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Uncertainty in Integrated Assessment Modelling: Can Global Sensitivity be of Help?

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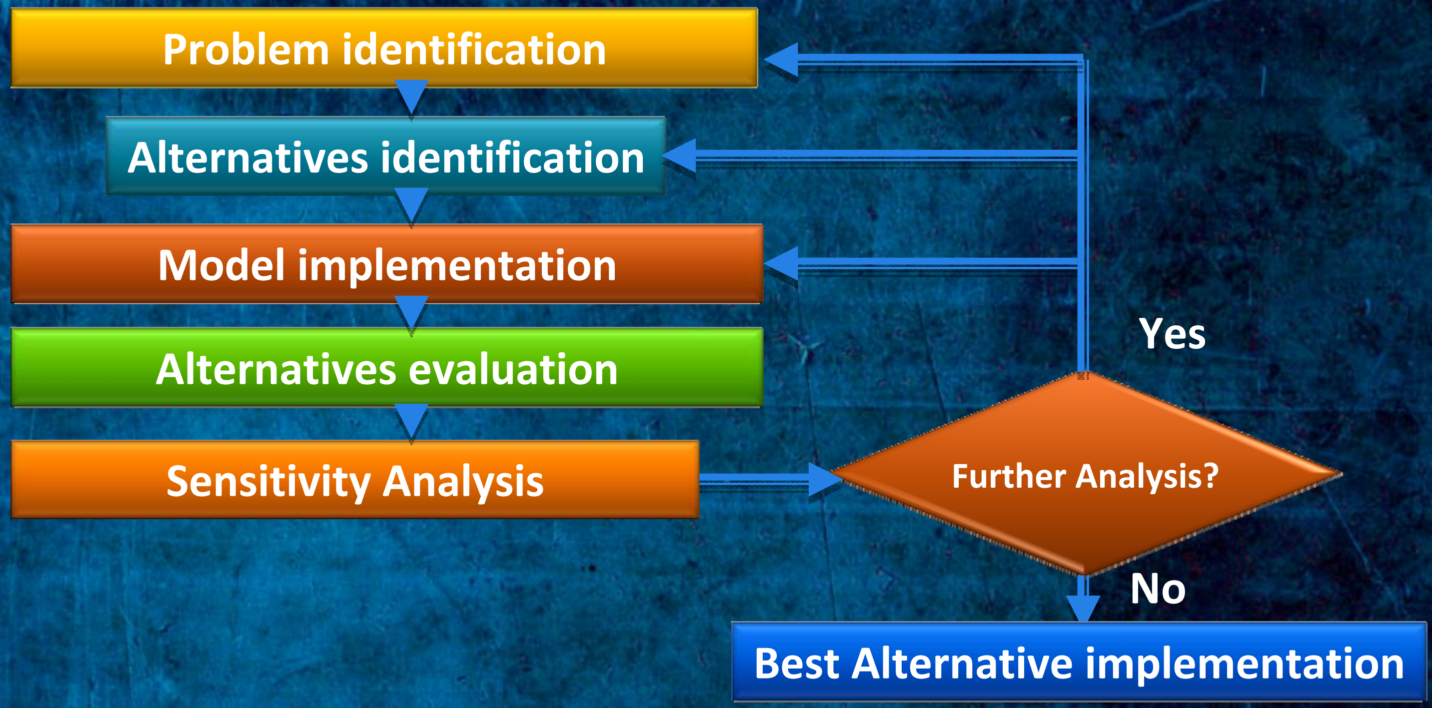
Decision-Making Process Steps



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"Forecasting is easy ....
... for the past"

Niels Bohr, Nobel Prize for Physics in 1922

The Problem I

- Uncertainties pervade forecasts in environmental and climate change sciences
- Environmental models are nowadays intermediaries between science and policy [Risbey et al (2005, EMA)].
- If uncertainties are not properly
  - Assessed
  - Communicated
- one runs the risk of model rejection by stakeholders (Saltelli and d’Hombres (2010), GEC); Stokstadt (2008; Science)).

