Stochastic capacity expansion models :

Risk exposure and Good-deal valuation

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Electricity markets : Investment models

Generation capacity expansion models have a long tradition in the power industry

- Models that optimally select the timing and the level of investment over a time horizon

- Well adapted for the regulated monopoly industry (from the 50^{ties} in EdF)
  - Risk was largely passed to the consumer through average cost pricing

- Still very popular after the restructuring of the sector
  - Interpretable as equilibrium in a competitive environment
  - What about the risk ?

Those models have been adapted to stochastic programming in order to accommodate wide ranges of uncertainties

- Estimation of the margin profit distribution of power plants
- Risk averse formulation using the good-deal risk measure

Outline

Generation Capacity Expansion Models

1. Introduction: optimization, equilibrium and risk
2. Estimation of the distribution of the margin profit
3. Risk averse formulation : good-deal risk measure

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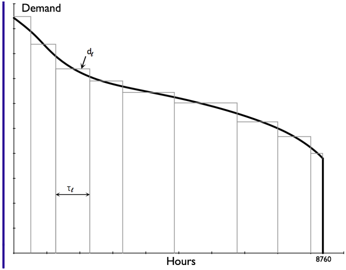
A basic capacity expansion model

- Two stages : invest in $t = 0$, operate in $t = 1$
- Price insensitive demand
- Invest $x(k)$ in K technologies (coal, CCGT, ...)
- Operate them at $y(k,\ell)$ in time segment ℓ
- Satisfy or curtail the demand $d(\ell)$



A deterministic optimization problem:

$$\min_{x\geq 0} \quad \sum_{k=1}^{K} I(k)x(k) + Q(x)$$

$$\begin{aligned} Q(x) = \quad & \min_{y,z} \sum_{\ell=1}^{L} \tau(\ell) \left(\sum_{k=1}^{K} c(k)y(k,\ell) + PCz(\ell) \right) \\ \textbf{s.t.} \quad & 0 \leq y(k,\ell) \leq x(k) & \tau(\ell)\mu(k,\ell) \\ & 0 \leq \textstyle\sum_{k=1}^{K} y_(k,\ell) + z(\ell) - d(\ell) & \tau(\ell)\pi(\ell) \end{aligned}$$

Parameters and units

$I(k)$	Investment cost (annuity) of technology k [EUR/MWyear]
$c(k)$	Operating cost of technology k [EUR/MWh]
PC	Value of Loss Load or price cap [EUR/MWh]
$d(\ell)$	Demand level [MW]
$\tau(\ell)$	Number of hours in time segment ℓ

$\pi(\ell)$	Electricity price in time segment ℓ [EUR/MWh]
$\mu(k,\ell)$	Gross margin of technology k in time segment ℓ [EUR/MWh]

From optimization to equilibrium

Equivalent to a perfect competitive equilibrium in a economy where

N agents maximize their profit ($\nu = 1, ...; N$ - given $\pi(\ell)$):

$$\begin{array}{ll} \max\limits_{x_\nu \geq 0} & \sum\limits_{k=1}^{K} \sum\limits_{\ell=1}^{L} \tau(\ell)\Big(\pi(\ell) - c(k)\Big) y_\nu(k,\ell) - I(k) x_\nu(k) \\ \text{s.t.} & 0 \leq y_\nu(k,\ell) \leq x_\nu(k) \end{array}$$

Market clearing conditions (with price cap) :

$$\begin{array}{lll} 0 \leq \pi(\ell) & \perp & \sum\limits_{\nu=1}^{N} \sum\limits_{k=1}^{K} y_\nu(k,\ell) + z(k,\ell) - d(\ell) \geq 0 \\ 0 \leq z(\ell) & \perp & PC - \pi(\ell) \geq 0 \end{array}$$

- Investment criterion

$$0 \leq x(k) \perp I(k) - \sum_{\ell=1}^{L} \tau(\ell) \mu(k,\ell) \geq 0$$

Introducing risk

- Uncertainties : demand, operating costs, but also regulatory risk
- Invest in $t = 0$ before the actual realization of parameters
- (Ω, P) a probability space where :
  - Scenario $\omega \in \Omega$: probability $p(\omega)$ and corresponding realization of :

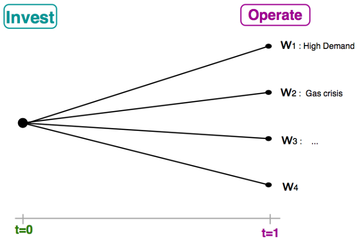
$$\big(c(k,\omega), d(\ell,\omega), PC(\omega)\big)$$

