When, and how, to be precautious

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The precautionary principle

- In the presence of physical uncertainty, society must take take robust preventive action to guard against worst-case outcomes
- The higher the degree of uncertainty, the more aggressive this preventive action should be
- We formalize “uncertainty” and investigate the PP in a stylized problem of reversible pollution control

Question: Do “optimal” adaptation and mitigation decisions agree with the PP?

Main result: Adaptation decisions are always consistent with the PP, mitigation not always

Uncertainty vs. risk

- Pollution control with **uncertain** dynamic structure
- Unlike situations featuring pure *risk*, here it is impossible to assign probabilities to events (Knightian uncertainty)
- Rather, we grant that a certain "benchmark" model is misspecified
- Controlling the degree of this misspecification, we search for robust solutions that perform "well" regardless of the true distribution
- Our normative goal is to maximize the utility of *worst-case* outcome that may occur

Who cares?

1. High structural uncertainty over the physics of environmental phenomena makes cost-benefit analysis based on expected utility untenable (Weitzman (RES, 2009))

2. High sensitivity of model outputs to seemingly ad hoc modeling assumptions (e.g., damage function, social discount rate) on which often little consensus exists, is problematic (Weitzman 2010)

$\Rightarrow$ Different models may arrive at dramatically different policy recommendations (e.g., DICE vs. Stern Report), generating heated debate and much confusion over the magnitude and timing of desirable policy

- Our approach tempers the above by making the degree of model misspecification (driven by the totality of available science) the most important model input

The literature we draw from

- Knightian uncertainty first discussed by (no surprise) Knight (1921). Ellsberg paradox (QJE, 1961) provided behavioral justification
- Gilboa and Schmeidler (JME, 1989) axiomatize **maxmin** expected utility over a set of uncertain priors
- Motivated by model misspecification in macro, Hansen and Sargent and coauthors (AER 2001; JET 2006) extend the G-S framework to dynamic settings, developing theory of **robust control**
- Nascent literature of applying robust control to environmental economics
  - Roseta-Palma and Xepapadeas (JRU, 2004) apply framework to water management
  - Gonzalez (ERE, 2008) studies PP for a different model of pollution control
  - Vardas and Xepapadeas (ERE, 2010) study PP for biodiversity management

Other models of ambiguity

- **Recursive Mutiple Priors:** Developed by Chen and Epstein (Econometrica 2002), Epstein and Schneider (JET 2004)
  - Asano (ERE, 2010) applies this model to study PP vis-a-vis timing of environmental policy
- **Smooth Ambiguity Model:** Developed by Klibanoff, Marinacci, and Mukerji (Econometrica, 2005; JET, 2009)
  - Gollier and Guerlinger (2008), Traeger (2009), Millner, Dietz and Heal (2010) apply the model to climate policy

Model description

- Adapt Dockner and Long (JEEM, 1993) infinite horizon dynamic model
- $E(t)$: emissions at time t
- $P(t)$: stock of pollution at time t
- Quadratic utility $u(E) = -\frac{1}{2}E^2 + AE$
- We modify the standard quadratic damage function $D(P) = sP^2, \;\; s > 0$ by allowing the possibility of investment in adaptation technology

Adaptation technology

- At time 0, the government sets a level of adaptation technology $z \in [0, 1]$ that alters the damage function in the following way

$$D(P, z) = zsP^2$$

- Thus, a *lower* level of z implies a *higher* investment in adaptation technology
- The cost of making an investment z is modeled by a strictly decreasing and convex function $\phi(z) : [0, 1] \mapsto \Re^+$ that satisfies

$$\phi(1) = 0, \quad \lim_{z \to 0} \phi(z) = \infty, \quad \lim_{z \to 0} \phi'(z) = -\infty.$$

- Possible candidates for $\phi(z)$ include $1/z^k - 1$, for $k > 0$.

