End-Use Residential Energy Demand: a MDCEV-GEV Model for the Joint Estimation of Perfect and Imperfect Substitute Goods

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Modeling demand systems with non-negativity constraints

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Wales and Woodland (1983) were the fisrt providing a solution to this problem.

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The model has been applied mainly in trasnportation and time allocation studies (among others: Bhat 2005, Bhat 2008, Ferdhous 2011).
Recent contributions to energy economics using this model are: Jong (2011), Zang (2011), Pinjari and Bhat (2011).

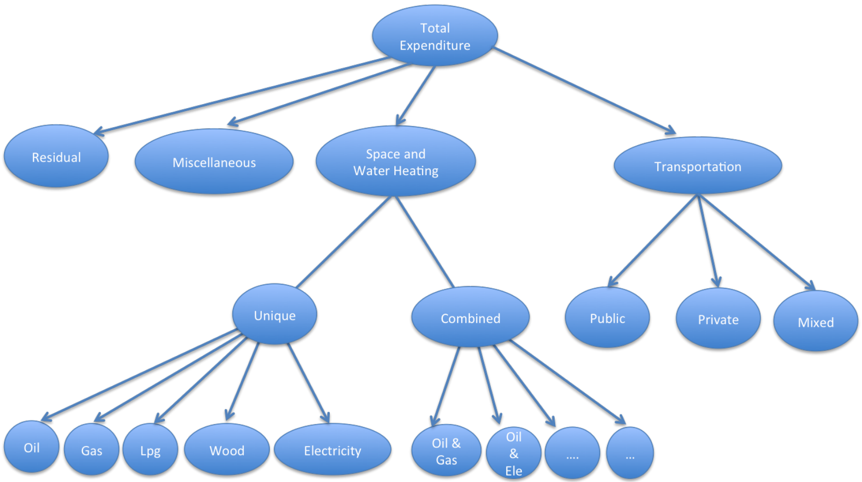
Modeling the simultaneous choice of energy expenditures and appliances

The second issue refers to the choice of how much to spend on space heating as simultaneous to the choice of the heating system (e.g. gas vs electricity based).
A path-breaking contribution is the model developed by Dubin and McFadden (1984) (a sort of generalization of the Tobit model to a case of multi-criteria selection).

Modeling the simultaneous choice of energy expenditures and appliances - cont.

I propose a refinement of the model of Bhat (2005) to the choice of expenditures allocated to different uses of energy (space heating, water heating, transportation) combining it with discrete choice models within each type of energy use to the choice of a specific technology (the alternatives are treated here as perfect substitutes).

Decision path



Model specification

Suppose there are M categories ($m = 1, 2, ..., M$; with m=space heating, water heating, etc.) and J_m fuels ($j = 1, 2, ..., J$; with j=electricity, natural gas, etc.). The household decides to spend some quantity of money e_m or zero in each category M and fuel J.

The general specification of U is:

$$U(x) = \frac{1}{\alpha_1}\psi_1(q_1)^{\alpha_1} + \sum_{m=2}^{M} \frac{\gamma_m}{\alpha_m}\psi_m \left\{ (\frac{q_m}{\gamma_m} + 1)^{\alpha_m} - 1 \right\};$$

$$\psi_m > 0, \qquad 0 \leq \alpha_m \leq 1, \qquad \gamma_m > 0$$

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I express the utility function in terms of expenditures (e_m) and prices (p_m) and I use a special form of the previous in which $\alpha \rightarrow 0$ ending up with a LES or a $\gamma -$ *profile* specification:

$$U = (\psi_{out})\frac{e_{out}}{p_{out}} + \sum_{m=2}^{M} \gamma_m \psi_m ln(\frac{e_m}{\gamma_m p_m} + 1)$$

The parameter γ acts as a satiation parameter through two main mechanisms:

1. γ allows for corner solutions shifting the point at which the indifference curves are asymptotic to the axes and the consumption point is tangential to the indifference curves (zero expenditure)

Go to Fig 1

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2. the indifference curves become steeper as γ increase meaning that our consumer has stronger preferences for the good (low satiation) Go to Fig 2