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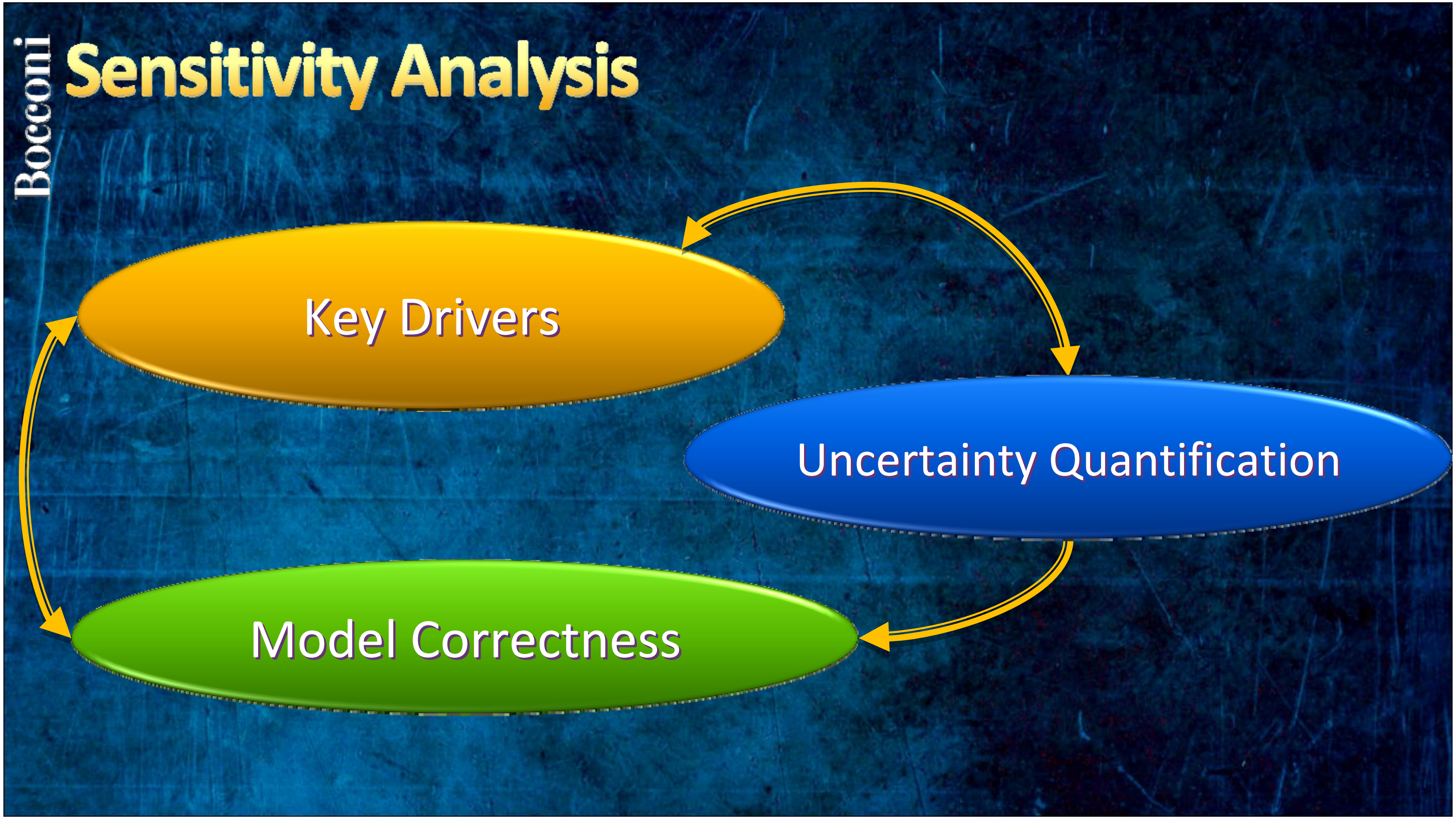
The Problem II

- *“Dealing consistently with risk and uncertainty across the Intergovernmental Panel on Climate Change (IPCC) reports is a difficult challenge”* (Swart et al, 2009; p.3; CC) and that *“observed differences in handling uncertainties by the three IPCC working groups emerge”* [ibidem](p.1).
- See also the work of Webster (2009, CC), Oppenheimer et al. (2007; Science).
- There is uncertainty in practically every factor that enters in an IAM.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Sensitivity Analysis
Key Drivers
Uncertainty Quantification
Model Correctness