A stochastic optimization problem:

$$\min_{x \geq 0} \sum_{k=1}^{K} I(k)x(k) + \mathbb{E}_P\big[Q(x,\omega)\big]$$

$$Q(x,\omega) = \min_{y,z} \sum_{\ell=1}^{L} \tau(\ell) \left(\sum_{k=1}^{K} c(k,\omega) y(k,\ell,\omega) + PC(\omega) z(\ell,\omega) \right)$$

$$\begin{array}{lll} \textbf{s.t.} & 0 \leq y(k,\ell,\omega) \leq x(k) & \tau(\ell)\mu(k,\ell,\omega) \\ & 0 \leq \sum_{k=1}^{K} y_(k,\ell,\omega) + z(\ell,\omega) - d(\ell,\omega) & \tau(\ell)\pi(\ell,\omega) \end{array}$$



Stochastic equilibrium

- Also amenable to a perfectly competitive stochastic equilibrium
- The dual variables $(\pi(\ell,\omega), \mu(k,\ell,\omega))$ are now contingent on ω
- The investment criterion becomes a NPV

$$0 \leq x(k) \perp I(k) - \mathbb{E}_P\Big[\sum_{\ell=1}^{L} \tau(\ell)\mu(k,\ell,\omega)\Big] \geq 0$$

- Interpretation :
  - Agents are risk neutral / there exist Arrow-Debreu securities (not represented explicitly)
  - All agents in the economy value their investment according to a common exogenous discount factor (CAPM)

Restrictions and extensions

Extensions :

- Multi-period problem
- Price sensitive demand (welfare optimization)
- Storage possibilities, electric grid,...
- Emissions constraints (CO_2,NOx,...)

Restrictions : Some feature are amenable to optimization but not to equilibrium

- Unit commitment characteristics (MIP)

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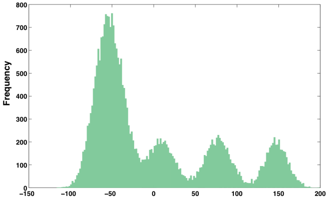


work done with Professor A. Shapiro, Georgia Tech.

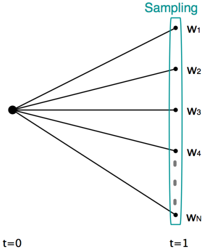
Motivations:

- Investors want to have the best estimation of the distribution of the gross margin
  - compute statistics, measuring the risk (VaR), evaluating hedging strategies,...

- When the risk factors have continuous distribution, the solution is obtained by sampling methods

Sample Average Approximation:

$$\tilde{\theta}^N \equiv \min_{x \geq 0} \sum_{k=1}^{K} I(k)x(k) + \frac{1}{N} \sum_{j=1}^{N} Q(x, \omega_j)$$



- The sample size is limited : one wants to refine it for the profit distribution

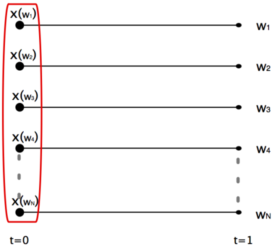
Nonanticipativity constraints

A Stochastic Program can be formulated as

$$\min_{\mathbf{x}\in\mathfrak{L}} \mathbb{E}[\sum_{k=1}^{K} I(k)\mathbf{x}(k,\omega) + Q(\mathbf{x}(\omega),\omega)]$$

where $\mathfrak{L}$ is the set of nonanticipativity constraints

$$\mathfrak{L} := \{\mathbf{x} \in \mathfrak{X} : \mathbf{x}(\omega) = x\}$$



The associated Lagrange multipliers :

$$\lambda(k,\omega) = \sum_{\ell=1}^{L} \tau(\ell)\mu(k,\ell,\omega) - I(k)$$

- $\lambda(k,\omega)$ is net margin of technology k for a given realization of ω

Estimating the distribution

We propose a sampling method :

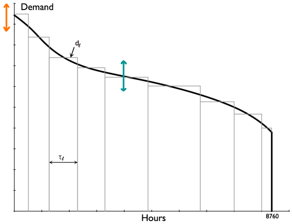
1. Solve the initial stochastic problem (SAA method - check quality)
   - Optimal investment $\Rightarrow \hat{x}(k)$
2. Sample again the parameters (large): $\omega_1, \omega_2, ..., \omega_M$
3. For each sample ω_j, and given $\hat{x}(k)$, compute

