Behaviour in Social Networks: Externalities, Altruism and Peer Effects

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FEEM, April 2013

Motivation

- **Peer effects**: behavioural complementarities due to
  - joint consumption
  - emulation
  - heard behaviour
  - ...
- Incentive to act increases in neighbors' actions.
- Common in risky or health related behavior:
  - adolescent groups (Powell et al., 2005)
  - smoking (Christakis and Fowler, 2007; Ali et al., 2011)
  - alcohol and drug consumption (Clark and Loheac, 2007, Mednick et al., 2010)
  - food disorders and obesity
- "social multiplier" - Glaeser, Sacerdote and Schenkman, 2002, Cutler and Glaeser, 2007).
- Identification and measurement of peer effects (Manski 1993, Bramouille et al. 2009, Lee 2010)

- Behaviors that generate peer effects often exert **negative externalities**:
  - passive smoke
  - driving while drunk
  - sexual transmission of disease and unintended pregnancies;
  - local pollution
- Externalities build up and the **stocks** affects welfare and incentives:
  - own stock: e.g., passive smoke affects risk of illness due to extra-cigarette;
  - neighbours'stocks: e.g., concern for friends' health or *altruism*).
- Externalities and stocks create **additional interaction channels**, as incentives to act may depend (negatively) on actions taken outside one's neighborhood.

Smoking networks

- Agents enjoy active smoke, and are affected by passive smoke;
- The stock of passive smoke increases the risk of illness due to an additional cigarette;
- Agents care for the effect of their smoke on their friends' health (altruism);
- Incentive to smoke decrease with their friends of friends' smoke.

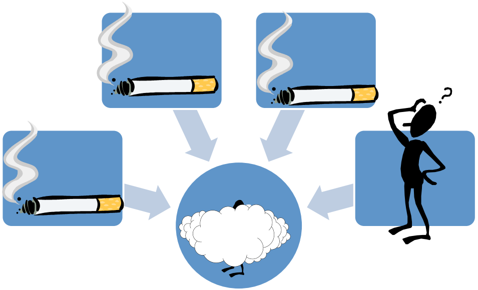


Figure : Indirect substitution effect

- Stylized facts about smoking our research contributes to account for (Christakis and Fowler, 2007):
  - Smokers tend to move towards periphery of the network.
  - Network affects behavior or behavior affects the network?
  - C-F argue that marginalization of smokers is due to link deletion - here we suggest that network affects behavior.
  - People tend to quit smoking in clusters. Who? Why? How does position in the network affect smoking behavior?
- Theoretical issues:
  - Estimation of peer effects and causality;
  - Choice of instrumental variables;
  - Identification: which networks?

Local complementarities in production

- Monopolists use neighbors' products as inputs;
- Increases in i's output increases demand for j's products.
- Increases in j's neighbor's output increase demand for j's output, j's price and i's marginal cost.

International Relations

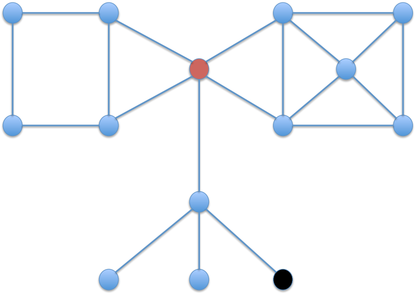
- Countries take actions that affect neighbors;
- A country decides to react if the aggregate actions taken by its neighbors exceeds a given threshold;
- If j and i are neighbors, and increase in j's neighbor's actions increases the probability that i's additional action meets j's threshold.

The Model

n agents in a fixed social network G with $g_{ij} = \{0, 1\}$.
Each agent i takes action $x_i \in R$.

