Likelihood of Environmental Coalitions --- An Empirical Testing of Shrinking Core Hypothesis in an Economy with Externality

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IEA: literature

- Barrett (1994,2003)
- Carraro and Siniscalco (1993)
- Chander and Tulkens (1995, 1997)
- Finus (2001)
- Yang (2008)

and many others

Core concepts

- In general equilibrium theory
- In the presence of externality (environmental problems)
- Core and stability of IEA

Shrinking core hypothesis

- In Arrow-Debreu economy: the core shrinks to the Walrasian equlilibrium (Debreu and Scarf, 1963)
- In the economy with externality: the core does not necessarily shrink to the Lindahl equilibrium (Muench, 1972)
- The question remains: shrinking or not?

Core and coalition

- Core solutions are the broadest solution concept of a cooperative game
- Core allocation is necessary to voluntary IEA
- The scope of core: a metaphor of randomly thrown dart

What does the core look like?

- Edgeworth box
- Higher dimension?
- Dynamic system?

Methodological principle here: mapping the core of a complicated game onto a simple metric space

Problem of efficient provision of stock externality

$$\underset{\{x_i(t)\}}{Max}\quad V=\sum_{i=1}^{N}\varphi_i W_i^E=\sum_{i=1}^{N}\int_0^{\infty}\varphi_i U^i(x_i(t),B(t))e^{-\delta t}dt,\qquad \sum_{i=1}^{N}\varphi_i=N. \qquad (1)$$

$$s.t.\quad F^i(x_i(t),b_i(t))=0,\qquad i=1,2,\ldots,N. \qquad (2)$$

$$\dot{B}(t)=\sum_{i=1}^{N}b_i(t)-\sigma B(t),\qquad \sigma>0. \qquad (3)$$

$$B(0)=B_0,\quad x_i(0)=x_{i,0}, \qquad (4)$$

$$\frac{\partial U^i}{\partial x_i}>0,\quad \frac{\partial U^i}{\partial B}<0,\quad \frac{\partial^2 U^i}{\partial x_i^2}<0,\quad \frac{\partial^2 U^i}{\partial B^2}<0,\quad \frac{\partial F^i}{\partial b_i}\Big/\frac{\partial F^i}{\partial x_i}>0 \qquad (5)$$

Problem of non-cooperative Nash equilibrium

$$\underset{\{x_i(t)\}}{Max} \quad W_i^M = \int_0^\infty U^i(x_i(t), B(t)) e^{-\delta t} dt, \qquad i = 1, 2, \ldots, N. \qquad (6)$$

$$s.t. \quad F^i(x_i(t), b_i(t)) = 0, \qquad i = 1, 2, \ldots, N. \qquad (7)$$

$$\dot{B}(t) = b_i(t) + \sum_{j \neq i}^{N} \overline{b}_j(t) - \sigma B(t), \qquad i = 1, 2, \ldots, N. \qquad \sigma > 0. \qquad (8)$$

Describing the cooperative game of providing stock externalities

- Triplet $V(\varphi_i, x_i(t), W^C_i)$ for $i = 1, 2, ..., N$
- First stage: bargain $\{\varphi_i\}$ on simplex $S = \{\{\varphi_i\} \mid \sum \varphi_i = N\}$.
- Second stage: all regions follow the solution paths $x_i(t)$ given by (1) under $\{\varphi_i\}$ and receive payoff W^C_i
- The nature of the game: bargaining initial emission/mitigation quota then acting cooperatively and efficiently
- The role of φ_i: crucial.