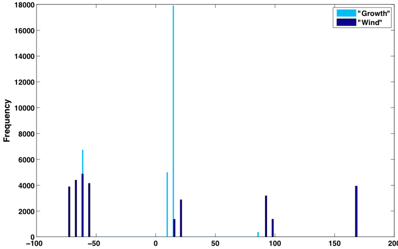
$$\lambda(k, \omega_j) = \sum_{\ell=1}^{L} \tau(\ell)\mu(k, \ell, \omega_j) - I(k)$$

4. Obtain the histogram of the distribution

Illustrative example

- 4 technologies: Nuclear, Coal, CCGT, OCGT
- Uncertainties on the load : "Wind" and "Growth"
- Sample 30.000 scenarios for the profit distribution
- *Badly represent the reality of wind (in progress)*

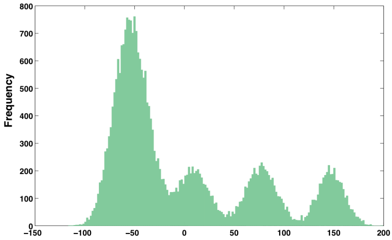




	$\hat{x}$ [MW] "Wind"	$\hat{x}$ [MW] "Growth"
Nuclear	4797.7	5048.1
Coal	0.0	0.0
CCGT	4811.0	4269.5
OCGT	897.4	659.46

Histogram of the gross margin of a CCGT unit [eur/MW]

Illustrative example



Histogram for a Nuclear unit [eur/MW] ("Wind" case)

- Risk and capital requirement :

Gross Profit	"Growth"	"Wind"
Nuclear : - $\text{CVaR}_{30\%}$	2.1%	8.8%
- StDev	35.22	73.78
CCGT : - $\text{CVaR}_{30\%}$	9%	53.5%
- StDev	33.6	86.82
OCGT : - $\text{CVaR}_{30\%}$	13.4%	81.3%
- StDev	31.14	78.74

◇ Capital requirement ≡ Capital needed for accepting the risk

- Measure by a CVaR
- Expressed in [%] of $I(k)$

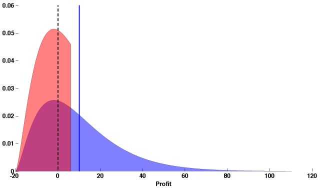


Illustration of the CVaR

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work done with A. Ehrenmann, CEEME, Gdf-Suez
E. Druenne, CEEME, Gdf-Suez

Motivations

- Power plants have quite different risk exposures
- Deterministic discount factor does not capture well the risk
- The CAPM or APT methodologies are difficult to apply
  - The market portfolio badly spans the systematic risks
  - More fundamentally : what are the systematic / idiosyncratic risks for a power company?

A. Ehrenmann and Y. Smeers, 2011: "Stochastic Equilibrium Models for Generation Capacity Expansion"

◇ Rather propose an approach based on risk measure

- **Risk averse** generation capacity expansion models
- Focus on the good-deal (introduced by Cochrane and Saa-Requejo)

Risk measures in short

Goals :

- Quantification of how risk affects the value of a portfolio
- Defined as **capital requirement**: minimal amount of capital which when added makes the position acceptable.
- For a random profit Z, $\rho(Z) + Z$ is acceptable
  - e.g: Mean-Variance, Exponential utility, CVaR,...

Received a lot of attention in the past decade:

- Desirable properties : axiom of coherence (by Artzner et al.)
  Theorem Any coherent risk measure has a dual representation :

$$\rho(Z) = \sup_{Q \in \mathcal{M}} \{\mathbb{E}_Q[-Z]\}$$

- Linked to ambiguity in economics

Risk-averse stochastic programming

Risk averse capacity expansion problem

$$\min_{x\geq 0} \sum_{k=1}^{K} I(k)x(k) + \rho\big(-Q(x,\omega)\big)$$

- Invest in a safe way to avoid large costs (unsatisfied demand, technology spillover,...)
- Interpretation :
  - A social planner (least cost for society)
  - An equilibrium in a complete market ? (work in progress)

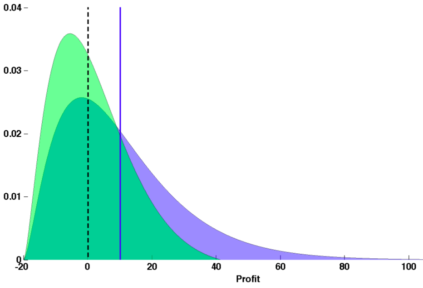
D. Ralph and Y. Smeers, 2010: "Pricing risk under risk measures: an introduction to stochastic-endogenous equilibria "

- Initial step to a full equilibrium model with risk averse investors

The good-deal risk measure (1)