Utility

$$U_i = \alpha_i x_i - \gamma_0 \frac{x_i^2}{2} + \phi \sum_{j \in N} g_{ij} x_i x_j$$



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where for all $h \in N$ we have defined

$$Q_h \equiv \left(\sum_{k \in N} g_{hk} x_k + x_h \right)$$

Setting $\bar{\phi} = \phi - \gamma_1 - \gamma_2$:

$$\alpha x_i - \frac{\gamma_0 + \gamma_1}{2} x_i^2 + \bar{\phi} \sum_{j \in N} g_{ij} x_i x_j - \gamma_2 \sum_k g_{ik}^{[2]} x_i x_k + h_{-i}$$

Equilibrium

FOC for agent i

$$\alpha - (\gamma_0 + \gamma_1)x_i + \bar{\phi}\sum_j g_{ij}x_j - \gamma_2 \sum_k g_{ik}^{[2]} x_k = 0$$

Eq. FOC's in matrix form:

$$\alpha \bar{1} = \left[(\gamma_1 + \gamma_0) I - [\bar{\phi} G - \gamma_2 G^2] \right] \bar{x}.$$

or

$$\frac{\alpha}{\gamma_1 + \gamma_0} \bar{1} = \left[I - \frac{\bar{\phi}}{\gamma_1 + \gamma_0} \left(G - \frac{\gamma_2}{\bar{\phi}} G^2 \right) \right] \bar{x}.$$

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Equilibrium Behaviour:

$$\frac{\alpha}{\gamma_1 + \gamma_0}\left[I - \frac{\bar{\phi}}{\gamma_1 + \gamma_0}\left(G - \frac{\gamma_2}{\bar{\phi}}G^2\right)\right]^{-1}\bar{1} = \bar{x}.$$

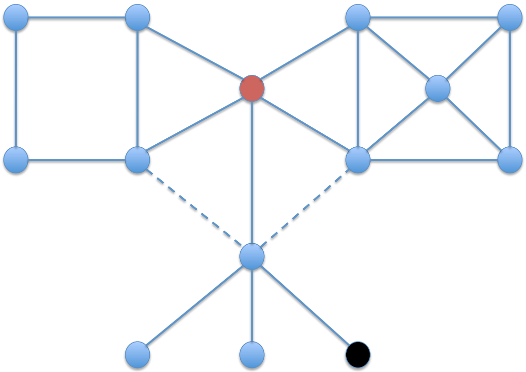

Figure : Peer effects and indirect substitution

Definition

The Bonacich centrality matrix of $\mathbf{g}$ with parameter a is given by:

$$\mathbf{M}(\mathbf{G}, a) \equiv (I - a\mathbf{G})^{-1}. \tag{1}$$

m_{ij} = discounted sum of walks from i to j in $\mathbf{G}$.

Definition

The vector of Bonacich centralities of $\mathbf{g}$ with parameter a is given by:

$$\mathbf{b}(\mathbf{G}, a) \equiv \mathbf{M}(\mathbf{G}, a) \cdot \bar{1}. \tag{2}$$

Proposition 1 (Adaptation of Ballester et al. 2006)

Let $\frac{\gamma_1+\gamma_0}{\bar{\phi}} > \mu(\mathbf{G} - \frac{\gamma_2}{\bar{\phi}}\mathbf{G}^2)$. The unique interior Nash Equilibrium of the game is given by:

$$x^* = \frac{\alpha}{\gamma_1 + \gamma_0}\mathbf{b}(\mathbf{G} - \frac{\gamma_2}{\bar{\phi}}\mathbf{G}^2, \frac{\phi - \gamma_1}{\gamma_1 + \gamma_0}). \tag{3}$$

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- Need to relate behavior to the original social network $\mathbf{G}$.
- How does the ranking of behaviors within a network differ?
- How does aggregate behavior in different networks $\mathbf{G}$ change?
- Who are the *key players* (those that if removed affect most the aggregate behavior)?

