An introduction to catastrophe models

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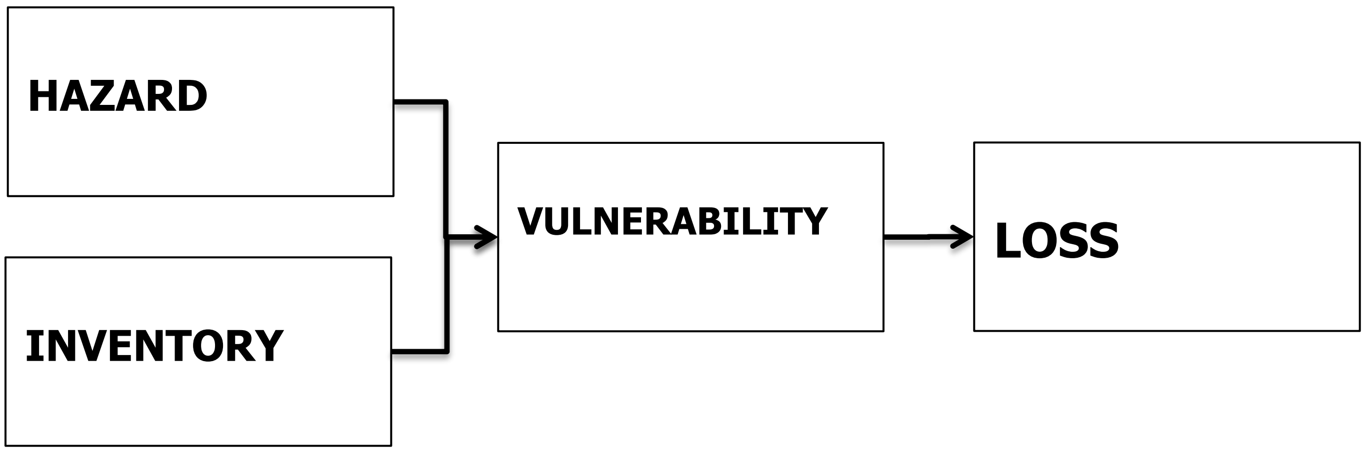
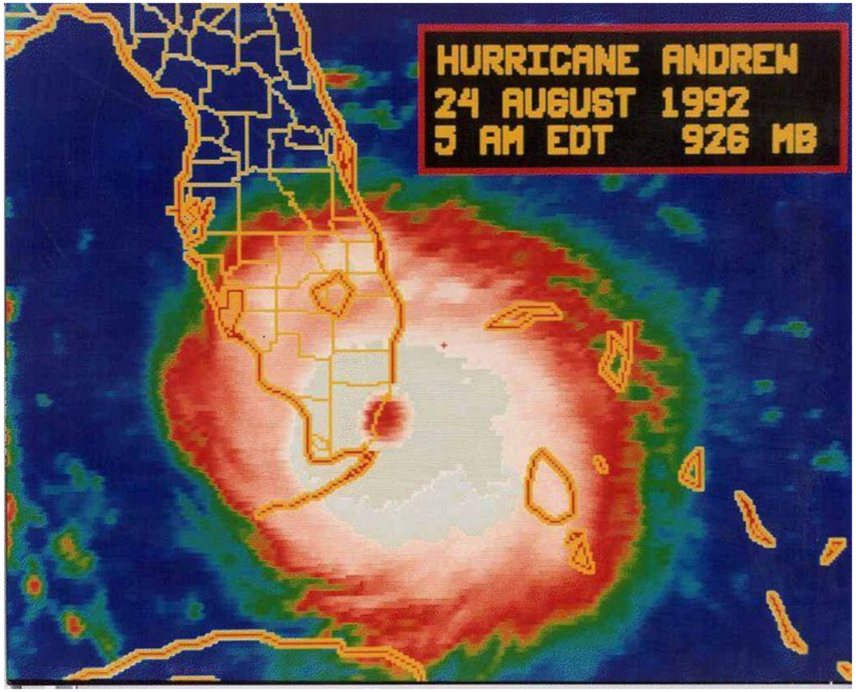
What are they?

