In particular, the cost structure facing a firm should not depend solely on level and variations in production and on factor prices. Costs are also affected by the stock of quasi-fixed factors, and adjustment costs depend on changes in these factors, and not on the change in production. A firm wishing to increase production can either change its fixed factors, incurring adjustment costs, or simply employ more variable inputs, incurring no adjustment costs (or both, of course). As we shall see shortly, using the change in output to capture adjustment costs may seriously obscure and even overturn the convexity of the cost function. Improving the specification of the technology in inventory models is thus a necessary step in accurately assessing the firm's incentive to smooth production.

Actually, this idea is not entirely new. Some authors have studied firms' decisions in the context of models with quasi-fixed factors. Maccini (1984) considers a model in which a firm simultaneously makes decisions regarding price, output, and capital stock, an input that is costly to adjust; and he discusses at the theoretical level the interaction between capital and inventory decisions. Maccini shows that it is optimal for the firm to use the excess of current capital stock relative to its desired level to produce output and to add to inventory. On the other hand, if the firm holds excess inventory, it will slow down the capital accumulation process because at least for a time it can meet

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<sup>5</sup> Guariglia and Schiantarelli (1995), using U.K. panel data, find evidence of concave costs.

future demand by inventory depletion. Other things equal, we should therefore expect a positive correlation between finished goods inventories and fixed capital. Maccini and Rossana (1984), using aggregate data, find empirical evidence of a strong interaction between inventories and stocks of quasi-fixed factors in an unrestricted specification based on a multivariate flexible accelerator model.<sup>6</sup> Their analysis focuses on whether allowing for these factors more precisely captures the cost of changing output, but does not address whether marginal costs are rising or falling. Miron and Zeldes (1988) do consider capital as a quasi-fixed factor and find that the model fails tests of overidentifying restrictions. However, their approach assumes a Cobb-Douglas production technology, which is unduly restrictive. Moreover, their proxy for the stock of capital is a poor one, as they model the growth in the capital stock as a function of the number of nonholiday weekdays in the month.

The empirical work contained in the papers cited above is based on aggregate data. We rely instead on firm level panel data. Hopefully, exploiting both the time series and cross-sectional variations will produce more precise estimates of the structural parameters. Moreover, one can investigate potential heterogeneity in inventory behavior among different types of firms (such as production-to-stock versus production-to-order firms). In terms of specification, we extend the simple linear-quadratic model to appropriately incorporate the role of quasi-fixed factors, while not imposing overly stringent assumptions on the nature of the technology. To this end, we rely on a flexible functional form that can be considered a second-order approximation to any cost structure. We assume that the firm takes the capital stock at the beginning of the period as quasi-fixed, while raw materials and labor are the variable inputs. At each point in time the short run restricted cost function depends on an index of technological change, variable factor costs, level of output, stock of capital, and gross change in capital itself (Berndt, Fuss, and Waverman, 1977):

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<sup>6</sup> The positive correlation between fixed capital and finished goods inventories is confirmed by Rossana (1990) who uses data for individual two-digit industries.



$$\begin{aligned}
C(w_t, K_{t-1}, I_t, Y_t, T_t) = & a_0 + a_T T_t + a_Y Y_t + a_w w_t + a_K K_{t-1} + \\
& a_I I_t + 0.5[a_{TT} T_t^2 + a_{YY} Y_t^2 + a_{ww} w_t^2 + a_{KK} K_{t-1}^2 + a_{II} I_t^2] + \\
& a_{wY} w_t Y_t + a_{wK} w_t K_{t-1} + a_{wI} w_t I_t + a_{KI} K_{t-1} I_t + \\
& a_{KY} K_{t-1} Y_t + a_{IY} I_t Y_t + u_{1,t} Y_t + u_{3,t} K_{t-1} + u_{4,t} I_t
\end{aligned} \tag{7}$$

where  $K_{t-1}$  is the capital stock at the end of the last period,  $I_t$  is gross fixed investment satisfying  $I_t = K_t - (1-d)K_{t-1}$ ,  $d$  is the capital depreciation rate,  $w_t$  is a column vector of variable input prices (normalized by the output price),  $T_t$  is an index of technological change, and  $u_{3,t}$  is an unobservable shock to the productivity of capital. The coefficients associated with input prices should be thought of as conformable row vectors; for simplicity, Hick's neutral technological change is assumed.<sup>7</sup> Adjustment costs are assumed to be internal and nonseparable and are captured by the terms involving gross investment (Galeotti, 1990), including the unobservable cost shock  $u_{4,t}$ . In addition to the costs specified in equation (7), the firm will pay a purchase price (in output units) for each unit of investment goods.