Definition

The weighted Bonacich centrality vector for **G** with parameter a and with weights vector $\bar{w}$ is defined as follows:

$$\mathbf{b}(\mathbf{G}, a, \bar{w}) = \mathbf{M}(\mathbf{G}, a) \cdot \bar{w} \tag{4}$$

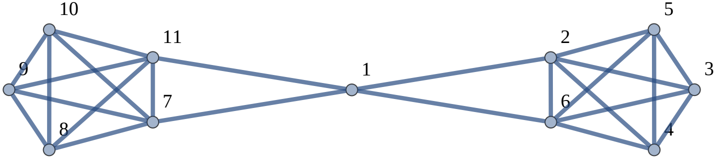
Proposition

The vector of equilibrium actions with altruism $\bar{x}$ is given by:

$$\frac{\alpha}{\gamma_1 + \gamma_0} \mathbf{b}(\mathbf{G}, \frac{(\phi - \gamma_1)}{\gamma_1 + \gamma_0}, \bar{w}) = \bar{x} \tag{5}$$

where the vector of weights is given by:

$$\bar{w} = [I + \frac{\gamma_2}{\gamma_1 + \gamma_0} \mathbf{M}(\mathbf{G} + \mathbf{G}^2)]^{-1} \cdot \bar{1}. \tag{6}$$



Three types of agents: 1, 2 and 3.

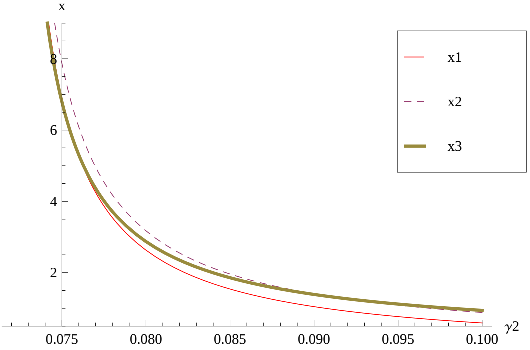
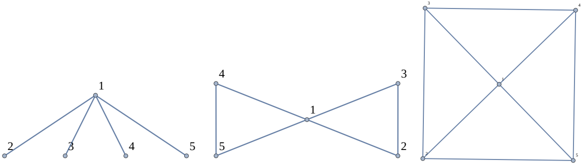


Figure : Equilibrium actions with varying degrees of altruism.

Proposition

The marginal effect of γ_2 on equilibrium behavior is given by:

$$\frac{\partial \bar{x}^*}{\partial \gamma_2} = -\frac{\partial}{\partial \phi}\bar{b}(\mathbf{G}, \frac{\phi - \gamma_1}{\gamma_1 + \gamma_0}, d) \qquad (7)$$



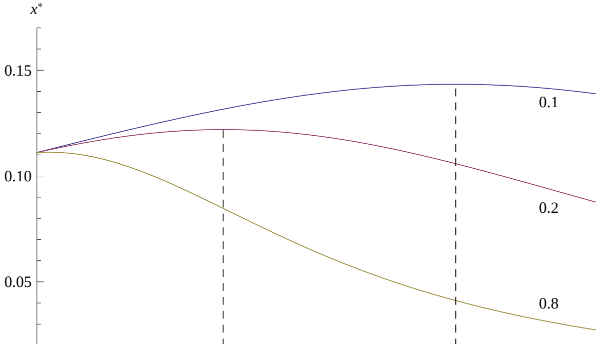
Network	Players	$\gamma_2 = 0$		$\gamma_2 = 0.06$	
		x^*	$\mathbf{x}^*$	x^*	$\mathbf{x}^*$
Star	1	3.38		2.01	
	2-5	2.59	13.77	1.82	9.31
Papillon	1	3.53		1.63	
	2-5	2.95	15.29	1.69	8.42
Connected Star	1	3.73		1.34	
	2-5	3.39	17.29	1.45	7.17

Changing the Network: Density

Look at degree in regular networks.