- Catastrophe models are loss estimation tools used to analyzing risks (manly) in the insurance industry.



Why do cat models exist?



- Landfall in Florida at **0905 UTC 24 August**
- CAT4 revised to **CAT5** in 2002
- Second landfall: CAT3 in Louisiana at 0830 UTC 26 August
- Min central pressure over Florida: **922hPa**
- 1992 insured loss: **15** billion$ in Florida (source: PCS)

Who are the key players?

- VENDORS:
  - Risk Management Solutions (RMS)
  - Applied Insurance Research (AIR)
  - EQUECAT
  - and few others
- Open Source projects:
  - Alliance for Global Open Risk Analysis
- Willis Research Network: partnership between academia and the insurance industry

A bit of actuarial math

- Maximum Loss:

$$M = \max(X_1, X_2, \ldots, X_N)$$

- Aggregate Loss:

$$L = X_1 + X_2 + \ldots + X_N$$

- *An important insurance problem is the find L and M distributions when X_i and N are random variables*

Random Max of one event

- Given one 'event' [i.e. (**N**, **X**)]

- Random Max:

$$\mathbf{M}_{\mathbf{N}} = \max(\mathbf{X}_1, \mathbf{X}_2, \cdots, \mathbf{X}_{\mathbf{N}})$$

- $$\Pr(\mathbf{M} \le l) = p_0 + p_1 \cdot F_{\mathbf{X}}(l) + p_2 \cdot (F_{\mathbf{X}}(l))^2 + \cdots$$

$$\Pr(\mathbf{M} \le l) = \sum_{n=0}^{\infty} p_n \cdot (F_{\mathbf{X}}(l))^n = P_{\mathbf{N}}(F_{\mathbf{X}}(l))$$

$$= P_{\mathbf{N}}(F_{\mathbf{X}}(l))$$, probability generating function

Random Max of two events

- Given two 'events' , (**N**, **X**) , (**Q**,**Y**)

- Random Max:

$$\mathbf{M}_{\mathbf{N}} = \max(\mathbf{X}_1, \mathbf{X}_2, \cdots, \mathbf{X}_{\mathbf{N}}, \mathbf{Y}_1, \mathbf{Y}_2, \cdots, \mathbf{Y}_{\mathbf{Q}})$$

- $$\Pr(\mathbf{M} \le l) = P_{\mathbf{N},\mathbf{Q}}(F_{\mathbf{X}}(l), F_{\mathbf{Y}}(l))$$

for k random variables...

- $$\Pr(\mathbf{M} \le l) = P_{Z_1,\ldots,Z_k}(F_{\mathbf{X}_1}(l),\ldots,F_{\mathbf{X}_k}(l))$$

- Independent random variables:

$$\prod_{j=1}^{k} P_{Zj}(F_{\mathbf{X}_j}(l))$$

- Correlated random variables:

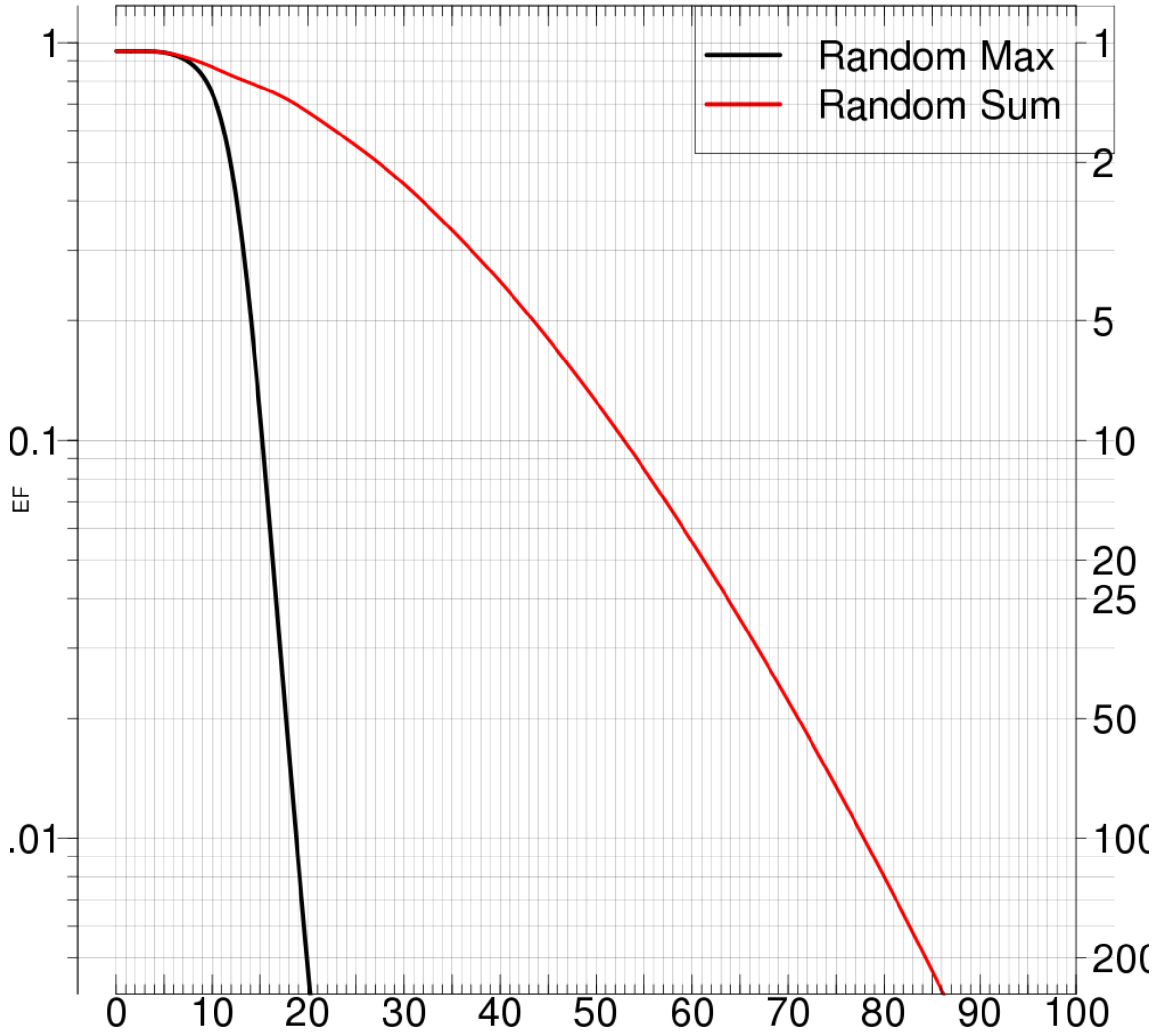
$$h^{-1}\left\{h \circ \sum_{j=1}^{k} P_{Zj}(F_{\mathbf{X}_j}(l))\right\}$$

Example

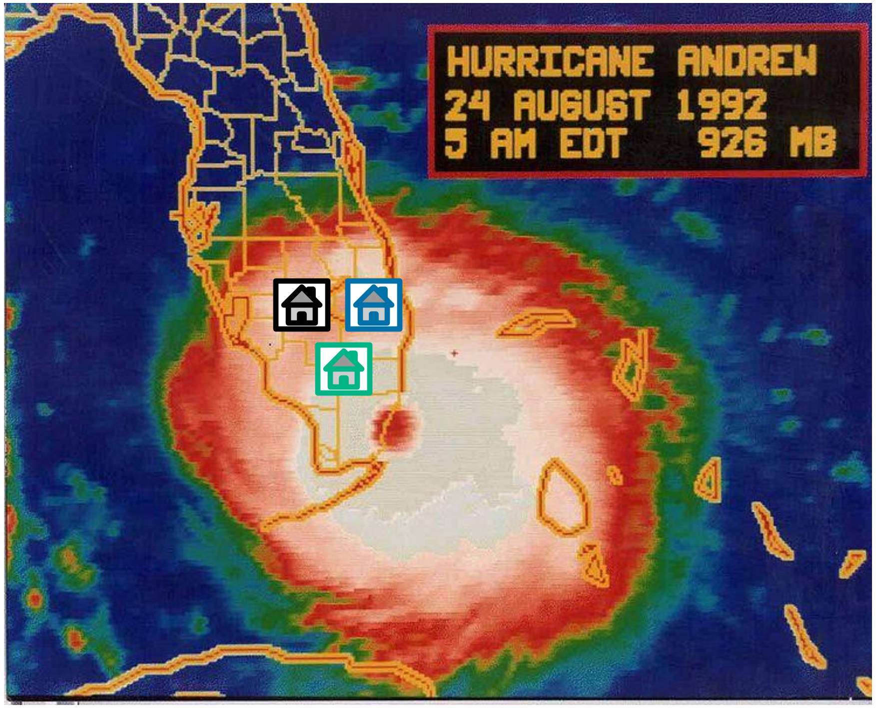
- An insurance underwrites:

Policy	Frequency	Severity
Client1	Poisson(λ=1)	logN(10,2)
Client2	Poisson(λ=0.8)	logN(12,2)
Client3	Poisson(λ=1.2)	logN(8,3)

Exeedance Probability

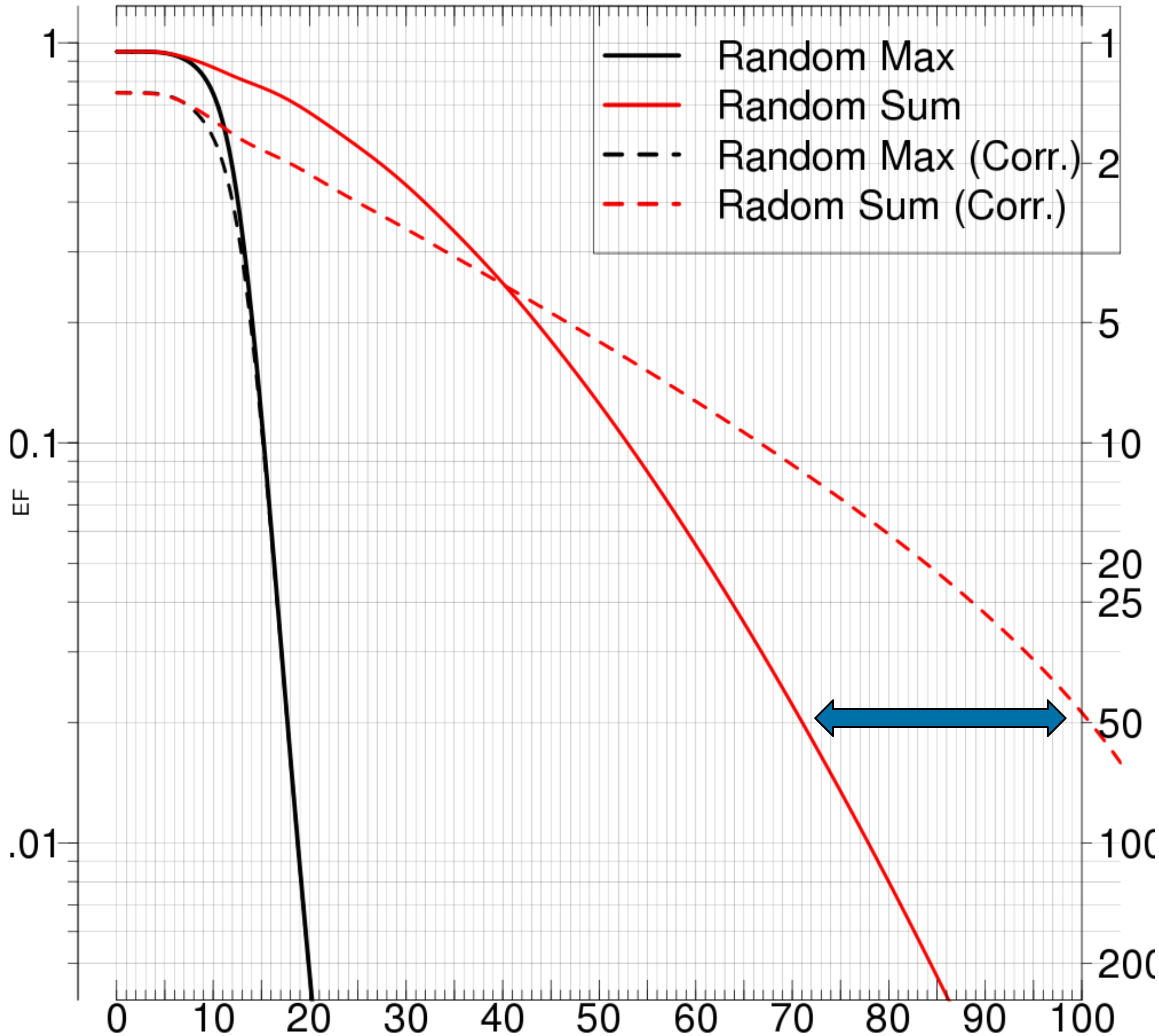


Are risks uncorrelated?



	Client1	Client2	Client3
Client1	ω_{11}	ω_{12}	ω_{13}
Client2	ω_{21}	ω_{22}	ω_{23}
Client3	ω_{31}	ω_{32}	ω_{33}

Exeedance Probability



Event Loss Table (ELT)

Policy	Frequency	Severity
Client1	Poisson($\lambda=1$)	logN(10,6) [$]
Client2	Poisson($\lambda=0.8$)	logN(12,4) [$]
Client3	Poisson($\lambda=1.2$)	logN(8,4) [$]

Event	Rate	Severity
Hurricane 1	$\lambda 1$	SUM over policies
Hurricane 2	$\lambda 2$	SUM over policies
Hurricane N	…	…

Event Loss Table

- Spatial correlation between risks is obtained exploiting the hazard spatial correlation
- The hazard module is an ensemble based representation of the climatology distribution of events for the next 1 to 5 years . *Quite often we assume stationary climate*.
- The severity distribution for each event is obtained as the sum of losses at each location that belongs to a given portfolio. Generally many approximations are required to get a readily available estimation of this quantity.
- Events are generally assumed to be *independent* (and *Poisson*)

Financing Catastrophe Risk in Developing Countries

Catastrophe Risk Financing in Developing Countries

- *The exposure of low and middle income countries to natural disasters is increasing.*
  - growing concentration of population and assets in risky areas
  - increases in climate variability
- Although the costliest disasters generally occur in developed countries low and middle-income countries have experienced the largest direct losses, in terms of annual average direct losses compared to GDP.