The firm maximizes the objective function (1), suitably modified to include investment expenditures, subject to the usual equation of motion for the capital stock and to the parametrization of production costs (7) and stockout costs (3). The optimal inventory and capital decisions are summarized by the following first order (cost minimizing) conditions:

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<sup>7</sup> Notice, however, that the so called bias of technical change, represented by the interaction terms between the  $T$  index and the other arguments of the cost function, could serve as a justification for the time dummies included in the estimated equations we present below.

$$E_t \left\{ (1-b)a_Y + a_{YY}(Y_t - bY_{t+1}) + a_{wY}(w_t - bw_{t+1}) + a_{KY}(K_{t-1} - bK_t) + \right. \\ \left. a_{IY}(I_t - bI_{t+1}) + ba_2(H_t - a_3S_{t+1}) + u_{1,t} - bu_{1,t+1} + u_{2,t} \right\} = 0 \quad (8)$$

$$E_t \left\{ a_I [1 - (1-d)b] + a_K b + a_{KI} K_{t-1} + a_{II} I_t + a_{IY} Y_t + a_{wI} w_t + [a_{wK} - \right. \\ \left. (1-d)a_{wI}] bw_{t+1} + [a_{KK} - (1-d)a_{KI}] bK_t + [a_{KI} - (1-d)a_{II}] bI_{t+1} + \right. \\ \left. [a_{KY} - (1-d)a_{IY}] bY_{t+1} + r_t + u_{4,t} - (1-d)bu_{4,t+1} + bu_{3,t+1} \right\} = 0 \quad (9)$$

where  $r_t$  is Jorgenson's user cost of capital. For the cost function (7) to be a valid representation of the firm's technology, the restricted cost function *cum* internal adjustment costs must meet several regularity conditions. Specifically, costs must be nondecreasing in variable factor prices, monotonically increasing in output, and nonincreasing in capital. Costs must also be concave in input prices and convex in level of capital.<sup>8</sup> In addition, because of adjustment costs the cost function must be increasing and convex in investment (Berndt, Fuss, and Waverman, 1977). As shown below, some of these properties can be directly verified on the basis of the estimated equations (8) and (9).

It is worth noting that, abstracting from input prices, the cost function specified in the standard linear-quadratic model (equation (2)) is itself a quadratic approximation of a cost function of the form  $C(DY, Y)$  with the interaction term suppressed.<sup>9</sup> It is difficult to find a rigorous justification for this choice in production theory. In fact, the

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<sup>8</sup> As is well known, restricted cost functions are simply special cases of restricted profit functions (Lau, 1976).

<sup>9</sup> Ramey's cost function, where interaction terms between the level of output and variable input prices are included, can also be considered a quadratic, albeit truncated, approximation to an arbitrary cost function. The quadratic flexible functional form for restricted technologies with internal adjustment costs has been widely used in empirical studies in the dynamic factor demand literature (e.g. Morrison and Berndt, 1981), thanks to its convenient properties for the analytical features of these models (Galeotti, 1996).

absence of quasi-fixed factor levels implies that there are no quasi-fixed inputs, but there is still a cost of adjustment. Presumably the exclusion of the stock of capital mainly reflects the lack of suitable data, but this omission becomes critical when the model is used to assess the curvature of the cost function with respect to output, since it is the change of the stock of quasi-fixed factors, not the change in output, that gives rise to adjustment costs.<sup>10</sup> For these reasons it is possible that estimation using the  $C(DY, Y)$  technology specification will result in incorrect inferences about the convexity of the cost structure. More specifically, assume that there is a favorable shock to demand so that the capital stock increases and that, even taking adjustment costs into account, this leads to a downward shift of the short-run cost function. In this case higher production may be associated with lower marginal costs, giving the misleading impression that short-run marginal costs are decreasing in output. In short, failure to take into account the shifting of the short-run cost function will affect the estimated shape of the curve and possibly give it the appearance of concavity. For these reasons it is useful to investigate whether or not the conclusions concerning the technological incentives to smooth production depend on the specification of the cost function, and in particular whether those drawn on the basis of the standard quadratic specification are confirmed when a more general cost function is employed.

#### **4. Data and Variables Definition**

The data are drawn from a balanced panel of 769 Italian manufacturing firms covering the period from 1982 to 1992, derived from reports to Italy's Company Accounts Data Service, which serves banks.<sup>11</sup>

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<sup>10</sup> This point was originally made by Maccini and Rossana (1984). Of course, fixed capital is likely not to be the only quasi-fixed factor characterizing the firm's technology. Also labor is often considered an input that is costly to adjust. In this case the measurement issues concerning its quantity (workers vs. hours) and user cost need to be addressed (see, e.g., Epstein and Denny, 1983; Shapiro, 1986).

