An Equilibrium Model of Habitat Conservation under Uncertainty and Irreversibility

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Milan, 18 November 2010

Aim and method

Title: "An Equilibrium Model of Habitat Conservation under Uncertainty and Irreversibility"

- Habitat Conservation $<=$ land/biodiversity
- Uncertainty and Irreversibility;
- Equilibrium Model: decentralized economy populated by a multitude of homogenous landholders and where the Government has introduced a payment scheme for conservation
- Method: Real Option Theory
- Goal: to determine the impact of uncertainty and optimal policy conversion dynamics; to analyse different policy scenarios on the basis of the relative long-run expected rate of deforestation.

Conservation/development dilemma

2 competing and mutually exclusive land destinations

1) Clearing land: irreversible reduction/loss of valuable ES (Environmental Services): biodiversity, carbon sequestration, etc.
2) Conserving land: cost opportunity of foregone profits from economic activities (e.g. agriculture, commercial forestry).
Irreversible development under uncertainty over future prospects may be later regretted:
- the possibility to pospone choice has an option value
Allocation?
- Welfare maximization under ROA

Land Policy

- **Command** mechanisms are costly in economic and political terms.
- **Decentralized** conversion decisions.
- Direct approach, PES (Payments for Environmental Services):

On a voluntary basis a provider delivers to a buyer a well-defined ES (or corresponding land use) in exchange for an agreed payment.

Which is the impact of payment schemes and/or command tools on dynamic land allocation?

Outline

- Literature
- Model set up.
- Conversion policies under different scenarios.
- Command approach vs voluntary approach.
- Conversion in the LR.

Optimal conservation decisions under irreversibility and uncertainty

Bulte et al., 2002, Kassar and Lassere, 2004; Leroux et al., 2009
-Our added value:
a) the complexity of challenges characterizing conservation policies;
b) the role that competition on markets for agricultural products may have on conversion decisions.

The basic model: landholder

-infinitesimally small landholders => competition
-each landholder owns one parcel.
- Land at $t = 0$: $L = A_0 + F$.
- L: Total Land;
- A_0 is the surface cultivated;
- F is the portion still in its pristine natural state covered by primary forest

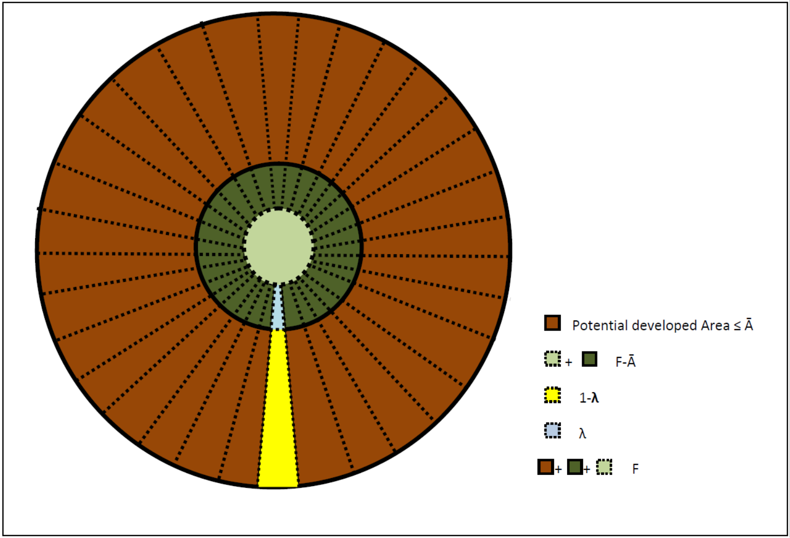
The basic model: Government

- $\bar{A} \leq F$: upper bound for conversion.
- $0 \leq \lambda \leq 1$: restriction on land development (buffer area)

Farmers:

- $A(t)$: total land developed at t.
- $N(t) = \frac{A(t)}{1-\lambda}$: the number of infinitesimally small farmers at t.
- $\bar{N} = \frac{\bar{A}}{1-\lambda} \leq F$: not everyone may become a farmer!