The unambiguous benchmark problem

- Risk is introduced to the standard model so that the stock of the pollutant accumulates according to the diffusion process

$$\mathrm{d}P_t = (E - mP_t)\,\mathrm{d}t + \sigma \mathrm{d}z_t \tag{1}$$

where $\{z_t : \; t \geq 0\}$ is a Brownian motion on an underlying probability space $(\Omega, \mathcal{F}, \; G)$

- Thus the government's objective, given a level of adaptation technology z, is to maximize individual welfare or

$$\begin{aligned} \max_{E} \quad & \mathbf{E} \int_0^\infty \mathrm{e}^{-\rho t} \left[AE - \frac{E^2}{2} - zsP^2 \right] \mathrm{d}t \\ \text{subject to:} \quad & (1), \;\; P(0) = P_0. \end{aligned} \tag{2}$$

Optimization problem (2) is referred to as the *benchmark* model.

Introducing ambiguity

- If there were no fear of model misspecification solving the benchmark problem would be sufficient
- The perturbed model is obtained by performing a change of measure from P to Q by replacing z_t by

$$\hat{z}_t + \int_0^t v_s \mathrm{d}s, \tag{3}$$

where $\{\hat{z}_t :, \ t \geq 0\}$ is a Brownian motion and $\{v_t : \ t \geq 0\}$ is a measurable drift distortion, so that $v_t = v_t(P(s) : \ s \leq t)$

- The distortions will be zero when $v_t \equiv 0$ and the two measures G and Q coincide.
- Pollution dynamics under model misspecification can be written as:

$$\mathrm{d}P = (E - mP + \sigma v)\,\mathrm{d}t + \sigma \mathrm{d}z \tag{4}$$

Restricting the degree of model misspecification

- The discrepancy between the two measures G and Q is measured through their relative entropy

$$R(Q) = \int_0^\infty \mathrm{e}^{-\rho t} \frac{1}{2} \mathbf{E}\left[v(t)^2\right] \mathrm{d}t, \tag{5}$$

- To express the idea that even when the model is misspecified the benchmark model remains a "good" approximation, the misspecification error is restrained so that

$$\int_0^\infty \mathrm{e}^{-\rho t} \mathbf{E}[v(t)^2] dt \leq \eta < \infty, \tag{6}$$

- By modifying the value of η in (6) the decision-maker controls the degree of model misspecification.

Robust control

- Under model misspecification benchmark pollution dynamics (1) are replaced by (4).
- Two robust control problems can be associated with the solution to our problem
- The **constraint** robust control problem

$$J(P_0, \eta, z) = \max_E \min_v \quad \mathbf{E} \int_0^\infty \mathrm{e}^{-\rho t} \left[AE - \frac{E^2}{2} - szP^2 \right] \mathrm{d}t$$
$$\text{subject to:} \quad (4), \quad (6), \quad P(0) = P_0. \tag{7}$$

- The **multiplier** robust control problem

$$J(P_0, \theta, z) = \max_E \min_v \quad \mathbf{E} \int_0^\infty \mathrm{e}^{-\rho t} \left[AE - \frac{E^2}{2} - szP^2 + \frac{\theta v^2}{2} \right] \mathrm{d}t$$
$$\text{subject to:} \quad (4), \quad P(0) = P_0. \tag{8}$$

Relationship between the two problems

- In the constraint problem (7), the parameter η is the maximum expected misspecification error that the decision-maker is willing to consider
- In the multiplier problem (8), the parameter θ can be interpreted as the lagrangean multiplier associated with entropy constraint $R(Q) \leq \eta$
- When $\theta \to \infty$ or, equivalently $\eta = 0$, then there are no concerns about model misspecification and the decision-maker may safely consider just the benchmark model

Relationship between the two problems

- The relationship between the two robust control problems is subtle. For instance, a particular θ can be associated with no, or even multiple, η's, while a particular η can map to multiple θ's
- **Proposition** [Hansen and Sargent (2001)]: *Suppose J is strictly decreasing in* η, $\theta^* \in (\underline{\theta}, +\infty]$, *and that there exists a solution* E^* *and* v^* *(corresponding to measure* Q^**) to the multiplier problem (8). Then, that* E^* *also solves the constraint problem (7) for* $\eta = \eta^* = R(Q^*)$.
- In what follows, we will focus on the multiplier problem (8), and relate it to the (more intuitive) constraint problem through the above result