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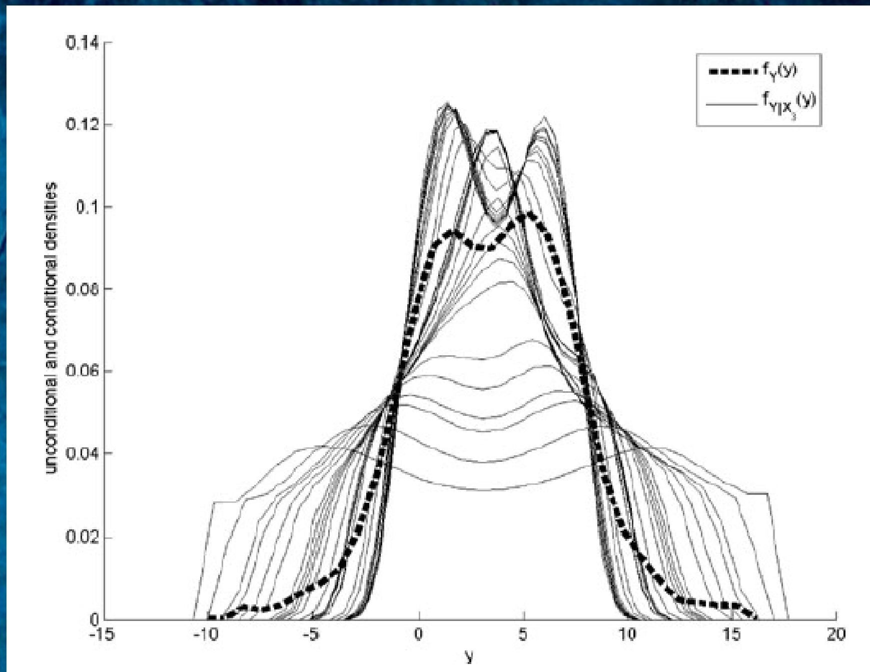
How to better understand our model

- Model structure.
  - IAMs are black boxes
  - Interactions or superimpositions?
- Key-uncertainty drivers
  - Aiding intuition
  - Identifying the factors on which to focus information collection or managerial attention
- Direction of change
  - Generalized comparative statics

Global Sensitivty Analysis



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Setting

- Ω_X set of possible values that the model inputs can assume
- $(\Omega_X, B(\Omega_X), \mathbf{P})$ corresponding probability space
- Input output mapping

$$y = g(\mathbf{x}) : \Omega_X \subseteq \mathbf{R}^n \to \mathbf{R}$$

Model Structure I

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- Efron and Stein (1981), Sobol' (1993) Rabitz and Alis (1999). g(x) integrable

$$g(\mathbf{x}) = g_0 + \sum_{i=1}^{n} g_i(x_i) + \sum_{i<j}^{n} g_{ij}(x_i, x_j) + \ldots + g_{1,2,\ldots,n}(x_1, x_2, \ldots, x_n)$$

$$g_0 = \mathbb{E}_{\mathbf{X}}[g(\mathbf{x})] = \int \ldots \int g(\mathbf{x}) \prod_{i=1}^{n} dF_i$$

$$g_i(x_i) = \mathbb{E}_{\mathbf{X}}[g(\mathbf{x}) \mid X_i = x_i] - g_0 = \int \ldots \int g(\mathbf{x}) \prod_{s=1, s\neq i}^{n} dF_s$$

$$g_i(x_i) = \mathbb{E}_{\mathbf{X}}[g(\mathbf{x}) \mid X_i = x_i, X_j = x_j] - g_i(x_i) - g_j(x_j) - g_0 = \int \ldots \int g(\mathbf{x}) \prod_{s=1, s\neq i, j}^{n} dF_s$$

……

Model Structure II

- By orthogonality

$$\mathrm{V}_{\mathbf{X}}[Y] = \sum_{i=1}^{n} V_i + \sum_{i<j}^{n} V_{ij} + \ldots + V_{1,2,\ldots,n}$$

- Sobol' Global sensitivity indices:

$$S_l^1 \equiv \frac{V_l}{\mathbb{V}_{\boldsymbol{X}}[Y]} = \frac{\mathbb{V}_{X_l}[\mathbb{E}\{Y \mid X_l\}]}{\mathbb{V}_{\boldsymbol{X}}[Y]}$$