<sup>11</sup> The Service was founded by a consortium of banks interested in pooling information on their clients. Each year the whole sample includes some 30,000 Italian nonfinancial firms. The sample, however, is not randomly drawn, as a firm enters only by borrowing from one of the banks in the

An important feature of the dataset is the availability of separate figures for inventories of raw materials and inventories of work-in-progress and finished goods.<sup>12</sup> However, not all the firms reported separate figures for the two types of inventory. In some years and for some firms, both were grouped into the finished goods inventories. In these instances, if separable figures for raw materials and finished goods inventories were available for a firm for at least three years, the average value has been used to partition total inventories for the missing years. If not, the firm was dropped from the sample.<sup>13</sup>

Application of the most general version of the model requires an estimate of the capital stock. The fixed capital figures at replacement cost have been calculated using the accounting value of the capital stock for a given year as benchmark and updating it using investment expenditure figures and investment deflators. In particular, we have assumed that the 1983 book value of capital correctly represents the replacement value. Such an assumption is reasonable because firms were allowed to make a one-time inflation adjustment of the accounting capital stock figure in 1983 for tax purposes (Visentini Law).

The data on sales, book value of capital stock, stock of inventories, and wage bill come from the company accounts. Supplementary information provided by the firms in the dataset includes figures for gross investment expenditure and number of employees. We have computed real output by adding the real change in finished goods inventories to real sales. Industry-specific output price indices have been used to calculate figures at constant prices. Capital stock and investment have been deflated using sectoral investment goods deflators. The real wage per employee-hour has been

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consortium. Balance sheets are reclassified to reduce the dependence of the data on the accounting conventions used by each firm. A nice feature of the dataset is that supplementary information (such as number of employees and gross investment expenditure) is often provided.

<sup>12</sup> Blinder (1986) suggests that work-in-progress should be grouped with finished goods.

<sup>13</sup> The number of firms in our sample was dictated primarily by the availability of suitable data on inventories.

obtained from the wage bill and the number of employees available at the firm level. Data on hours worked are available at the manufacturing level and the product price indices are at the two-digit sectoral level. For the discount factor  $\beta$  we have used the average nominal rate of interest on bank loans less the average output inflation rate in each sector, and for the capital depreciation rate  $\delta$  the aggregate sector level rate as used by the National Statistical Institute (ISTAT) in the estimation of sectoral net capital stock data.<sup>14</sup> Firm-specific data on the price of raw materials and on the user cost of capital were not available; their role in estimation has been captured by time and industry dummies.

All nominal values are in millions of lire, real values are in 1980 prices. Some relevant descriptive statistics and the detail of the branches of total manufacturing to which the firms in the sample belong are provided in the Appendix.

## 5. Empirical Results

In this section we first present estimates of the standard linear-quadratic models found in the literature and then discuss the econometric results for our more general model. The Generalized Method of Moments (Hansen and Singleton, 1982) is used to estimate all equations. Estimation is performed in first differences to eliminate any firm-specific component of the error term. In all estimated equations we include year dummies interacted with sectoral dummies to allow for, respectively, technical progress with different effects for each industry, sector-specific macroeconomic shocks, and cost shocks (price of materials and user cost of capital) with an industry-specific dimension.<sup>15</sup>

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<sup>14</sup> The average value of  $\beta$  is over 9 percent in real terms. This roughly corresponds to the average real interest rate on T-bills over the sample period plus a risk premium of 4-5 percentage points.

<sup>15</sup> The firms in the sample belong to the following sectors: ferrous and non-ferrous ores and metals; non-metallic minerals and mineral products; chemical products; metal products except machinery and transport equipment, agricultural and industrial machinery; office and data processing machines, precision and optical instruments; electrical goods; transport equipment; food, beverages and tobacco; textile and clothing, leather and footwear; paper and printing products, rubber and plastic products.

Appropriately lagged values of the predetermined variables of the various models are used as instruments. We use the Hansen-Sargan test of overidentifying restrictions to check for instrument validity and general model misspecification.