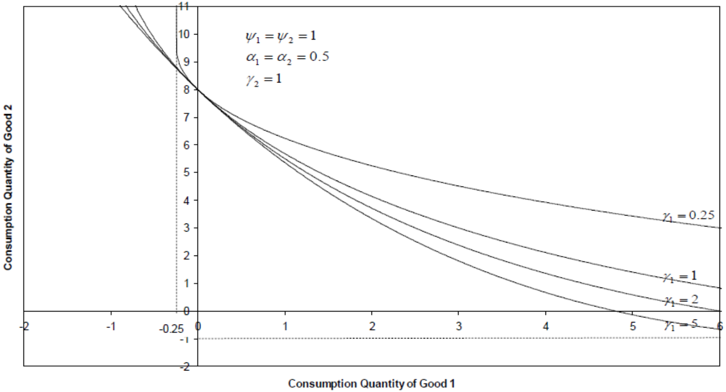
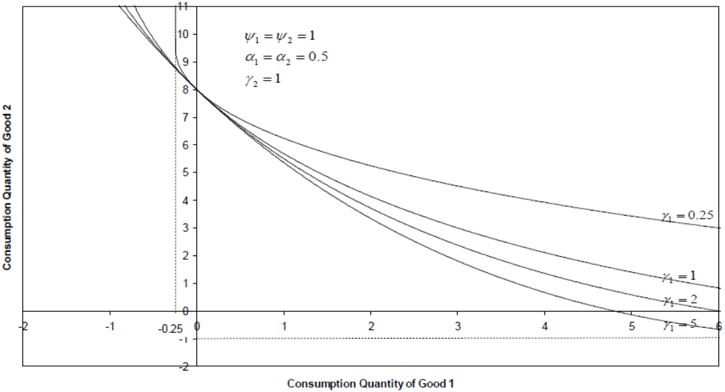


Figure : Role of γ.*Source* : *Bhat*, 2005*p*.280

Go back



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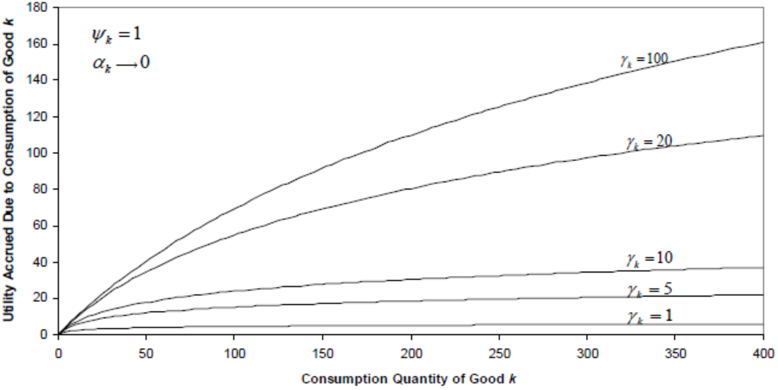

Figure : Role of γ.*Source* : *Bhat*, 2005*p*.280

The randomness comes in to the model because of the difficulty for the analyst to describe the quality of each alternative. The random term is introduced as a multiplicative element in ψ:

$$\psi(x_m, \epsilon_m) = exp(\beta' x_m + \epsilon_m)$$

In order to accommodate the presence of perfect substitutes we rewrite the overall utility function as:

$$\tilde{U} = [exp(\beta' x_{res} + \epsilon_{res})]\frac{e_{res}}{p_{res}} + [exp(\beta' x_{res} + \epsilon_{mis})]\frac{e_{mis}}{p_{mis}} +$$
$$+ \sum_{m=sh/wh,tr}^{B} \gamma_m [exp(max_{j \in N_j}\ W_{jm})] ln(\frac{e_m}{\gamma_m p_m} + 1)$$

And B the subset of M containing the alternatives presenting a discrete choice model. We can write W_{jm} as:

$$W_{jm} = \beta' x_m + \delta' z_{jm} + \eta_{jm}$$

The Kuhn-Tucker conditions

The Lagrangian function for the maximization of the utility function subject to the budget constraint is:

$$\mathscr{L} = \tilde{U} - \lambda \left[\sum_{m=1}^{M} e_m - E \right]$$

λ is the lagrangian multiplier and the f.o. Kuhn-Tucker conditions:

$$\begin{array}{lll}
\psi_1(\frac{e_1}{p_1})^{-1} - \lambda = 0 & \mathit{if} & e_1^* > 0 \\
\psi_m(\frac{e_m}{\gamma_m p_m} + 1)^{-1} - \lambda = 0 & \mathit{if} & e_{m \notin B}^* > 0 \\
\psi_m(\frac{e_m}{\gamma_m p_m} + 1)^{-1} - \lambda < 0 & \mathit{if} & e_{m \notin B}^* = 0 \\
exp(max_{j \in N_j} \ W_{jm})[(\frac{e_m}{\gamma_m p_m} + 1)^{-1} - 1] - \lambda = 0 & \mathit{if} & e_{m \in B}^* > 0 \\
exp(max_{j \in N_j} \ W_{jm})[(\frac{e_m}{\gamma_m p_m} + 1)^{-1} - 1] - \lambda < 0 & \mathit{if} & e_{m \in B}^* = 0
\end{array}$$

The Kuhn-Tucker conditions (cont.)

For the first category the lagrangian multiplier is:

$$\lambda = exp[\beta' x_1 + \epsilon_1)](\frac{e_1}{p_1} + 1)^{-1}$$

substituting for λ in the f.o.c., normalizing all prices and taking logarithmic transformation:

$$\begin{aligned} V_m + \epsilon_m &= V_1 + \epsilon_1 \qquad if \quad e_m^* > 0 \\ V_m + \epsilon_m &< V_1 + \epsilon_1 \qquad if \quad e_m^* = 0 \end{aligned}$$

where

$$V_1 = \beta' x_1 - ln e_1^*,$$

$$V_m = \beta' x_m - ln(\frac{e_m^*}{\gamma_m} + 1) \qquad if \quad m \notin B$$

$$V_m = \beta' x_m + Max_{j \in N}[\delta' z_{jm} + \eta_{jm}] - ln(\frac{e_m^*}{\gamma_m} + 1) \qquad if \quad m \in B$$

The Kuhn-Tucker conditions (cont.)

$$V_m = \beta' x_m + Max_{j \in N}[\delta' z_{jm} + \eta_{jm}] - ln(\frac{e_m^*}{\gamma_m} + 1)$$

is for the MNL equal to:

$$V_m = \beta' x_m + \theta_j ln \sum_{j \in N} exp(\frac{\delta' z_{jm}}{\theta_j}) - ln(\frac{e_m^*}{\gamma_m} + 1)$$

and for the Nested Logit:

$$V_m = \beta' x_m + \sum_{k=1}^{K} (\sum_{m \in D} [exp(\delta' z_{jm})]^{\frac{1}{\rho_k}})^{\frac{\rho_k}{\theta_j}} - ln(\frac{e_m^*}{\gamma_m} + 1)$$

Let's assume ϵ to be identically standrad extreme value distributed. The second assumption refers to η that is decomposed in two parts: $\eta_{mj} = \lambda_m + \lambda_{mj}$, the first element is an unobserved component common to the alternatives and the second is extreme value identically distributed with a scale parameter θ_m. The terms in λ_{mj} are indipendent of ϵ_m, but correlated among them and we assume a general correlation structure as follow:

$$F_m(\lambda_{m1}, \lambda_{m2}, ..., \lambda_{mJ}) = exp\left[-G_m(e^{\lambda_{m1}}, e^{\lambda_{m2}}, ..., e^{\lambda_{mJ}})\right]$$

where G_m is a non-negative, homogenous function.

Marginal Choice Probabilities

The marginal choice probabilities to participate to the first K alternatives of the M categories of consumption $(K \geq 1)$ with positive expenditure allocations as:

$$P(e_1^*, e_2^*, e_3^*, ..., e_K^*, 0, 0, ..., 0) = |J| \left[\frac{\prod_{m=1}^{K} e^{V_m}}{\left(\sum_{m=1}^{M} e^{V_m}\right)^K} \right] (K-1)!$$

J is the jacobian:

$$J_{im} = \frac{\partial \left[V_1 - V_{i+1} + \epsilon_1\right]}{\partial e_{m+1}^*}; \qquad i, m = 1, 2, 3, ..., K-1$$

whose determinant is defined as:

$$|J| = \left[\prod_{m=1}^{K} r_m\right] \left[\sum_{m=1}^{K} \frac{1}{r_m}\right], \qquad where \qquad r_m = \left(\frac{1}{e_m^* + 1}\right)$$

