Climate Change and Event Uncertainty in a Dynamic Model with Overlapping Generations.

Michele Moretto[*] Roberto Tamborini[†]

February 1997

Abstract

In this paper the climate change effect is an unforeseen earth temperature level above which a negative externality on technology and hence on society's welfare is exerted. We use a dynamic overlapping generations model to develop a positive analysis of the growth path of an economy with the negative temperature spillover leading to a structural breakdown in capital productivity. Two scenarios for the impact of climate change on intergenerational equity are analyzed: the first is consistent with a state-of-nature framework in which atomistic agents cannot influence the probability that a particular event (productivity collapse) will occur. In the second, according to the maxim "that everybody does what I am doing....", agents recognize that their choices may influence the probability of productivity collapse occurring in their lifetime.

Key words: Global Warming, Event Uncertainty, Overlapping Generations.
JEL: C61, D81, Q25

---

[*] Department of Economics, University of Padova, via del Santo 22, 35100 Padova, E-mail moretto@decon.unipd.it.

[†] Department of Economics, University of Trento, via Inama 5, 38100 Trento, E-mail rtambori@gelso.unitn.it

1 Introduction

Evolution of human life and the development of particular social attitudes and economic skills have largely been conditioned by climatic conditions and their permanent changes (Nougier, 1977). Until the scientific and industrial revolution of the XVIII century, if on the one hand it was recognized that the climate had some influence on human activity, on the other climatic conditions were regarded as a token of Nature unaffected by, and unrelated to, human activity. We are now aware that large-scale industrial production is likely to have long-term irreversible effects on climate, mainly through "greenhouse effects", which in turn may dramatically limit the overall productivity of land and physical capital as well as future generations' ability to satisfy their needs through production.

Hence the problem of intergenerational equity is at the centre of the debate over sustainability of growth. Often, growth itself is defined in terms of equity between generations. That is, increasing concern for the negative environmental consequences of economic activity calls into question the desiderableness of a process of economic growth that overemphasizes the production of goods for the present generation and does not discount the growth opportunities left over for future generations. In this respect, some authors (see for instance Pearce, Barbier and Markandya, 1990) propose that the target of economic policies should be to maintain natural capital (i.e. environmental assets and natural resources) intact. That is, economic growth is “sustainable” if it can proceed ad infinitum at a constant rate consistent with the availability and renewability of natural resources. Other authors (i.e. Solow, 1974, 1986, Tietenberg, 1988, Nordhaus, 1991a, Pezzy, 1989, 1992) define sustainability as a more general obligation to give future generations a general option for well-being which should at least not be lower than that of the current generation.

Intergenerational equity is a moral category. As such, it is not obvious, at least from the viewpoint of the self-interest postulate of positive economics, that present generations should care about future ones more than the consumer of a commodity should care about other consumers. As is well known, a condition ensuring that individual self-interest does not prevent society from allocating resources optimally is that its members exchange private endowments in competitive markets. Since the markets for transactions with future generations are simply non existent, the potential welfare conflict between self-interested generations, and the ensuing intertemporal misuse of resources, cannot be ignored.

Suppose we agree on the moral imperative of intergenerational equity. Hence we may also agree to delegate a social planner to allocate resources in a way that maximizes intergenerational welfare or, opting for a decentralized "altruistic" solution, each of us agrees to include future generations' welfare in his/her own utility function to be maximized intertemporally. Ideally, the outcomes of the two agreements should be equivalent. However, the key problem is that both hinge on a "consequentialist" moral attitude, that is one which is legitimated by the alleged ability of the individual to discount the consequences of his/her own actions. If we are largely ignorant of the future consequences of our actions, not only is it "technically" difficult to solve the intertemporal allocation problem correctly but, what is more important in the present context, the very foundation of the moral imperative for intergenerational equity is flimsy.[1]

Thus, in addition to the "missing market" problem, the second crucial problem is that we know very little about the reciprocal effects between industrial production activity and climate changes (Lave, 1991, Chichilnisky and Heal, 1993). The general presumption is that a correlation exists between the scale of production with known technologies, emission of greenhouse gases (GHGs), increase in the earth's average temperature and other significant climate changes that are likely to negatively affect most human activities including production itself.[2] However,

> there is no agreement on how rapidly greenhouse gases will be emitted over the next century, how rapidly they will accumulate in the atmosphere, what will be the cost of abatement, how large the climate change will be, or even whether the change will be predominantly beneficial or harmful (Lave, 1991, pag.9)

As argued by Chichilnisky and Heal (1993), this kind of uncertainty is not the probabilistic risk that insurance markets are used to dealing with. We are not simply uncertain over the possible outcomes of a known process: we lack

---

[1] While correct valuation of non-market environmental goods is an essential step for efficient intertemporal use of resources by an economy, it does not guarantee that the economy will develop sustainably (Howarth and Norgaard, 1992).

[2] The global warming phenomenon relates to the stock of GHGs in the atmosphere, some of which, like CO_2; are persistent over a long time-span. Moreover, as there is no available technology for removing emissions already emitted (i.e. there is no way of getting "negative emissions"), this makes GHGs emissions irreversible.