The basic model



- $B(t)$: per-parcel value of ES randomly fluctuating according to

$$dB(t) = \alpha B(t)dt + \sigma B(t)dz(t)$$

where α, σ, $z(t)$: drift, volatility, Wiener process.
- c: conversion unit cost.
- Once forest is cleared, land enters as an input in agricultural production and landholder $\longrightarrow$ farmer.
- Let **D**emand for agricultural products be specified as

$$P_A(t) = \delta A(t)^{-\gamma}$$

- No price-making!

The basic model: Government (2)

Payments:

- $\eta_1 B(t)$ with $\eta_1 \in [0, 1]$: compensation paid if the entire plot is conserved.
- $\eta_2 \lambda B(t)$ with $\eta_2 \in [0, 1]$: compensation for the buffer area.

The basic model: market for agricultural products

Competing among symmetric farmers $\longrightarrow A(t)$ is determined by the entry zero π condition at each t.

- **Farmer**, instantaneous profit function:

$$\pi(A(t), B(t); \bar{A}) = (1-\lambda)P_A(t) + \eta_2 \lambda B(t)$$

- **Landholder**; ENPV:

$$E_0\left[\int_0^T \eta_1 B(t)e^{-rt}dt \ + \int_T^\infty [(1-\lambda)P_A(t) + \eta_2\lambda B(t)]e^{-rt}dt \ \mid B(0) = B\right]$$

$$\frac{\eta_1}{r-\alpha}B + E_0\left[\int_T^\infty (1-\lambda)\delta A(t)^{-\gamma} + (\eta_2\lambda - \eta_1)B(t)e^{-r(t-T)}dt\ \right]$$

where r is the constant risk free interest and T is the stochastic conversion time.

- **Only on the 2nd term matters**: excess profit from development must at least equal the deforestation cost, $(1-\lambda)c$.

Optimal stopping time problem: equilibrium

Given $(A(t), B(t))$: should I convert right now (stopping) or wait over next dt (continuation)?

$$V(\bullet) = \max_{\tau} E_0\left[\int_0^{\infty} [(1-\lambda)P_A(t) + \eta_2 \lambda B(t)] e^{-rt} dt - I_{[t=\tau]}(1-\lambda)c\right]$$

Solution: a barrier B^C delimiting the conversion and conservation set of values over the space $(A(t), B(t))$.
- $dA(t) \uparrow$, $P_A(t) \downarrow$ and conditions for conservation are restored.
- Symmetric Nash equilibrium in entry strategies!

Equilibrium

Case $\eta_1 > \eta_2\lambda$: only reductions in B can induce conversion!

$$V(A, B; \bar{A}) = Z_2(A)B^{\beta_2} + (1-\lambda)\frac{\delta A^{-\gamma}}{r} + (\eta_2\lambda - \eta_1)\frac{B}{r-\alpha}$$

where $Z_2(A) \leq 0$ and $\beta_2 < 0$.
Proposition 1 *Provided that each agent rationally forecasts the future dynamics of the market for agricultural goods for land to be converted the following condition must hold*

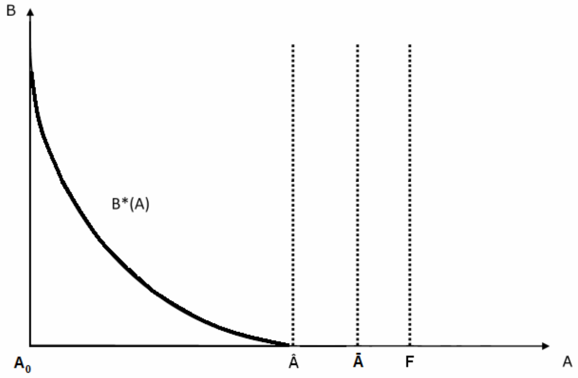
$$V(A, B^*(A); \bar{A}) = (1-\lambda)c$$

and $\hat{A} = (\frac{\delta}{rc})^{1/\gamma}$ and $A^+ = [\frac{(\beta_2-1)\bar{A}^{-\gamma}+\hat{A}^{-\gamma}}{\beta_2}]^{-\frac{1}{\gamma}}$.
$\hat{A} = (\frac{\delta}{rc})^{1/\gamma}$ is the last parcel for which conversion makes economic sense $\left(\frac{\delta A^{-\gamma}}{r} - c = 0\right)$