### 5.1. *Standard Linear-Quadratic Models*

We estimate two versions of the standard linear-quadratic model, a basic version and a modified one, that adds a cubic term in output and observable cost shocks in the cost function.<sup>16</sup> Transforming the first order condition (4) into an equation to be estimated involves a normalization, since all coefficients except  $a_3$  are identified only up to a constant of proportionality. Normalizing by the coefficient on  $H_t$ ,  $a_2$ , using the specification of the modified version, the first order condition (6) yields the following equation for each firm (indexed by  $i$ ):

$$\begin{aligned}
 H_{it} = & \text{b}^{-1} \frac{a_0}{a_2} (-\Delta Y_{it} + 2\text{b}\Delta Y_{it+1} - \text{b}^2 \cdot \Delta Y_{it+2}) + \text{b}^{-1} \frac{a_1}{a_2} (-Y_{it} + \text{b}Y_{it+1}) + \\
 & a_3 S_{it+1} + \text{b}^{-1} \frac{a_4}{a_2} (-Y_{it}^2 + \text{b}Y_{it+1}^2) + \text{b}^{-1} \frac{a_5}{a_2} (-w_{it} + \text{b}w_{it+1}) + v_{it}
 \end{aligned} \tag{10}$$

In the basic specification both  $a_4$  and  $a_5$  equal zero. In equation (10),  $v_{it}$  is a composite error term of the form:

$$v_{it} = e_{it} - \text{b}^{-1} \frac{1}{a_2} (u_{1,it} + u_{2,it} - \text{b}E_t u_{1,t+1})$$

where  $e_{it}$  represents an expectational error that, in principle, may have an MA(1) structure since it also involves forecast errors of variables two periods ahead. The error

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<sup>16</sup> If we disregard the different timing of the variables in the stockout cost function, these two versions correspond to the models implemented by West (1986) and Ramey (1991) respectively.

term also involves cost shocks that are unobservable to the econometrician but that we assume to be observed by the firm. This assumption implies that the current level of production and inventories will be affected by  $u_{1,it}$  and  $u_{2,it}$ , resulting in non-orthogonality between contemporaneous firm level variables and the error term, and thus excluding these variables as potential instruments. Under the assumption that the unobservable cost shocks are white noise, variables dated t-2 or earlier are acceptable instruments in the first-difference equation.

As a proxy for observable cost shocks we have included the average hourly real wage per employee. To proxy for material costs and user cost of capital, for which only an aggregate index at the manufacturing level is available, we have included year dummies interacted with eleven industry dummies, as explained above.

Estimates of the parameters of equation (10) (with and without the restriction  $a_4=a_5=0$ ), using the total sample of firms and using the regressors lagged two and three times as instruments, are reported in Table 1.<sup>17</sup> In Table 2 and 3 we report the results of estimating equation (10) when firms are classified according to size (more or fewer than 100 employees) and financial strength summarized by the coverage ratio (defined as post-tax income from operations divided by interest payments). Technology and adjustment costs may differ between small and large firms. Moreover, small firms are more likely to face information problems and to suffer from financial constraints. Firms with low coverage ratios are also likely to pay a higher premium on external finance.

The significant and positive coefficient  $a_3$  implies that stockout motives are strong. Adjustment costs, captured by the coefficient  $a_0$ , also have the correct sign in both specifications but are significant only in the modified version of the model. The coefficient in front of the squared output term in the cost function,  $a_7$ , is always negative

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<sup>17</sup> Fuhrer, Moore, and Schuh (1995) argue that although consistent, GMM estimates can be biased and imprecise in small samples in a time series context and are sensitive to the normalization chosen. This may not be a serious problem here, as the total number of observations is 7,690 rather than just a few hundred (which they consider typical). Furthermore, our results are not sensitive to different normalizations, as we show later.

and significant. The coefficient of the cubic term in the cost function and that of the wage variable are correctly signed and significant. In both cases the second derivative of the cost function is negative (and significantly so), which confirms Ramey's finding of declining marginal costs of production. Finally, the Hansen-Sargan test of overidentifying restrictions does not suggest a rejection of the model at conventional significance levels.

The results on the concavity of the cost function in output also hold for the two subsamples of Tables 2 and 3. Note, however, that adjustment costs have the wrong sign (and significantly so) when firms are classified according to size. The same remark applies to the impact of the wage rate. Moreover, both  $a_0/a_2$  and  $a_1/a_2$  are very imprecisely determined for firms with high coverage ratios. The overall impression is that allowing for these two forms of heterogeneity does not improve the performance of the model and leaves the conclusions on concavity unaltered.

