Modeling asymmetric dependence using copula functions: an application to Value-at-Risk in the energy sector

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Outline of the talk

- Motivations & Empirical questions
- Data
- Copulas & Value-at-Risk
- Results

Motivations & Empirical questions

Introduction

- VaR measures the worst expected loss of a financial position over a specific time interval at a given confidence level.
  - Simplicity: market risk is reduced to a single number.
  - Legal aspects: Basel Capital Accord (1988 & 1995) sets the minimum capital requirements that banks must meet to guard against credit and market risks as a function of VaR.
- Possible extensions:
  - VaR as indicator for building portfolios or hedging financial positions.
  - VaR measures the probability of extreme price swings $\longrightarrow$ short-run energy security indicator to hedge economic disruptions due to erratic fluctuations in the price of energy products.
- Ill parametric assumptions can lead to over/underestimation of VaR:
  - e.g. if risk is underestimated banks are penalized.

Motivations (1/2)

- **Stock returns are nonnormal:**
  - *Excess Skewness*: large stock returns are often negative [Hansen (1994), Harvey and Siddique (1999, 2000)]
  - *Excess Kurtosis*: extreme returns occur more often than expected [Hull and White (1998)]
  - *Asymmetric dependence*: correlation is higher in bear than in bull markets [Hong et al. (2007), Longin & Solnik (2001)]
  - *Non-zero tail dependence*: higher (than expected under the Normal) probability of joint extreme events [Jondeau and Rockinger (2003)]

Motivations (2/2)

- **Forecasting and Risk Management:**
  - *Flexible models vs simple models*: often simple (misspecified) models outperform complicated models [Hansen and Lunde (2005), Swanson & White (1995-1997), Zaffaroni (2008)]
  - *Data Snooping*: good forecasting performance or just luck? [Gonzalez et al. (2004), White (2000)]

Contributions

1. Forecasting VaR with copulas in the context of energy economics.
    - Usually the focus is on methodology rather than on data.
2. In-depth analysis of VaR forecasts.
    - The norm is to rank forecasts, nothing is said about their statistical significance.

Empirical questions

- **Explore the forecasting performance of copula functions**
  1. **Are there nonnormalities at the univariate level?** (Yes)
  2. **Are there nonnormalities at the multivariate level?** (Yes)
  3. **Is it worth modeling them for Risk Management?** (Yes)
  4. **Do forecasts from flexible models outperform simple models?** (Yes)

Copulas can be used to create flexible multivariate distributions.

- Sklar's Theorem: decompose the joint distribution of (x, y) into three parts [Sklar (1959)]:

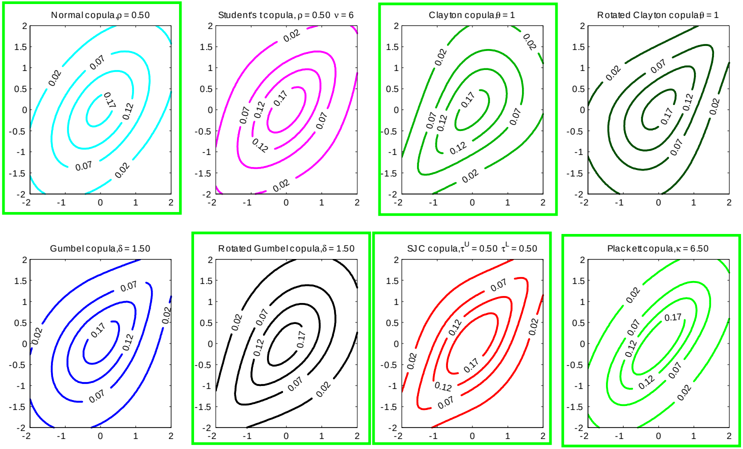
$$Joint(x, y) = Copula(marginal\,(x)\,, marginal\,(y))$$

- **Copulas describe how the marginals are linked.**
- Sklar's theorem can be written as [Patton 2006]:

$$\mathcal{L}_{XY}\left(\theta\right) = \mathcal{L}_{C}\left(\kappa\right) + \mathcal{L}_{X}\left(\varphi\right) + \mathcal{L}_{Y}\left(\gamma\right) \tag{1}$$

- Can use (1) to get $\hat{\theta} \equiv \left[\widehat{\kappa} \quad \widehat{\varphi} \quad \widehat{\gamma}\right]'$ with 2-stage QML:
  - Consistency & asymptotic normality [Newey & McFadden (1994)].
  - Avoid curse of dimensionality.

Flexible modeling with copulas



Notes: Normal variates linked with copula functions.