Proposition

Consider a regular network, with $\gamma_1 + \gamma_2 < \phi$. Equilibrium behaviour is a *non monotonic function of degree*, increasing for low degrees and decreasing for high degrees. ▸ See here



Adding and Severing Links

- We know from Ballester et al. (2006) that if we increase the entries of a network G, the resulting equilibrium is characterized by a larger aggregate behavior (due to stronger complementarities).
- But how do we increase the terms of $\tilde{G}$ by adding and severing links to G?

Proposition

Take r integer such that

$$(\phi - \gamma_1 - \gamma_2) \leq r\gamma_2.$$

Consider G' obtained by fully connecting an independent set of nodes Z of size z in G, with $z = r + 2$.
Then $\tilde{G}' < \tilde{G}$ and $x^*(\tilde{G}) > x^*(\tilde{G}')$.

Key-Players

Theorem (Ballester et al. 2006)

If $\frac{\eta(\gamma_1+\gamma_0)}{\lambda\gamma_2} > \mu(\mathbf{C})$, the key player is the agent with the highest intercentrality index, measured by $c_i = b_i^2/m_{ii}$.

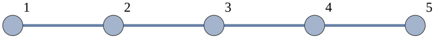


Figure : Line network

Table : Key Player - Line network

γ_2	b_i	c_i
0	$3 > 2 > 1$	$3 > 2 > 1$
0.05	$1 > 2 > 3$	$1 > 2 > 3$

Parametrization: $\phi = 1$, $\gamma_0 = 0$, $\gamma_1 = 0.9$, $\alpha = 2$

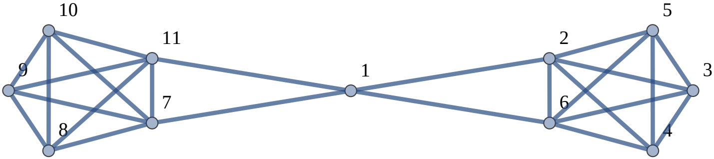


Table : Key Player - Ballester2006 network

γ_2	b_i	c_i
0	$2 > 1 > 3$	$2 > 1 > 3$
0.002	$2 > 1 > 3$	$2 > 3 > 1$
0.01	$2 > 3 > 1$	$2 > 3 > 1$
0.02	$3 > 2 > 1$	$3 > 2 > 1$

Parametrization: $\phi = 1$, $\gamma_0 = 0$, $\gamma_1 = 0.95$, $\alpha = 2$

Policy-specific Key-Players

Definition

The α-key player is the agent i such that $\frac{\partial \mathbf{x}}{\partial \alpha^i}$ is maximal.

Proposition

The α-key player is the agent with the highest centrality in the network $\mathbf{C}$.

Homophily, Polarization and Smoking

- Two types: $\alpha^H > \alpha^L$.
- Pattern of connections matters for behaviour.
- Degree of **Homophily** q = share of same type neighbors.
- Assume q is the same across agents and types.

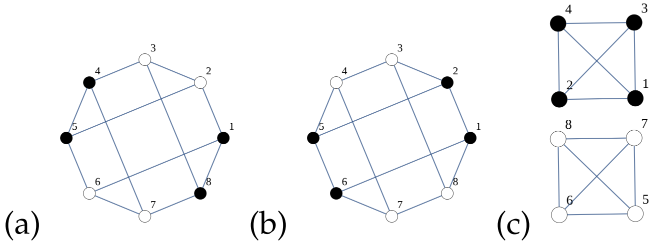


Figure : Three networks with increasing degrees of homophily.