In order to ensure that the results are robust with respect to the normalization chosen, the same analysis was performed using the Legendre-Clebsch normalization. Parameter estimates are asymptotically invariant to the choice of normalization but may be affected in small samples. The normalization is based on a second-order necessary condition requiring that the second partial of the firm's objective function with respect to  $H_t$ , designated  $c$ , be non-negative. According to this normalization, the estimating equation is:

$$\begin{aligned}
H_{it} = & \frac{a_0}{c} [-(\Delta S_{it} - 2H_{it-1} + H_{it-2}) + 2b(\Delta S_{it+1} + H_{it+1} + H_{it-1}) - \\
& b^2(H_{it+2} - 2H_{it+1} + \Delta S_{it+2})] + \frac{a_1}{c} [b(H_{it+1} + S_{it+1}) + H_{it-1} - S_{it}] + \\
& b \frac{a_2 a_3}{c} S_{it+1} + \frac{a_4}{c} (-Y_{it}^2 + bY_{it+1}^2) + \frac{a_5}{c} (-w_{it} + bw_{it+1}) + v_{it}
\end{aligned} \tag{11}$$

where  $c = a_0(1 + 4b + b^2) + a_1(1 + b) + ba_2$ . The results of estimation for the full sample of data are presented in Table 4. It emerges that the general conclusions



previously reached on the basis of equation (10) are not affected by the choice of the normalization rule, since in this case too marginal costs appear to be decreasing, and significantly so. In summary, our panel data estimates show that the cost function is significantly concave, regardless of how the sample is divided and of the choice of normalization.

## 5.2. Capital as a Quasi-fixed Factor

When, as discussed in the preceding section, we extend the model to allow for a generalized quadratic restricted cost function with capital as a quasi-fixed factor, the equations to be estimated corresponding to (8) and (9) are (omitting the constants):

$$a_{YY}(Y_{it} - bY_{it+1}) + a_{wY}(w_{it} - bw_{it+1}) + a_{KY}(K_{it-1} - bK_{it}) + \quad (12)$$

$$a_{IY}(I_{it} - bI_{it+1}) + a_2 b(H_{it} - a_3 S_{it+1}) = v_{1,it}$$

$$a_{KL}K_{it-1} + a_{II}I_{it} + a_{IY}Y_{it} + a_{wI}w_{it} + [a_{wK} - (1-d)a_{wI}]bw_{it+1} + \quad (13)$$

$$[a_{KK} - (1-d)a_{KI}]bK_{it} + [a_{KI} - (1-d)a_{II}]bI_{it+1} + [a_{KY} -$$

$$(1-d)a_{IY}]bY_{it+1} + r_{it} = v_{2,it}$$

The disturbance terms here include both expectational errors and the unobservable shocks to the cost function. If this is the correct specification, one can appreciate the potential for biased estimates resulting from the Euler equation implied by the standard linear-quadratic model (see equation (10)). One possible source of bias is the misspecification of adjustment costs. The second source is omitted variables. In fact, leaving out the capital stock is likely to bias coefficient  $a_1$  in equation (10) downward if the coefficient of the capital stock  $a_{KY}$  is negative, because downward shifts of the cost

function, associated with increases in the capital stock, could be confused with movements along it.

The results of estimating the system of equations (12) and (13) simultaneously are reported in Table 5. Note that if we had firm level data about the user cost of capital we could identify all the parameters. However, since we are proxying for the cost of capital with time and industry dummies interacted, we need to impose a normalization. For consistency with most of the results presented so far, we have set  $a_2$  equal to one in equation (12). The two most important results, given the objective of this paper, are that  $a_{YY}$  is positive and  $a_{KY}$  is negative. The positive (and significant) coefficient  $a_{YY}$  suggests that the restricted cost function is convex in output. The negative and significant  $a_{KY}$  coefficient indicates that higher levels of capital decrease the marginal cost of output, thereby supporting the hypothesis that the omission of the capital stock from standard Euler equations for inventories may explain the finding of decreasing marginal costs.

In this model, as well as in the standard one, there is a significant role for holding inventories in order to avoid stockout costs, as coefficient  $a_3$  is positive and significant. The importance of stockout costs and of cost shocks explains why, even in the presence of a technological incentive to smooth production, the variance of production may exceed that of sales.

Returning to technological considerations, the restricted cost function is convex in the stock of capital and in investment ( $a_{KK}$  and  $a_{II}$  both positive), as required by the regularity conditions for a well-behaved cost function with increasing marginal costs of adjustment.

Finally, the test of overidentifying restrictions does not reject the model specification, and most of the parameters are estimated with great precision. Only the parameters  $a_{KK}$  and  $a_{wY}$  are not statistically significant, although they do exhibit the expected sign.