- If sum of S_l is close to unity, then the model is additive

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Monotonicity

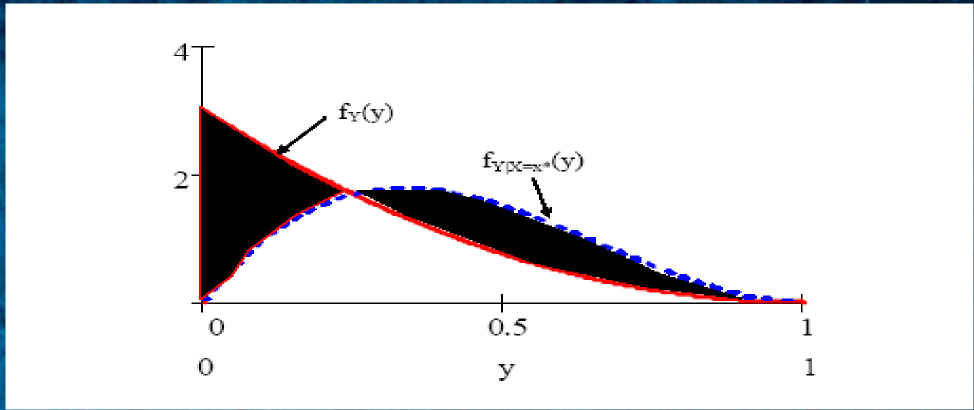
- By knowledge of the functions

$$E[Y \mid X_i = x_i] = g_i(x_i) - g_0$$

- Average behavior over the entire variation range of X_i
- Difference with respect to partial derivatives
- Exact behavior if g(x) is additive

The intuition

- Conditioning on $X_i=x_i^*$



- If Y is independent of xi then its distribution will not change. However, if its distribution changes significantly, then the decision maker view of Y changes significantly.

The δ importance measure

- We then define the importance of an Individual parameter (Borgonovo, 2006, 2007)

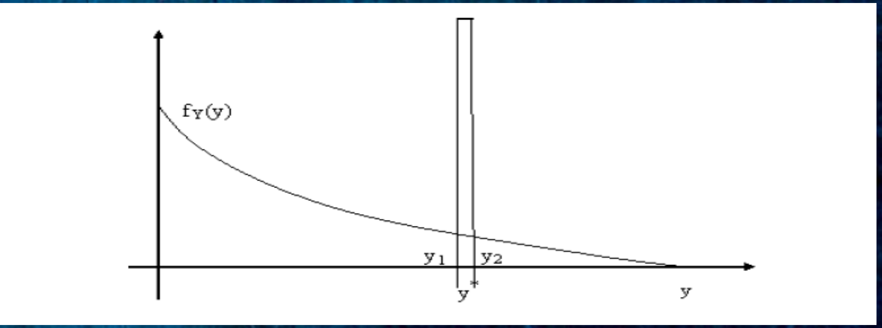
$$\delta_i = \frac{1}{2}\mathrm{E}[\int \left| f_Y(y) - f_{Y|X_l = x_l}(y) \right| dy]$$

Properties

1. $0 \leq \delta_l \leq 1$
   - Proof: based on the triangle inequality

2. $\delta_l = 0$ if Y is independent of X_l
   - Proof: $f_Y(y) = f_{Y|Xl(y)}$

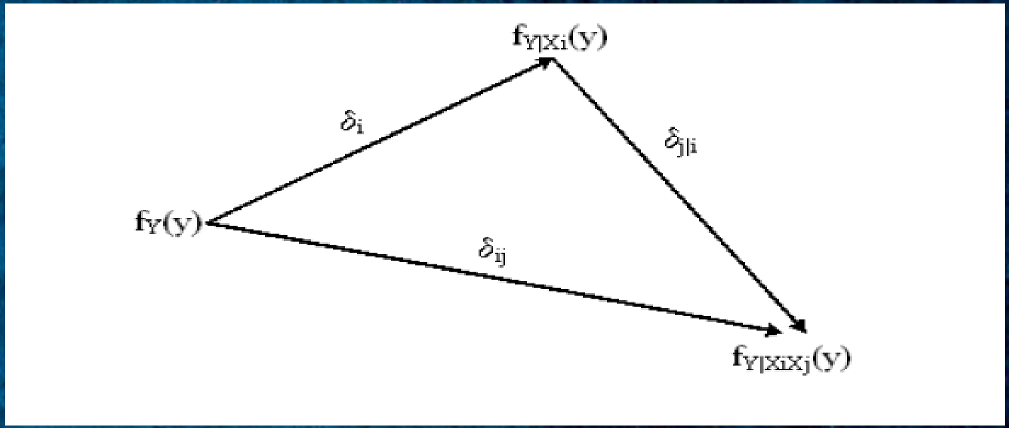
3. $\delta_{1,2,\ldots,n} = 1$
   - Proof: limit, Dirac-δ