Copulas and VaR: ingredients of VaR (3/4)

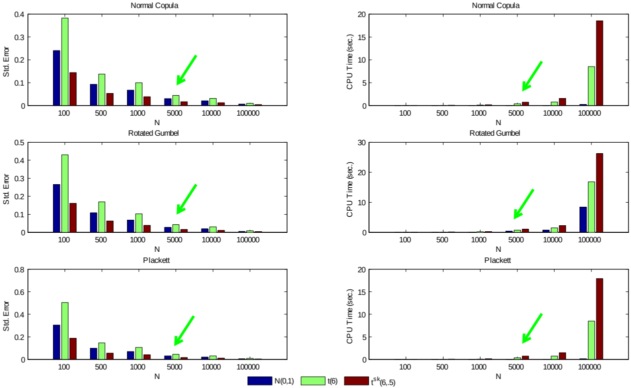
- Volatilities and co-volatilities are the main ingredients of VaR

$$VaR_t = E\left(r_t | \Omega_{t-1}\right) - q\sigma_t \quad (2)$$

$$VaR^{portfolio} = \sqrt{\sum_{i=1}^{N} VaR_i^2 + 2\sum_{i \neq j} \rho_{ij} VaR_i VaR_j}. \quad (3)$$

- Volatilities: measured with GARCH models
- Co-volatilities: use copulas instead of correlation.
  - No closed-form equations to forecast VaR with copulas: Monte Carlo methods to generate the loss distribution.

Evaluation of 5% VaR simulations from copula functions.



Notes: S.E. = f(N) (1st col.); CPU time = f(N) (2nd col.); N = no. simulations.

Data

1. Daily data from Jan 1992 to May 2008 on the DJ's U.S. Oil & Gas Producers (90% coverage of the industry):
   - Eq. Weighted Portfolio: a Small Cap (SC) index (high risk/high return) and Large Cap (LC) index (low risk/low return).
   - Full sample: 4260 observations.
2. Model selection is done once with in-sample data (3500 obs.).
3. Recursive estimation & forecasting using out-of-sample data (760 obs.).

Question 1: Are there nonnormalities in the marginals?

Descriptive Statistics			
	Small Caps		Large Caps
Mean	15.0984	>	9.1980
Min	-8.9679		-7.7561
Max	9.5355		9.6772
5% VaR	-1.6055		-1.6179
Std. Dev.	22.83855	>	19.9336
Skewness	-0.1037		0.1311
Kurtosis	5.8686		6.4749
Jarque-Bera	1203.5038		1767.2001
P-value	0.0000		0.0000
Correlation		.57	

Notes: The table displays the annualized mean and the annualized volatility. The null hypothesis of the Jarque-Bera statistic is that returns are unconditionally normal.

Q1: Marginal distribution models (2/4)

T^{sk} distribution

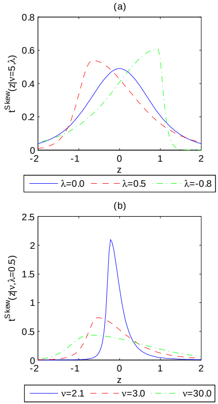
- Find correct specifications:

r_t^{SC} ∼AR(1) - GARCH(1,1)
r_t^{LC} ∼ARMA(1,1) - GARCH(1,1)

- Start with a general model [Hansen (1994)]:

$$T^{sk}(\upsilon, \lambda) = \begin{cases} N(0,1) \text{ if } \upsilon \longrightarrow \infty \text{ \& } \lambda = 0 \\ t(\nu) \text{ if } \lambda = 0 \end{cases}$$

- Then use std. residuals to test parametric assumptions.



Answer to Q1: Yes, marginal distributions of the data are not normal (4/4)

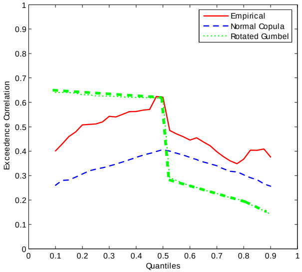
Which distribution is correct? The Student's T distribution seems to provide the best fit

- The parameter controlling the skewness of the T^{sk} density (λ) is never statistically significant;
- QQ plots: $T^{sk} \succ N,\ T$
- AIC & BIC: $N \succ T, T^{sk}$ but $T \succeq T^{sk}$
- LR test: restrictions on T^{sk} :
  - $T \succ T^{sk} \succ N$
- PIT test: $H_0 : u \sim i.i.d.U\left(0,1\right)$:
  - KS tests: reject $H_0 : u \sim U\left(0,1\right)$ for N and T
  - ACFs: never reject $H_0 : u \sim i.i.d.$

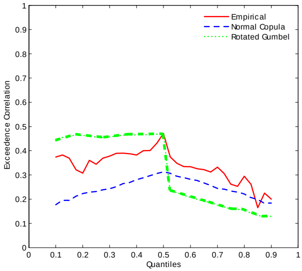
Question 2: Are there nonnormalities in the dependence structure?