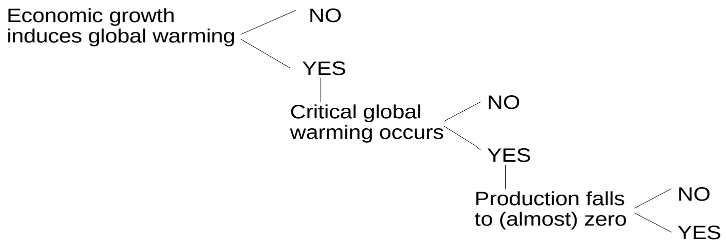

Figure 1:

knowledge of the process itself. Moreover, we also typically lack experience of repeated occurrences on which frequency estimates of probabilities may be based. This state of knowledge entails that a framework to study the impact of climate change on intergenerational equity should include some form of "structural uncertainty". The best we can do is to devise a tree of events on which a measure of our level of confidence in our scientific knowledge may be attached to each branch. A very simple example is shown in figure 1.

How uncertainty interferes with the moral approach to intergenerational equity is easily understood with the help of this outline. Suppose the present generation adheres to the intergenerational equity principle; therefore, the chosen growth path depends on assessment of the state of nature of future generations, given available knowledge. For instance, the level of confidence in the final event, that is in the (irreversible) negative temperature effect on production activity, may be negligible. Otherwise, beliefs may differ in society in such a way that only some agents discount the event of "falling productivity" while others do not. In either case, the actual growth path will be different than it would have been if all agents were certain of a fall in productivity. Now suppose that productivity does fall to zero as a consequence of previous economic growth: ex post, the growth path chosen by previous generations turns out to be unsustainable, but those generations can hardly be blamed for their choice, if they maximize intergenerational utility consistently with their available knowledge.

A further critical uncertain dimension is missing in this scheme: time. Even though we may be prepared to say "yes" confidently at each node, our behaviour may still be conditioned by our assessment of the time horizon of the event. In a strictly moral approach to intergenerational equity, the time horizon should be of no value if all generations are to be treated equally. However, a zero time discount rate may give rise to well-known paradoxes in this context (Beltratti, 1995).

Since uncertainty seriously limits the viability of the moral content of intergenerational equity, we think it is interesting to develop a positive analysis of the growth path of an economy that uses a technology with negative spillover onto climate conditions (temperature), leading to a structural breakdown in capital productivity, where self-interested generations are uncertain as to the time of the breakdown.

2 Climate change and overlapping generations

The model developed here is based on a number of simplifications of the many nuances of the picture drawn in the previous paragraphs. Firstly, we consider the case where global warming only affects technology[3]. In particular there exists a critical temperature level beyond which the production process exhibits zero (negative) capital productivity. Second, the warming effect of technology and the negative cumulative spillover of temperature on capital productivity are deterministic processes; thus we concentrate on the time of the productivity breakdown as the only source of uncertainty. The uncertainty is structural, i.e. agents do not know the exact value of the temperature threshold, which is discovered only when it is reached, as agents alive at that time observe the quantity produced. Finally, we utilize the two-period overlapping generations framework of Samuelson (1958) and Diamond (1965), and we assume that agents in each generation are self-interested. Such a demographic structure allows analysis of situations where

---

[3] Most models of the greenhouse effect specify the feedback of pollution on future welfare through both amenity and production effects, where the former typifies the fact that environmental quality is seen as a luxury good that does not influence consumption as such but can effect the well-being of individuals in society. However, unlike with most polluting gases, GHGs such as carbon dioxide, methane and nitrous oxides are typically benign for individuals, whilst their negative impact on climate change in the long-run possibly lead to economy-wide disruptions. Therefore a deliberate choice is made to assume that the only effect of the stock of GHGs is on productive capacity (Cesar and Zeeuw, 1995).

agents' actions have consequences that outlive them and are not discounted. However, as we shall see, time uncertainty, in the form of each generation's probabilistic assessment of the productivity breakdown occurring in its own lifetime, is a way in which this future state of nature does come to affect the economy's growth path.

As to assimilation of emissions in the atmosphere, many authors facing the problem of not having a nice quantitative (deterministic) assimilation function take the linear approximation as a convenient proxy (Foster, 1975, Dasgupta, 1982, Barbier and Markandya, 1990, Pethig, 1990). Others consider different, non linear specifications of the assimilation function to give an idea of the consequences of slight variations in the steady state levels of pollution and consumption (Barbier and Markandya, 1990, Cesar and Zeeuw, 1995).

Since if a climate change occurs it is likely to be irreversible, a crucial problem for most sustainable growth models is the ability of achieving sustainable development before the process of assimilation of Greenhouse gases in the environment breaks down. Such sustainable development relies on different specifications of the assimilation function of Greenhouse gases (Cesar and Zeeuw,1995).

As regards the scientific relationship between gaseous emissions and global average temperature, nobody can safely predict rises in temperature associated with the level of economic activity. However, most models assume a linear relationship between accumulation of carbon dioxide (CO_2) and other GHGs and the increase in temperature, ignoring the possibility of nonlinearity and the existence of threshold levels beyond which the ecosystem will break down (Matteucci, 1990, Beltratti, 1993a). The same holds in our case, after assuming a relationship between temperature and GHGs concentration.