Properties (continued)

4. $\delta_{ij}=\delta_i$ if Y depedendent on X_i, indep. of X_j

5. $\delta_i \le \delta_{ij} \le \delta_i + \delta_{j|i}$



6. Invariant for Monotonic Transformation of Y

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Properties

- Normalization: $0 \leq \delta_i \leq 1$
- Joint normalization: $0 \leq \delta_{1,2,\ldots,n} \leq 1$
- Scale Invariance

For any X and Y on measurable spaces $(\mathcal{X},\mathcal{A})$ *and* $(\mathcal{Y},\mathcal{B})$, *it holds that*

$$\delta(Y,X) = \delta(X,Y) = \frac{1}{2}\int_{\mathcal{X}\times\mathcal{Y}} |f_X(x)f_Y(y) - f_{XY}(x,y)| \, \mathrm{d}y \, \mathrm{d}x.$$

Plischke, Borgonovo and Smith (2012)

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Estimation

- Several ways for estimating these sensitivity measures in the literature
- We utilize a method that allows one to perform a simple Monte Carlo Simulation and extract the sensitivity measures directly from such dataset (post-processing)
- For variance-based sensitivity measures Ziehn and Tomlin (2009)
- For density-based Plischke et al (2012)
- Notable computational burden reduction

Applications in IAM

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State of the art

- Monte Carlo simulation part of best practices in the IAM literature.
- Uncertainty analysis of the DICE model (Nordhaus, 1994, 2008) and PAGE model (Hope, 2006).
- Dietz (2011) assessment of catastrophic climate change based on the PAGE model
- Popp (2004) using ENTICE, an adaptation of the DICE model.
- Three studies:
  - van Vuuren et al. (2008) apply a probabilistic approach to an energy model,
  - Hof et al. (2008) use the FAIR IAM
  - Anthoff and Tol (2011) effects of uncertainty on the social cost of carbon (current damages caused by each unit of emissions) using the FUND model.
- Raw correlations or standardized regression coefficients. The SA literature describes specifically the weaknesses of using correlations or standardized regression coefficients

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Literature Review

- We reviewed more than 1000 works from 1980 to 2010.
- Sensitivity is mainly a synonym of dependence
- OFAT methods most widely utilized
- No systematic use of SA methods

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The DICE Model

- We refer to Nordhaus (1994) and Nordhaus (2008) for details
- 51 factors (model inputs)
- The model consists of a set of interconnected equations and outputs the following quantities of interest:
  - Inter-generational welfare (utility),
  - social cost of carbon in 2005,
  - global atmospheric temperature in 2105,
  - global emission level in 2105,
  - optimal carbon tax for 2015