What is the empirical evidence when the system of equations (12) and (13) is estimated separately for large and small firms and for credit-constrained and unconstrained firms? Tables 6 and 7 provide the answer when the sample is divided according to firm size and coverage ratio. The main finding concerning the curvature of the cost function and the existence of technological incentives to smooth production is confirmed for all subsamples. In fact  $a_{YY}$  is positive and significant and  $a_{KY}$  negative and significant in all cases. Note also that, here, in contrast with other studies (Miron and Zeldes, 1988; Ramey, 1991), the parameter in front of the wage rate,  $a_{wY}$ , is now statistically significant, thus suggesting a discernible impact of observable cost shocks. The cost function is well-behaved, as the sign and significance of  $a_{II}$  suggest for both cuts of the dataset. The main weakness of the model concerns the  $a_{KK}$  parameter, which is negative in three cases out of four and significantly so for large firms. At any rate, the Hansen-Sargan test supports the model specification in all cases. On the whole, the evidence from the disaggregated analysis strongly supports our earlier conclusions for the full sample.

## 6. Conclusions

The starting point of this paper is the absence of consensus and precise conclusions about the shape of the cost function facing the firm and about the existence of incentives that lead to production smoothing.

The first novel aspect of our contribution is the investigation of the issue of the concavity/convexity of this cost function on the basis of panel data. Exploiting both the time series and the cross sectional variation in the data should enhance the precision of the estimates. Estimation of standard linear-quadratic models that incorporate a stockout avoidance motive for holding inventories as well as both observable and unobservable cost shocks yields the result of a significantly decreasing marginal cost schedule.

The second, and most important element of novelty of this paper concerns the specification of the firm's cost function. In particular, we make the point that omitting the stock of quasi-fixed factors from the firm's cost function may give rise to misleading conclusions regarding the behavior of marginal production costs. Indeed, when we generalize the (restricted) cost function to take a quadratic flexible form with capital as a quasi-fixed input, the result of declining marginal costs is reversed. The restricted short-run cost function is, instead, significantly convex in output. Moreover, marginal costs are decreasing in the level of the capital stock. This may explain why previous studies have found short-run marginal costs to be decreasing in output: this occurs because downward shifts of the cost function due to an increase of the capital stock may be misinterpreted as movements along the same cost function. Our findings support the existence of technological incentives to production smoothing as a reason for holding inventories. These conclusions also hold when we group firms in the sample according to their size and financial coverage ratio. It remains true that, even when the generalized version of the cost function is used, there is a significant role for stockout costs.

Before concluding, a few remarks concerning the limitations of our empirical analysis are in order. The first concerns the issue of temporal aggregation. In fact, while the amount of inventories in finished goods is likely to change quite frequently, capital stock decisions are typically low-frequency. The problems that may arise when low-frequency data are used to analyze high-frequency decisions has been studied by Christiano and Eichenbaum (1989) for the case of inventories alone. We believe that this issue is potentially relevant but very difficult to address in our case, as it is hard or impossible to obtain high-frequency panel data (e.g. quarterly or even monthly), especially for fixed assets. The second issue concerns the impact and relevance of other quasi-fixed inputs, besides fixed capital, such as labor. This is certainly a topic that deserves further investigation. Still, in our view the estimation on panel data of the cost function that allows for the quasi-fixity of capital represents a significant advance that

yields important results on the controversial issue of the shape of marginal production costs.

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**Table 1: Standard Linear-Quadratic Model - Full Sample**

	Basic Version	Modified Version
$a_0 / a_2$	0.007 (1.14)	0.032 (3.99)
$a_1 / a_2$	-0.043 (-1.61)	-0.152 (-3.51)
$a_3$	0.059 (6.38)	0.053 (4.77)
$a_4 / a_2$	-	0.54E-04 (3.34)
$a_5 / a_2$	-	112.27 (1.92)
Slope of Marginal Cost	-0.074 (-1.23)	-0.060 (-3.81)
Hansen-Sargan Test	25.69 (0.69)	27.91 (0.47)
Sample Period	1985-90	1985-90
Number of Firms	769	769

Footnotes to the table:

- (1)  $t$ -statistics are reported in parenthesis. The Sargan-Hansen test tests the overidentifying restrictions of the model. P-values are reported in parenthesis.
- (2) Year dummies interacted with industry dummies for eleven sectors are included.
- (3) The slope of marginal cost is computed according to the formula given in the main text. Both this value and its t-statistics are computed assuming discount rate and output to be equal to their respective overall sample means.
- (4) Instruments include year and industry dummies plus real production, sales and wage rate lagged two and three times in addition to the year-industry dummies.
- (5) In the modified version of the model we allow for a cubic term in output and for observable cost shocks.