2.1 Climate change effects on production

Firms produce a single non storable good using capital and temperature as production factors according the following linearly separable production function:

$$f(k_t; T_t; T^*) = g(k_t) + \psi \min(0; T^* - T_t) \qquad \text{with } \psi \geq 0 \tag{1}$$

Technology $f(\cdot)$ requires installation of capital one period before production, and it is twice continuously differentiable and strictly concave in k. T_t

is the average current temperature level of the earth, while T^* stands for the upper average bound of the temperature above which it exerts a negative externality on production. The parameter ψ measures the incidence of the temperature level on production; $\psi = 0$ typifies the case where the temperature does not affect production.

The second part on the r.h.s of equations (1) reflects the costs, in terms of production loss, that the firms must sustain because of a level of temperature above the critical threshold T^*. As mentioned by Beltratti (1993a), such costs may be imputed to the effects of extreme climate conditions, such as hurricanes and floods.[4]

On the assumption that the environmental factor has no stock feedback productive effects, the link between economic activity and the environmental factor is represented as follows:

$$T_t = \gamma k_t \qquad \text{with } \gamma > 0. \tag{2}$$

That is to say, the temperature level is endogenous. Emissions are a nonplanned joint output of production; emissions determine GHGs concentration, which eventually increases the average level of temperature. The above equation (2) has, therefore, to be thought of as a reduced form of a more general production structure based on fixed capital and fossil fuels, with strong complementarity between the two factors of production.[5]

Finally, the reduced form (2) allows us to rewrite production function (1)

---

[4] Equation (1) implies that as long as $T_t \leq T^*$ the temperature does not have any (positive or negative) effect on (average and marginal) capital productivity. This specification seems justified on the ground that it might be the speed of temperature change that determines how badly ecosystems are affected (see for example Tahvonen, von Storch and Xu, 1992, and Beltratti, 1993a). On the other hand, we may consider how the level of temperature influences production (and/or the utility) as suggested by Nordhaus (1991b) and Beltratti (1993b). In this case our production function should be written as:

$$f(k_t; T_t; T^*) = g(k_t) + \psi \min(T_t; T^*) \equiv g(k_t) + \psi T_t + \psi \min(0; T^* - T_t)$$

If $\psi > 0$ the temperature presents a growing positive externality effect at least till the upper level T^*; beyond this level the externality stops. On the other hand if $\psi < 0$ the temperature presents a negative increasing externality which reaches its maximum effect when the upper level T^* is reached.

[5] As suggested by Beltratti (1993a), we can assume that the stock of emissions of CO_2 and other GHGs influence temperature through out a linear relationship:

$$T_t = \beta P_t \quad ;$$

in terms of capital only. That is:

$$f(k_t, , k^*) = g(k_t) + \psi\gamma \min(0, k^* - k_t) \qquad \text{with } \psi\gamma \geq 0, \tag{3}$$

or

$$f(k_t, , k^*) = \begin{cases} g(k_t) & \text{if } k_t \leq k^* \\ g(k_t) - \psi\gamma(k_t - k^*) & \text{if } k_t > k^* \end{cases}$$

Now k^* plays the role of an upper bound to the productive capital stock induced by the environmental factor temperature. Capital productivity is $g'(k_t)$ as long as $k_t \leq k^*$ and drops to $g'(k_t) - \psi\gamma$ when accumulation reaches the upper level k^*. To simplify the model and make the scenario more dramatic, we introduce the following assumption on capital productivity.

Assumption 1. $g'(k_t) < \psi\gamma$ for all $k_t > k^*$.

As the ecosystem breaks down after T^* is crossed, some irreversible consequences like melting of ice caps, desertifications, species extinctions etc. lead to immediate collapse of the economy expressed by a negative (zero) capital productivity.[6] Under the above assumption, the general shape of (3) is as in fig.2 below.

Fig.2 About here

2.2 Structural uncertainty

In our model, structural uncertainty arises because the economy does not know the exact temperature threshold of collapse of capital productivity. We call this the economy's "disaster point". The threshold can be discovered only

---

where Greenhouse gas concentration evolves over time according to the following process:

$$P_t = E_t + (1 - h)P_{t-1}$$

This means that P increases over time due to emissions and gradually declines according to the assimilative capacity of nature h, i.e. the average atmospheric lifetime of GHGs is $\frac{1}{h}$. As long as current emissions depend linearly on the level of activity expressed by the use of capital, $E_t = \varphi k_t$, and by assuming for sake of simplicity $h = 1$ we get the reduced form (2), with $\gamma = \varphi\beta$.

[6] Results would not change much if we allowed for the more general case where k^* is seen as a sign that the ecosystem will soon approach a breakdown. In this case, reaching k^* does not prevent having positive capital productivity, with production continuing to grow at least over a small range $\bar{k} - k^*$, with $\bar{k} > k^*$.

by increasing the quantity of capital, that is by "getting there". Formally, given the technical relationship (2), we assume that the economy does not know the exact value of k^* as long as $k_t < k^*$ and discovers it only when $k_t \geq k^*$, after observing the quantity produced. We also assume that the agents treat k^* as a random variable with all the relevant information summarized in the probability distribution $H(k) = \Pr(k^* \leq k)$, with density $h(k)$ for $k > 0$. As the accumulation process evolves over time, the assessment at time t of the probability of k^* occurring at $t+1$ must be modified. Given that the disaster has not yet occurred, at each time the distribution of k^* depends on the history of k up to time t. In particular, it depends on $\hat{k}_t = \sup_{0<s<t}(k_s)$, as it is known that k^* must lie above this value, for otherwise the disaster would have occurred at some time before t (Tsur and Zemel, 1994, 1996). By the property of the truncated conditional distributions, if k^* has not yet been reached at time t its probability distribution is updated to:

$$H(k \mid \hat{k}_t) = \begin{cases} \frac{H(k) - H(\hat{k}_t)}{1 - H(\hat{k}_t)} & \text{for} \quad k^* > \hat{k}_t \\ 1 & \text{for} \quad k^* \leq \hat{k}_t \end{cases} \tag{4}$$

where $H(k \mid \hat{k}_t) = \Pr(k^* \leq k \mid k^* > \hat{k}_t)$.