Solution of robust control problem with fixed z

- Solve the Bellman-Isaacs equation to determine value function

$$\rho V = \max_{E} \min_{v} \left\{ AE - \frac{E^2}{2} - szP^2 + \frac{\theta v^2}{2} + V_P(E - mP + \sigma v) + \frac{\sigma^2}{2} V_{PP} \right\}$$

- After some algebra

$$V(P, \theta, z) = \alpha_1(\theta, z)P^2 + \alpha_2(\theta, z)P + \alpha_3(\theta, z)$$

- Max-min optimal emissions E^* are linear in P and satisfy

$$E^*(P, \theta, z) = A + \alpha_2(\theta, z) + \alpha_1(\theta, z)P,$$

- Worst-case misspecification v^* is given by

$$v^*(P, \theta, z) = -\frac{\sigma}{\theta}(2\alpha_1(\theta, z)P + \alpha_2(\theta, z)).$$

Properties of the fixed-z optimal solution

- **Lemma:** *The value function $V(P;\theta,z)$ is*
  (a) *Strictly increasing and concave in θ*
  (b) *Strictly decreasing and convex in z. Moreover, the partial derivative V_z is increasing in θ.*
- $\alpha_1(\theta,z)$ and $\alpha_2(\theta,z)$ are negative and increasing in θ
  - Given fixed z, the **more** uncertainty over pollution dynamics, the **more** one chooses to mitigate emissions at a given pollution level P (Straight-up PP)
- $\alpha_1(\theta,z)$ and $\alpha_2(\theta,z)$ are decreasing in z
  - Given fixed θ, the less we invest in adaptation technology, the more we mitigate emissions

Characterizing the worst-case pollution accumulation

- **Proposition.** *Worst-case pollution accumulation P^* is a Gaussian diffusion process with a stationary distribution that is* $N\left(\frac{4A(m+\rho)}{4m^2+4m\rho+8sz(1-\frac{\sigma^2}{\theta})}, \frac{\sigma^2}{\sqrt{(2m+\rho)^2+8sz(1-\frac{\sigma^2}{\theta})}-\rho}\right)$

- In steady state, the expected value and variance of P^* are decreasing in θ and z

- Since we also know the transient distribution of P^*, the worst-case entropy of our misspecified model

$$R(P^*(\theta, z), \theta, z) \quad = \quad \int_0^{\infty} \mathrm{e}^{-\rho t} \mathbf{E}[v^*(t)^2] \mathrm{d}t,$$

can be explicitly calculated

Solving the optimal investment problem

- Suppose that at time 0 a policy maker wants to decide how much to invest in adaptation technology (choose z)
- Statistical evidence and climate science suggests a possible model misspecification that corresponds to an entropy of η
- The policy maker takes this misspefication seriously, so that a maxmin criterion is adopted over future welfare
- Recall that $V(P_0, \theta, z)$ denotes the maxmin value of a multiplier problem θ. Thus, at time 0, the policy maker wishes to solve the following optimization problem

$$\max_{z \in [0,1]} \qquad V(P_0, \theta, z) - \phi(z) \tag{9}$$

Adaptation vs. Mitigation

- Recall that pollution damages are given by $D = zsP^2$
- There are two basic, non mutually exclusive ways to guard against catastrophic damages.

  1. Improve adaptation technology (i.e., lower z)
  2. Mitigate emissions (i.e., lower P)

  The PP we wish to investigate is the following:

  When uncertainty grows, we must increase both our levels of (a) adaptation investment, and (b) emissions mitigation

Optimal adaptation and mitigation

Theorem 1 [Adaptation] *Optimal adaptation investment strictly increases in model uncertainty. In other words, $z^*(\theta)$ is strictly increasing in θ.*

- The situation is not so simple with mitigation. Indeed, it is possible for mitigation go up or down as model uncertainty increases.
- The responsiveness of our optimal adaptation decision $z^*(\theta)$ to changes in θ measured through

$$\frac{\mathrm{d}z^*}{\mathrm{d}\theta}(\theta)$$

  will play a critical role
- The interplay of adaptation and mitigation complicates the rationality of a PP