Equilibrium

If $\hat{A} \leq \bar{A}$ then

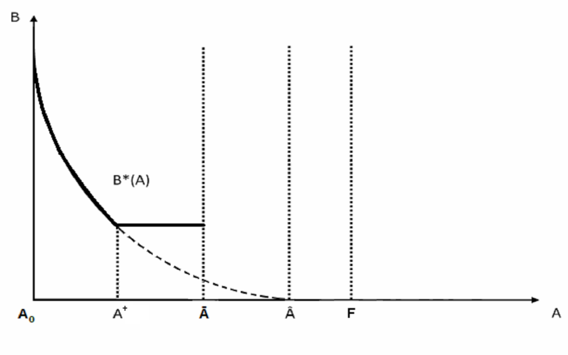
$$B^*(A) = \frac{\beta_2}{\beta_2 - 1}(r - \alpha)\frac{1-\lambda}{\eta_1 - \eta_2 \lambda}\left[\left(\frac{\hat{A}}{A}\right)^{\gamma} - 1\right] c \quad \text{for } A_0 < A \leq \hat{A} \text{ (a)}$$



Equilibrium

if $\hat{A} > \bar{A}$ then

$$B^*(A) = \begin{cases} \frac{\beta_2}{\beta_2 - 1}(r-\alpha)\frac{1-\lambda}{\eta_1-\eta_2\lambda}\left[\left(\frac{\hat{A}}{A}\right)^{\gamma}-1\right]c, & \text{for } A_0 < A \leq A^+ \quad \text{(b)} \\ (r-\alpha)\frac{1-\lambda}{\eta_1-\eta_2\lambda}\left[\left(\frac{\hat{A}}{A}\right)^{\gamma}-1\right]c, & \text{for } A^+ < A \leq \bar{A} \quad \text{(c)} \end{cases}$$



Equilibrium

Not surprisingly, $\frac{\partial A^+}{\partial \hat{A}} < 0$ and $\frac{\partial A^+}{\partial \bar{A}} > 0$.

$\Delta \hat{A}$	δ	c	r	γ
	> 0	< 0	< 0	< 0

Table 1: *Changing profit from agriculture*

$\Delta B^*(A)$	δ	c	r	γ	
	≥ 0	≤ 0	≤ 0	≤ 0	
	α	σ^2	η_1	η_2	λ
	≤ 0	≤ 0	≤ 0	≥ 0	< 0 $(\frac{\eta_1}{\eta_2} > 1)$ ≥ 0 $(\frac{\eta_1}{\eta_2} \leq 1)$

Table 2: *Payment fluctuation and policy parameters*

Equilibrium

Case $\eta_1 < \eta_2 \lambda$: Increase in B can induce conversion!

$$V(A, B; \bar{A}) = Z_1(A) B^{\beta_1} + (1 - \lambda) \frac{\delta A^{-\gamma}}{r} + (\eta_2 \lambda - \eta_1) \frac{B}{r - \alpha}$$

where $Z_1(A) \leq 0$ and $\beta_1 > 0$.

Proposition 2 *Provided that each agent rationally forecasts the future dynamics of the market for agricultural goods for land to be converted the following condition must hold*

$$V(A, B^{**}(A); \bar{A}) = (1 - \lambda) c$$

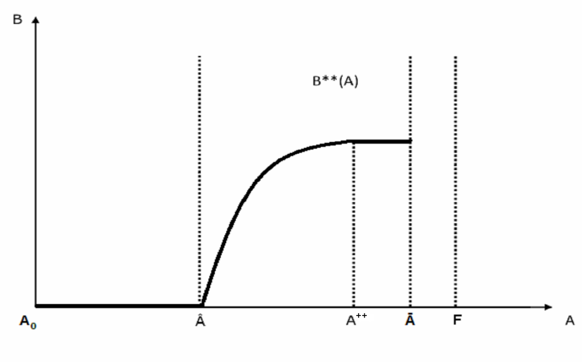
Equilibrium

If $\hat{A} \leq \bar{A}$ then

$$B^{**}(A) = \begin{cases} 0, \quad \text{for } A_0 < A \leq \hat{A} \quad \text{(a)} \\ \frac{\beta_1}{\beta_1 - 1}(r-\alpha)\frac{1-\lambda}{\eta_2\lambda - \eta_1}\left[1-(\frac{\hat{A}}{A})^\gamma\right]c, \quad \text{for } \hat{A} < A \leq A^{++} \quad \text{(b)} \\ (r-\alpha)\frac{1-\lambda}{\eta_2\lambda - \eta_1}\left[1-(\frac{\hat{A}}{\bar{A}})^\gamma\right]c, \quad \text{for } A^{++} < A \leq \bar{A}, \quad \text{(c)} \end{cases}$$