2.3 An overlapping generations model

To capture the interaction in markets between individuals at different stages of their life-cycles, and the presence of some environmental externalities on production due to climate change, we present a simplified version of a standard Samuelson (1958) and Diamond (1965) overlapping generations model.

Consider an infinite horizon market economy consisting of finitely living individuals and perfectly competitive firms. At each date $t = 1, 2, 3, \ldots$ a new generation of individuals is born who live for two periods. For the sake of simplicity, we assume that the population does not grow over time and is normalized to one, while all individuals are identical.[7] Since in this economy each generation lives for two periods, an individual born at time t consumes c_t^0 in period t and c_{t+1}^1 in period $t+1$. The purpose of such individuals is to maximize their life utility function:

$$u(c_t^0) + \theta E_t \left[u(c_{t+1}^1) \right], \tag{5}$$

---

[7] Considering population growth does not change qualitatively the results with substantial complication of the model.

where µ 2 [0; 1] is the discount factor due to the existence of time preference, and $E_t(:)$ is the expectation operator, taken with respect to the information available at time t on the "disaster point" when the consumption decision is made. We choose u(:) to be a standard increasing differentiable function with the following properties:

Assumption 2. $u^0(c) > 0; u^{00}(c) < 0; \lim_{c\,!\,1} u^0(c) = 0; \lim_{c\,!\,0} u^0(c) = 1$; and $u(0) = u_0$ ¿ 0

When young, the individual supplies labour inelastically, earning a fixed real wage normalized to 1, and supplies his savings to firms, earning the gross return $R_{t+1} = 1 + r_{t+1}$ when old: Firms use savings to build up productive capital, which becomes productive after one period. Thus the individual is subject to the following budget constraint:

$$c_t^0 = 1 ¡ s_t \quad (6)$$

$$c_{t+1}^1 = R_{t+1}s_t \quad (7)$$

where s_t is saving at time t: As capital accumulation increases the stock of GHGs emissions which, in the end, influences the temperature and hence the production technology; agents' savings degrade the environment bequeathed to future generations. However, to make more dramatic the relation between generations' allocative decisions and growth sustainability, we deliberately rule out the possibility of young people allocating their wages between investment in capital and investment in environmental quality improvement.[8]

Firms maximize one period's profits by renting capital to the point where the marginal product of capital is equal to its rental rate $½_t$ = ± + r_t. Given the production function (3) and assumption 1, profit maximization implies:

$$\begin{array}{lll} ½_t = g^0(k_t) & \text{if} & k_t < k^¤ \\ ½_t = 0 & \text{if} & k_t ¸ k^¤ \end{array} \quad (8)$$

Finally, capital, in turn, lasts for one period only so the rate of depreciation is ± = 1:[9]

---

[8] As already pointed out, there are very few real possibilities of reducing the Greenhouse Effect. The only way to reduce the stock of GHGs is to diminish fossil fuel use and hence to decrease production. See also footnotes 2 and 5.

[9] As is usual in this setting, we assume that there exists an initial generation of old agents who posses the capital stock in the first period, k_1;as endowment. These old agents supply the capital to the firms and consume $c_1 = (1 + r_1)k_1$; and the firms in the first period have technology $g(k_1)$ with $k_1 < k^¤$:

3 Competitive equilibrium:the atomistic individual

A competitive equilibrium for this economy is represented by a sequence $\{k_t; c_t; s_t; r_t\}$; for $t \geq 1$; such that at each date: (i) individuals maximize (5) subject to (6), (7) and (4) is taken as given, i.e. this is consistent with a state-of-nature framework in which the atomistic individual cannot influence the probability that a particular state (productivity collapse or non-collapse) will occur; (ii) firms maximize profits taking the rental rate r_t as given; (iii) markets clear, and (iv) k_1 and T_1 are given.

Since each individual maximizes independently of previous and future generations, if at time $t+1$ the equilibrium stock of capital $k_{t+1} > k^*$ by (8) the individual's second period consumption drops to zero with, by assumption 2, associated utility much lower than its prior value. Then, equation (5) becomes:

$$u(c_t^0) + \mu \left[u(c_{t+1}^1) \Pr(k^* > k_{t+1} \mid k^* > \hat{k}_t) + u_0 \Pr(k^* \leq k_{t+1} \mid k^* > \hat{k}_t) \right]; \quad (9)$$

Making use of (8) and assumption 2, the individual optimal saving decision is derived from the following first order condition:

$$u'(c_t^0) = \mu(1 + r_{t+1}) u'(c_{t+1}^1) \Pr(k^* > k_{t+1} \mid k^* > \hat{k}_t) \qquad (10)$$

According to equation (10), the individual equates marginal utilities across periods of time, after considering the interest rate, his time preference and his probability assessment that the equilibrium aggregate stock of capital does not exceed the upper level k^*: As $\lim_{k^* \to \infty} H(k^*) = 1$ then also $\lim_{k^* \to \infty} \Pr(k^* > k_{t+1} \mid k^* > \hat{k}_t) = 1$; and we get the nonevent case.