Q2: Are there nonnormalities in the dependence structure? (2/6)

Exceedence correlation: (a) unconditional distribution of returns and (b) unconditional copula of transformed std. residuals



(a)



(b)

Q2: Copula modeling (1/3)

- Use 5 different copulas with Normal & Student's T marginals
  - Best models: T+(a)symmetric copulas;
  - Worst Models: Clayton copula
  - SJC $\tau^L > \tau^U$

Copula Functions

	Normal	Plackett	Clayton	Rotated Gumbel	SJC
Symmetric	Y	Y	N	N	Y/N
Tail dependence	N	Y	Y	Y	Y

Q2: Copula modeling (2/3)

Copula Functions: estimation & ranking

	Coefficient		Rank	$\mathcal{L}_C$
N+Normal	ρ	0.5463	5	-620.4050
N + Clayton	θ	0.7622	15	-463.9627
N + Plackett	κ	6.4716	4	-624.3161
N + Rotated Gumbel	δ	1.5316	11	-580.4840
N + SJC	τ^U	0.2776	12	-574.8843
	τ^L	0.3407		
T + Normal	ρ	0.5450	7	-612.6361
T + Clayton	θ	1.0669	13	-536.7095
T + Plackett	κ	7.2299	2	-636.9853
T + Rotated Gumbel	δ	1.6894	3	-631.8730
T + SJC	τ^U	0.4011	1	-648.6506
	τ^L	0.4529		

Answer to Q2: Yes, there're nonnormalities at the multivariate level (3/3)

1. Asymmetric dependence:
   - Large cap and small cap indices display excess correlation (i.e. $\rho_t^- > \rho_t^+$).
2. Tail dependence:
   - Best copula models have non-zero tail dependence.

Question 3: Is it worth modeling nonnormalities for VaR?

Q3: Are nonnormalities important for VaR? (1/5)

- Use statistical tests and loss functions for backtesting copula models, MA models, RiskMetrics^{TM}
  - MA models: (Moving Average no ARMA(0,q)!) single index (SI) models $\Longrightarrow$ no diversification.
  - RiskMetrics: both SI and portfolio (P) model

Which models provide correct coverage? (i.e. % of VaR violations $\sim$ 5%?)

- Test VaR Violations $= r_t < VaR_t$
  - Conditional coverage test [Christoffersen (1998)]
  - Dynamic Quantile test [Engle & Manganelli (2002)] :

$$H_0 : \text{independence} + \text{correct coverage}$$

Q3: Correct coverage (2/5)

	%	LR^{UC}	LR^{IND}	LR^{CC}	DQ
RiskMetrics^{TM} (SI)*	6.32	2.5942	1.1463	3.7404	12.9224
p-value		0.1073	0.2843	0.1541	0.0443
MA(20)*,†	6.71	4.2843	0.0793	4.3637	11.7127
p-value		0.0385	0.7782	0.1128	0.0687
MA(60)*	6.71	4.2843	0.6686	4.9529	17.4732
p-value		0.0385	0.4135	0.0840	0.0077
N + Normal	7.50	8.7809	0.5445	9.3254	13.2330
p-value		0.0030	0.4606	0.0094	0.0395
T + Normal*,†	5.00	0.0001	0.6166	0.6167	8.8593
p-value		0.9934	0.4323	0.7347	0.1816
T + Rotated Gumbel*,†	4.21	1.0348	0.2566	1.2915	8.0954
p-value		0.3090	0.6124	0.5243	0.2312
T + Plackett*	5.00	0.0001	0.5419	0.5420	12.9546
p-value		0.9934	0.4617	0.7626	0.0438
T + SJC*,†	4.61	0.2476	0.0705	0.3181	6.1299
p-value		0.6188	0.7905	0.8530	0.4088
RiskMetrics^{TM} (P)†	2.50	12.1042	1.0278	13.1320	11.1186
p-value		0.0005	0.3107	0.0014	0.0848

Q3: Are nonnormalities important for VaR? (3/5)

Which is the best model among those with correct coverage?