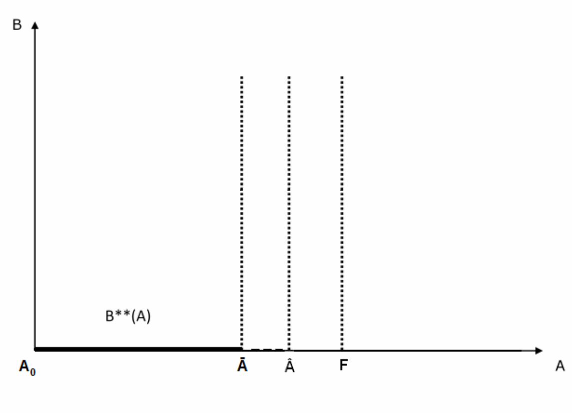
with $\hat{A} = (\frac{\delta}{rc})^{1/\gamma}$ and $A^{++} = [\frac{(\beta_1 - 1)\bar{A}^{-\gamma} + \hat{A}^{-\gamma}}{\beta_1}]^{-\frac{1}{\gamma}}$.

Equilibrium



Equilibrium

if $\hat{A} > \bar{A}$ then



Equilibrium

Again not surprisingly, $\frac{\partial A^{++}}{\partial \bar{A}} > 0$ and $\frac{\partial A^{++}}{\partial \hat{A}} > 0$.

$\Delta B^{**}(A)$	δ	c	r	γ	
	≤ 0	≥ 0	≥ 0	≥ 0	
	α	σ^2	η_1	η_2	λ
	≥ 0	≥ 0	≥ 0	≤ 0	< 0

Table 3: *Payment fluctuation and policy parameters*

Expected long-run rate of forest conversion

Consider $\eta_1 > \lambda\eta_2$. Rearranging the conversion threshold yields

$$\xi = \frac{\beta_2}{\beta_2 - 1}(1-\lambda)\frac{P_A(A)}{r} - \frac{\eta_1 - \lambda\eta_2}{r-\alpha}B \quad \text{for } \xi < \hat{\xi}$$

where $\hat{\xi} = \frac{\beta_2}{\beta_2 - 1}(1-\lambda)c$. For an initial $(\widetilde{B}, \tilde{A})$ such that $\xi(\widetilde{B}, \tilde{A}) = \hat{\xi}$ the average or expected growth rate of deforestation is given by:

$$\frac{1}{dt}E[d\ln A] \simeq -\frac{\alpha - \frac{1}{2}\sigma^2}{\gamma}\left(1 - \left(\frac{\tilde{A}}{\hat{A}}\right)^{\gamma}\right) \quad \text{for } \alpha < \frac{1}{2}\sigma^2$$

Note that.

- the rate is increasing in σ.

- the rate is decreasing in c: higher conversion cost and lower cost opportunity in the future.

Expected long-run rate of forest conversion

Consider $\eta_1 < \lambda\eta_2$. Rearranging the conversion threshold yields

$$\varsigma = \frac{\beta_1}{\beta_1 - 1}(1-\lambda)\frac{P_A(A)}{r} + \frac{\lambda\eta_2 - \eta_1}{r-\alpha}B \qquad \text{for} \quad \varsigma < \hat{\varsigma}$$

where $\hat{\varsigma} = \frac{\beta_1}{\beta_1 - 1}(1-\lambda)\,c$. For an initial $(\widetilde{B}, \tilde{A})$ such that $\varsigma(\widetilde{B}, \tilde{A}) = \hat{\varsigma}$ the average or expected growth rate of deforestation is given by:

$$\frac{1}{dt}E\left[d\ln A\right] \simeq \frac{\alpha - \frac{1}{2}\sigma^2}{\gamma}\left(\left(\frac{\tilde{A}}{\hat{A}}\right)^{\gamma} - 1\right) \qquad \text{for } \alpha > \frac{1}{2}\sigma^2$$

Note that.