Optimal mitigation

Theorem 2 [Mitigation] *Consider a neighborhood of θ, say $[\theta_{\min}, \theta_{\max}]$. If $z^*(\theta)$ satisfies*

(a)

$$\frac{\mathrm{d}z^*}{\mathrm{d}\theta}(\theta) > (<) \frac{-\frac{\partial \alpha_1}{\partial \theta}(\theta, z^*(\theta)) - \frac{2\alpha_1^2(\theta, z^*(\theta))\sigma^2}{\theta^2(\rho+m)}}{\frac{\partial \alpha_1}{\partial z}(\theta, z^*(\theta))}, \qquad (10)$$

then emissions are unambiguously **decreasing** *(increasing) in θ in $[\theta_{\min}, \theta_{\max}]$;*

(c)

$$\frac{-\frac{\partial \alpha_1}{\partial \theta}(\theta, z^*(\theta))}{\frac{\partial \alpha_1}{\partial z}(\theta, z^*(\theta))} < \frac{\mathrm{d}z^*}{\mathrm{d}\theta}(\theta) < \frac{-\frac{\partial \alpha_1}{\partial \theta}(\theta, z^*(\theta)) - \frac{2\alpha_1^2(\theta, z^*(\theta))\sigma^2}{\theta^2(\rho+m)}}{\frac{\partial \alpha_1}{\partial z}(\theta, z^*(\theta))} \qquad (11)$$

then emissions will be decreasing in θ in $[\theta_{\min}, \theta_{\max}]$ if and only if current pollution levels are **high enough***.*

Optimal mitigation (in english...)

- **Corollary** *Emissions will be decreasing in* θ *if levels of optimal adaptation investment are high enough (i.e.,* $z^*(\theta)$ *is low enough) and the rate of change of* $z^*(\theta)$ *is high enough*
- In such cases there is so much investment in adaptation, and adaptation decisions are so sensitive to uncertainty, that mitigation is counter-productive
- The above is interesting when we recall the opposite result we obtain in the model with fixed adaptation technology

The effect of adaptation cost on precaution

- Focus on the following family of cost functions

$$\phi(z;k) = \frac{1}{z^k} - 1, \quad k > 0, \tag{12}$$

- **Proposition** *Fix a level of uncertainty θ and consider a family of optimization problems (9), parametrized according to Eq. (12).*
  (a) *Optimal values of $z^*(\theta;k)$ are increasing in k. In other words, optimal levels of adaptation investment are decreasing in the cost of adaptation technology.*
  (b) *Optimal emissions are decreasing in k. In other words, optimal levels of mitigation are increasing in the cost of adaptation technology.*

- The more expensive an adaptation technology is, the less we can expect to invest in it and the more we expect to mitigate

Numerical experiments – Optimal adaptation (1)

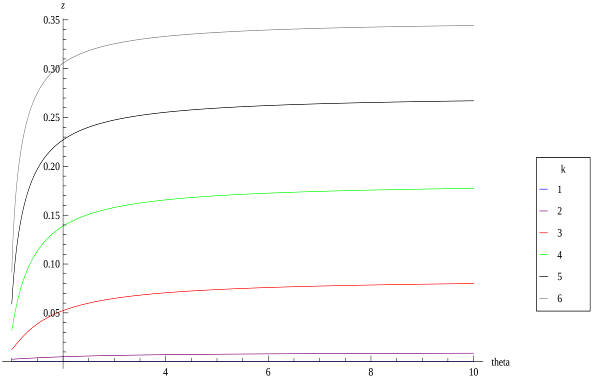

Figure: $z^*(\theta; k)$ as a function of θ for $k = 1, 2, .., 6$.