**Table 2: Standard Linear-Quadratic Model - Classification Criterion: Size  
(Modified Version)**

	Small Firms	Large Firms
$a_0 / a_2$	-0.020 (-3.83)	-0.032 (-4.25)
$a_1 / a_2$	-0.009 (-1.00)	-0.006 (0.12)
$a_3$	0.029 (3.45)	0.058 (4.02)
$a_4 / a_2$	0.17E-04 (2.05)	0.20E-04 (2.00)
$a_5 / a_2$	-40.380 (-2.88)	-101.591 (-2.05)
Slope of Marginal Cost	-0.106 (5.26)	-0.067 (-1.41)
Hansen-Sargan Test	29.26 (0.35)	30.88 (0.32)
Sample Period	1985-90	1985-90
Number of Firms	450	319

Footnotes:

- (1)  $t$ -statistics are reported in parenthesis. The Sargan-Hansen test tests the overidentifying restrictions of the model. P-values are reported in parenthesis.
- (2) Year dummies interacted with industry dummies for eleven sectors are included.
- (3) The slope of marginal cost is computed according to the formula given in the main text. Both this value and its t-statistics are computed assuming discount rate and output to be equal to their respective overall sample means.
- (4) Instruments include year and industry dummies plus real production, sales and wage rate lagged two and three times in addition to the year-industry dummies.
- (5) A firm is classified as large (small) if it has more (fewer) than 100 employees on average.

**Table 3: Standard Linear-Quadratic Model - Classification Criterion:  
Financial Coverage Ratio (Modified Version)**

	Low Coverage Ratio	High Coverage Ratio
$a_0 / a_2$	0.017 (4.03)	-0.007 (-1.46)
$a_1 / a_2$	-0.101 (-6.37)	0.007 (0.25)
$a_3$	0.034 (5.69)	0.063 (14.16)
$a_4 / a_2$	0.53E-04 (6.78)	-0.23E-04 (-3.84)
$a_5 / a_2$	21.456 (1.57)	9.720 (0.83)
Slope of Marginal Cost	-0.068 (-5.91)	-0.001 (-1.21)
Hansen-Sargan Test	86.66 (0.13)	29.98 (0.36)
Sample Period	1985-90	1985-90
Number of Firms	384	385

Footnotes:

- (1)  $t$ -statistics are reported in parenthesis. The Sargan-Hansen test tests the overidentifying restrictions of the model. P-values are reported in parenthesis.
- (2) Year dummies interacted with industry dummies for eleven sectors are included.
- (3) The slope of marginal cost is computed according to the formula given in the main text. Both this value and its t-statistics are computed assuming discount rate and output to be equal to their respective overall sample means.
- (4) Instruments include year and industry dummies plus real production, sales and wage rate lagged two and three times in addition to the year-industry dummies.
- (5) Firms with low (high) coverage ratios are those that have a coverage ratio below (above) 2.96. This figure is the median value over the sample.

**Table 4: Linear-Quadratic Model under Legendre Normalization - Full Sample**

	Basic Version	Modified Version
$a_0 / c$	-0.016 (-1.41)	-0.044 (-2.75)
$a_1 / c$	-0.003 (-0.08)	0.015 (0.18)
$a_2 a_3 / c$	-0.055 (-2.23)	-0.070 (-3.86)
$a_4 / c$	-	-0.30E-04 (-1.20)
$a_5 / c$	-	56.032 (0.81)
Slope of Marginal Cost	0.028 (1.54)	-0.069 (1.78)
Hansen-Sargan Test	23.00 (0.52)	12.42 (0.95)
Sample Period	1986-90	1986-90
Number of Firms	769	769

## Footnotes:

- (1)  $t$ -statistics are reported in parenthesis. The Sargan-Hansen test tests the overidentifying restrictions of the model. P-values are reported in parenthesis.
- (2) Year dummies interacted with industry dummies for eleven sectors are included.
- (3) The slope of marginal cost is computed according to the formula given in the main text. Both this value and its  $t$ -statistics are computed assuming discount rate and output to be equal to their respective overall sample means.
- (4) Instruments include year and industry dummies plus real production, sales and wage rate lagged two and three times in addition to the year-industry dummies.

**Table 5: Euler Equations for Inventories and Capital Stock Derived from the Generalized Quadratic Cost Function - Full Sample**

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$a_2$	1
$a_3$	0.116 (25.46)
$a_{YY}$	0.239 (16.49)
$a_{KY}$	-0.55E-03 (-6.84)
$a_{IY}$	-0.49E-02 (-19.62)
$a_{wY}$	7.099 (0.12)
$a_{KI}$	0.21E-04 (14.50)
$a_{II}$	0.39E-04 (20.36)
$a_{KK}$	0.60E-06 (0.64)
$a_{wI}$	-7.20 (-5.06)
$a_{wK}$	-12.99 (-6.65)
Hansen-Sargan Test	111.78 (0.62)
Sample Period	1985-91
Number of Firms	769

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Footnotes (see also the footnotes to Table 1):

- (1) To achieve identification a normalization rule has been imposed by setting the coefficient  $a_2$  equal to 1.
- (2) In addition to those previously specified, the list of instruments also includes two and three times lagged values of the capital stock and gross investment.