Nordhaus's Results

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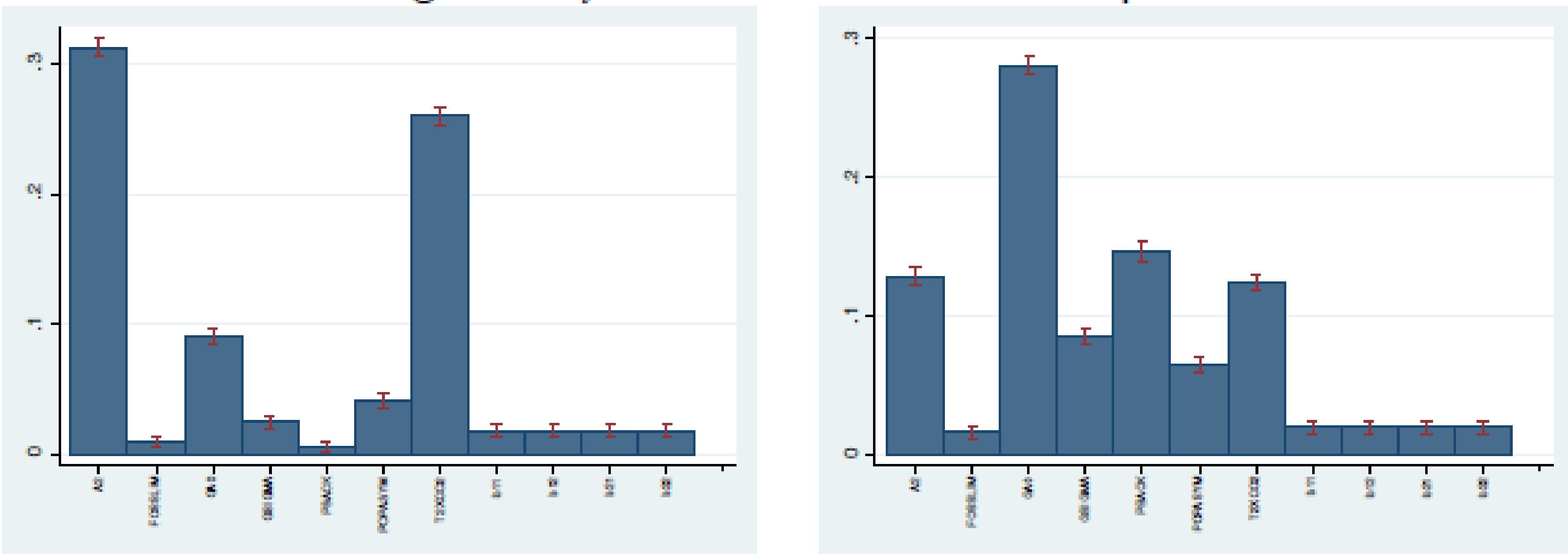
- Performs a seires of OFAT Analysis to lower the number of inputs
- Then assesses uncertainty on 8 factors.
- Weaknesses:
  - Ranking is dependent on the variation range
  - No systematic identification
- We use 10000 model runs on these 8 factors and display the Bootstrap-Bias-corrected estimator used in Plischke et al (2012)

On Nordhaus' 8 factors

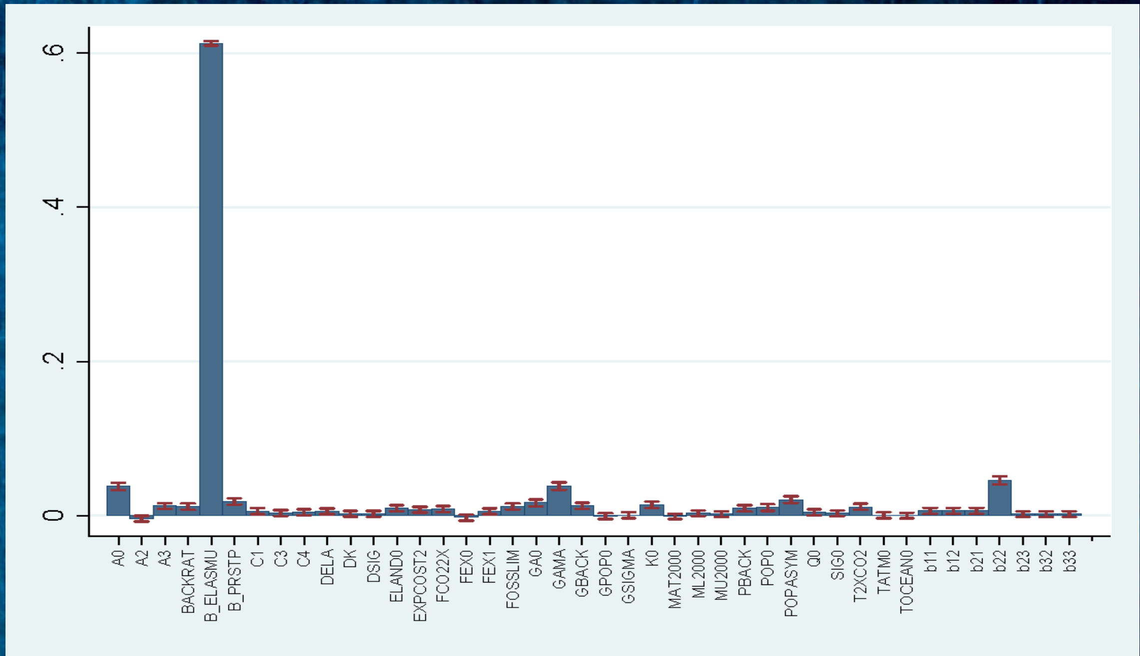
Figure 1: δ_i's with Pre-Selected Model Inputs