Clearly, as $\Pr(k^* > k_{t+1} \mid k^* > \hat{k}_t) < 1$ period two's marginal utility of consumption decreases, and period-one consumption increases. That is, the optimal first period consumption level is greater than the nonevent case as a result of the possibility of an agent experiencing a capital productivity collapse when old.

We take the individual behaviour modelled so far as representative of his generation. The first order condition (10) ensures the existence of a saving function:

$$s_t = s \left[r_{t+1}; \Pr(k^* > k_{t+1} \mid k^* > \hat{k}_t) \right]$$

The market equilibrium requires that the demand for goods in each period be equal to the supply, or, in other words, that the investment at time $t+1$ is fully determined by a saving decision at time t. Since, for increasing accumulation trajectories, $\hat{k}_t \equiv k_t$ holds; taking account of (4) and (8) we get:

$$k_{t+1} = s\left[g'(k_{t+1}), \frac{1 - H(k_{t+1})}{1 - H(k_t)}\right] \tag{11}$$

Equation (11) implicitly defines k_{t+1} as a function of the stock of capital at time t, k_t. On the other hand, for nonincreasing paths of accumulation, it is known with certainty that the disaster will never occur, and the above equilibrium condition reduces to the nonevent one.

Generally, the dynamic behaviour of this economy depends on the properties of the saving function and on the characteristics of the production function. Unfortunately, we can say relatively little about the evolution of k for the general case (Galor and Ryder, 1989, Azzariadis 1993). We therefore simply discuss a parametric case.

Assumption 3. $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, for $\sigma > 0\ (\neq 1)$; and $u(c) = \ln c$ for $\sigma = 1$.

Assumption 4. $g(k) = k^{\alpha}$, $0 < \alpha < 1$.

Assumption 5. $H(k) = 1 - e^{-k/\mu}$, $\mu > 0$

Under the above assumptions, for increasing accumulation paths, equation (11) is expressed in the form:

$$k_{t+1} = 1 - \theta^{-\frac{1}{\sigma}} \alpha^{\frac{\sigma-1}{\sigma}} k_{t+1}^{\frac{\alpha(\sigma-1)+1}{\sigma}} \left[\frac{1 - H(k_{t+1})}{1 - H(k_t)}\right]^{-1/\sigma}; \quad \text{for } \sigma > 0\ (\neq 1) \tag{12}$$

and

$$k_{t+1} = \frac{\theta\frac{1-H(k_{t+1})}{1-H(k_t)}}{1 + \theta\frac{1-H(k_{t+1})}{1-H(k_t)}}; \quad \text{for } \sigma = 1 \tag{13}$$

This specification has the advantage of pointing out the exclusive role that learning has on the dynamic path of capital, and hence on the economy's speed towards the disaster point. Moreover, it is easy to show that the economy has a well-defined steady state which is equivalent to the nonevent case:

$$\overset{1}{k} = 1 ¡ ®^{\frac{¾ ¡ 1}{¾}} µ^{¡\frac{1}{¾}} \overset{1}{k}^{\frac{®(¾ ¡ 1)+1}{¾}};\ \text{for } ¾ > 0\,(\neq 1);\ \text{and } \overset{1}{k} = \frac{µ}{1+µ};\ \text{for } ¾ = 1 \quad (14)$$

Some numerical solutions will help to illustrate these results. Let us assume the intertemporal discount factor equal to $µ = 0{:}9$; and set $® = 0{:}5$ and $¹ = 1$: Figures 3 and 4 show the accumulation function (12) for $¾ = 0{:}8$ and $1{:}8$.

Fig:3 About here

Fig:4 About here

As in our setting each generation is endowed with a fixed stock of resource at the beginning of its life, the effect of learning on the dynamic path of capital produces a redistribution of consumption within each generation between the first and the second period which, although leaving the amount of resources unchanged for future generations, produces a reduction in the profitability of capital accumulation. Thus, the economy slows down its accumulation and hence its speed towards the disaster point.

Does the unobservability of the temperature threshold have consequences for equity between generations? To understand this we have somehow to specify the effect of crossing this threshold on the remaining generations. Since a climate change above a critical level is likely to be irreversible we may suppose, without forcing the reality, that the inability to reach sustainable development before the process of GHGs assimilation in the environment breaks down, i.e. $k^{¤} < \overset{1}{k}$; means that all remaining generations may lose a large part (or all) of their endowment or may experience a permanent loss of utility (i.e. $u_0 ¿ 0$).

Whilst when $k^{¤}$ is known each generation finds it optimal to keep capital accumulation in a small neighborhood below $k^{¤}$ to prevent productivity falls, when the disaster point is uncertain the path of accumulation is no longer constant but increases along generations starting well below $k^{¤}$; then falling dramatically after this level is reached. Therefore, generations in between k_0 and $k^{¤}$ will receive more resources and utilities than generations after $k^{¤}$ is discovered.[10] Individuals clearly cannot provide for the climate change

---

[10] Obviously, if after $k^{¤}$ is discovered all the remaining generations keep the same endowment and the capital threshold does not adversely effect the ability of the ecosystem to recover, only the generation which observes $k^{¤}$ will suffer a welfare loss.

caused by their offspring acting individually and, hence, ex-post the growth path chosen by the previous generations turns out to be unsustainable.