- the rate is decreasing in σ.

- the rate is increasing in c: higher conversion cost but also higher conservation cost opportunity.

Data:

- The total extent of originally forested area is $\bar{A} = 320000$ hectares.
- $A_0 = 25000$ hectares.
- We draw demand for agricultural products as in Bulte et al. (2002) by setting $\delta = \$6990062$ (in 1998 US\$) and $\gamma = 0.887$.
- α takes values 0, 0.025, and 0.05 while σ varies within the interval $[0, 0.325]$.
- $r = 0.07$. Differently from Bulte et al. (2002) where $c = 0$ we consider also different levels of costly deforestation.

Scenario 1	$\eta_1 = 1$	$\eta_2 = 0$	$\lambda = 0$
Scenario 2	$\eta_1 = 0.7$	$\eta_2 = 0$	$\lambda = 0$
Scenario 3	$\eta_1 = 1$	$\eta_2 = 0$	$\lambda = 0.3$
Scenario 4	$\eta_1 = 0.7$	$\eta_2 = 0.5$	$\lambda = 0.3$
Scenario 5	$\eta_1 = 0.7$	$\eta_2 = 1$	$\lambda = 0.3$

Table 4: policy scenarios

In the scenarios we provide the optimal forest stock which should be held, $\bar{A} - \tilde{A}$;
the average deforestation rate (Def rate) at which such stock should be optimally exhausted in the long-run

Case $\eta_1 > \lambda\eta_2$
Scenario 1

	α=0.00		α=0.025		α=0.05	
σ	Ã-	Def rate	Ã-	Def rate	Ã-	Def rate
0	0	0	76684	-	222474	-
0,025	0	0	79996	-	223155	-
0,05	0	0	88790	-	225127	-
0,075	461	0,0032	100809	-	228196	-
0,1	23495	0,0056	114211	-	232103	-
0,125	44778	0,0088	127886	-	236574	-
0,15	64431	0,0127	141240	-	241361	-
0,175	82567	0,0173	153977	-	246265	-
0,2	99297	0,0225	165955	-	251135	-
0,225	114723	0,0285	177125	0,0004	255866	-
0,25	128945	0,0352	187484	0,0070	260389	-
0,275	142053	0,0426	197056	0,0144	264662	-
0,3	154136	0,0507	205879	0,0225	268664	-
0,325	165273	0,0595	213998	0,0314	272388	0,0032

Table 5: Optimal forest stock and long-run average rate of deforestation in scenario 1 with $c = 0$

Case $\eta_1 > \lambda\eta_2$
Scenario 1 with costs

	α=0.00		α=0.025		α=0.05	
σ	Ã-	Def rate	Ã-	Def rate	Ã-	Def rate
0	138990	-	138986	-	138986	-
0,025	149250	0,0003	140887	-	139960	-
0,05	159060	0,0011	145990	-	142796	-
0,075	168411	0,0025	153098	-	147262	-
0,1	177304	0,0046	161212	-	153037	-
0,125	185739	0,0072	169705	-	159775	-
0,15	193725	0,0106	178218	-	167148	-
0,175	201268	0,0145	186547	-	174877	-
0,2	208381	0,0192	194577	-	182737	-
0,225	215077	0,0245	202243	0,0003	190556	-
0,25	221371	0,0305	209514	0,0060	198207	-
0,275	227278	0,0372	216376	0,0124	205601	-
0,3	232816	0,0446	222827	0,0196	212678	-
0,325	238004	0,0527	228876	0,0274	219400	0,0027

Table 7: Optimal forest stock and long-run average rate of deforestation in scenario 1 with $c = 500$.