Equilbrium Behaviour is symmetric around common mean:

$$\begin{cases} x^H = \frac{1}{2}\left[\frac{\alpha^H+\alpha^L}{\gamma_1+\gamma_0-d(\phi-\gamma_1-\gamma_2)+\gamma_2 d^2} + \frac{\alpha^H-\alpha^L}{\gamma_1+\gamma_0+\gamma_2 d^2(1-2q)^2+d(1-2q)(\phi-\gamma_1-\gamma_2)}\right] \\ x^L = \frac{1}{2}\left[\frac{\alpha^H+\alpha^L}{\gamma_1+\gamma_0-d(\phi-\gamma_1-\gamma_2)+\gamma_2 d^2} + \frac{\alpha^L-\alpha^H}{\gamma_1+\gamma_0+\gamma_2 d^2(1-2q)^2+d(1-2q)(\phi-\gamma_1-\gamma_2)}\right] \end{cases} \quad (8)$$

Proposition

Let d^* be the degree for which equilibrium behavior is maximal.

- When $d < d^*$, polarization is monotonically increasing in q.
- When $d > d^*$, polarization is non monotone in q, reaching its maximum at $\bar{q} \in (1/2, 1]$, where $\bar{q}$ is decreasing in γ_1, γ_2 and d.
- The maximal polarization is independent of the degree.

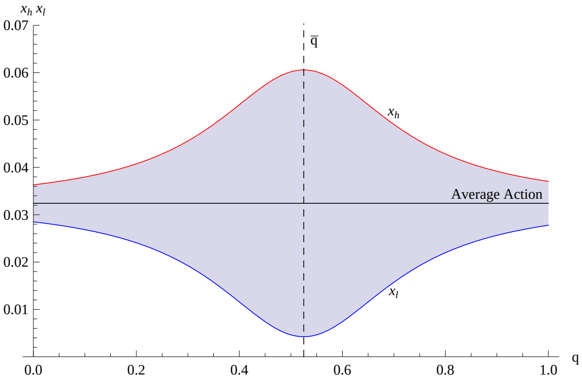


Figure : x_H and x_L

Empirical Analysis

From FOC we get

$$x = \beta_1 \mathbf{G}x + \beta_2 \mathbf{G}^{\mathbf{2}} x + \rho z + \zeta \mathbf{G}^* z + \epsilon$$

where

$$\beta_1 = \frac{(\phi - \gamma_1 - \gamma_2)}{\gamma_1 + \gamma_0} \; and \; \beta_2 = \frac{\gamma_2}{\gamma_1 + \gamma_0}$$

Biases in the estimation of peer effect

Bias I

If $\gamma_2 = 0$ then $\beta_1 = \frac{(\phi - \gamma_1)}{\gamma_1 + \gamma_0}$: downward bias

Bias II

If γ_2 is omitted then

$$x = \beta_1 \mathbf{G}x + \rho z + \zeta \mathbf{G}^* z + \epsilon$$

and

$$\frac{Cov(x, \mathbf{G}x)}{Var(\mathbf{G}x)} = \beta_1 + \beta_2 \delta_{\mathbf{G^2}x, \mathbf{G}x} < \beta_1$$

Identification

- We analyze conditions of network so that the parameters are identified depending on $\mathbf{G} = \mathbf{G}^*$ and $\mathbf{G} \neq \mathbf{G}^*$, with and without altruism
- If $\mathbf{G} = \mathbf{G}^*$, all regular networks are excluded
- If $\mathbf{G} \neq \mathbf{G}^*$, a larger class of networks are excluded (e.g., star network).
- We derive an optimal set of instrument to solve the endogeny problem derived from the presence of endogeneity of $\mathbf{G}x$ and $\mathbf{G^2}x$.

Conclusions

- Model with peer effects, negative externalities and altruism
- Altruism generates new strategic interdependencies of the substitute type with distance two neighbors.
- Central agents in a network need not be those with largest behaviour.
- Adding links may reduce behavior. In regular networks: **inverted bell pattern** w.r.t. degree
- Heterogeneous agents: behaviour is largest for large - but not complete - segregation of types
- Implications for empirical work:
  - Peer effects may be underestimated;
  - Relation degree-behaviour non linear;
  - Two-distance neighbours may fail to be valid instrument for estimating peer-effects.