**Table 6: Euler Equations for Inventories and Capital Stock Derived from the Generalized Quadratic Cost Function - Classification Criterion: Size**

	Small Firms	Large Firms
$a_2$	1	1
$a_3$	0.015 (5.37)	0.112 (12.74)
$a_{YY}$	0.046 (8.42)	0.35 (15.36)
$a_{KY}$	-0.19E-03 (-3.08)	-0.84E-03 (-7.78)
$a_{IY}$	-0.94E-03 (-8.38)	-0.007 (-17.85)
$a_{wY}$	16.274 (2.31)	179.629 (1.73)
$a_{KI}$	0.73E-05 (0.53)	0.24E-04 (12.76)
$a_{II}$	0.24E-04 (1.80)	0.33E-04 (10.46)
$a_{KK}$	-0.44E-05 (-0.84)	-0.75E-05 (-3.59)
$a_{wI}$	-3.312 (-9.23)	-8.241 (-3.51)
$a_{wK}$	-6.105 (-10.23)	-4.042 (-1.50)
Hansen-Sargan Test	106.46 (0.52)	100.58 (0.52)
Sample Period	1985-91	1985-91
Number of Firms	450	319

Footnotes (see also the footnotes to Table 2):

- (1) To achieve identification a normalization rule has been imposed by setting the coefficient  $a_2$  equal to 1.
- (2) In addition to those previously specified, the list of instruments also includes two and three times lagged values of the capital stock and gross investment.

**Table 7: Euler Equations for Inventories and Capital Stock Derived from  
the Generalized Quadratic Cost Function -  
Classification Criterion: Financial Coverage Ratio**

	Low Coverage Ratio	High Coverage Ratio
$a_2$	1	1
$a_3$	0.114 (22.61)	0.091 (29.31)
$a_{YY}$	0.194 (26.63)	0.530 (22.17)
$a_{KY}$	-0.47E-03 (-6.91)	-0.96E-03 (-13.12)
$a_{IY}$	-0.40E-02 (-31.19)	-0.011 (-24.97)
$a_{wY}$	58.141 (2.34)	-271.135 (-4.38)
$a_{KI}$	0.17E-04 (17.33)	0.30E-04 (17.44)
$a_{II}$	0.31E-04 (16.63)	0.38E-04 (22.12)
$a_{KK}$	-0.12E-05 (-1.15)	0.75E-05 (6.65)
$a_{wI}$	-5.604 (-9.70)	-10.090 (-5.29)
$a_{wK}$	-8.772 (-9.13)	-18.483 (-6.11)
Hansen-Sargan Test	99.54 (0.47)	134.95 (0.09)
Sample Period	1985-91	1985-91
Number of Firms	384	385

Footnotes (see also the footnotes to Table 3):

- (1) To achieve identification a normalization rule has been imposed by setting the coefficient  $a_2$  equal to 1.
- (2) In addition to those previously specified, the list of instruments also includes two and three times lagged values of the capital stock and gross investment.

## Appendix: Sample Descriptive Statistics

Selected Variables	Total Sample	Small Firms	Large Firms	Low Coverage Ratio	High Coverage Ratio
Number of Firms	769	450	319	384	385
Sample Period	1985-91	1985-91	1985-91	1985-91	1985-91
Production					
mean	155.88	65.47	282.74	125.18	186.49
median	75.95	46.09	162.78	66.51	88.95
st.dev.	258.92	86.37	350.82	185.11	312.85
Sales					
mean	155.12	65.21	281.28	124.58	185.59
median	75.72	46.01	161.66	66.36	88.78
st.dev.	257.26	86.28	348.32	184.13	310.70
Inventories					
Stock					
mean	15.76	6.04	29.43	15.40	16.12
median	7.15	3.94	17.03	7.88	6.49
st.dev.	30.83	6.68	43.62	24.97	35.73
Capital Stock					
mean	8397.80	3354.59	15473.02	6130.64	10659.07
median	3629.42	2308.47	7641.81	3213.75	4221.37
st.dev.	17493.27	3414.08	25167.78	10106.35	22343.52
Gross Investment					
mean	1237.76	486.03	2292.00	826.91	1647.53
median	436.58	248.72	1026.56	341.72	573.17
st.dev.	2943.83	733.13	4262.61	1549.12	3818.65

Note: Variables in millions of 1980 lire.