4 Competitive equilibrium: the social individual

In the previous section we have modelled the behaviour of a representative atomistic individual, that is, an individual who takes market variables as given, including the aggregate stock of capital. In this section we wish to give a different characterization of individual behaviour. We now assume that the representative young recognizes, when taking optimal saving decisions, the possibility of influencing the probability that the disaster point may be reached in the second period. In our single-agent general-equilibrium framework this sort of self-protecting behaviour implies assuming that the second period's expected utility is evaluated taking account that the capital stock in period $t + 1$ is just the amount saved by the young in period t:[11] We may call this agent a "social individual", in the sense that he maximizes his own utility but discounts the aggregate consequences of his choice, as if he were applying the maxim: "suppose that everybody does what I am doing...". We think that this characterization of representative behaviour is, in at least one respect, more consistent than the "atomistic" standard one. Combining competitive atomistic behaviour with the use of individual functions as representative of the whole economy, as in current practice, is ambiguous in that the individual's decision variables (e.g. quantities) are at the same time "negligibly small" and are the actual variables on which the market equilibrium values (e.g. prices) are computed.[12] To put it in game-theoretic terms, competitive behaviour is characterized by each agent taking the best action of all the others as given. But if homogeneity of all agents is common knowledge, it seems rational for each representative agent to take his own best action as representative of all the others'. Therefore, the first order condition for a maximum now becomes:

---

[11] For a general theory of self-protection in a general equilibrium framework see Laffont (1980).

[12] This ambiguity is clear in our setup, where the representative individual chooses s_t taking $\Pr(k^* > k_{t+1} | k^* > \hat{k}_t)$ as given, as if s_t were negligible on aggregate capital, whereas $k_{t+1} = s_t$.

$$u(c_t^0) + \beta\left[u(c_{t+1}^1)\Pr(k^* > s_{t+1} \mid k^* > \hat{s}_t) + u_0 \Pr(k^* \le s_{t+1} \mid k^* > \hat{s}_t)\right]; \quad (15)$$

where $\hat{s}_{t-1} = \sup_{0<s<t-1}(s_s)$. Making use of (8), (4) and assumption 2, the optimal saving decision is now given by:

$$u'(c_t^0) = \beta\left[(1 + r_{t+1})u'(c_{t+1}^1)\frac{1 - H(s_{t+1})}{1 - H(\hat{s}_t)} - \left(u(c_{t+1}^1) - u_0\right)\frac{h(s_{t+1})}{1 - H(\hat{s}_t)}\right] \quad (16)$$

Besides the usual equalization of marginal utilities across periods of time, now the saving decision taken as of time t by the young has to be corrected for the reduction in the second-period utility due to an increase in the disaster probability induced by positive accumulation. This negative effect on utility is measured by the term $\left(u(c_{t+1}^1) - u_0\right)$, which is generally much less than zero, on the r.h.s. of (16).

After applying the equilibrium condition, $k_{t+1} = s_t$; and making use of assumptions 3 and 4, for increasing trajectories, we obtain:[13]

$$k_{t+1} = 1 - \beta^{-\frac{1}{\sigma}}\alpha^{\frac{\sigma-1}{\sigma}}k_{t+1}^{\frac{\alpha(\sigma-1)}{\sigma}}\left[k_{t+1}^{-1}\frac{1 - H(k_{t+1})}{1 - H(k_t)} - \frac{h(k_{t+1})}{1 - H(k_t)}\right]^{-1/\sigma}; \text{for } 0 < \sigma < 1. \quad (17)$$

Moreover, the steady state is lower than the one in the nonevent case:

$$\bar{k}^s = 1 - \beta^{-\frac{1}{\sigma}}\alpha^{\frac{\sigma-1}{\sigma}}\bar{k}^{s\frac{\alpha(\sigma-1)}{\sigma}}\left[\frac{1}{\bar{k}^s} - 1\right]^{-1/\sigma} < \bar{k}; \quad \text{for } 0 < \sigma < 1. \quad (18)$$

Continuing with the example of the previous section, Fig. 5 below shows the accumulation function (17) for $\sigma = 0.8$.

Fig.5 About here

---

[13] It is common practice to assume for the utility a functional form: $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$, which implies that $u(0) = -\frac{1}{1-\sigma} < 0$. Inclusion of the term -1 is convenient as it entails that $u(c)$ approaches to $\log(c)$ as $\sigma \to 1$. While this term can be omitted as the agent's decision is invariant with respect to linear transformation of the utility function, this is not the case for positive monotonic transformations like the one implied by the agent's ability to influence the probability that a particular event will occur: $u(c_t^0) + \beta \Pr(c_t^0)u(c_{t+1}^1)$.

The increase in the disaster probability induced by positive accumulation reinforces the redistribution of consumption favouring the first period, further reducing the profitability of accumulation and hence the economy's speed towards the disaster point. Intergenerational altruism, then, occurs through this under-saving result. Specifically, if $\bar{k}^s < k^* < \bar{k}$ no collapse will occur, and the optimal consumption and capital accumulation converge to a sustainable steady state which guarantees a welfare equity between generations. However, if $k^* < \bar{k}^s$; as accumulation increases along generations much more slowly than the atomistic case, "future-oriented" behaviour is able to postpone the catastrophe further (even though it cannot be eliminated completely).