Fuel Choice Probabilities

The probability to choose fuel j conditional to the allocation of positive expenditure to category m is obtained from a General Extreme Value Distribution:

$$P(j|e_m^* > 0; j \in J_j) = \frac{e^{\gamma' z_{jm}} \cdot G_{jm}(e^{\gamma' z_{1m}}, e^{\gamma' z_{2m}}, ..., e^{\gamma' z_{Jm}})}{\frac{1}{\theta_m} \times G_m(e^{\gamma' z_{1m}}, e^{\gamma' z_{2m}}, ..., e^{\gamma' z_{Jm}})}$$

Unconditional Probabilities

We are interested in estimate the unconditional probability that the individual chooses to participate in consumption of fuel *1* in category *2* for an amount e^*_{12}, and of fuel *2* in category *3* for an expenditure e^*_{23}, etc.:

$$P(e^*_1, e^*_{12}, e^*_{23}, .., 0, ..., 0) = P(e^*_1, e^*_2, e^*_3, ..., e^*_M, 0, .., 0) \times P(1|e^*_2 > 0) \times P(2|e^*_3 > 0) \times ...$$

NOTE: we have the product of the marginal and conditional probabilities $(P(B) = P(A)(B|A))$ as in the case of a standard Nested Logit.

ISTAT: Households Consumption Survey

Year: 2010

Observations: 22.009

Prices: Price data comes from AEEG and EUROSTAT for gas and electricity. For the others fuels from the Bulletins of energy prices of "Camere di Commercio".

Descriptive statistics - Participation rates

Participation rates

Expenditure category	Total number (%) of individual participating	Mean Expenditure*
Residual	22,009 (100%)	2198.02
Miscellaneous	22,009 (100%)	38.85
Space and Water Heating	20,153 (93.20%)	67.31
Transportation	16,463 (74.80%)	176.91

*The mean expenditure is measured only for individuals with non zero consumptions.

Space and Water Heating

Participation rates

Expenditure category	Total number (%) of individual participating	Mean Expenditure*
Unique system		
Oil	810 (3.68%)	87.49
Natural Gas	14,503 (65.90%)	75.40
Lpg	593 (2.69%)	87.04
Wood (solid)	534 (2.43%)	6.32
Electricity	487 (2.21%)	45.02
Combined system		
Nat. Gas- Electricity	996 (4.53%)	48.15
Lpg - Electricity	598 (2.72%)	34.89
Wood - Electricity	553 (2.51%)	15.42
Oil - Electricity	4.67 (2.12%)	58.73
Electricity - Nat.Gas	109 (0.50%)	22.26
Lpg - Nat.Gas	436 (1.98%)	63.93
Wood - Nat.Gas	234 (1.06%)	7.85
Oil -Nat.Gas	193 (0.88%)	32.83

*The mean expenditure is measured only for individuals with non zero consumptions.

Transportation

Participation rates

Expenditure category	Total number (%) of individual participating	Mean Expenditure*
Public	874 (3.97%)	41.17
Private	13,171 (59.84%)	173.52
Mixed	2418 (10.99%)	244.45
No transportation	5546 (25.20%)	*

*The mean expenditure is measured only for individuals with non zero consumptions.

MDCEV -GEV results

MDCEV model

	(I) Residual	(II) Miscellaneous	(III) Space and Water Heating	(IV) Transportation
Household's components	-	-0.026 (0.04)	-0.088*** (0.03)	-0.09* (0.05)
Children	-	-0.036 (0.11)	-0.117 (0.09)	-
High Educ.	-	-0.057 (0.12)	-0.036 (0.10)	-
Rooms	-	0.018 (0.02)	0.06*** (0.01)	-
Home owners	-	0.026 (0.07)	-0.137** (0.05)	-
Renewal	-	-0.126 (0.12)	-0.377** (0.11)	-
New Houses	-	-0.046 (0.01)	-0.151*** (0.07)	1.452*** (0.075)
Temperature	-	-	-0.007** (0.004)	-0.05*** (0.008)
Gender head	-	-0.048 (0.08)	0.075 (0.07)	-0.045*** (0.006)
Suburbs	-	0.094 (0.11)	0.083 (0.09)	0.114 (0.10)
Campaign	-	0.156 (0.15)	0.029 (0.11)	0.1327 (0.15)
Detached House	-	-0.105 (0.18)	-0.47*** (0.14)	-0.036*** (0.22)
Popular house	-	-0.181 (0.143)	-0.246*** (0.10)	-0.912*** (0.17)
Rural house	-	-0.190 (0.22)	-0.6064*** (0.17)	-0.059 (0.29)
Number of cars	-	-	-	0.576*** (0.07)
Constant	-	-2.042*** (0.21)	-4.04*** (0.16)	-3.450*** (0.20
$\gamma_{space/water}$	-	-	3.66*** (0.04)	-
γ_{tra}	-	-	-	2.85*** (0.25)