The core allocations w.r.t. Nash equilibrium

- Core allocation of (1) or cooperative game solution of (1): the grand coalition under $\{\varphi_i\}$ cannot be blocked by any sub-coalitions, including the Nash equilibrium (individual rationality)
- Sub-coalitions: cooperate internally under the same $\{\varphi_i\}$ and interact with outsiders strategically in the Nash way (‘hybrid’ Nash equilibrium or P.A.N.E. in Chander & Tulkens)
- Core allocations are determined by initial endowments (Bergstrom, 1976) or by the unique Nash equilibrium

Agenda of this research

- Probing the ‘size’ of the core constructively and empirically
- Studying the relationship b/w the ‘size’ of the core and number of regions (agents)
- Answering ‘shrinking core’ hypothesis
- Policy implications of core ‘size’

RICE-m model

- RICE model with m regions, m = 2, 3, 4, 5, 6
- Similar (and simpler?) to FEEM's WITCH model
- Obtaining the scope of the core allocations for each m
- Constructing comparable metrics for the scope of the core

Breaking down of regions

- RICE-6: the United States (USA), European Union (EU), other high-income countries (OHI), China (CHN), Eastern European countries and former Soviet Union (EEC), and the rest of world (ROW)
- RICE-5: USA, [EU+ OHI], CHN, EEC, ROW
- RICE-4: [USA+EU+ OHI], CHN, EEC, ROW
- RICE-3: [USA+EU+OHI], [CHN+EEC], ROW
- RICE-2: [USA+EU+OHI], [CHN+EEC+ROW]

Methodologies

(1) Identifying the core allocation (cooperative game solution):

Incentive checking on all 'hybrid' Nash equilibriums by searching proper $\{\varphi_i\}$ on simplex $S = \{\{\varphi_i\} \mid \sum \varphi_i = N\}$.

Two checking tables for each RICE-m

open file:

Sufficient conditions of core properties based on these two tables:

Inner points of the core in RICE-m

	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	$\sum\varphi_i$
m = 2	1.802	0.198					2
m = 3	2.033	0.745	0.222				3
m = 4	1.737	0.353	1.752	0.158			4
m = 5	1.922	1.376	0.238	1.359	0.105		5
m = 6	1.574	1.805	1.094	0.202	1.229	0.096	6

- Under those social welfare weights, the two incentive checking tables have sweeping homogenous sign.

(2) Probing the ‘borders’ of core properties:

Increase or decrease of single φ_i from the inner point identified in (1) while keep other φ_j fixed on the simplex until core properties are violated. (the process requires solving ‘hybrid’ Nash equilibriums repeatedly).

RICE-m has 2m such ‘border’ points on the simplex. They are called ‘vertex’ (a clean geometric term) .

Vertexes of the core in RICE-4

	φ_1	φ_2	φ_3	φ_4
Inner P.	1.737	0.353	1.752	0.158
1 up	2.056	0.303	1.505	0.136
1 down	1.441	0.399	1.981	0.179
2 up	1.645	0.546	1.659	0.150
2 down	1.811	0.198	1.826	0.165
3 up	1.246	0.253	2.387	0.113
3 down	2.165	0.440	1.198	0.197
4 up	1.702	0.346	1.717	0.235
4 down	1.752	0.356	1.767	0.124

(3) Calculating the volumes of core:

Span a convex hull with 2m vertexes of border points with core properties. All points inside this convex hull have core properties (or inside the core).

The approximate ‘size’ of the core:

volume ratio b/w convex hull and simplex.

$$CS(m) = Vol(H(m))/Vol(S(m))$$

Definition of volume: Vol(S) --- simplex + origin
Vol(H) --- convex hull + origin

or, “pyramid” vs. “thin prism cone”

Volumes of the cores in RICE-m

	m = 2	m = 3	m = 4	m = 5	m = 6
V(H)	0.025797	1.40493E-3	3.45035E-5	1.71412E-7	1.58499E-9
V(S)	0.5	1.66667E-2	4.16667E-2	8.33333E-3	1.38889E-3
CS(m)	5.1594E-2	8.429E-3	8.2808E-4	2.0569E-5	1.1412E-6

- The core is shrinking as m increase !
- The core is very small

Policy implications

- Forming voluntary IEA is not easy (based on dart throwing metaphor)
- Some policy interventions may be necessary to boost opportunities of IEA (enlarging the size of the core)
- Any connections b/w shrinking core phenomena and coalition size vs. coalition stability examined by Carraro ? (still thinking)

- Thanks. Comments and suggestions are welcome.