Finally, as long as $u_0 = -\infty$ with $\sigma \geq 1$; we get:

$$k_{t+1} = 0^+; \quad \text{for } \sigma \geq 1 \tag{19}$$

The crucial assumption used to obtain this result is that the utility of zero consumption equals minus infinity.[14] As any distribution policy leads, with a positive probability, to zero consumption in the second period, an individual who anticipates the influence of his own saving decision on the probability of disaster is induced to consume the entire endowment during his youth. The agent simply assumes the worst event with certainty in the second period and optimizes against this "cautious" deterministic problem. Hence, consumption is not random.[15]

5 Final remarks

Climate changes are paradigmatic of long-term, uncertain, negative externalities of current economic activity on future generations' welfare. In this paper we have addressed this issue in its most dramatic form, where capital accumulation raises temperature, which in turn exerts a negative effect on

---

[14] In a representative agent model this assumption may be justified on the equivalence between zero consumption and non-survival or on the bases of an extreme fear of being left with no resources in old age.

[15] Under the assumption that the utility of zero consumption is $-\infty$; Nyarko and Olson (1996), in a one-good stochastic model with unobservable resource stock, show that the optimal policy follows a completely deterministic "cautious" or "minimax" behavior that assumes the worst in each period and optimizes against it. However, they find that with unobservable resources a form of over-saving occurs.

productivity, and overlapping generations of self-interested individuals determine the growth path of the economy under uncertainty about the level of capital k^* beyond which productivity collapses (structural uncertainty). We have stressed that in this setup intergenerational equity may be a moral imperative which however cannot be made compelling since the individuals in each generation do not have a clear understanding of the consequences of their actions. Leaving a benevolent and almighty social planner aside, are there in-built economic mechanisms that can alleviate the threat on future generations?

Our answer hinges on the role of structural uncertainty. Were k^* known with certainty, each generation would find it optimal to keep the capital stock constant in a small neighbourhood below k^*. Structural uncertainty about k^* has two effects: i) at the individual level, each generation of youngs discounts the probability of productivity collapse occurring in their own old age (destroying their lifetime savings), so that they consume more and save less; ii) at the economy level, the accumulation path is no longer constant but increasing towards k^* from below. Unless the steady state is reached below k^*, this accumulation mechanism cannot prevent some future generations from being hurt by the productivity loss. Our first conclusion is that structural uncertainty, combined with generational self-interest, plays an ambiguous role: on the one hand, it may set the economy on a harmful path for future generations, on the other hand it decelerates the accumulation rate.

We have pointed out this latter effect of structural uncertainty by comparing a standard competitive setup, where the representative individual chooses optimally by observing the market equilibrium value of the real interest rate and updating the probability of productivity collapse implied by the new capital stock, and one where each individual is, in a sense, aware of being representative, and hence knows that saving increases social capital and the probability of productivity collapse. We think that this characterization of individual ”social” behaviour may be regarded as consistent with the assumption of the representative agent (the standard ”atomistic” characterization entailing a loss of awareness of the fact that ”everyone is doing what I am doing”). In fact , our characterization leads to a different result which enhances the ”slowdown” effect of structural uncertainty in two ways: i) the accumulation path is flatter than in the ”atomistic” setup, and ii) the steady state is at a lower capital stock. Consequently, our second conclusion is that individual awareness of ”social” behaviour may still be insufficient to rescue future generations from productivity collapse, but reduces the speed at which the economy approaches the collapse point (enlarging the number of generations which enjoy positive productivity) and make it less likely.

References

[1] Azariadis C., (1993), Intertemporal Macroeconomics, Blackwell, Oxford.

[2] Barbier E.B., and A. Markandya, (1990), "The Conditions for Achieving Environmentally Sustainable Development", European Economic Review, 34, pp.659-675.

[3] Beltratti A., (1993a), "Le decisioni di politica ambientale in presenza di incertezza sul modello fisico del clima", in I. Musu (ed.) Economia e Ambiente, Il Mulino, Bologna, pp.139-156.

[4] Beltratti A., (1993b), "Climate Change and Option Values", FEEM Working Paper n.60-93, Milan.

[5] Beltratti A., (1995), "Intergenerational Equity and Environmental Preservation", FEEM Working Paper n.42.95, Milan.

[6] Cesar H., and A. de Zeeuw, (1995), "Sustainability and the Greenhouse Effect: Robustness Analysis of the Assimilation Function", in C. Carraro and J.A. Filar (eds.), Control and Game-Theoretic Models of the Environment, Birkhäuser, Boston, pp.25-45.

[7] Chichilnisky G., and G., Heal, (1993), "Global Environmental Risks", Journal of Economic Perspectives, 7, pp.65-86.

[8] Dasgupta P., (1982), Control of Resources, Basil Blackwell, Oxford.

[9] Diamond P.A., (1965), "National Debt in a Neoclassical Growth Model", American Economic Review, 5, pp.1126-1150.

[10] Foster B.A., (1975), "Optimal Pollution Control with a Nonconstant Exponential Rate of Decay", Journal of Environmental Economics and Management, ?, pp.1-6.

[11] Galor O., and H. Ryder, (1989), "On the Existence of Equilibrium in an Overlapping Generations Model with Productive Capital", Journal of Economic Theory, 49, pp.360-375.