MDCEV -GEV results

NL for space and water heating

	(I) Oil	(II) Gas	(III) Lpg	(IV) Wood	(V) Electricity
Household's components	-	0.06***	0.43	0.12***	-0.003
		(0.01)	(0.02)	(0.03)	(0.03))
Children	-	0.034	0.254***	0.129	-0.003
		(0.05)	(0.08)	(0.08)	(0.03)
High Educ.	-	0.123*	-0.11	-0.28**	0.167***
		(0.16)	(0.10)	(0.12)	(0.10)
Rooms	-	0.018	-0.055***	-0.06	-0.108***
		(0.02)	(0.01)	(0.01)	(0.02))
Home owners	-	0.126**	0.225**	0.16**	-0.20***
		(0.04)	(0.07)	(0.07)	(0.06)
Renewal	-	0.189***	0.204*	-0.04	-0.18
		(0.08)	(0.12)	(0.14)	(0.21)
Temperature	-	-0.015***	-0.007**	-0.01***	0.017
		(0.002)	(0.004)	(0.003)	(0.004)
Gender head	-	-0.048	0.151**	-0.05	-0.02
		(0.04)	(0.07)	(0.07)	(0.09)
Suburbs	-	-0.213***	0.523***	0.114	-0.26***
		(0.04)	(0.07)	(0.10)	(0.09)
Campaign	-	-0.61	0.089***	0.30	-0.198
		(0.05)	(0.08)	(0.08)	(0.12)
Detached House	-	0.23***	-0.28**	-0.027***	-0.1
		(0.08)	(0.11)	(0.13)	(0.18)
Popular house	-	-0.391	-0.45***	0.397***	-0.07
		(0.05)	(0.07)	(0.08)	(0.148)
Rural house	-	-0.143	-0.029***	-0.28**	-0.017
		(0.10)	(0.12)	(0.13)	(0.20)
Price space sys	-	-2.05***	-0.44***	-0.04	-0.05
		(0.10)	(0.07)	(0.06)	(0.15)

MDCEV -GEV results

NL for space and water heating - cont.