Social Cost of Carbon 2005 (left) and Global Emissions in 2105 (right)

the coefficient in the damage function (A2), climate sensitivity (T2XCO2) and the growth rate of total factor productivity (GA0) are the most influential inputs

When all factors are considered



- Appears in the DICE model as the variable μ of Ramsey's equation
- μ determines the shape of the utility function and the relationship between consumption increases and utility or welfare

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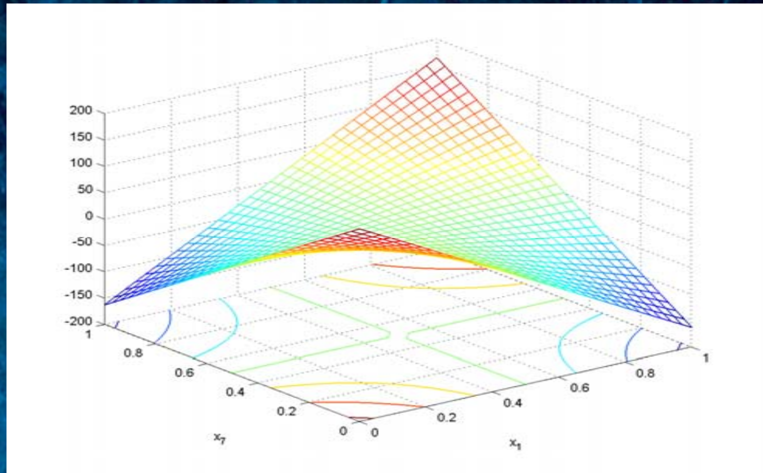
Were the 8 rightly selected?

Table 4: ?pre-selected parameters SA rank

Pre-selected vars.	Soc. Cost Carb. (2005)	Glob. Emiss. (2105)	Atmos. Temp. (2105)	Carbon Tax (2015)
A2	22	5	6	10
GA0	7	11	11	3
FOSSLIM	42	26	30	34
GSIGMA	16	36	26	6
PBACK	17	32	29	12
POPASYM	10	12	8	8
T2XCO2	2	3	5	7
b12	12	13	12	21

Interactions



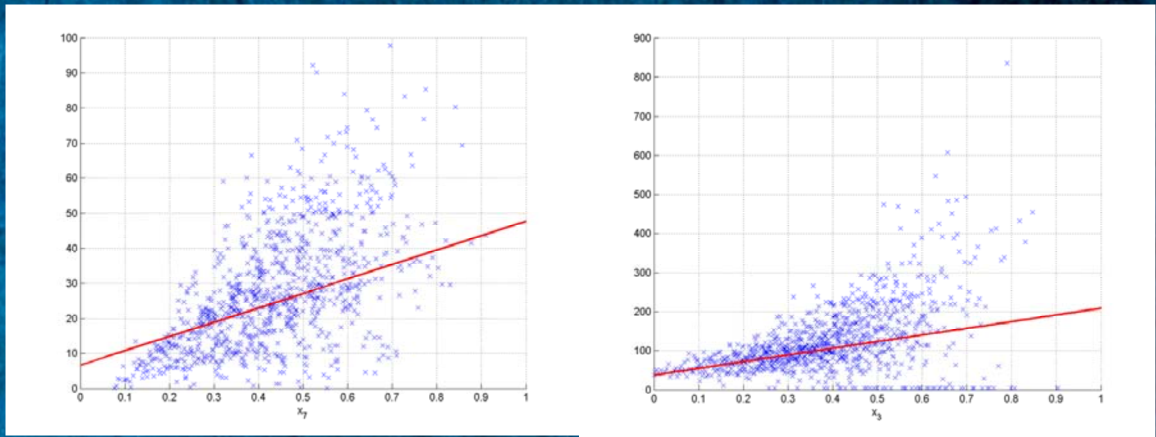
$$g_{A2,T2XC02}\left(A2, T2XC02\right)$$

- Interaction effects are relevant

$$\sum_{i,j=1}^{n} S_{i,j} = 0.4233$$

Monotonicity

- Plotting the first order functions one gains insights of what happens to the model when factors vary individually



- SCC vs T2XCO2 and Global emission vs GA0 (right)

Conclusions

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- Systematic approach for inferring insights from models
- Several sensitivity questions can be asked
  - Model structure
  - Monotonicity
  - Key-Uncertainty Drivers
- Approach rigorous, avoids pitfalls
- Computationally Convenient, because in a post-processing mode

Thank you for your attention!

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