[12] Howarth R.B., and R.B. Norgaard, (1992), "Environmental Valuation under Sustainable Development", American Economic Review Papeper and Proceedings, 82, pp.473-477.

[13] Laffont J.J., (1980), Essays in the Economics of Uncertainty, Harvard University Press, Cambridge MA.

[14] Lave L.B., (1991), "Formulating Greenhouse Policies in a Sea of Uncertainty", The Energy Journal, 12, pp.9-21.

[15] Matteucci G., (1990), “Analysis of the Probability Distribution of the Late Pleistocene Climate Record: Implications for Model Validation”, Climate Dynamics, 5, pp.35-52.

[16] Nougier L.R. (1977), L’économie prehistorique, Paris, Presse Universitaire de France.

[17] Nordhaus W., (1991a), “To Slow or Not to Slow: The Economics of the Greenhouse Effect”, The Economic Journal, 101, pp.920-937.

[18] Northhaus W., (1991b), “Economic Approaches to Greenhouse Warming”, in R. Dornbusch and J. Poterba (eds.), Economic Policy Responses to Global Warming, MIT Press, Boston, pp.?

[19] Nyarko Y., and L.J. Olson, (1996), "Optimal Growth with Unobservable Resources and Learning", Journal of Economic behaviourand Organization, 29, pp.465-491.

[20] Pearce D., E., Barbier and A., Markandya, (1990), Sustainable Development. Economics and Environment in the Third World, London Environmental Economics Centre, Earthscan Publications, London.

[21] Pethig R., (1990), “Optimal Pollution Control, Irreversibilities and the Value of Future Information”, Discussion Paper n.6-90, University of Siegen.

[22] Pezzey J., (1989), “Economic Analysis of Sustainable Growth and Sustainable Development”, Environment Department Paper n.15, Word Bank, Washington D.C.

[23] Pezzey J., (1992), "Sustainable Development Concepts: An Economic Analysis”, Environment Department Paper n.2, World Bank, Washington D.C.

[24] Samuelson P., (1958), “An Exact Consumption-Loan Model of Interest With or Without the Social Contrivance of Money”, Journal of Political Economy, 66, pp.467-482.

[25] Solow R.M., (1974), “Intergenerational Equity and Exhaustible Resources”, Review of Economic Studies, Symposium on the Economics of Exhaustible Resources, pp.29-46.

[26] Solow R.M., (1986), “On the Intergenerational Allocation of Natural Resources”, Scandinavian Journal of Economics, 88, pp.141-149.

[27] Tahvonen O., H., von Storch and J., Xu, (1992), “Optimal Control of CO_2 Emissions” Paper presented at the Annual Conference of the EAERE, Cracow, Poland, June 16-19.

[28] Tietenberg T.H., (1988), Environmental and Natural Resource Economics, Second edition, Scott, Foresman and Co, Glenview.

[29] Tsur Y., and A., Zemel, (1994), ”Endangered Species and Natural Resource Exploitation: Extinction vs. Coexistence”, Natural Resource Modelling, 8, pp.389-413.

[30] Tsur Y., and A., Zemel, (1996), ”Accounting for Global Warming Risk: Resource Management under Event Uncertainty”, Journal of Economic Dynamic and Control, 20, pp.1289-1305.

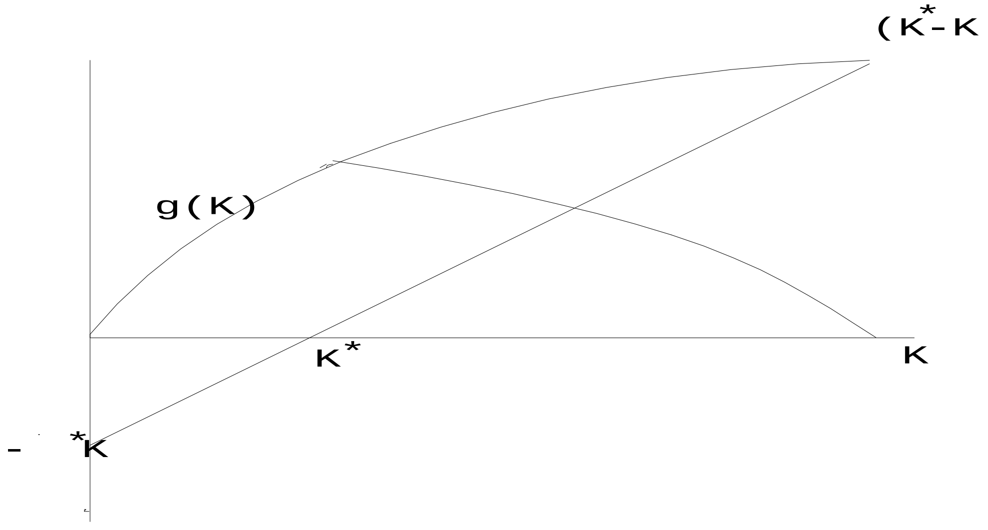

Figure 2: Production Function

Figure 3: Dynamic path of capital with ¾ = 0:8, $\overset{1}{k}$ = :44896

Figure 4: Dynamic path of capital with ¾ = 1; 8, $\overset{1}{k}$ = :52674

Figure 5: Dynamic path of capital with $\sigma = 0.8; \bar{k}^s = .35339$