	(VI) Gas-El	(VII) Lpg-El	(VIII) Wood-El	(IX) Oil- El	(X) El-Gas	(XI) Lpg-Gas	(XII) Wood-Gas	(XIII) Oil-Gas
Household's components	0.04	0.085**	0.09	0.018	0.139***	0.177***	0.062	0.03
	(0.03)	(0.03)	(0.03)	(0.04)	(0.03)	(0.05)	(0.06)	(0.04)
Children	-0.20***	-0.129	-0.08	-0.11	0.10	-0.04	0.17***	0.074
	(0.09)	(0.10)	(0.11)	(0.127)	(0.08)	(0.25)	(0.10)	(0.14)
High Educ.	0.42	-0.28*	-0.37*	0.33	0.03	-0.035	-0.179	0.22
		(0.10)	(0.17)	(0.20)	(0.11)	(0.25)	(0.13)	(0.25)
Rooms	-0.08***	-0.179***	-0.11***	0.013	-0.33***	-0,.019	-0.07***	-0.036
	(0.01)	(0.02)	(0.02)	(0.02)	(0.07)	(0.02)	(0.03)	(0.03)
Home owners	0.03	-0.23***	0.11	-0.03	-0.08	0.01	0.042	0.10
	(0.06)	(0.08)	(0.08)	(0.08)	(0.17)	(0.09)	(0.13)	(0.14)
Renewal	-0.18	0.11	-0.23	-0.09	-0.10	-0.04	-0.004	-0.046
	(0.13)	(0.28)	(0.19)	(0.23)	0.57	(0.18)	(0.22)	(0.32)
Temperature	0.0025	0.008**	-0.0002	-0.01**	-0.01	0.001***	-0.009	-0.018***
	(0.004)	(0.005)	(0.004)	(0.006)	(0.01)	(0.01)	(0.08)	(0.08)
Gender head	-0.125***	-0-07	0.04	-0.19***	-0.001	0.008	0.03	0.07
	(0.06)	(0.07)	(0.08)	(0.08)	(0.18)	(0.09)	(0.13)	(0.14)
Suburbs	-0.44***	-0.16**	0.38**	-0.27***	-0.20	0.33***	0.096	-0.03
	(0.11)	(0.10)	(0.11)	(0.12)	(0.45)	(0.11)	(0.13)	(0.16)
Campaign	-0.78***	-0.07	0.16	-0.49***	-0.09	0.379***	-0.093	-0.14
	(0.159)	(0.12)	(0.12)	(0.18)	(0.37)	(0.14)	(0.17)	(0.21)
Detached House	0.58***	-0.30	-0.23	0.05	-0.03	-0.012	-0.022	0.166
	(0.26)	(0.25)	(0.22)	(0.23)	(0.51)	(0.165)	(0.24)	(0.24)
Popular house	0.43***	0.27***	-0.09	0.43***	-0.03	-0.21	-0.56***	-0.25
	(0.142)	(0.14)	(0.14)	(0.19)	(0.40)	(0.28)	(0.024)	(0.20)
Rural house	-0.3	-0.01	0.46***	-0.29	-0.08	0.039	-0.14	-0.11
	(0.22)	(0.20)	(0.18)	(0.13)	(0.72)	(0.47)	(0.36)	(0.35)
Price space sys	0.0025	-0.05*	-0.07**	-0.4	-0.23	-0.54*	-0.66*	-0.37**
	(0.28)	(0.25)	(0.24)	(0.31)	(1.41)	(0.52)	(0.54)	(0.69)
Price water sys	-0.07	0.10	-0.147***	-0.53***	-0.03	-0.65***	-0.20*	-0.518*
	(0.55)	(0.10)	(0.08)	(0.29)	(0.92)	(0.14)	(0.12)	(0.41)
Theta	-	-	-	-	-	-	-	0.37***
Rho	-	-	-	-	-	-	-	0.62***

MDCEV -GEV results

MNL for transportation

	(I) Private	(II) Public	(III) Mixed
Household's components	-	-0.04*	0.304***
		(0.03)	(0.04)
Children	-	-0.21*	-0.34**
		(0.10)	(0.11)
High Educ.	-	-0.178*	0.264
		(0.10)	(0.11)
Number of cars	-	0.825***	0.804***
		(0.06)	(0.06)
Temperature	-	0.07***	-0.007**
		(0.005)	(0.004)
Gender head	-	0.511***	0.13
		(0.07)	(0.08)
Detached House	-	0.293*	0.54***
		(0.14)	(0.12)
Popular house	-	1.01***	-0.36*
		(0.12)	(0.22)
Rural house	-	0.403*	-0.029***
		(0.19)	(0.12)
Price Gasoline	-	0.76***	-0.50***
		(0.08)	(0.01)
Theta	-	-	0.98***

Estimation on subsamples

Using climate differences and house categories, as proxy of familiy wealth, I verify the results on subsamples:

- **Temperature**: average temeperature is used to build do groups. Families living in cold areas lower satiation parameter for space and water heating and higher for transportation. The preference for Natural Gas to heat the houses is weaker in this case and private transportation is preferred.
- **Wealth**: the model is estimated just families living in popular houses. In this case the preference for the outside good (residual) is higher (lower constants and higher satiation in MDCEV). Combined systems for space heating are and low cost transportation more preferred.

Contributions: A new way to model total energy demand is proposed and implemented within the class of Multiple Discrete Continuous Models.
The results of the empirical application confirm the goodness of this modeling approach (both $\gamma' s$ and θ are statistically different from one).

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Agenda for future research:

- Non-constant prices issue. How can we model non linear budget constraints in the case with more than two or three goods?
- Adopt a forecasting procedure (as in Pinjari et al. 2010) to suitably evaluate public policies and to perform scenario analysis.