The good-deal risk measure (from Cochrane and Saá-Requejo,2000):

$$\rho^{\mathrm{GD}}(Z) = \sup_{\mathcal{Q}} \mathbb{E}_{\mathcal{Q}}\big[-Z(\omega)\big]$$
$$\mathbb{E}_{\mathcal{Q}}\big[f_1(i,\omega))\big] = f_0(i)$$
$$\mathbb{E}_P\left[\left(\frac{dQ}{dP}(\omega)\right)^2\right] \leq \frac{1+h^2}{(R^f)^2}$$



Good-deal risk measure

- A coherent risk measure in the sense of Artzner et al. (1999)
- Lead to a "risk-neutral" probability $\rightarrow$ no arbitrage
- Incorporate the price of related risk factor f $\rightarrow$ CAPM, APT
- Parameter h is linked to the admissible Sharpe ratio

The good-deal risk measure (2)

Problem The Lagrangian dual formulation of the good-deal :

$$\rho^{\mathrm{GD}}(Z) = \inf_{\mathsf{w},\eta} A\left(\mathbb{E}_P[\eta^2]\right)^{\frac{1}{2}} - \sum_i f_0(i)\mathsf{w}(i)$$

$$\eta(\omega) \geq \textstyle\sum_i f_1(i,\omega)\mathsf{w}(i) - Z(\omega)$$

Interpretation : replicating portfolio in an incomplete market

- Optimality conditions give

$$\eta(\omega) = \max\left(0, \sum_i f_1(i,\omega)\mathsf{w}(i) - Z(\omega)\right)$$

- η mesures the hedge, is called a *regret*
- When $A \to \infty$: super-hedge (the value of Z is the value of a worser portfolio)

Risk-averse stochastic capacity expansion

Using the dual formulation of the good-deal measure :

$$\min_{\mathbf{u},\mathsf{w},\eta} \quad \sum_{k=1}^{K} I(k)x(k) + \mathsf{w}_0(i) + A\mathbb{E}[\eta(\omega)^2]^{\frac{1}{2}}$$

$$\begin{aligned}
\text{s.t.} \quad & x(k) \geq 0 \\
& 0 \leq y(k,\ell,\omega) \leq x(k) \\
& \sum_{k\in K} y_(k,\ell,\omega) + z_(\ell,\omega) - d(\ell,\omega) \geq 0 \\
& (R_f)\mathsf{w} + \eta(\omega) \geq \sum_{\ell=1}^{L} \tau_\ell \left(\sum_{k=1}^{K} c(k,\omega)y(k,\ell,\omega) + \mathtt{PC}\ z_(\ell,\omega) \right)
\end{aligned}$$

- Second Order Cone Program (efficient solver)
- Extension to multistage problems : time consistent
- Current work: investments that mitigate the risk due to the electrical grid, high learning rates and technological acceptance

Illustrative example

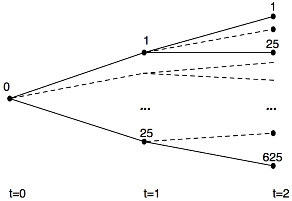
- Three stages example: 1/25/125
- Three technologies: coal, CCGT, OCGT
- 5 years time period and 2 risk factors

α:	- 5%	0%	5%	10%	15 %
Probability	0.1	0.15	0.30	0.25	0.2

Demand Growth

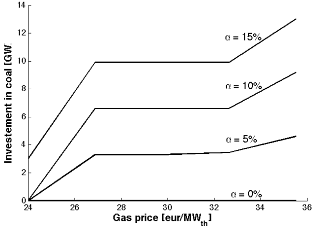
β:	0%	12%	24%	36%	48 %
Probability	0.1	0.4	0.2	0.25	0.1

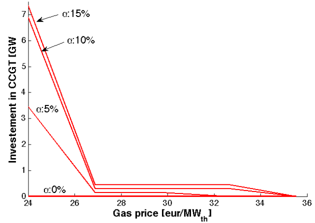
Gas price growth



- PC= 500 eur/Mwh
- $R^f = 1.02$ (for computing annual cost)
- $A^2 = 1.5$ (A^2 = 1 + Sharpe ratio^2)

Illustrative example : investement





Investment [GW]	Coal	CCGT	OCGT
$x_0(k,\omega_0)$	66	3	23
$\mathbb{E}[x_1(k,\omega_1) \mid \mathcal{F}_0]$	4.4	0.6	1.4

Impact of risk aversion:

	A=1.05	A=1.22	A=2.23
Coal	63	65	69
CCGT	6	3	0
OCGT	19	23	23

Initial investment ($t = 0$) for different Sharpe ratio