Numerical experiments – Optimal adaptation (2)

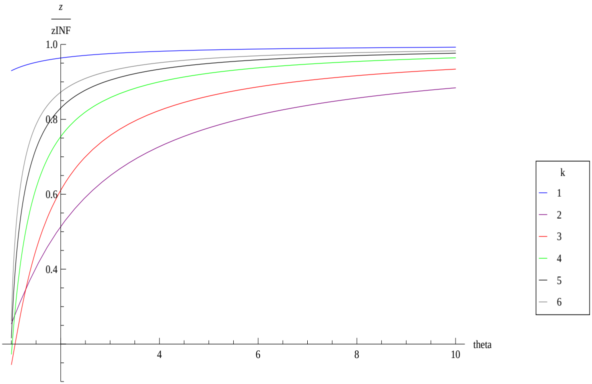

Figure: $\frac{z^*(\theta;k)}{z^*(\infty;k)}$ as a function of θ for $k = 1, 2, .., 6$.

The effect of adaptation cost on mitigation

- Numerical results confirm theoretical findings on adaptation: I.e., optimal adaptation decisions comport with PP regardless of the cost function
- But what happens to mitigation decisions as we vary k?
- Turns out that all three cases of Theorem 2 can occur
- Numerical results suggest that, when the cost of adaptation technology is **low enough** (for e.g., when $k = 1.5$), the PP can be **unambiguously irrational**

Numerical experiments– Optimal mitigation (1)

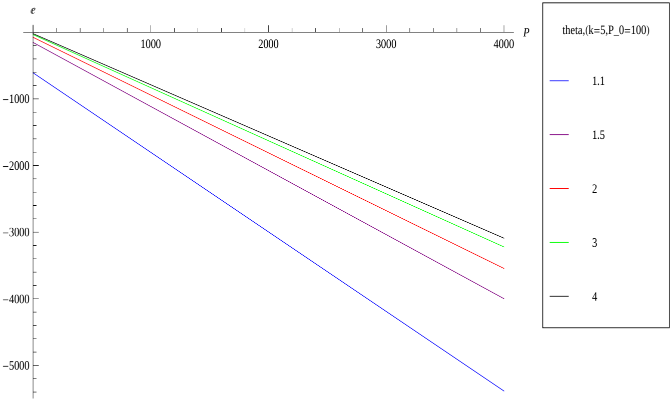

Figure: Emissions as a function of P for different θ: ($k = 5, P_0 = 100$). This case corresponds to a conventional PP.

Numerical experiments– Optimal mitigation (2)

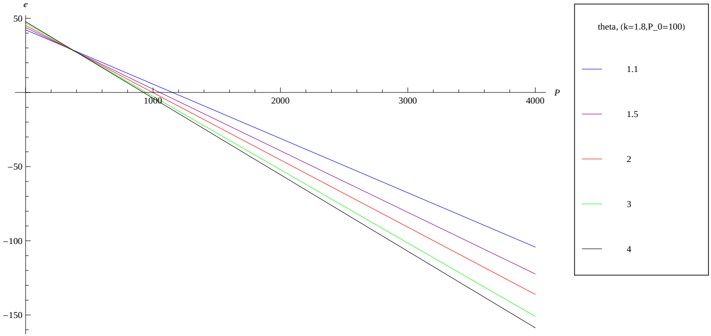

Figure: Emissions as a function of P for different θ: $(k = 1.8, P_0 = 100)$. This case corresponds to part (b) of Theorem 2, i.e., for high enough pollution we observe a reversal of the PP

Numerical experiments– Optimal mitigation (3)

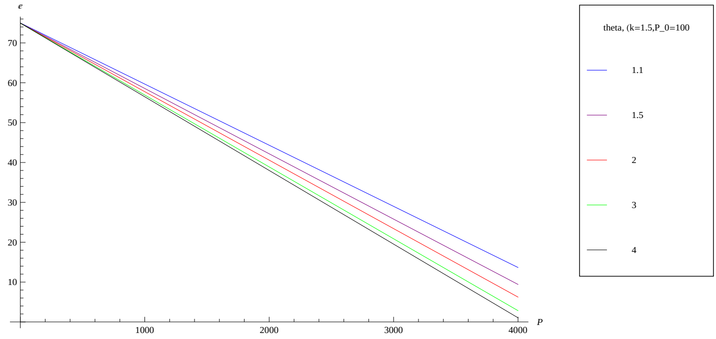

Figure: Emissions as a function of P for different θ: ($k = 1.5, P_0 = 100$). This case corresponds to an unambiguous reversal of the PP .

Thank You