- Loss functions (i.e. best model = min(Loss)):
  - Lopez (1998): penalty increases with magnitude of violations.
  - Blanco & Ihle (BI, 1999): focuses on the avg. size of the violations.
  - Quantile Loss (Q) [Kroenker & Basset (1978)]: penalizes more heavily violations.

Models with correct conditional coverage

Rank	# Violations	Lopez	BI	Q
1	**T+RG**	**T+SJC**	**T+RG**	**T+SJC**
2	**T+SJC**	**T+RG**	**T+SJC**	**T+PL**
3	T+N	T+PL	T+PL	T+N
4	T+PL	T+N	T+N	T+RG
5	MA(20)	MA(20)	MA(20)	MA(20)
6	MA(60)	MA(60)	MA(60)	MA(60)

Notes: The table uses the following notation: T = Student's T marginal distribution; RG = Rotated Gumbel copula; N = Normal copula; PL = Plackett Copula.

Answer to Q3: Yes, nonnormalities are important when forecasting VaR (5/5)

- Best models (i.e. correct coverage & min loss):
  - T marginals: excess kurtosis, but no asymmetry.
  - SJC and Rotated Gumbel copulas: asymmetric & non-zero tail dependence
- The "Normal-Normal" model does not provide correct VaR forecasts (i.e. too many VaR violations)
- RiskMetrics does not provide correct VaR forecasts (i.e. too few VaR violations)
- MA models correctly forecast VaR but maximize the loss functions.

Question 4: Do flexible models outperform simple models?

Is the performance of different models statistically different or is it just luck?

Pairwise comparison:

- DM test [Diebold and Mariano (1995)]:
    - $H_0 : d_t = 0$, where $d_t = Q_t^i - Q_t^j$ for model $i \neq j$
    - *DM* has low power in small samples: use *MDM*[Harvey et al. (1997)]

Q4: Do flexible models outperform simple models? (2/5)

Modified DM test

	T+N	T+PL	T+RG	T+SJC
MA(20)	-1.9121	-1.9048	-1.6956	-1.8440
p-value	0.0562	0.0572	0.0904	0.0656
MA(60)	-2.0547	-2.0591	-1.8819	-1.9819
p-value	0.0403	0.0398	0.0602	0.0479
T+N		-0.3683	0.2820	-0.6734
p-value		0.7127	0.7780	0.5009
T+PL			0.4950	-0.3624
p-value			0.6207	0.7172
T+RG				-1.1529
p-value				0.2493

Notes: The table uses the following notation: T = Student's T marginal distribution; RG = Rotated Gumbel copula; N = Normal copula; PL = Plackett Copula.

Q4: Do flexible models outperform simple models? (3/5)

Is the performance of different models statistically different or is it just luck?

Joint comparison:

- White's Reality Check (RC) test [White (2000)]:
  - H_0 : best of l models $\sim$ benchmark model
  - H_1 : $\exists$ at least one model $\succ$ benchmark model
  - relies on sample averages of empirical and bootstrapped loss differentials
    - use stationary bootstrap [Politis and Romano (1994)]
  - the RC test generates high p-values when a poor model is introduced
  - use Hansen (2005) lower bound (p-value$_L$)

Q4: Do flexible models outperform simple models? (4/5)

Reality Check p-values

	White	$Hansen_L$
MA(20)	0.1440	0.0390
MA(60)	0.0720	0.0460
RiskMetrics^{TM} (SI)	0.2090	0.0690
N+N	0.3050	0.1030
T+N	0.9260	0.4430
T+PL	0.9270	0.4610
T+RG	0.8880	0.3430
T+SJC	0.9820	0.6480
RiskMetrics^{TM} (P)	0.0230	0.0230

Notes: The table uses the following notation: T = Student's T marginal distribution; RG = Rotated Gumbel copula; N = Normal copula; PL = Plackett Copula.

Answer to Q4: Yes, flexible models provide better VaR forecasts (5/5)

- **Pairwise comparison of models:**
  - Copula models $\succ$ MA models.
  - Can't discriminate among copula models.
- **Joint comparison of models:**
  - MA models and RiskMetrics specifications are the worst models (p-values ~ 0).
  - "Normal+Normal" model has the lowest p-value among copula models.
  - T+SJC copula has the highest p-value among copula models.

Conclusions and directions for future research

Conclusions

1. Nonnormalities are present at the univariate as well as at the multivariate level.
2. Modeling nonnormalities allows for better VaR forecasts than under the assumption of normality.
3. Flexible models provide better forecasts than simple models (MA models and RiskMetrics).

Directions for future research

- Time varying correlations.
- Copulas VS multivariate GARCH.
- Other economic problems: hedging & portfolio allocation.