- *Post-disaster financing strategies generally have high opportunity costs*

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Post-disaster reconstruction

- Raising new debt in an expensive post-event capital market may
  - significantly affect the country's debt service
  - raising taxes may discourage new private investments that are central to restarting the economy.
- Post-disaster assistance from the international donor community may be slow and unreliable
- ***There is a critical need to develop ex-ante funding programs that are more efficient in meeting disaster needs***

Catastrophe bonds

- Catastrophe (cat) bonds are risk-linked securities that transfer a specified set of risks from a sponsor to investors. They are often structured as floating rate bonds whose principal is lost if specified trigger conditions are met. If triggered the principal is paid to the sponsor.

- The triggers are linked to major natural catastrophes. Catastrophe Bonds are typically used by insurers as an alternative to traditional catastrophe reinsurance.

Common Trigger Types

- **Indemnity**
  - Bond payout determined by (re)insurers portfolio loss
- **Parametric Index**
  - Bond payout determined by a mathematical formula

$$0.025 \times \sum_i w_i \times \left(\max(0, (WS_i - 22.5) \times \beta_i)\right)^4$$

- **PCS / PERILS Index**
  - Bond payout based on reported losses and factors (by country/state/county)
- **Modelled Loss**
  - Bond payout based on output from a catastrophe model

Why did Andrew take everybody by surprise ?