Scenario 4

	α=0.00		α=0.025		α=0.05	
σ	Ã-	Def rate	Ã-	Def rate	Ã-	Def rate
0	0	0	663	-	192003	-
0,025	0	0	5010	-	192896	-
0,05	0	0	16552	-	195484	-
0,075	0	0	32326	-	199513	-
0,1	0	0	49915	-	204641	-
0,125	0	0	67862	-	210508	-
0,15	0	0	85390	-	216791	-
0,175	8385	0,0173	102105	-	223227	-
0,2	30341	0,0225	117826	-	229619	-
0,225	50588	0,0285	132486	0,0004	235829	-
0,25	69252	0,0352	146082	0,0070	241764	-
0,275	86457	0,0426	158644	0,0144	247372	-
0,3	102314	0,0507	170223	0,0225	252625	-
0,325	116931	0,0595	180879	0,0314	257512	0,0032

Table 11: Optimal forest stock and long-run average rate of deforestation in scenario 4 with $c = 0$

Scenario 4 with costs

	α=0.00		α=0.025		α=0.05	
σ	Ã-	Def rate	Ã-	Def rate	Ã-	Def rate
0	98233	-	98228	-	98228	-
0,025	110097	0,0003	100421	-	99351	-
0,05	121511	0,0011	106319	-	102625	-
0,075	132458	0,0024	114566	-	107792	-
0,1	142927	0,0044	124024	-	114495	-
0,125	152914	0,0069	133977	-	122345	-
0,15	162419	0,0101	144007	-	130974	-
0,175	171443	0,0139	153874	-	140064	-
0,2	179994	0,0184	163436	-	149353	-
0,225	188080	0,0236	172613	0,0003	158642	-
0,25	195713	0,0294	181359	0,0058	167776	-
0,275	202908	0,0359	189653	0,0120	176647	-
0,3	209680	0,0432	197485	0,0189	185178	-
0,325	216046	0,0511	204860	0,0265	193320	0,0026

Table 13: Optimal forest stock and long-run average rate of deforestation in scenario 4 with $c = 500$

Case $\eta_1 < \lambda\eta_2$

	α=0.00		α=0.025		α=0.05	
σ	Ã-	Def rate	Ã-	Def rate	Ã-	Def rate
0	0	0	0	0	0	0
0,025	0	0	0	0	0	0
0,05	0	0	0	0	0	0
0,075	0	0	0	0	721	0,0032
0,1	0	0	0	0	1657	0,0030
0,125	0	0	0	0	2727	0,0027
0,15	0	0	0	0	3874	0,0023
0,175	0	0	0	0	5048	0,0020
0,2	0	0	0	0	6214	0,0016
0,225	0	0	0	0	7348	0,0012
0,25	0	0	0	0	8434	0,0009
0,275	0	0	0	0	9462	0,0005
0,3	0	0	0	0	10427	0,0002
0,325	0	0	0	0	11328	-

Table15: Optimal forest stock and long-run average rate of deforestation in with $c = 1400$.

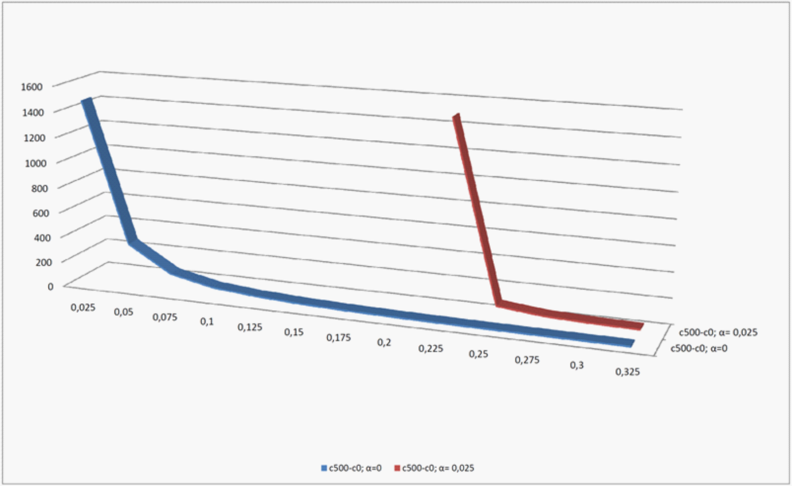


Figure 5: Difference in expected time for total conversion, between c=500 and c=0 with $\alpha = 0$ and $\alpha = 0.025$.

Conclusions

We contribute to the vast literature on irreversible land conversion under uncertainty studying:
- the role of competition in conversion dynamics,
- the impact of policy (command and voluntary approach) on landholder decision making,
and deriving a robust approximation for the long-run expected deforestation rate.

Research for the Future

Competition versus Centralized equilibrium
Endogeneity of the optimal policy