- Matrix $\mathbf{G^{[k]}}$ identifies walks of order k
- $m^{ij} = \sum_{k=0}^{\infty} a^k \mathbf{G^{[k]}}$

$$\mathbf{M} = \begin{bmatrix} m^{ii} & m^{ij} & m^{ik} & \cdots & m^{iw} \\ m^{ji} & m^{jj} & m^{jk} & \cdots & m^{jw} \\ m^{ki} & m^{kj} & m^{kk} & \cdots & m^{kw} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ m^{wi} & m^{wj} & m^{wk} & \cdots & m^{ww} \end{bmatrix}$$

$$\bar{\mathbf{b}} = \begin{bmatrix} m^{ii} & m^{ij} & m^{ik} & \cdots & m^{iw} \\ m^{ji} & m^{jj} & m^{jk} & \cdots & m^{jw} \\ m^{ki} & m^{kj} & m^{kk} & \cdots & m^{kw} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ m^{wi} & m^{wj} & m^{wk} & \cdots & m^{ww} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\alpha - x^*[\gamma_1 + \gamma_0 - d(\phi - \gamma_1 - \gamma_2) + \gamma_2 d^2] = 0.$$

$$x^* = \frac{\alpha}{\gamma_1 + \gamma_0 - d(\phi - \gamma_1 - \gamma_2) + \gamma_2 d^2}.$$

$$\frac{\partial x^*}{\partial d} = \frac{\alpha(\phi - \gamma_1 - \gamma_2 - 2d\gamma_2)}{Den^2}.$$

$$\begin{cases} d < \frac{\phi-\gamma_1-\gamma_2}{2\gamma_2} & \Rightarrow & \frac{\partial x^*}{\partial d} > 0 \\ d = \frac{\phi-\gamma_1-\gamma_2}{2\gamma_2} & \Rightarrow & \frac{\partial x^*}{\partial d} = 0 \\ d > \frac{\phi-\gamma_1-\gamma_2}{2\gamma_2} & \Rightarrow & \frac{\partial x^*}{\partial d} < 0 \end{cases}$$

Let:

- $\bar{c} = max\{\tilde{g}_{ij}\} = 1$
- $\underline{c} = min\{\tilde{g}_{ij}\}$
- $\theta = -min\{0, \underline{c}\} > 0$
- $\lambda = \bar{c} + \theta$
- Define the matrix **C** as:

$$c_{ij} = \frac{\tilde{g}_{ij} + \theta}{\lambda} \in [0, 1]. \tag{9}$$

Proposition

Consider **G** *and the matrix* **C** *defined as in (9). Let* $\frac{\eta(\gamma_1+\gamma_0)}{\lambda\gamma_2} > \mu(C)$*. The unique Nash equilibrium of the game is given by:*

$$\bar{x} = \frac{\alpha\eta \mathbf{b}(\mathbf{C}, \frac{\lambda\gamma_2}{\eta(\gamma_1+\gamma_0)})}{\eta(\gamma_1 + \gamma_0) + \gamma_2\theta b(\frac{\lambda}{\gamma_1}, \mathbf{C})}. \tag{10}$$

Rewrite FOC's as:

$$\bar{x} = \frac{\alpha}{\gamma_1 + \gamma_0}\mathbf{b}(\mathbf{G}, \frac{\phi - \gamma_1}{\gamma_1 + \gamma_0}) - \frac{\gamma_2}{\gamma_1 + \gamma_0}\mathbf{M}\underbrace{(\mathbf{G} + \mathbf{G}^2)\bar{x}}_{z}$$

The term:

$$z_i = \sum_j [(g_{ij} + g_{ij}^{[2]})x_j]$$

measures the aggregate equilibrium actions that agents in *i*'s neighborhood are exposed to.

The correction of equilibrium behavior is higher for those agents whose high centrality (in **G**) comes from paths that lead to agents who are exposed to large amounts of externalities in equilibrium.